Intuitionistic Fuzzy Similarity and Information Measures with Physical Education Teaching Quality Assessment

Arunodaya Raj Mishra, Divya Jain and D.S. Hooda

Abstract Information and similarity measures have a vital place in the fuzzy set theory. It has been investigated by many researchers with different aspects. In this paper, new intuitionistic fuzzy similarity and information measures based on sine function are proposed. Comparison of proposed information measures with the existing ones is listed. Numerical results clearly indicate the efficiencies of these measures over others. New technique for multi-criteria decision-making (MCDM) quandaries to rank the alternatives is introduced. This technique is developed on the application intuitionistic fuzzy information measure and weighted averaging operator (IFWAO). A case of five colleges ranking of a district region is studied and discussed.

Keywords Intuitionistic fuzzy set · Intuitionistic fuzzy information measure · Similarity measure · MCDM · TOPSIS · Physical education teaching quality

1 Introduction

Fuzziness is the imperfection and skepticality, if the limits and margins of the set under the concern is not well defined, i.e., whether the element belongs to or does not belong to a set. The incipient approaches and theories treating imprecision and dubiousness were introduced by Zadeh [1]. Grzegorzewski [2] discussed the promising and indispensable inclusion of IFS followed [3] who introduced the

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concept of intuitionistic fuzzy set (IFS). Mishra et al. [4, 5] proposed exponential and trigonometric intuitionistic fuzzy information measures and applied these to assessment of service quality and rating of township development. Castineira [6] recognized the incompatibility measure in the framework of the IFS of Atanassov in the form of axioms.

While Liu and Wang [7] introduced incipient methods to solve multi-criteria decision-making quandaries in an intuitionistic fuzzy environment. An approach in group decision-making was developed by Xu [8, 9] on intuitionistic preference relations and also proposed clustering algorithm for IFS. The distance between IFS was investigated by Szmidt and Kacprzyk [10], Wang and Xin [11], and many other researchers further studied the distance measures in IFS and applied it to many real-life problems of pattern recognition.

The degree of homogeneous attribute or similarity of fuzzy sets is known as similarity measures. It is a consequential concept in the fuzzy set theory. It was introduced by Wang [12] who also provided computation formula. Thereafter it occupied the main focus of the various research findings. Many investigations have been done and extensively applied in fuzzy clustering, image processing, fuzzy reasoning, and fuzzy neural networks. Comparative study of similarity measures was conducted by Wang et al. [13]. Many studies fixate on the similarity measure and the information measure for IFS [11, 14] and the relationship between them, particularly, on the organized conversion of the information measure to the similarity measure and similarity measure to information measure for IFSs.

In today's scenario, it is common to make decision in the presence of multiple conflicting criteria, which is known as multiple criteria (or attribute) decision-making (MCDM/MADM) quandaries. For example, if one has to buy a refrigerator, he has to look at many parameters like prize, capacity, safety, power consumption, technology, etc. A decision in multi-criteria decision-making quandaries can be taken by selecting the best suitable alternatives based on quantitative and qualitative assessment. Required option can be chosen by providing the preference set in the terms of numerical value or interval. But, in real-life situation the data set or preference values are not always precise, it can be vague or imprecise, which can be great dispute for the decision taking authorities and hence was a leading factor toward MCDM techniques in fuzzy scenarios.

The technique for ordering the preferable option by similarity to an ideal elucidation (TOPSIS) was developed by Hwang and Yoon [15]. In this technique, the concept is that the chosen option should be the closest from the positive ideal elucidation and the farthest from the negative ideal elucidation. Joshi and Kumar [16] proposed intuitionistic fuzzy information and distance measure-based TOPSIS technique for multi-criteria decision-making (MCDM). Likewise various authors introduced IFS with TOPSIS to give a hybrid technique for MCDM problems. Some methods were based on IFS using entropy weights and linear programming given by Liu and Wang [7, 17, 18].

The present communication is arranged as follows: Sect. 2 introduces the fundamental conceptions of fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy aggregation operator, and intuitionistic fuzzy information and similarity measures, followed by the study of new similarity and intuitionistic fuzzy information measures and their validity in Sect. 3. In Sect. 4, empirical illustration is listed, and comparison is tabulated between new and existing intuitionistic fuzzy information measures, followed by the application of the proposed intuitionistic fuzzy information measure in physical education teaching assessment in Sect. 5.

2 Prerequisite

In this section, authors discuss some fundamental conceptions of fuzzy sets and intuitionistic fuzzy sets, and information and similarity measures for IFSs. Throughout this paper, FSs and IFSs represent the fuzzy sets and intuitionistic fuzzy sets in X, respectively.

Definition 2.1 ([1]) Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse and $\tilde{A} \subset X$. Then $\tilde{A}$ is a fuzzy set defined by

$$\tilde{A} = \left\{ \left(x_i, \mu_{\tilde{A}}(x_i) \right) : \mu_{\tilde{A}}(x_i) \in [0,1]; \forall\, x_i \in X \right\}, \tag{1}$$

where $\mu_{\tilde{A}}(x_i)$ is the membership function $\mu_{\tilde{A}}(x_i) : X \to [0,1]$ such that

$$\mu_{\tilde{A}}(x_i) = \begin{cases} 0, & \text{if } x_i \notin \tilde{A} \text{ and there is no ambiguity} \\ 1, & \text{if } x_i \in \tilde{A} \text{ and there is no ambiguity} \\ 0.5, & \text{there is maximum ambiguity whether } x_i \notin \tilde{A} \text{ or } x_i \in \tilde{A}. \end{cases}$$

Definition 2.2 ([3]) The intuitionistic fuzzy set $A \subset X = \{x_1, x_2, \ldots, x_n\}$ is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{2}$$

where $\mu_A : X \to [0,1]$ is the degree of membership and $\nu_A : X \to [0,1]$ is the degree of nonmembership of $x \in X$ in A, respectively, such that

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.$$

The intuitionistic index (or hesitancy degree) of an element $x \in X$ in A is as

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

It implies $0 \leq \pi_A(x) \leq 1, \forall x \in X$ [3].

If $\pi_A(x) = 0$ then IFSs can formed FSs, i.e., $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$, with $\pi_A(x) = 0, \forall x \in X$.

The complement set of A is A^c and defined as

$$A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}. \tag{3}$$

[19] represents $\alpha = \langle a,\ b\rangle$ an intuitionistic fuzzy number (IFN), if $0 \le a + b \le 1$.

Definition 2.3 Let $\alpha_1 = \langle a_1,\ b_1\rangle$ and $\alpha_2 = \langle a_2, b_2\rangle$ be two IFNs, three intuitionistic fuzzy aggregation operators are defined by

$$\begin{aligned} &\alpha_1 \oplus \alpha_2 = \langle a_1 + a_2 - a_1 a_2,\ b_1 b_2\rangle, \alpha_1 \otimes \alpha_2 = \langle a_1 a_2, b_1 + b_2 - b_1 b_2\rangle \text{ and} \\ &\quad w\alpha_1 = \left\langle 1 - (1 - a_1)^w, b_1^w\right\rangle, w > 0. \end{aligned} \tag{4}$$

Definition 2.4 ([20]) Let $h : IFS(X) \to [0, 1]$ be an information measure defined on IFSs. Then it is valid, if it holds the given postulate:

(P1). $h(A) =$ (minimum) $\Leftrightarrow A$ is a crisp set;
(P2). $h(A) = 1$ (maximum) $\Leftrightarrow \mu_A(x_i) = \nu_A(x_i)$ for any $x_i \in X$;
(P3). $h(A) \le h(B)$ and if A is less fuzzy than B, i.e., $\mu_A(x_i) \le \mu_B(x_i)$ and $\nu_A(x_i) \ge \nu_B(x_i)$ for $\mu_B(x_i) \le \nu_B(x_i)$ or $\mu_A(x_i) \ge \mu_B(x_i)$ and $\nu_A(x_i) \le \nu_B(x_i)$ for $\mu_B(x_i) \ge \nu_B(x_i)$ for any $x_i \in X$;
(P4). $h(A) = h(A^c)$.

Definition 2.5 The function $Sim : IFS(X) \times IFS(X) \to [0, 1]$ is said to be a valid measure of similarity on $IFS(X)$, if Sim holds

(S1). $Sim(A, A^c) = 0$ if A is a crisp set;
(S2). $Sim(A, B) = 1$, iff $A = B$;
(S3). $Sim(A, B) = Sim(B, A)$;
(S4). $Sim(A, C) \le Sim(A, B)$ and $Sim(A, C) \le Sim(A, B)$, For all $A, B, C \in IFS(X)$ and $A \subseteq B \subseteq C$.

3 Similarity and Information Measures for IFSs

Here, similarity and information measures for IFSs are proposed and their validity are proved.

Let $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in X\}$ be two IFSs in X defines

$$Sim(A, B) = 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left[\frac{\{\max(|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|)\}\pi}{2}\right]. \tag{5}$$

Theorem 1 *The mapping Sim, defined by* (5) *is a similarity measure for IFSs.*

Proof It is obvious that $Sim(A, B)$ holds the postulate (S1)–(S3). Hence only (S4) has to be proved. For $A \subset B \subset C$, we have $\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$, and $v_A(x_i) \ge v_B(x_i) \ge v_C(x_i), \forall x_i \in X$. It implies

$$\max(|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|) \\ \le \max(|\mu_A(x_i) - \mu_C(x_i)|, |v_A(x_i) - v_C(x_i)|), \forall x_i \in X.$$

$$\sin\left[\frac{\{\max(|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|)\}\pi}{2}\right] \\ \le \sin\left[\frac{\{\max(|\mu_A(x_i) - \mu_C(x_i)|, |v_A(x_i) - v_C(x_i)|)\}\pi}{2}\right], \forall x_i \in X.$$

These imply

$$1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left[\frac{\{\max(|\mu_A(x_i) - \mu_B(x_i)|, |v_A(x_i) - v_B(x_i)|)\}\pi}{2}\right] \\ \ge 1 - \frac{1}{n}\sum_{i=1}^{n} \sin\left[\frac{\{\max(|\mu_A(x_i) - \mu_C(x_i)|, |v_A(x_i) - v_C(x_i)|)\}\pi}{2}\right], \forall x_i \in X.$$

This means that

$$Sim(A, B) \ge Sim(A, C). \tag{6}$$

Similarly,

$$Sim(A, B) \ge Sim(A, C). \tag{7}$$

Let $A = \{(x, \mu_A(x), v_A(x)) : x \in X\}$ be an IFS in X defines

$$h(A) = \frac{1}{n}\sum_{i=1}^{n} \sin\left\{\left(\frac{\mu_A(x_i) \wedge v_A(x_i)}{\mu_A(x_i) \vee v_A(x_i)}\right)\frac{\pi}{2}\right\}. \tag{8}$$

Theorem 2 *The mapping* h, *defined by* (8) *is information measure on IFSs.*

Proof Let $h_i(A) = \sin\left\{\left(\frac{\mu_A(x_i) \wedge v_A(x_i)}{\mu_A(x_i) \vee v_A(x_i)}\right)\frac{\pi}{2}\right\}$, for $i = 1, 2, \ldots, n$. From $0 \le \mu_A(x_i) \le 1$, $0 \le v_A(x_i) \le 1$, we have $0 \le \left(\frac{\mu_A(x_i) \wedge v_A(x_i)}{\mu_A(x_i) \vee v_A(x_i)}\right)\frac{\pi}{2} \le \frac{\pi}{2}$. Thus $0 \le h_i(A) \le 1$.

(P1). For $\mu_A(x_i) = 0$, $v_A(x_i) = 1$ or $\mu_A(x_i) = 1$, $v_A(x_i) = 0$ for any $x_i \in X$. It implies $h_i(A) = 0$. Hence $h(A) = 0$.

Conversely, suppose that $h(A) = 0$ for $i = 1, 2, \ldots, n$. Since $0 \le h_i(A) \le 1$, therefore, it follows that $h_i(A) = 0$. Also since $0 \le \left(\frac{\mu_A(x_i) \wedge v_A(x_i)}{\mu_A(x_i) \vee v_A(x_i)}\right)\frac{\pi}{2} \le \frac{\pi}{2}$, we have

$$\left(\frac{\mu_A(x_i) \wedge \nu_A(x_i)}{\mu_A(x_i) \vee \nu_A(x_i)}\right)\frac{\pi}{2} = 0 \text{ or } \left(\frac{\mu_A(x_i) \wedge \nu_A(x_i)}{\mu_A(x_i) \vee \nu_A(x_i)}\right)\frac{\pi}{2} = \frac{\pi}{2}.$$

Hence A is a crisp set.

(P2). Let $\mu_A(x_i) = \nu_A(x_i)$ for each $x_i \in X$. Applying this condition to (8), we easily obtain $h(A) = 1$.

Conversely, suppose that $h(A) = 1$. From (8) and $0 \leq h_i(A) \leq 1$, we obtain that $h_i(A) = 1$ for each $x_i \in X$. Also from $0 \leq \left(\frac{\mu_A(x_i) \wedge \nu_A(x_i)}{\mu_A(x_i) \vee \nu_A(x_i)}\right)\frac{\pi}{2} \leq \frac{\pi}{2}$, we have $\mu_A(x_i) = \nu_A(x_i)$ for each $x_i \in X$.

(P3). Let $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_B(x_i) \geq \nu_A(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$, i.e., $\mu_A(x_i) \geq \mu_B(x_i) \geq \nu_B(x_i) \geq \nu_A(x_i)$. Thus

$$\left(\frac{\mu_A(x_i) \wedge \nu_A(x_i)}{\mu_A(x_i) \vee \nu_A(x_i)}\right)\frac{\pi}{2} \leq \left(\frac{\mu_B(x_i) \wedge \nu_B(x_i)}{\mu_B(x_i) \vee \nu_B(x_i)}\right)\frac{\pi}{2}. \tag{9}$$

It follows that $h_i(A) \leq h_i(B)$.

Similarly, when $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_B(x_i) \leq \nu_A(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$, i.e., $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$. we can also prove that $h_i(A) \leq h_i(B)$.

Hence we have

$$h(A) \leq h(B). \tag{10}$$

(P4). For $A^c = \{(x_i, \nu_A(x_i), \mu_A(x_i)) : x_i \in X\}$, we can easily get that

$$h(A^c) = h(A). \tag{11}$$

Hence $h(A)$ is valid intuitionistic fuzzy information measure.

4 Numerical Comparisons

In present section, the efficiency of information measure through comparisons with some existing information measures for IFSs is demonstrated.

Example 1 Let

$$A_1 = \{(x, 0.2, 0.4) : x \in X\}, A_2 = \{(x, 0.3, 0.4) : x \in X\},$$
$$A_3 = \{(x, 0.4, 0.5) : x \in X\}, A_4 = \{(x, 0.5, 0.5) : x \in X\}.$$

We first recall these information measures.

Vlachos and Sergiagis [21] proposed an entropy measure E_{VS} according to cross entropy measure:

Table 1 Comparisons with existing entropies

	A_1	A_2	A_3	A_4
h	0.7071	0.9239	0.9511	1
E_{VS}	0.9510	0.9897	0.9920	1
E_{WZ}	0.9749	0.9927	0.9898	1

$$E_{VS} = -\frac{1}{n \ln 2}\sum_{i=1}^{n}[\mu_A(x_i)\ln\mu_A(x_i) + \nu_A(x_i)\ln\nu_A(x_i) \\ - (1-\pi_A(x_i))\ln(1-\pi_A(x_i)) - \pi_A(x_i)\ln 2]. \quad (12)$$

Wei and Zhang [22] developed information measure based on cosine function for IFSs:

$$E_{WZ}(A) = \frac{1}{n}\sum_{i=1}^{n}\cos\left\{\frac{(\mu_A(x_i)-\nu_A(x_i))\pi}{2(1+\pi_A(x_i))}\right\}. \quad (13)$$

These two information measures satisfy the set of requirements in Definition 2.4.

From Table 1, it can be seen that h and E_{VS}, the nearer the degree of membership and nonmembership, the higher the information measure. Particularly, when $\mu_A(x_i) = \nu_A(x_i)$, the information measure reaches its maximum value. The results obtained by using the information measures h and E_{VS} are in accordance with the intuition.

Table 1 shows that $h(A_1) < h(A_2) < h(A_3) < h(A_4)$ and $E_{VS}(A_1) < E_{VS}(A_2) < E_{VS}(A_3) < E_{VS}(A_4)$, which are consistent with the intuition. But, from the above table it is visible that $E_{WZ}(A_1) < E_{WZ}(A_2) > E_{WZ}(A_3) < E_{WZ}(A_4)$. It implies that the information measure E_{WZ} is not in accordance with the intuition. And hence our measure is more efficient and authentic than the information measure E_{WZ}.

5 Physical Education Teaching Quality Assessment

So far, researchers concentrated on the empirical as well as theoretical evidences for the reform and development of the teaching quality assessment to deal with confounding quandaries of expansion and quality improvement of physical education. The association of teaching quality system and the proposal for the fundamental supervision is favorable to the growth and the suitable progress of the undergraduate physical education majors [23, 24].

Xu [19, 25] dealt with triangular intuitionistic fuzzy information measure in the multi-criteria attribute decision-making quandaries, while solving the problems of physical education teaching quality in the universities and colleges. Here, in this section, we develop an approach on intuitionistic fuzzy information measure to evaluate the physical education teaching quality in the universities and colleges. The developed IFMWAO method is as follows:

Step 1. Let $C = \{C_1, C_2, \ldots, C_m\}$ be a discrete set of colleges of a district region, and $L = \{L_1, L_2, \ldots, L_n\}$ be the set of options. Let $\tilde{D} = \left(\tilde{r}_{ij}\right)_{m\times n} = \left(\mu_{ij}, v_{ij}\right)_{m\times n}$ be the intuitionistic fuzzy judgment matrix, where μ_{ij} represents the extend up to which C_i holds L_j and v_{ij} reflects C_i does not support the option L_j.

Step 2. By using information measure formula (8), we can easily compute the information of each intuitionistic fuzzy value in the judgment matrix and obtain the information matrix of this judgment matrix as $D = \left(h_{ij}\right)_{m\times n}$, where $h_{ij} = E\left(\tilde{r}_{ij}\right)$.

Step 3. Normalize the information values in the above decision matrix by the following equation

$$\overline{h}_{ij} = \frac{h_{ij}}{\max h_{ij}}, j = 1(1)n; i = 1(1)m. \quad (14)$$

And expressed it as $\overline{D} = \left(\overline{h}_{ij}\right)_{m\times n}$.

Step 4. Compute weight vector by applying the given formula

$$w_i = \frac{1 - \sum_{j=1}^{n} \overline{h}_{ij}}{m - \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{h}_{ij}}, i = 1(1)m. \quad (15)$$

Step 5. Chen and Tan [26] defines the score function δ of x as

$$\delta(x) = \mu_A(x) - v_A(x), \quad (16)$$

where $\delta(x) \in [-1, 1]$. For higher values of deviation $\delta(x)$ the membership value $\mu_A(x)$ should be high, while the nonmembership value should be low. The accuracy function h defined by Hong and Choi [27] to compute the degree of precision of the intuitionistic fuzzy value is

$$\eta(x) = \mu_A(x) + v_A(x), \quad (17)$$

where $\eta(x) \in [0, 1]$. The greater value of $\eta(x)$ results in the superior degree of membership of IFS. Let $\alpha_1 = (\mu_1, v_1)$, $\alpha_2 = (\mu_2, v_2)$ be two intuitionistic fuzzy values, then (Fig. 1)

1. for $\delta(\alpha_1) < \delta(\alpha_2)$, we obtain $\alpha_1 < \alpha_2$;
2. for $\delta(\alpha_1) = \delta(\alpha_2)$, we get if $\eta(\alpha_1) < \eta(\alpha_2)$, then $\alpha_1 < \alpha_2$; otherwise if $\eta(\alpha_1) = \eta(\alpha_2)$, then $\alpha_1 = \alpha_2$.

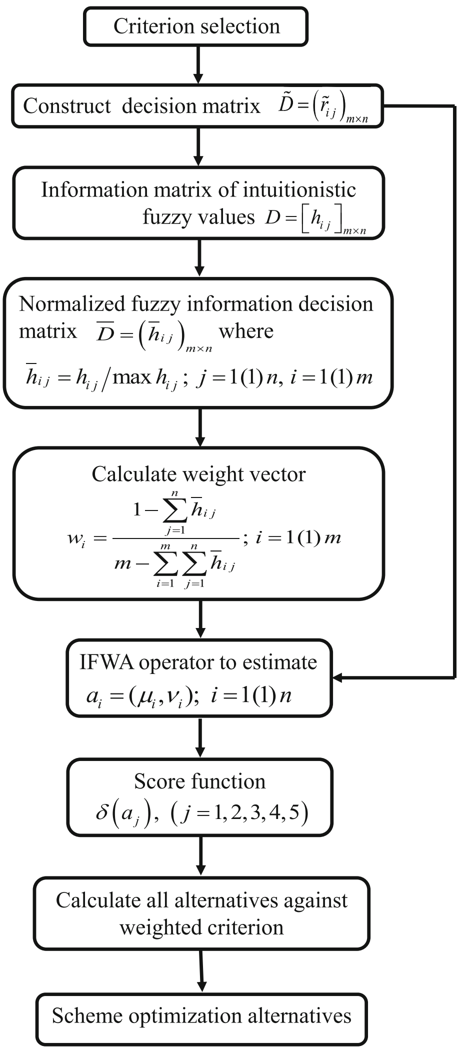

Fig. 1 Implementation flowchart of IFMWAO method

Step 6. Wu and Zhang [28] proposed the intuitionistic fuzzy weighted averaging operator as

$$IFWA_{\omega}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\left(1-\prod_{i=1}^{n}(1-\mu_i)^{\omega_i},\ \prod_{i=1}^{n}v_i^{\omega_i}\right), \qquad (18)$$

where ω_i is the weight of α_i, $\omega_i\in[0,1]$ and $\sum_{i=1}^{n}\omega_i=1$.

5.1 A Real Case Study

The above-defined method is illustrated mathematically in this subsection. Let there be a panel of five colleges of a district region $C_j(j = 1, 2, \ldots, 5)$. The education institution has to take a decision based on the following four options: L_1 is the sports basic theoretical knowledge evaluation; L_2 is the physical fitness and sports skill evaluation; L_3 is the learning attitude evaluation; L_4 is the effective expression. The ranking of five colleges of a district region $C_j(j = 1, 2, \ldots, 5)$ are calculated by the triangular intuitionistic fuzzy set on the above four options, and the decision matrix is constructed as (Table 2)

Step 1. The information measure is computed using formula (8) in the above judgment matrix and get the following information matrix:

$$D = \left(h_{ij}\right)_{4\times 5} = \begin{bmatrix} 0.7071 & 0.8090 & 0.7071 & 0.4343 & 0.6228 \\ 0.9511 & 0.7071 & 0.5090 & 0.8090 & 0.8655 \\ 0.6228 & 0.9511 & 0.5090 & 1.0000 & 0.8655 \\ 0.8090 & 1.0000 & 0.9511 & 0.7071 & 0.8090 \end{bmatrix}.$$

Step 2. Using (14), the above information matrix is transformed to the normalized information matrix as

$$\overline{D} = \left(\overline{h}_{ij}\right)_{4\times 5} = \begin{bmatrix} 0.7840 & 1.0000 & 0.8740 & 0.5386 & 0.7698 \\ 1.0000 & 0.7435 & 0.5352 & 0.8506 & 0.9100 \\ 0.6228 & 0.9511 & 0.5090 & 1.0000 & 0.8655 \\ 0.8090 & 1.0000 & 0.9511 & 0.7071 & 0.8090 \end{bmatrix}.$$

Step 3. Compute weight vector by utilizing the normalized information matrix and by the formula (14), we have

$$w_i = \frac{1 - \sum_{j=1}^{n} \overline{h}_{ij}}{m - \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{h}_{ij}}, i = 1(1)m.$$

Hence, obtained as $W = (0.2455, 0.2475, 0.2401, 0.2669)$.

Table 2 Weighted intuitionistic fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
L_1	(0.6, 0.3)	(0.5, 0.3)	(0.3, 0.6)	(0.7, 0.2)	(0.7, 0.3)
L_2	(0.5, 0.4)	(0.6, 0.3)	(0.6, 0.2)	(0.5, 0.3)	(0.4, 0.6)
L_3	(0.7, 0.3)	(0.5, 0.4)	(0.6, 0.2)	(0.4, 0.4)	(0.6, 0.4)
L_4	(0.5, 0.3)	(0.5, 0.5)	(0.4, 0.5)	(0.6, 0.4)	(0.5, 0.3)

Table 3 Ranking order alternatives for different methods

Method	Ranking	Best alternative
TOPSIS proposed by Grzegorzewski and Mrowka [29]	$C_1 \succ C_4 \succ C_3 \succ C_5 \succ C_2$	C_1
Intuitionistic fuzzy TOPSIS proposed by Joshi and Kumar [16]	$C_1 \succ C_3 \succ C_4 \succ C_5 \succ C_2$	C_1
Proposed IFMWAO method	$C_1 \succ C_3 \succ C_4 \succ C_5 \succ C_2$	C_1

Step 4. Obtain score function

By applying $IFWA_{\omega}$ operator (18), the set of intuitionistic fuzzy values $\alpha_j = (\mu_j, v_j)$, where $j = 1(1)n$, is obtained as

$$\alpha_1 = (0.5812, 0.3222), \alpha_2 = (0.4770, 0.3684), \alpha_3 = (0.5458, 0.3345),$$
$$\alpha_4 = (0.4596, 0.3142), \alpha_5 = (0.4941, 0.3816).$$

By (16), score function $\delta(\alpha_j), (j = 1, 2, 3, 4, 5)$ is as

$$\delta(\alpha_1) = 0.2590, \delta(\alpha_2) = 0.1086, \delta(\alpha_3) = 0.2113, \delta(\alpha_4) = 0.1454, \delta(\alpha_5) = 0.1125.$$

Step 5. Desirable ranking

The options are ranked in the descending order by comparing the score values. Here, we obtained the following ordering as $C_1 \succ C_3 \succ C_4 \succ C_5 \succ C_2$. Thus the most desirable college is C_1.

The ranking of these five alternatives are also shown in the following Table 3.

It is found that there is no conflict in the preference ordering of all the alternatives by intuitionistic fuzzy TOPSIS method and developed technique. There is only one conflict found in deciding the preference ordering of C_3 and C_4.

6 Conclusions

In this work, new intuitionistic fuzzy similarity and information measures are developed and analyzed the features of developed measures. Numerical results illustrate the proposed similarity and information measures are sensible and effective supplement of measures of IFSs. An intuitionistic fuzzy information method for MCDM problems has been proposed. A real case study to the rank five colleges of a district region based on four attributes using the proposed method is done.

Moreover, the results of the developed technique and the already existing method are compared. Consequently, it is concluded that our method reduces the complexity, while evaluating the results and hence it is more effective and efficient.

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