Chapter 6
Mathematical Analysis of Radiation Pattern of RDRA

Abstract In this chapter, detailed study using mathematical analysis for radiation pattern of RDRA has been described. RF excitation with proper impedance match can generate *J*-current density into surfaces of RDRA, which leads to produce *A*-magnetic vector potential and finally *E*-electric intensity or *H*-magnetic field intensity. Acceleration or deceleration of charge carriers causing current is mandatory phenomenon for radiations. Wave can only propagate if wave vector $k > k_c$, where k_c is cutoff frequency. The lowest resonance can be termed as dominant mode and second and third resonances are higher-order modes. Propagation constant $k_x = n\pi/a$, and propagation takes place if $k_x > n\pi/a$, while no propagation takes place if $k_x < n\pi/a$. Thus, standing waves inside the resonator are formed and energy storing will take place. Hence, mode spectrum will result into corresponding resonant frequency generation. Wave propagation can be well defined by Helmholtz equation. The Maxwell's equations describe the behavior of electromagnetic fields and form the basis of all EM classical phenomena. P_{rad} (power radiated) can be evaluated using Parseval's power theorem. The radiated power is produced by oscillating dipole moments. The current varying in time can be analyzed by Fourier analysis. If medium is inhomogeneous, wave possesses exponential growth or decay in some direction. Thus, Poynting vector "*S*" shall give the magnitude and phase of the radiated fields in particular direction.

Keywords Impedance match · Current density · Magnetic vector potential · Power radiated · Poynting vector · Persvals power theorem · Moat-shaped DRA

6.1 Introduction

RF excitation with proper impedance match can generate *J*-current density into surfaces of RDRA, which leads to produce *A*-magnetic vector potential and finally *E*-electric intensity. Acceleration or deceleration of charge carriers causing current is mandatory phenomenon for radiations. Wave can only propagate if wave vector

R.S. Yaduvanshi and H. Parthasarathy, *Rectangular Dielectric Resonator Antennas*, DOI 10.1007/978-81-322-2500-3_6

$k > k_c$, where k_c is cutoff frequency and the lowest resonance can be termed as dominant mode and second and third resonances are higher-order modes. Propagation constant $k_x = n\pi/a$. Propagation takes place if $k_x > n\pi/a$, while no propagation takes place $k_x < n\pi/a$. Standing waves inside the resonator are formed and energy storing will take place. Hence, mode spectrum will result into corresponding resonant frequency generation due to equivalent RLC circuit formation. Wave propagation can be well defined by Helmholtz equation. The Maxwell's equations describe the behavior of electromagnetic fields and form the basis of all EM classical phenomenon. P_{rad} (power radiated) can be evaluated using Parseval's power theorem. The radiated power is produced by oscillating dipole moments. The current varying in time can be analyzed by Fourier analysis. If medium is inhomogeneous, wave possesses exponential growth or decay in some direction. Thus, Poynting vector "*S*" shall give the magnitude and phase of the radiated fields in particular direction.

Finally, the radiation pattern produced by the surface electric and magnetic current densities on the RDRA surfaces is computed. PEC walls, the surface electric current density is $J_s = \hat{n} \times E$.

Then, the far-field magnetic vector and electric vector potentials are determined by the usual reactance potential formulae as follows:

$$\underline{A}(\omega, \underline{r}) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{\sigma}\int_s \underline{J}_s(\omega, \underline{r}')\ \exp(jk\hat{r}\cdot\underline{r}')\ \mathrm{d}s(\underline{r}'); \tag{6.1a}$$

and

$$\underline{F}(\omega, \underline{r}) = \frac{\epsilon}{4\pi}\frac{e^{-jkr}}{\sigma}\int_s \underline{M}_s(\omega, \underline{r}')\ \exp(jk\hat{r}\cdot\underline{r}')\ \mathrm{d}s(\underline{r}'). \tag{6.1b}$$

Lorentz force conditions are applied to determine the far-field electric scalar and magnetic scalar potentials as follows:

$$\begin{aligned}\underline{\overline{\phi}}_e(\omega, \underline{r}) &= \frac{j}{\omega\epsilon\mu}\operatorname{div}\underline{A}(\omega, \underline{r})\\ &= \frac{k}{\omega\epsilon\mu}(\hat{r}, \underline{A}(\omega, \underline{r}))\end{aligned} \tag{6.2a}$$

$$\begin{aligned}\underline{\overline{\phi}}_m(\omega, \underline{r}) &= \frac{j}{\omega\epsilon\mu}\operatorname{div}\underline{F}(\omega, \underline{r})\\ &= \frac{k}{\omega\epsilon\mu}(\hat{r}, \underline{F}(\omega, \underline{r}))\end{aligned} \tag{6.2b}$$

The far-field electric and magnetic fields (i.e., up to Order (r^{-1})) are then determined as follows:

$$\underline{E} = -\nabla\underline{\bar{\phi}_e} - j\omega\underline{A} + \frac{1}{\epsilon}\underline{\nabla} \times \underline{F}; \tag{6.3}$$

$$\begin{aligned}\underline{H} &= -\frac{1}{\mu}\underline{\nabla} \times \underline{A} - \underline{\nabla\overline{\phi}_m} - j\omega\underline{F};\\ &= \frac{jk^2}{\omega\epsilon\mu}\hat{r}(\hat{r}, \underline{A}) - j\omega\underline{A\phi}\frac{jk}{\epsilon}\hat{r} \times \underline{F};\\ &= -j\omega\underline{A}_{\perp}\underline{\phi_m}\frac{jk}{\epsilon}\hat{r} \times \underline{F};\end{aligned} \tag{6.4}$$

where

$$\begin{aligned}\underline{A}_{\perp} &= A_\theta\hat{\theta} + A_\phi\hat{\phi};\\ \underline{H}_{\perp} &= \frac{jk}{\mu}\hat{r} \times \underline{A} - j\omega\underline{F}_1;\end{aligned}$$

Finally, we derive expression for the Poynting vector as follows:

$$\underline{S} = \frac{1}{2}\mathrm{Re}\{E \times H^*\}.$$

Up to order $\left(\frac{1}{r^2}\right)$ i.e., value $1/r^2$ is taken into account from where, the RDRA radiation resistance is evaluated:

$$\frac{1}{2}I^2R_r = \lim_{r\to\infty}\int S \cdot \hat{r} \cdot r^2 \cdot \mathrm{d}\Omega;$$

when I is the input current to the RDRA, R_r or $R_r(\omega)$ is radiation resistance and depends on the frequency.

6.2 Radiation Pattern of RDRA Due to Probe Current *i*(*t*) and Probe Length d*l*

$$\frac{\mu I\overrightarrow{dl}e^{-jkr}}{4\pi r} = \overrightarrow{\underline{A}}; \quad \text{where } A \text{ is magnetic vector potential} \tag{6.5}$$

From Helmholtz equation $|\vec{\underline{A}}|$

$$\underline{E} = -j\omega\vec{\underline{A}}$$

Radiated power can be given as follows:

$$\frac{|\underline{E}|^2}{2\eta} = \frac{\omega^2 \left|\vec{\underline{A}}\right|^2}{2\eta}, \quad \sqrt{\frac{\mu}{\epsilon}} = \eta = \text{characteristic impedance.}$$

$$\vec{\underline{A}} = \frac{\mu}{4\pi} \int\limits_{\text{Volume}} \frac{J(\underline{r}', \omega) e^{-jk|\underline{r}-\underline{r}'|}}{|\underline{r} - \underline{r}'|} \mathrm{d}^3 r'; \text{ at source.} \tag{6.6}$$

We know that radiation pattern can be defined by the electrical field intensity E_θ, E_ϕ:

$$E_\theta = -j\omega A_\theta \quad \text{and} \quad \underline{A}_\theta = \hat{\theta} \cdot \underline{A}$$

Antenna surface current density can be expressed as follows:

$$J(\underline{r}', \omega) = \sum_{mnp} \underline{J}_s[mnp, \underline{r}'] e^{j\omega(mnp)t}; \quad \text{where, } r = (x, y, z) \tag{6.7}$$

The magnetic vector potential in terms of J can be written as follows:

$$\begin{aligned} \underline{A} &= \frac{\mu}{4\pi} \sum_{mnp} \int \frac{\underline{J}_s[mnp, \underline{r}'] e^{j\omega(mnp)\left(t - \frac{|\underline{r}-\underline{r}'|}{c}\right)}}{|\underline{r} - \underline{r}'|} \mathrm{d}s(\underline{r}'); \quad \text{where, d}s \text{ is surface of RDRA} \\ &= \frac{\mu}{4\pi} \frac{e^{jkn}}{|\underline{r} - \underline{r}'|} \sum_{mnp} \int\limits_s \underline{J}_s[mnp, \underline{r}'] e^{j\omega(mnp)\hat{\underline{r}}\cdot\underline{r}'} \mathrm{d}s(\underline{r}') \end{aligned} \tag{6.8}$$

$$H_\phi = E_\theta/\eta, \quad H_\theta = -{}^{E_\phi}\!/_{\eta}.$$

Hence radiated power can be given as:

$$P_{\text{rad}} = \frac{1}{2\eta}\left(|E_\theta|^2 + \left|E_\phi\right|^2\right)$$

$$\begin{cases} \hat{\theta} = \hat{x}\cos\varphi\cos\theta + \hat{y}\sin\varphi\cos\theta - \hat{z}\sin\theta \\ \hat{\phi} = -\hat{x}\sin\varphi + \hat{y}\cos\varphi \end{cases}$$

$$\begin{aligned} E_\theta = &\frac{\mu}{4\pi r^2} \mathrm{Re} \sum_{mnp} \int\limits_s \left\{ J_{sx}[mnp, \underline{r}'] \cos\varphi\cos\theta + J_{sy\left[mnp,\underline{r}'\right]} \sin\phi\cos\theta - J_{sz\left[mnp,\underline{r}'\right]} \sin\theta \right\} \\ &\exp(j\omega \frac{(mnp)}{c}\}(x'\cos\phi\sin\theta + y'\sin\phi\sin\theta + z'\cos\theta)\ \mathrm{d}s(\underline{r}')]\ e^{j\omega(mnp)t}; \end{aligned} \tag{6.9a}$$

$$E_\varphi = \mathrm{Re}\sum_{mnp}\int_s \left\{-J_{sx}(mnp,\underline{r}')\sin\phi + J_{sy[mnp,\underline{r}']}\cos\phi\right\}e^{j\frac{\omega(mnp)}{c}} \quad (6.9b)$$
$$(x'\cos\varphi\sin\theta + y'\sin\phi\sin\theta + z'\cos\theta)\,\mathrm{d}s(\underline{r}')\,e^{j\omega(mnp)t}\mathrm{d}s(\underline{r}').$$

Radiated power P_{rad}, x, y, z component wise, can thus be defined as follows:

$$P_x[\hat{r}|mnp] = \int_s J_{sx}(mnp,\underline{r}']e^{j\frac{\omega(mnp)\hat{r}\cdot\underline{r}'}{c}}\mathrm{d}s(\underline{r}') \quad (6.10a)$$

$$P_y[\hat{r}|mnp] = \int_s J_{sy}(mnp,\underline{r}']e^{j\frac{\omega(mnp)\hat{r}\cdot\underline{r}'}{c}}\mathrm{d}s(\underline{r}') \quad (6.10b)$$

$$P_z[\hat{r}|mnp] = \int_s J_{sz}(mnp,\underline{r}']e^{j\frac{\omega(mnp)\underline{r}\cdot\underline{r}'}{c}}\mathrm{d}s(\underline{r}') \quad (6.10c)$$

$$\hat{r}(\theta,\phi) = \hat{x}\cos\phi\sin\theta + \hat{y}\sin\phi\sin\theta + \hat{z}\cos\theta.$$

Let $s = mnp$ for convenience then

$$\begin{aligned} E_\theta &= \mathrm{Re}\sum_s\left\{P_x[\hat{r}|s]\cdot\cos\phi\cos\theta + P_y[\hat{r}|s]\sin\phi\cos\theta - P_z[\hat{r}|s]\sin\theta\right\}e^{j\omega(s)t} \\ &= \mathrm{Re}\sum_s E_{s_\theta}e^{j\omega(s)t} \end{aligned} \quad (6.11)$$

where $s = (mnp) = \begin{bmatrix}000\\001\\010\end{bmatrix}$ and so on till $s = [111]$, similarly

$$E_\phi = \mathrm{Re}\sum_s(-P_x[\hat{r}|s]\sin\phi + P_y[\hat{r}|s]\cos\phi)e^{j\omega(s)t} \quad (6.12)$$

6.2.1 Radiation Pattern

Now, power radiation pattern can be defined as follows:

$$\begin{aligned} \frac{|E_\theta|^2+|E_\phi|^2}{2\eta} &= \frac{1}{2}\left\{\sum_s\left(E_{s\theta}e^{j\omega(s)t} + E_{s\phi}e^{j\omega(s)t}\right)\right\} \\ &\quad\times\frac{1}{2}\left\{\sum_s H_{s\theta}e^{j\omega(s)t} + \sum_s H_{s\phi}e^{j\omega(s)t}\right\} \end{aligned} \quad (6.13)$$

$$
\begin{aligned}
&= \frac{1}{4}\left(\sum_s E_s \times H_m^* \, e^{j(\omega_s-\omega_m)t} + \sum_s E_s^* \times H_m \, e^{j(\omega_m-\omega_s)t}\right) \\
&= \frac{1}{4}\sum_s \left[E_{s\theta} \times H_{s\text{ø}}^* + E_{s\text{ø}}^* \times H_{s\theta}\right] \\
&= \frac{1}{2}\mathrm{Re}\sum_s \left(E_{s\theta} \times H_{s\text{ø}}^*\right) \\
&= \frac{1}{2}\left(E_{s\theta}\hat{\theta} + E_{\text{ø}}\hat{\text{ø}}\right) \times \left(\frac{E_{s\theta}^*}{\eta}\hat{\phi} - \frac{E_{s\text{ø}}}{\eta}\hat{\theta}\right) \\
&= \sum_s \frac{|E_{s\theta}|2}{2\eta}\hat{r} + \frac{|E_{s\text{ø}}|2}{2\eta}\hat{r};
\end{aligned}
\tag{6.14}
$$

6.3 Poynting Vector

Poynting vector is defined as radiated power flux per unit solid angle or power radiated in particular direction in specified angular zone.

$H = \nabla \times A$

$E = -\nabla \text{ø} - \frac{\mathrm{d}A}{\mathrm{d}t}$; scalar and magnetic vector potential from Lorentz gauge conditions.

$S = (E \times H^*)$; S is Poynting vector (energy flow or flux).

$$Z = \frac{P_{\mathrm{rad}}}{|I|^2} = \text{Input impedance}$$

$$
\begin{aligned}
\underline{S}\cdot\hat{r} = \frac{1}{2\eta}\sum_{mnp}\Big[&\omega(s)^2|P_x(\hat{r}|s)\cos\phi\cos\theta + P_y(\hat{r}|s)\sin\phi\cos\theta - P_z(\hat{r}|s)\sin\theta\big|^2 \\
&+ \omega(s)^2|P_x(\hat{r}|s)\sin\phi - P_y(\hat{r}|s)\cos\phi\big|^2\Big]
\end{aligned}
\tag{6.15}
$$

$$
\begin{aligned}
\underline{S}\cdot\hat{r}(r,\theta,\phi) = \frac{1}{2\eta}\sum_{mnp}\omega(mnp)^2\{&|P_x(\theta,\phi|mnp)\cos\phi\cos\theta \\
&+ P_y(\theta,\phi|mnp)\sin\phi\cos\theta - P_z(\theta,\phi|mnp)\sin\theta\big|^2 \\
&+ |P_x(\theta,\phi|mnp)\sin\phi \pm P_y(\theta,\phi|mnp)\cos\phi\big|^2\}
\end{aligned}
\tag{6.16}
$$

6.4 Moat-Shaped RDRA Radiation Pattern

Moat-shaped RDRA is shown in Fig. 6.1a with x, y, and z coordinates, and feed is given at $a/2$ position.

In Fig. 6.1b, rectangular moat-shaped RDRA is covered with r copper plate to reduce resonant frequency.

$E(t, x, y, z)$ is electric field intensity of RDRA to be computed in time domain and $E(\omega, x, y, z)$ in frequency domain having a, b, and d dimensions, excited with feed probe at $\frac{a}{2}, \frac{a}{2}, 0$ point by $I_0 \cos \omega t$ RF current.

$A = A_z\hat{z}$ (due to RF excitation current $I_0 \cos \omega t$ along length d inserted into the RDRA).

Hence, magnetic vector potential can be written as follows:

$$A_z(\omega, x, y, z) = \frac{\mu I_0}{4\pi} \int_0^d \frac{e^{-jk|r - a/2\hat{x} - b/2\hat{y} - \xi\hat{z}|}}{|r - a/2\hat{x} - b/2\hat{y} - \xi\hat{z}|} \mathrm{d}\xi, \tag{6.17}$$

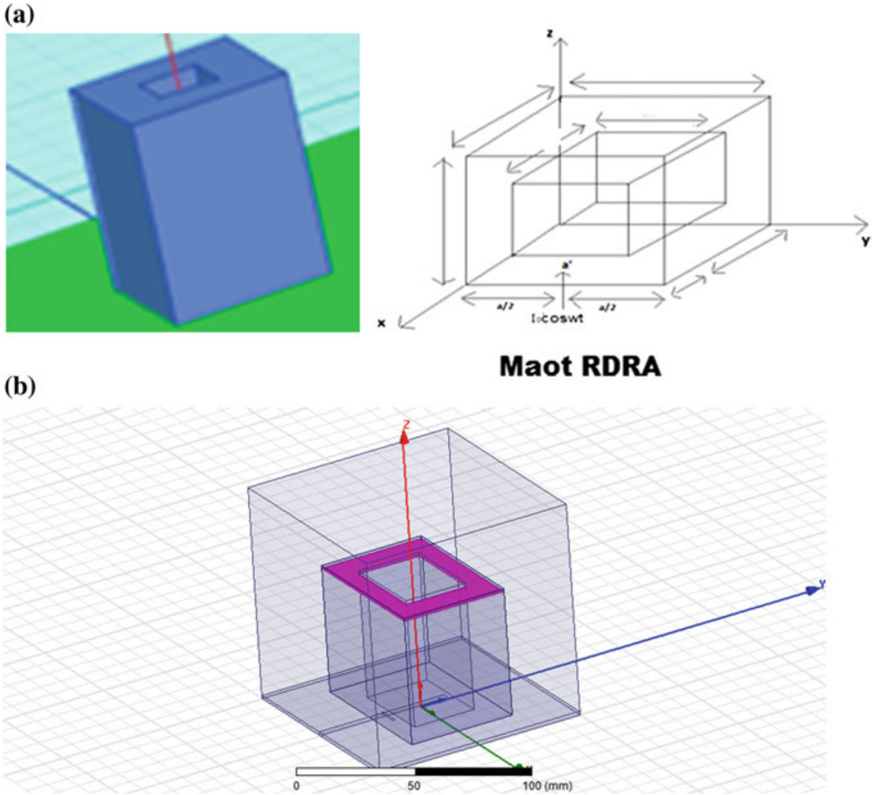

Fig. 6.1 **a** Moat-shaped RDRA. **b** RDRA moat cover with rectangular copper plate to reduce resonant frequency

Let $C = \frac{\mu}{4\pi}, k = \omega/c$ and ξ = variable probe length.

$$A_z = CI_0 \int_0^d \frac{\exp\{-jk\left((x - a/2)^2 + (y - a/2)^2 + (z - \xi)^2\right)}{\left((x - a/2)^2 + (y - a/2)^2 + (z - \xi)^2\right)^{1/2}}\Bigg\}\mathrm{d}\xi \tag{6.18}$$

Far-field approximation can be determined as follows:

$$A_z = \frac{CI_0 e^{-jkr}}{r} P(\theta_0, \varnothing_0), \tag{6.19}$$

where $P(\theta_0, \varnothing_0)$ is radiation pattern.

Here, it is assumed that probe is very small as compared to RDRA.

$$\begin{aligned}
&(x - a/2)^2 + (y - a/2)^2 + (z - \xi)^2 \\
&\quad = (x - a/2)^2 + \left(y - a/2\right)^2 + z^2 - 2z\xi \\
&(x - a/2)^2 + \left(y - a/2\right)^2 + z^2 \gg d^2 \\
&\quad r = \left((x - a/2)^2 + (y - a/2)^2 + z^2\right)^{1/2}
\end{aligned}$$

where r = distance from the points (x, y, z) in the center of the feed probe $\left(a/2, a/2, 0\right)$

$$\begin{aligned}
&\left((x - a/2)^2 + (y - a/2)^2 + (z)^2\right)^{1/2} \\
&\quad = (r^2 - 2z\xi)^{1/2} = r(1 - z\xi/r_0{}^2) = r - z\xi/r_0.
\end{aligned}$$

Hence, magnetic vector potential due to source inside RDRA can be computed as follows:

$$\begin{aligned}
A_z &= \frac{CI_0 e^{-jkr_0}}{r_0} \int_0^d \exp\left\{\frac{jkz\xi}{r_0}\right\}\mathrm{d}\xi, \quad \text{where } I_0 \text{ probe RF current.} \\
&= \frac{CI_0 e^{-jkr_0}}{r_0} \frac{\exp\left(jkz\xi/r_0\right)}{\left(jkz/r_0\right)} \left| \begin{matrix} \xi = d \\ \xi = 0 \end{matrix} \right| \quad \text{i.e., variable probe length.} \\
&= \frac{CI_0}{r_0} e^{-jkr_0} \frac{\exp\left(jkzd/r_0\right) - 1}{\left(jkz/r_0\right)} \\
&= \frac{CI_0}{r_0} e^{-jkr_0} \frac{\exp\left(jkzd/2r_0\right) 2j \sin\left(kzd/2r_0\right)}{\left(jkz/r_0\right)}
\end{aligned} \tag{6.20}$$

$$A_z = 2CI_0 \exp\left\{-jk\left(r_0 - {zd}/{2r_0}\right)\right\}\frac{\sin\left({kzd}/{2r_0}\right)}{kz}$$

where, $z = r\cos\theta$.

$$A_z(\omega, x, y, z) = CI_0 \exp(-jkr_0)\exp\frac{\left(\frac{jkd}{2}\ \cos\theta_0\right)\sin\left(\frac{kd\ \cos\theta_0}{2}\right)}{kr_0\ \cos\theta_0} \tag{6.21}$$

here, (r, θ, Φ) are spherical polar coordinates of (x, y, z) so as to relate $\left({a}/{2}, {a}/{2}, 0\right)$, the probe insertion point. Hence, magnetic vector potential can be expressed as follows:

$$\begin{aligned} E(t, x, y, z) =& \frac{\omega}{r_0^3}|P(\theta_0)|\sin(\omega t - kr_0 + \Psi(\theta_0))\left(\left(x-{a}/{2}\right)^2+\left(y-{a}/{2}\right)^2\right) + \left(x - \frac{a}{2}\right)z\hat{x} \\ &+ \left\{-\frac{\omega\left(\left(y-{a}/{2}\right)z\right)}{{r_0}^3}|P(\theta_0)|(\sin(\omega t - kr_0 + \Psi(\theta_0)))\right\}\hat{y} \end{aligned} \tag{6.22}$$

$$\begin{aligned} B(t, x, y, z) &= -\frac{k\left(x - \frac{a}{2}\right)}{r_0^2}|P(\theta_0)|\sin(\omega t - kr_0 + \Psi(\theta_0))\hat{y} \\ &+ \frac{k\left(y - \frac{a}{2}\right)}{{r_0}^2}|P(\theta_0)|\sin(\omega t - kr_0 + \Psi(\theta_0))\hat{x} \\ &= k\frac{P(\theta_0)}{{r_0}^2}\sin(\omega t - kr_0 + \Psi(\theta_0))\left(\left(y - \frac{a}{2}\right)\hat{x} + \left(x - \frac{a}{2}\right)\hat{y}\right) \end{aligned} \tag{6.23}$$

Finally, we derive expression for the Poynting vector as follows:

$$\underline{S} = \frac{1}{2}\mathrm{Re}\{E \times H^*\}$$

Up to $\mathrm{O}\left(\frac{1}{r^2}\right)$ from where the radiator resistance is evaluated as

$$\frac{1}{2}I^2R_r = \lim_{r\to\infty}\int S\cdot\hat{r}\cdot r^2\cdot \mathrm{d}\Omega$$

where I is the input current to the RDRA. R_r or $R_r(\omega)$ depends on the frequency.

Hence, this completes the solution for radiation pattern of RDRA.

6.5 Quality Factor of RDRA

The quality factor Q of the RDRA can be evaluated by comparing the power radiated $P_{\text{rad}} = \frac{1}{2}I^2R_r$ with the average electromagnetic energy (W) stored with the RDRA as follows:

$$W(\omega) = \frac{1}{4} \int_{[0,a]\times[0,b]\times[0,c]} (\epsilon(\underline{E}, E^*)) + \mu(\underline{H}, H^*)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \tag{6.24}$$

The average energy stored per unit cycle with the RDRA is

$$P(\omega) = \frac{W(\omega)}{2\pi/_{\omega}} = \frac{\omega}{2\pi} W(\omega) \tag{6.25}$$

The quality field factor of the RDRA is thus

$$Q(\omega) = \frac{2\omega W(\omega)}{|I(\omega)|^2 \cdot R_r(\omega)};$$

where ω corresponds to resonant frequency.

The quality factor of a resonant mode measures how sharp its resonance is. As per conservation of energy,

$$\int |E|^2 \mathrm{d}v = \int |H|^2 \mathrm{d}v$$

(time) average magnetic energy will be equal to electric energy inside the resonator.

The time-averaged energy dissipated in the walls of RDRA in unit time can be calculated as of energy into walls from the electromagnetic fields in the cavity normal component of energy based on the boundary conditions as energy flux density as follows:

$$\underline{S} = \left(\frac{C}{8\pi}\right)\mathrm{Re}(E \times H^*) \tag{6.27}$$

Hence, total energy dissipated is given by

$$\frac{c}{8\pi} \oint \mathrm{Re}|H|^2 \mathrm{d}f$$

Change in resonant frequency due to dielectric material used in RDRA:

The resonant frequency is reduced by $\sqrt{\mu\epsilon}$

If $\omega \to \omega\sqrt{\mu\epsilon}$

$\omega a, \omega b$ are orthogonal frequencies, and Ea and Eb are orthogonal fields.
$\frac{\omega'}{2|\omega''|}$ = quality factor (Q), ω' is real frequency, and ω'' is imaginary frequency.
Complex freq $\omega = \omega' + j\omega''$

$$\int Ea \cdot Eb^* \mathrm{d}v = \int Ha \cdot Hb^* \mathrm{d}v = 0$$

Resonator filled with non-absorbing dielectric, for which ϵ and μ differ from unity by replacing ω by $\omega\sqrt{\mu\epsilon}$ and E by ϵE, and H by μH.

The (time) average energy flux through surface is

$$\underline{S} = \frac{c}{8\pi}\mathrm{Re}\left(E_t \times H_t^*\right) \tag{6.28}$$

where $S = \dfrac{c}{4\pi}(E \times H)$.

If Q of heat evolved per unit time and volumes

$$Q = \frac{\omega}{4\pi}\left(\epsilon''\overline{E^2} = +\mu''\overline{H^2}\right) \tag{6.29}$$

Bar denotes time-average exciting frequency, must be exactly equal to the chosen resonance frequency, and is required to establish field configuration inside resonator. This results in dissipation of energy in the cavity walls and dielectric filling of the cavity resonator. A measure of the sharpen of response of the cavity to external excitation is quality of the cavity. This is defined as 2π times the ratio of the time-averaged energy stored in the cavity to the energy dissipated.

$$Q = \omega_0 \frac{\text{stored energy}\{W(\omega)\}}{\text{power loss}(I \cdot I \cdot R_r)} \tag{6.30}$$

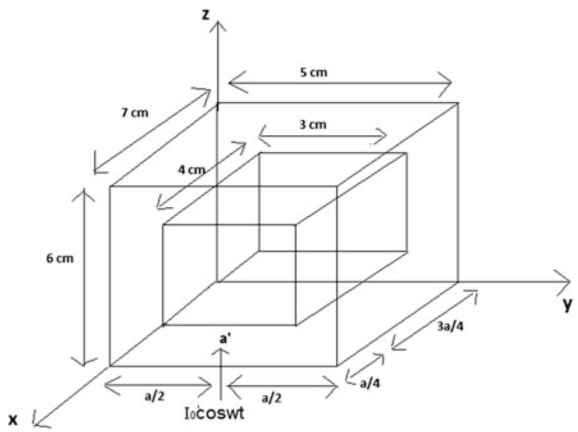

Fig. 6.2 Rectangular RDRA moat

where ω_0—Resonant frequency oscillations of fields are damped and time dependent. Change in frequency $\Delta\omega$ to occur based on superposition of frequencies:

$$\omega = \omega_0 + \Delta\omega$$

$$E(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(\omega) e^{-j\omega t} \mathrm{d}\omega$$

Q. No. 1 Compute resonant frequency and propagation constant of given RDRA shown in Fig. 6.2 and also compute quality factor of a RDRA having dimensions $10 \times 10 \times 10\ \mathrm{mm}^3$ with dielectric constant 10 and probe current 10 mA.