Application of Fuzzy Soft Multi Sets in Decision-Making Problems

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Abstract Alkhazaleh and Salleh presented a fuzzy soft multi set theoretic approach to solve decision-making problems using the Roy-Maji Algorithm, which has some limitations. In this research work, we have proposed an algorithm to solve fuzzy soft multi set based decision making problems using the Feng's algorithm, which is more stable and more feasible than the Alkhazaleh–Salleh Algorithm. The feasibility of our proposed algorithm in practical applications is illustrated by a numerical example.

Keywords Decision making · Soft set · Level soft set · Fuzzy soft set · Fuzzy soft multi set · Fuzzy soft multi set part

1 Introduction

In 1999, Molodstov [12] initiated the notion of soft set theory as a general mathematical tool for dealing with vagueness, uncertainties and not clearly defined objects. Research works on the soft set theory are progressing rapidly. Some new algebraic operations and results on soft set theory defined in [2, 10]. Adding soft sets [12] with fuzzy sets [15], Maji et al. [9] defined fuzzy soft sets and studied their basic properties. As a generalization of soft set, Alkhazaleh and others [1, 4–6, 14] proposed the notion of a soft multi set and its basic algebraic and topological structures were studied. Alkhazaleh and Salleh [3] initiated the notion of fuzzy soft multi set theory and presented its application in decision making using Roy-Maji

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Algorithm [13]. Maji et al. [11] first proposed the application of soft sets for solving the decision making problems and thereafter in 2007, they also presented an application on fuzzy soft sets based decision making problems in [13]. Kong et al. [8] mentioned that the Roy-Maji algorithm [13] was wrong and they introduced a revised algorithm. Feng et al. [7] studied the validity of the Roy-Maji algorithm [13] and mentioned that the Roy-Maji Algorithm [13] has some limitations. Also, they proposed an adjustable approach to fuzzy soft sets based decision making problems by using thresholds and choice values.

In fact, all these concepts have a good application in some real life problems. But, it is seen that all these theories have their own difficulties that is why in this paper, we are going to propose an algorithm to fuzzy soft multi set based decision making problems using Feng's algorithm, which is another one new mathematical tool for solving some real life applications of decision making problems. The feasibility of our proposed algorithm in practical applications is illustrated by a numerical example.

2 Preliminary Notes

In this current section, we briefly recall some basic notions of soft sets, fuzzy soft multi sets and level soft sets.

Definition 2.1 [12] Suppose that, U be an initial universe and $\hat{E}$ be a set of parameters. Also, let $\tilde{P}(U)$ denotes the power set of the universe U and $\hat{A} \subseteq \hat{E}$. A pair $(\tilde{F}, \hat{A})$ is said to be a soft set over the universe U, where $\tilde{F}$ is a mapping given by $\tilde{F} : \hat{A} \to \tilde{P}(U)$.

Definition 2.2 [3] Suppose $\{U_i : i \in \Lambda\}$ be a set of universes, such that $\cap_{i\in\Lambda} U_i = \phi$ and let for each $i \in \Lambda$, E_i be a sets of decision parameters. Also, let $\tilde{U} = \prod_{i\in\Lambda} FS(U_i)$ where $FS(U_i)$ is the set of all fuzzy subsets of U_i, $\hat{E} = \prod_{i\in\Lambda} E_{U_i}$ and $\hat{A} \subseteq \hat{E}$. A pair $(\tilde{F}, \hat{A})$ is said to be a fuzzy soft multi set over the universe $\tilde{U}$, where $\tilde{F}$ is a function given by $\tilde{F} : \hat{A} \to \tilde{U}$.

Definition 2.3 [3] For any fuzzy soft multi set $(\tilde{F}, \hat{A})$, where $\hat{A} \subseteq \hat{E}$ and $\hat{E}$ is a set of parameters. A pair $\left(e_{U_i,j}, \tilde{F}_{e_{U_i,j}}\right)$ is said to be a U_i—fuzzy soft multi set part of $(\tilde{F}, \hat{A})$ over U, $\forall e_{U_i,j} \in a_k$ and $\tilde{F}_{e_{U_i,j}} \subseteq \tilde{F}\left(\hat{A}\right)$ is an approximate value set, for $a_k \in \hat{A}$, k $\in \{1, 2, 3, .., \mathrm{m}\}$, $i \in \{1, 2, 3, .., \mathrm{n}\}$ and $j \in \{1, 2, 3, .., \mathrm{r}\}$.

Definition 2.4 [7] Let $\varpi = (\tilde{F}, \hat{A})$ is a fuzzy soft set over the universe U, where $\hat{A} \subseteq \hat{E}$ and $\hat{E}$ is a set of parameters. For $t \in [0, 1]$, the t-level soft set of ϖ is a crisp soft set $L(\varpi; t) = \left(\tilde{F}_t, \hat{A}\right)$ defined by $\tilde{F}_t(e) = \left\{u \in U : \mu_{\tilde{F}(e)}(u) \geq t\right\}$, $\forall e \in \hat{A}$.

Definition 2.5 [7] Suppose $\varpi = (\tilde{F}, \hat{A})$ be a fuzzy soft set over U, where $\hat{A} \subseteq \hat{E}$ and $\hat{E}$ is the parameter set. Let $\lambda : \hat{A} \to [0,1]$ be a fuzzy set in $\hat{A}$, which is called a threshold fuzzy set. The level soft set of the fuzzy soft set ϖ with respect to the fuzzy set λ is a crisp soft set $L(\varpi; \lambda) = \left(\tilde{F}_{\lambda}, \hat{A}\right)$ defined by $\tilde{F}_{\lambda}(e) = \left\{u \in U : \mu_{\tilde{F}(e)}(u) \geq \lambda(e)\right\}, \forall e \in A$.

Definition 2.6 [7] Let $\varpi = (\tilde{F}, \hat{A})$ is a fuzzy soft set over a finite universe U, where $\hat{A} \subseteq \hat{E}$ and $\hat{E}$ is the set of parameters. The mid-threshold of the fuzzy soft set ϖ define a fuzzy set $mid_{\varpi} : \hat{A} \to [0,1]$ by $\forall e \in \hat{A}$, $mid_{\varpi}(e) = \frac{1}{|U|} \sum_{u \in U} \mu_{\tilde{F}(e)}(u)$ and the level soft set of ϖ with respect to the mid-threshold fuzzy set mid_{ϖ}, namely $L(\varpi; mid_{\varpi})$ is said to be mid-level soft set of ϖ.

Definition 2.7 [7] Let $\varpi = (\tilde{F}, \hat{A})$ be a fuzzy soft set over a finite universe U, where $\hat{A} \subseteq \hat{E}$ and $\hat{E}$ is the parameter set. The max-threshold of the fuzzy soft set ϖ define a fuzzy set $\max_{\varpi} : \hat{A} \to [0,1]$ by $\forall e \in \hat{A}$, $\max_{\varpi}(e) = \max_{u \in U} \mu_{\tilde{F}(e)}(u)$ and the level soft set of ϖ with respect to the max-threshold fuzzy set $\max_{\varpi}$, namely $L(\varpi; \max_{\varpi})$ is said to be top-level soft set of ϖ.

3 An Adjustable Approach Based on Feng's Algorithm

3.1 Feng's Algorithm Using Choice Values

The details of Feng's Algorithm [7] for solving a decision-making problem based on a fuzzy soft set are as follows:

Algorithm 1 (*Feng's Algorithm*)

1. Input the fuzzy soft set $\varpi = (\tilde{F}, \hat{A})$
2. Input a threshold fuzzy set $\lambda : \hat{A} \to [0,1]$ (or select a threshold value $t \in [0,1]$ or select mid-level decision criterion or select top-level decision criterion) for solving decision making problem.
3. Obtain the level soft set $L(\varpi; \lambda)$ of ϖ with respect to the threshold fuzzy set λ (or $L(\varpi; t)$ or $L(\varpi; mid)$ or $L(\varpi; \max)$).
4. Present the level soft set $L(\varpi; \lambda)$ (or $L(\varpi; t)$; or $L(\varpi; mid)$; or $L(\varpi; \max)$) as in tabular form and also, obtain the choice value s_i of $u_i \in U, \forall i$.
5. The final optimal decision to be select u_k if $s_k = \max_i s_i$.
6. If k has more than one value, then any one of u_k may be chosen.

3.2 Application of Fuzzy Soft Multi Sets in Decision-Making Problems

In this section, we propose an algorithm (Algorithm 2) for fuzzy soft multi sets based decision making problems, using Feng's Algorithm [7], as described above. In the following, we have to show our algorithm (Algorithm 2):

Algorithm 2

(1) Input the (resultant) fuzzy soft multi set $(\tilde{F}, \hat{A})$
(2) Apply the Algorithm 1 (Feng's Algorithm) to the first fuzzy soft multi set part in $(\tilde{F}, \hat{A})$, to obtain the decision S_{k_1}.
(3) Modify the fuzzy soft multi set $(\tilde{F}, \hat{A})$, by taking all values in each row, where the choice value of S_{k_1} is maximum and changing the values in the other rows by 0 (zero), to get $(\tilde{F}, \hat{A})_1$.
(4) Apply the Algorithm 1 (Feng's Algorithm) to the second fuzzy soft multi set part in $(\tilde{F}, \hat{A})_1$, to obtain the decision S_{k_2}
(5) Modify the fuzzy soft multi set $(\tilde{F}, \hat{A})_1$, by taking the first two parts fixed and apply the method as in step (3) to the next part, to get $(\tilde{F}, \hat{A})_2$
(6) Apply the Algorithm 1 (Feng's Algorithm) to the third fuzzy soft multi set part in $(\tilde{F}, \hat{A})_2$, to obtain the decision S_{k_3}.
(7) Finally, we have the optimal decision for decision maker is $(S_{k_1}, S_{k_2}, S_{k_3})$.

3.3 Application in Decision-Making Problems

Let us consider three universes $U_1 = \{h_1, h_2, h_3, h_4\}$, $U_2 = \{c_1, c_2, c_3\}$ and $U_2 = \{v_1, v_2, v_3\}$ are sets of houses, cars and hotels respectively and let $E_{U_1} = \{e_{U_1,1}, e_{U_1,2}, e_{U_1,3}\}$, $E_{U_2} = \{e_{U_2,1}, e_{U_2,2}, e_{U_2,3}\}$, $E_{U_3} = \{e_{U_3,1}, e_{U_3,2}, e_{U_3,3}\}$ be the sets of respective decision parameters related to the above three universes.

Let $\tilde{U} = \prod_{i=1}^{3} FS(U_i)$, $\tilde{E} = \prod_{i=1}^{3} E_{U_i}$ and $\hat{A} \subseteq \hat{E}$, such that

$$\begin{aligned}\hat{A} =\{ & a_1 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,1}), a_2 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,1}), a_3 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,1}), a_4 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,1}),\\ & a_5 = (e_{U_1,1}, e_{U_2,1}, e_{U_3,2}), a_6 = (e_{U_1,1}, e_{U_2,2}, e_{U_3,2}), a_7 = (e_{U_1,2}, e_{U_2,3}, e_{U_3,3}), a_8 = (e_{U_1,3}, e_{U_2,3}, e_{U_3,3})\}.\end{aligned}$$

Assume that, Mr. X wants to buy a house, a car and rent a hotel with respect to the three sets of decision parameters as in above. Suppose the resultant fuzzy soft multi set be $(\tilde{F}, \hat{A})$ given in Table 1.

First, we apply the Algorithm 1 (Feng's Algorithm) to the U_1—fuzzy soft multi set part in $(\tilde{F}, \hat{A})$ to obtain the decision from the first fuzzy soft multi set part U_1. Now we represent the U_1—fuzzy soft multi set part in $(\tilde{F}, \hat{A})$ as in Table 2.

Table 1 The tabular representation of the fuzzy soft multi-set $(\tilde{F}, \hat{A})$

U_i		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_1	h_1	0.3	0.8	1	0.8	0.4	0.9	1	0.8
	h_2	0.4	0.9	0.8	0.6	0.6	0.6	0.9	0.7
	h_3	0.9	0.3	0.7	0.1	0.8	0.7	0.8	1
	h_4	0.7	0.8	0	0.5	0.7	0.5	0.4	0.9
U_2	c_1	0.8	0.8	0.8	0.5	1	0.8	0.8	0.8
	c_2	0.6	0.8	0.6	0.3	0.9	0.8	0.8	0.8
	c_3	0.6	0.5	0.3	0.1	0.9	0.5	0.5	0.5
U_3	v_1	0.9	0.7	0.5	0.5	0.8	0.8	0.5	0.8
	v_2	0.7	0.6	0.5	0.3	0.5	0.8	0.6	0.9
	v_3	0.9	0.5	0.7	0.4	0.4	1	0.8	0.9

Table 2 Tabular representation of U_1—fuzzy soft multi set part of $(\tilde{F}, \hat{A})$

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_1	h_1	0.3	0.8	1	0.8	0.4	0.9	1	0.8
	h_2	0.4	0.9	0.8	0.6	0.6	0.6	0.9	0.7
	h_3	0.9	0.3	0.7	0.1	0.8	0.7	0.8	1
	h_4	0.7	0.8	0	0.5	0.7	0.5	0.4	0.9

In the Table 3, we see that the maximum choice value (s_k) is $s_3 = 6$ and scored by h_3. So we modify the fuzzy soft multi set $(\tilde{F}, \hat{A})$, by taking all values in each row are fixed, where the choice value of h_3 is maximized and changing the values in other rows by 0 (zero), to get $(\tilde{F}, \hat{A})_1$ as in Table 4.

We apply the Algorithm 1 (Feng's Algorithm) to the U_2—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_1$, to obtain the decision from U_2—fuzzy soft multi set part in $(\tilde{F}, \hat{A})$. Now we represent the U_2—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_1$ as in Table 5.

In Table 6, we see that the maximum choice value (s_k) is $s_1 = 3$ and scored by c_1. Therefore, we modify the fuzzy soft multi set $(\tilde{F}, \hat{A})_1$ by taking all values in each row are fixed, where the choice value of c_1 is maximized and changing the values in other rows by 0 (zero), to get $(\tilde{F}, \hat{A})_2$ (Table 7).

Similarly, we apply the Algorithm 1 (Feng's Algorithm) to the U_3—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_2$, to obtain the decision from U_3—fuzzy soft multi set part in $(\tilde{F}, \hat{A})$. Now we represent the U_3—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_2$ as in Table 8.

In Table 9, we see that the maximum choice value (s_k) is 2, scored by v_1 and v_2.

Thus, the final optimal decision for decision maker Mr. X is (h_3, c_1, v_1) or (h_3, c_1, v_2), i.e. Mr. X may chose (h_3, c_1, v_1) or (h_3, c_1, v_1).

Remark 1 In the step (7) of our algorithm (Algorithm 2), if there are too many optimal choices obtained, then decision maker may go back to the step (2) as in our algorithm (Algorithm 2) and replace the level soft set (decision criterion) that he/she once used to adjust the final optimal decision in the fuzzy soft multi set based decision making problems.

Table 3 Mid-level soft set of the U_1—fuzzy soft multiset part in $(\tilde{F}, \hat{A})$, with choice values

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Choice value (s_k)
U_1	h_1	0	1	1	1	0	1	1	0	$s_1 = 5$
	h_2	0	1	1	1	0	0	1	0	$s_2 = 4$
	h_3	1	0	1	0	1	1	1	1	$s_3 = 6$
	h_4	1	1	0	1	1	0	0	1	$s_4 = 5$

Table 4 The tabular representation of the fuzzy soft multi-set $(\tilde{F}, \hat{A})_1$

U_i		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_1	h_1	0.3	0.8	1	0.8	0.4	0.9	1	0.8
	h_2	0.4	0.9	0.8	0.6	0.6	0.6	0.9	0.7
	h_3	0.9	0.3	0.7	0.1	0.8	0.7	0.8	1
	h_4	0.7	0.8	0	0.5	0.7	0.5	0.4	0.9
U_2	c_1	0.8	0	0	0	1	0	0	0.8
	c_2	0.6	0	0	0	0.9	0	0	0.8
	c_3	0.6	0	0	0	0.9	0	0	0.5
U_3	v_1	0.9	0	0	0	0.8	0	0	0.8
	v_2	0.7	0	0	0	0.5	0	0	0.9
	v_3	0.9	0	0	0	0.4	0	0	0.9

Table 5 Tabular representation of the U_2—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_1$

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_2	c_1	0.8	0	0	0	1	0	0	0.8
	c_2	0.6	0	0	0	0.9	0	0	0.8
	c_3	0.6	0	0	0	0.9	0	0	0.5

Table 6 Mid-level soft set of the U_2—fuzzy soft multi set part in $(\tilde{F}, \hat{A})_1$, with choice values

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Choice value (s_k)
U_2	c_1	1	0	0	0	1	0	0	1	$s_1 = 3$
	c_2	0	0	0	0	0	0	0	1	$s_2 = 1$
	c_3	0	0	0	0	0	0	0	0	$s_3 = 0$

Table 7 Tabular representation of the fuzzy soft multi set $(\tilde{F}, \hat{A})_2$

U_i		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_1	h_1	0.3	0.8	1	0.8	0.4	0.9	1	0.8
	h_2	0.4	0.9	0.8	0.6	0.6	0.6	0.9	0.7
	h_3	0.9	0.3	0.7	0.1	0.8	0.7	0.8	1
	h_4	0.7	0.8	0	0.5	0.7	0.5	0.4	0.9
U_2	c_1	0.8	0	0	0	1	0	0	0.8
	c_2	0.6	0	0	0	0.9	0	0	0.8
	c_3	0.6	0	0	0	0.9	0	0	0.5
U_3	v_1	0.9	0	0	0	0.8	0	0	0.8
	v_2	0.7	0	0	0	0.5	0	0	0.9
	v_3	0.9	0	0	0	0.4	0	0	0.9

Table 8 Tabular representation of U_3—fuzzy soft multi set part in $(\tilde{F},\hat{A})_2$

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
U_3	v_1	0.9	0	0	0	0.8	0	0	0.8
	v_2	0.7	0	0	0	0.5	0	0	0.9
	v_3	0.9	0	0	0	0.4	0	0	0.9

Table 9 Mid-level soft set of the U_3—fuzzy soft multi set part in $(\tilde{F},\hat{A})_2$, with choice values

		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Choice value (s_k)
U_3	v_1	1	0	0	0	1	0	0	0	$s_1 = 2$
	v_2	0	0	0	0	0	0	0	1	$s_2 = 1$
	v_3	1	0	0	0	0	0	0	1	$s_3 = 2$

Advantages of our algorithm (Algorithm 2) are as follows:

(1) From the above illustration, we have seen that our algorithm (Algorithm 2) is too simple and less computation than Alkhazaleh–Salleh Algorithm [3]. Because instead of computing comparison tables and calculating scores as in Alkhazaleh–Salleh Algorithm [3], we have to consider only choice values of objects form the level soft sets of fuzzy soft multi set parts in the fuzzy soft multi set.

(2) Also, our algorithm (Algorithm 2) is an adjustable algorithm, because the level soft set (decision rule) used by decision makers, which are changeable. For example, if we take top-level decision criterion in step (2) of our algorithm (Algorithm 2), then we have the choice value of each object in the top-level soft set of fuzzy soft multi set parts in the fuzzy soft multi set, if we take another decision rule such as the mid-level decision criterion, then we have choice values from the mid-level soft set of fuzzy soft multi set parts in the fuzzy soft multi set.

4 Conclusion

In [8], Kong et al. mentioned that the Roy-Maji Algorithm [13] was wrong and Feng et al. [7] mentioned that the Roy-Maji Algorithm [13] has some limitations and Alkhazaleh and Salleh [3] presented an application of fuzzy soft multi set based decision-making problems using Roy-Maji Algorithm [13], so Alkhazaleh–Salleh Algorithm [3] is not sufficient to solve fuzzy soft multi set based decision making problems. In this study, we have proposed an algorithm for fuzzy soft multi set based decision making problems using Feng's algorithm [7], which is more stable and more feasible than the Alkhazaleh–Salleh Algorithm [3] for solving decision-making problems based on fuzzy soft multi sets.

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