Chapter 9
Sensitivity Analysis of Rectangular DRA

Abstract Sensitivity analysis of rectangular DRA depending on dielectric material, and *a*, *b*, and *d* dimensions. These dimensions decides resonant frequency of RDRA. The resonant modes are formed when realized with excitation. The resonant frequency solution is worked with MATLAB and HFSS software. When these dimensions are changed, resonant frequency of RDRA also changes. Variance method has been tried out to evaluate error.

Keywords Isolated RDRA · Ground plane RDRA · Resonant frequency · Sensitivity analysis · Variance · Error minimization

Rectangular DRAs of dielectric material having *a*, *b*, and *d* dimensional length have been analyzed for frequency and resonant modes. RDRA is shown in Fig. 9.1. These have been solved based on MATLAB and HFSS. Figure 9.2 presented rectangular DRA with *a*, *b*, and *d* dimensions. Table 9.1 has shown RDRA dimensions and their corresponding resonant frequencies. Figure 9.3 indicated resonant modes with RDRA height. Plot of frequency versus length "*a*" variation is shown in Figs. 9.4, 9.5 and 9.6. HFSS simulated modes in RDRA with S_{11} parameters are shown in Figs. 9.8 and 9.9 (Fig. 9.7).

$\delta a, \delta b, \delta d$ are (small change in length) random variables, and computed functions are $f\left(\delta_{mnp}\right)$ and ω_{mnp}. The variance functions are $\sigma_a, \sigma_b, \sigma_d$. These are mainly dependent on *a*, *b*, and *d*. Taylor's expansion is restricted to second-order variable. Hence, $\delta a, \delta b, \delta d$ are mapped in terms of $\sigma_a, \sigma_b, \sigma_d$ using diagonal matrix. C_{mnp}, D_{mnp}, are amplitude coefficients which depend on the RDRA *a*, *b*, or *d*.

Frequency relationship can be determined based on *a*, *b* and *d* length variation as given below:

$$\frac{\delta}{\delta d}, \frac{\delta}{\delta b}, \frac{\delta\omega(mnp|a,b,d)}{\delta a}$$

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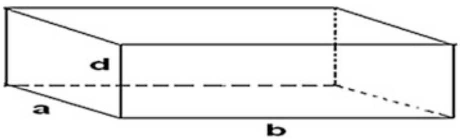

Fig. 9.1 Isolated RDRA

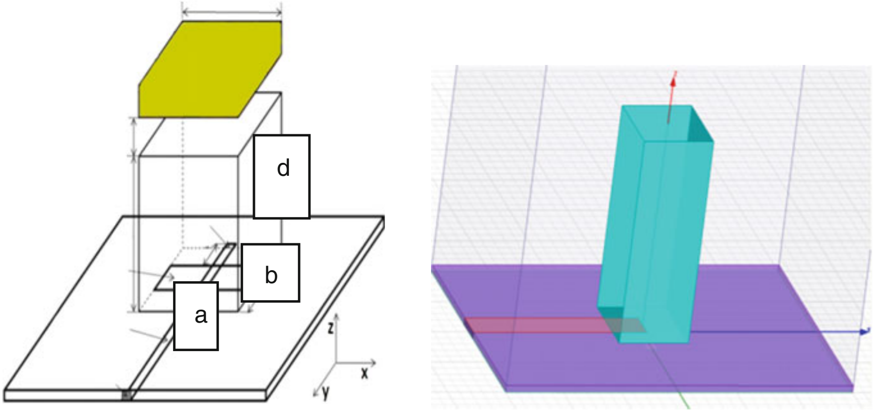

Fig. 9.2 Rectangular DRA with *a*, *b*, and *d* dimensions

Table 9.1 RDRA dimensions

	x (mm)	y (mm)	z (mm)	ε_r	Material used
RDRA	$a = 7$ $a = 6$ $a = 5$	$b = 7$ $b = 6$ $b = 5$	$d = 10$ $d = 15$ $d = 30$	10	Sapphire
Substrate	20	30	0.5	3.38	$Arlon_{25}N$(tm)
Ground plane	20	30	–	–	–
Microstrip feed line	15	1.11	–	–	–

DRA dimensions (mm)			Resonant frequencies (GHz) simulated
a	b	d	
7	7	10	$f_1 = 13.46$
6	6	15	$f_2 = 13.85$
5	5	30	$f_3 = 14.21$

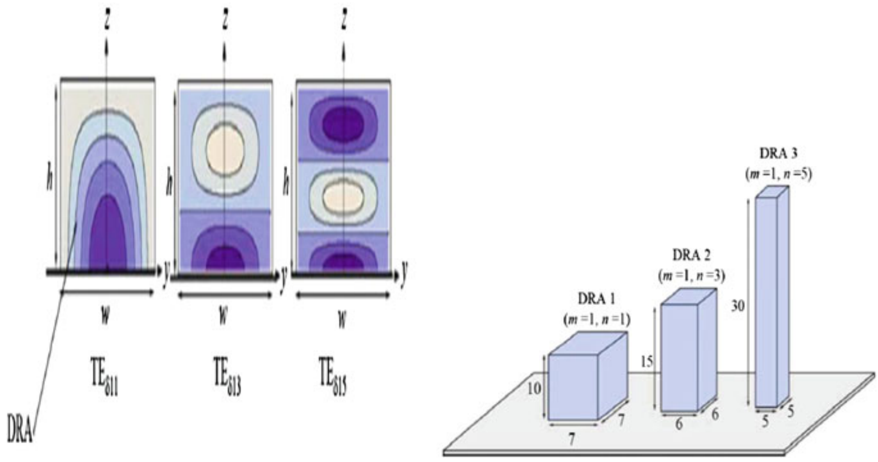

Fig. 9.3 Resonant mode and RDRA height relationship

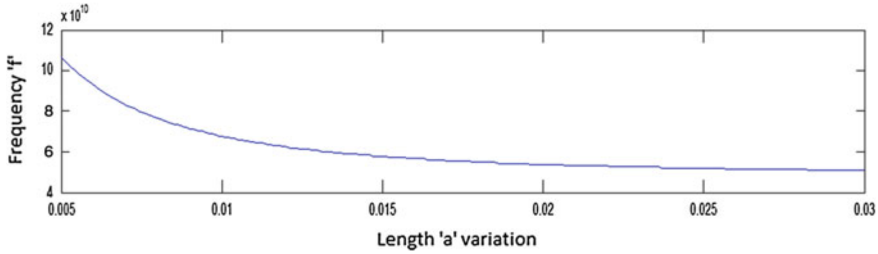

Fig. 9.4 Plot of frequency versus length "*a*" variation

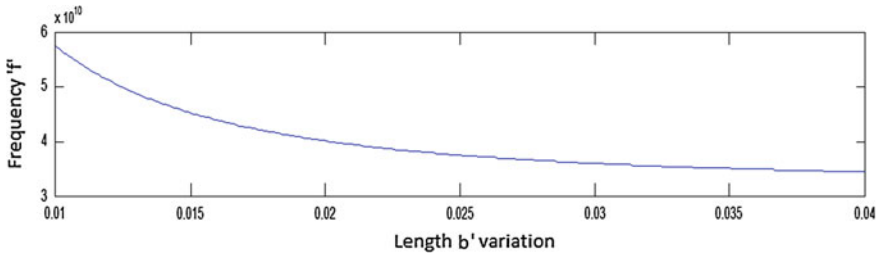

Fig. 9.5 Plot of frequency versus length "*b*" variation

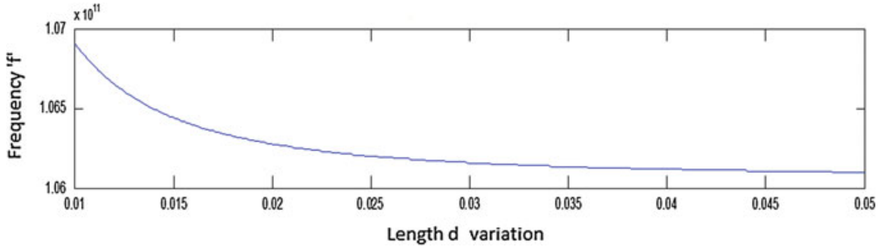

Fig. 9.6 Plot of frequency versus length "*d*" variation

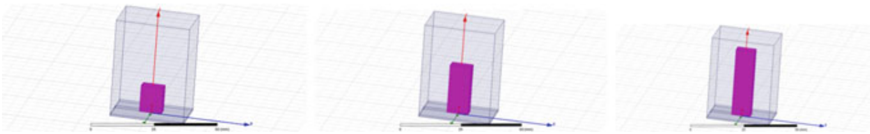

Fig. 9.7 RDRA having three different heights for increasing resonant modes

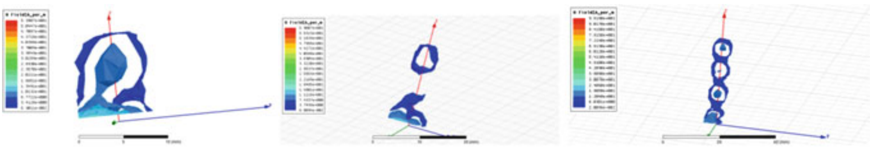

Fig. 9.8 Modes' pattern

H_z and E_z fields are expressed based on principle of orthogonality as:

$$E_z(x, y, z, t) = \sum_{mnp} \mathrm{Re}\left(e^{j\omega(mnp)t} C(mnp)\right) u_{mnp}(x, y, z) \tag{9.1a}$$

$$H_z(x, y, z, t) = \sum_{mnp} \mathrm{Re}\left(e^{j\omega(mnp)t} C(mnp)\right) v_{mnp}(x, y, z) \tag{9.1b}$$

At $z = 0$, E_z field

$$E_z(t, x, y, 0) = \sum_{mnp} \mathrm{Re}\left(e^{j\omega(mnp)t} C(mnp)\right) u_{mn}(xy) \sqrt{\frac{2}{d}} \tag{9.2}$$

and

$$H_x, H_y = -J_{sy}, J_{sx}$$

C_{mnp}, D_{mnp} are amplitude coefficients.

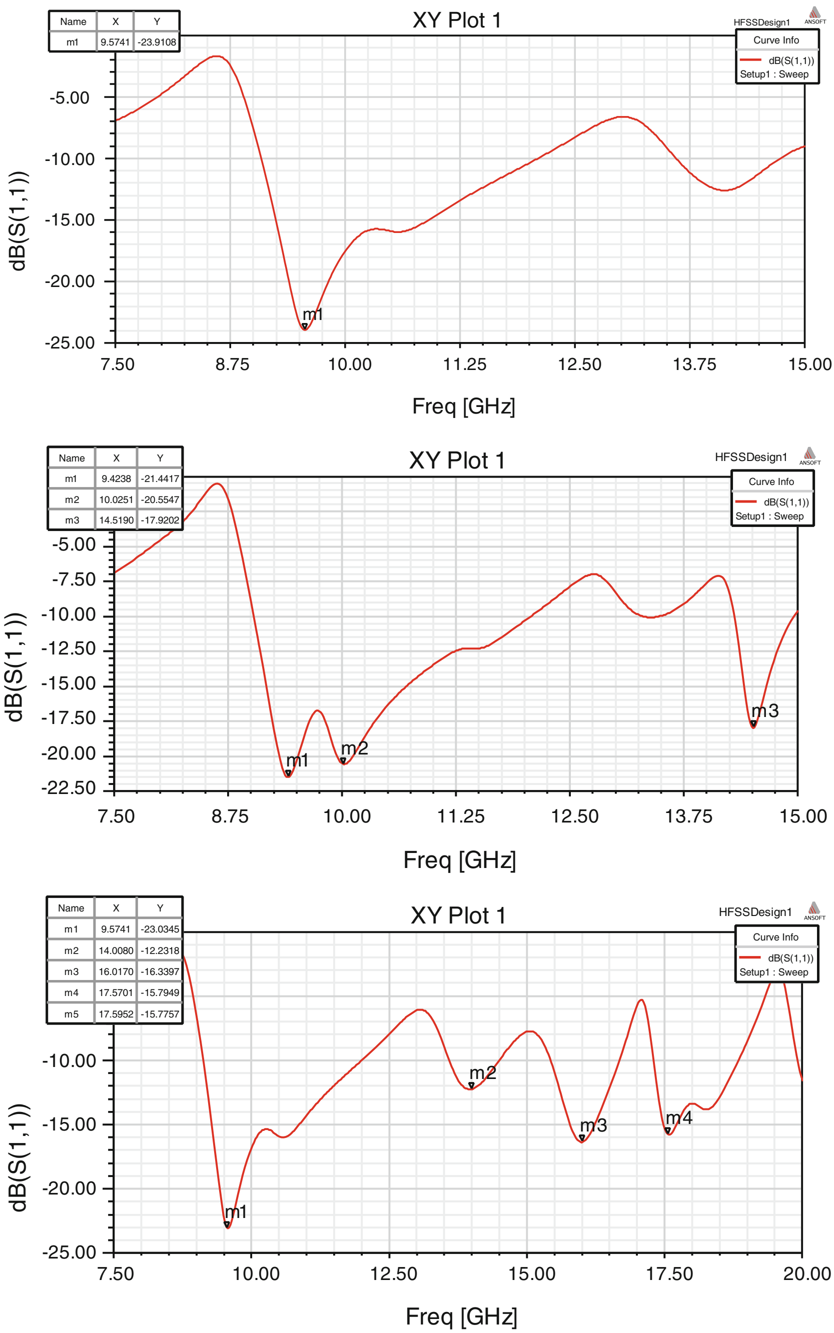

Fig. 9.9 Resonant frequency based on RDRA height

We have computed

$$C_{mnp} = \frac{2\sqrt{2}}{\sqrt{abd}} \int J_{sx}(X, Y) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \mathrm{d}x\mathrm{d}y \tag{9.3}$$

Now if

$$\begin{aligned} a &\to a + \delta a \quad (a \text{ is increased to } a + \delta a) \\ b &\to b + \delta b \quad (b \text{ is increased to } b + \delta b) \\ d &\to d + \delta d \quad (d \text{ is increased to } d + \delta d) \end{aligned}$$

We need to compute

$$C_{(mnp|a + \delta a, b + \delta b, d + \delta d)}$$

and similarly

$$\omega_{(mnp|a + \delta a, b + \delta b, d + \delta d)}$$

This can be approximated by mean variance method and Taylor's expansion

N normal distribution with mean zero
σ variance function

To compute error

$$\begin{bmatrix} \delta a \\ \delta b \\ \delta d \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_b^2 & 0 \\ 0 & 0 & \sigma_d^2 \end{bmatrix}\right)$$

(By Taylor's expansion)

$$\begin{aligned} C_{(mnp|a + \delta a, b + \delta b, d + \delta d)} = {} & C_{(mnp|a,b,d)} + \frac{\delta C(mnp)}{\delta a}\delta a + \frac{\delta C(mnp)}{\delta b}\delta b + \frac{\delta C(mnp)}{\delta d}\delta d \\ & + \frac{1}{2}\left(\frac{\delta^2 C(mnp)}{\delta a^2}\delta a^2 + \frac{\delta^2 C(mnp)}{\delta b^2}\delta b^2 + \frac{\delta^2 C(mnp)}{\delta d^2}\delta d^2 \right. \\ & \left. + 2\frac{\delta^2 C(mnp)}{\delta a \delta b}\delta a \delta b + 2\frac{\delta^2 C(mnp)}{\delta a \delta d}\delta a \delta d + 2\frac{\delta^2 C(mnp)}{\delta b \delta d}\delta b \delta b\right) \end{aligned} \tag{9.4}$$

Hence, variance or error can be written as

$$\begin{aligned} \left\langle C_{(mnp|a + \delta a, b + \delta b, d + \delta d)} - C^2_{(mnp|a,b,d)} \right\rangle &= \left|\frac{\delta C(mnp)}{\delta a}\right|^2 \left\langle |\delta a|^2 \right\rangle \\ &+ \left|\frac{\delta C(mnp)}{\delta b}\right|^2 \left\langle |\delta b|^2 \right\rangle + \left|\frac{\delta C(mnp)}{\delta d}\right|^2 \left\langle |\delta d|^2 \right\rangle \\ &= \sigma_a^2 \left|\frac{\delta C(mnp)}{\delta a}\right|^2 + \sigma_b^2 \left|\frac{\delta C(mnp)}{\delta b}\right|^2 + \sigma_d^2 \left|\frac{\delta C(mnp)}{\delta d}\right|^2 \end{aligned} \tag{9.5}$$

Similarly, we can compute:

$$\left\langle \omega_{(mnp|a+\delta a,b+\delta b,d+\delta d)} - \omega_{(mnp|a,b,d)} \right\rangle$$

Error value

$$\omega_{(mnp)} + \delta\omega_{(mnp)} - \left\langle |\delta\omega|^2 \right\rangle$$

Error or variance:

$$= \left|\frac{\delta\omega(mnp)}{\delta a}\right|^2 \sigma_a^2 + \left|\frac{\delta\omega(mnp)}{\delta b}\right|^2 \sigma_b^2 + \left|\frac{\delta\omega(mnp)}{\delta d}\right|^2 \sigma_d^2$$

$$\omega = \pi\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{d^2}}$$

The change in frequency based on RDRA change in dimension in x, y, and z direction is given below:

$$\frac{\partial\omega}{\partial a} = \frac{\frac{-m^2\pi}{a^3}}{\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{d^2}}} \tag{9.6a}$$

$$\frac{\partial\omega}{\partial b} = \frac{\frac{-n^2\pi}{b^3}}{\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{d^2}}} \tag{9.6b}$$

$$\frac{\partial\omega}{\partial d} = \frac{\frac{-p^2\pi}{d^3}}{\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2} + \frac{p^2}{d^2}}} \tag{9.6c}$$

This gives the complete solution of RDRA sensitivity analysis. The higher-order modes of a rectangular DRA were used to produce radiation patterns with enhanced gain. The advantage of this approach is for enhancing gain. The maximum achievable gain on mode $m = 1$, $n = 7$ to increase Directivity to 13.7 dBi. Such DRA designed at 11 GHz with height 35 mm, this investigation focused on rectangular DRAs, for excitation of the appropriate higher-order modes in RDRAs.

9.1 MATLAB Simulation

Matlab Program for Sensitivity Analysis

```
clear all;

clc;

close all;

c=3*10^8;

m=1;

n=1;

p=1;

E=10;


a=5*10^-3:.1*10^-3:30*10^-3;

b=10*10^-3;

d=15*10^-3;

for i=1:length(a)

f(i)=c/(2*pi)*sqrt(E)*sqrt((m*pi/a(i))^2+(n*pi/b)^2+(pp*pi/(2*d)^2));

end

a1=15*10^-3;

b1=10*10^-3:.1*10^-3:40*10^-3;

d1=20*10^-3;
```

```
for k=1:length(b1)

f1(k)=c/(2*pi)*sqrt(E)*sqrt((m*pi/a1)^2+(n*pi/b1(k))^2+(pp*pi/(2*d1)^2));

end



a2=10*10^-3;

b2=5*10^-3;

d2=10*10^-3:.1*10^-3:50*10^-3;

for t=1:length(d2)

f2(t)=c/(2*pi)*sqrt(E)*sqrt((m*pi/a2)^2+(n*pi/b2)^2+(pp*pi/(2*d2(t))^2));

end



subplot(3,1,1);plot(a,f);title('plot a vs f when a is varying');

subplot(3,1,2);plot(b1,f1);title('plot b vs f when b is varying');

subplot(3,1,3);plot(d2,f2);title('plot d vs f when d is varying');
```

9.2 HFSS Simulations

Now using HFSS software, we shall verify.

This gives the complete solution for RDRA sensitivity analysis.

It has been observed that resonant modes have been increasing based on increase in dipole moment, i.e., modes are proportional to the height of RDRA.

9.2.1 HFSS Result

See below Figs. 9.10, 9.11, 9.12, 9.13 and Table 9.2.

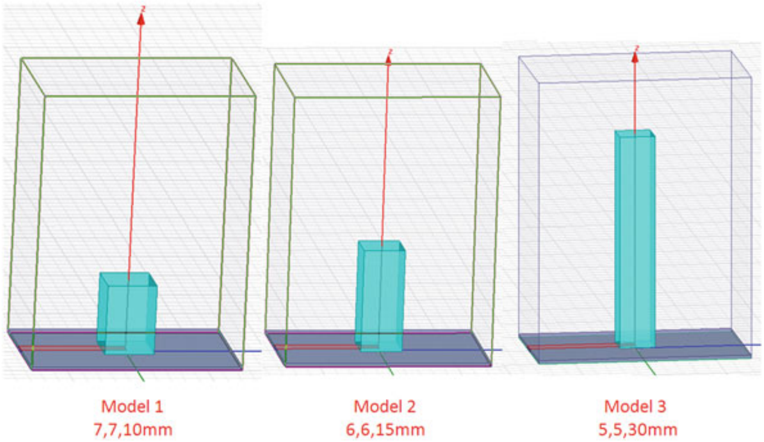

Fig. 9.10 HFSS models of RDRA

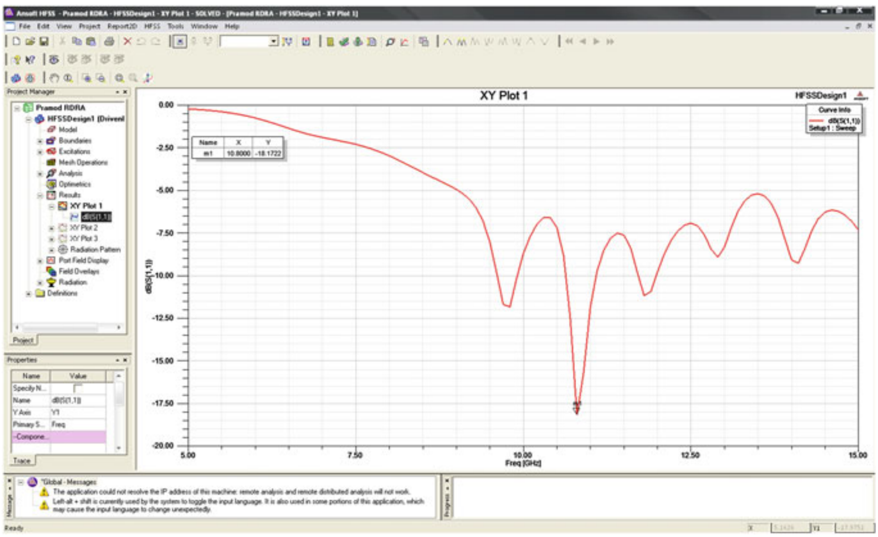

Fig. 9.11 Return loss versus frequency with dimensions $a = 5$ mm, $b = 5$ mm, $d = 30$ mm shows Return loss 18 dBi at $f = 10.95$ GHz

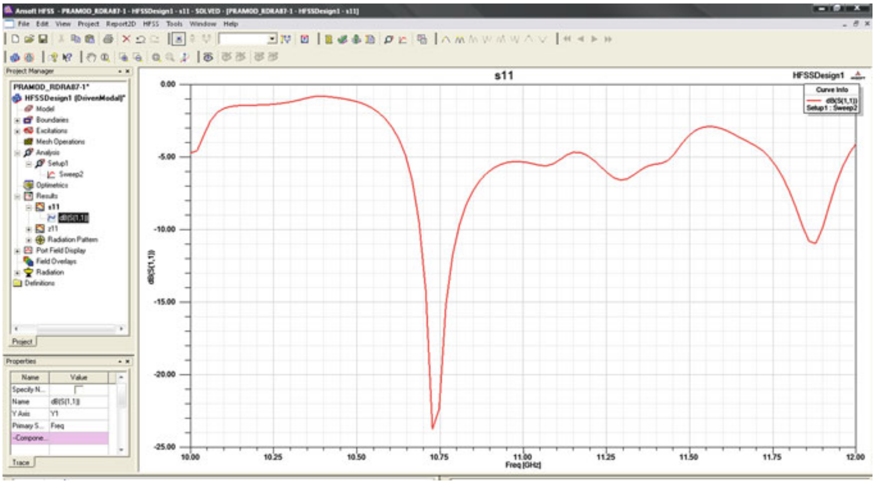

Fig. 9.12 Return loss versus frequency with dimensions $a = 6$ mm, $b = 6$ mm, $d = 15$ mm shows return loss 24 dBi at $f = 10.95$ GHz

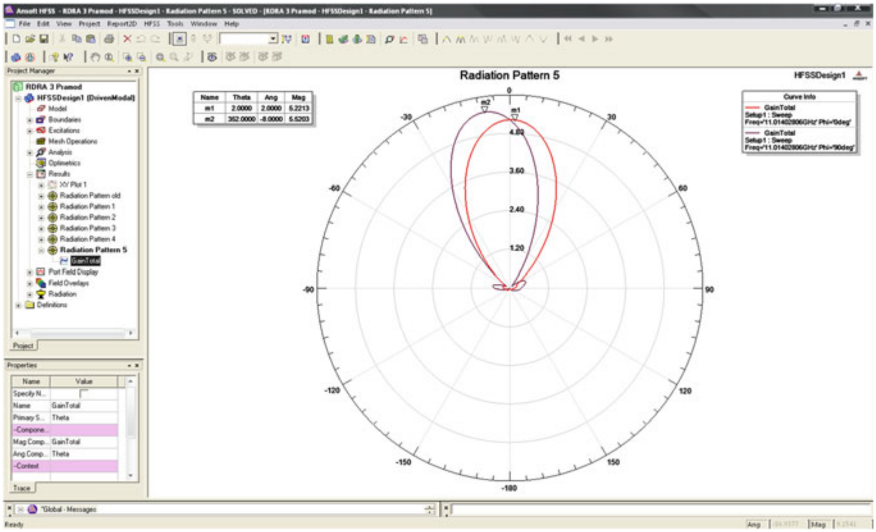

Fig. 9.13 Gain = 5.5 at $f = 11$ GHz for DRA 1

Table 9.2 Dimensions table

	x (mm)	y (mm)	z (mm)	ε_r	Material used
RDRA	$a = 7$ $a = 6$ $a = 5$	$b = 7$ $b = 6$ $b = 5$	$d = 10$ $d = 15$ $d = 30$	10	TMM10i
Substrate	20	30	0.5	3.38	Arlon$_{25}$N(tm)
Ground plane	20	30	–	–	–
Microstrip feed line	19.2	1.1672	–	–	–
Lumped element	1.1672	0.5	–	–	–

9.3 Radiation Pattern

See below Figs. 9.14 and 9.15.

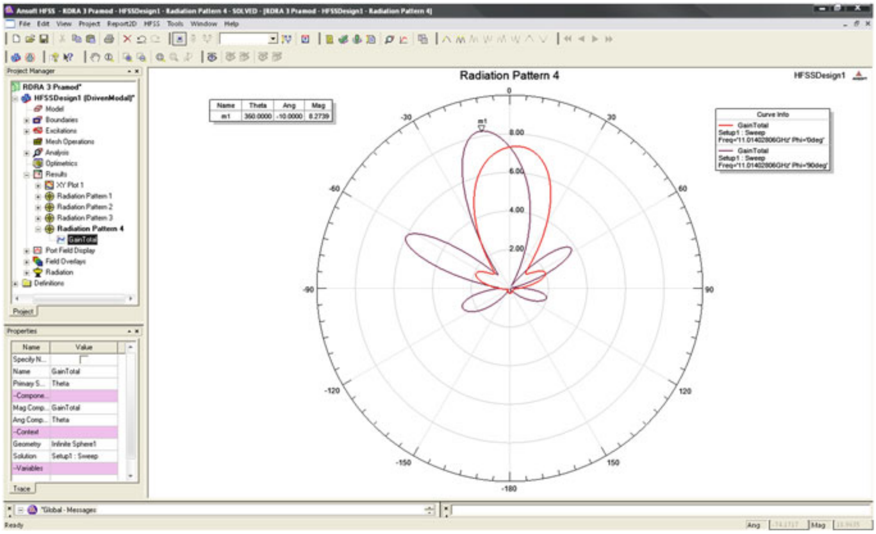

Fig. 9.14 Gain = 8.2 at f = 11 GHz for DRA 2

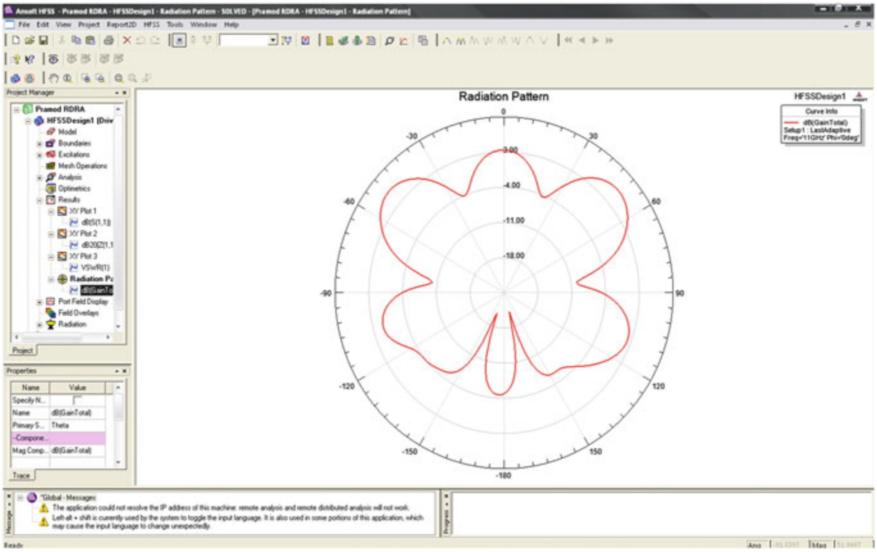

Fig. 9.15 Gain = 9.5 at f = 11 GHz for DRA 3