Chapter 4
Mathematical Analysis of Transcendental Equation in Rectangular DRA

Abstract Mathematical analysis of transcendental equation in rectangular DRA has been derived. Transcendental equation of rectangular DRA provides complete solution of propagation constants, i.e., k_x, k_y, and k_z. The propagation constant gives rise to resonant frequency with the help of characteristic equation. The wave numbers k_x, k_y, and k_z are in x, y, and z direction, respectively. The free space wave number is k_0. The exact solution of RDRA resonant frequency can be determined from combined solution of transcendental equation and characteristic equation. These equations have unique solution. RDRA depends upon boundary conditions. MATLAB-based simulation has been worked for RDRAs. They have been depicted with examples. This chapter has given a complete design solution of rectangular DRAs.

Keywords Mathematical analysis · Transcendental equation · Rectangular DRA · Propagation constant · Eigen vectors · Effective electrical length · Characteristic equation

Transcendental equation of rectangular DRA provides complete solution of propagation constants, i.e., k_x, k_y, and k_z. The propagation constant gives rise to resonant frequency with the help of characteristic equation. The wave numbers k_x, k_y, and k_z are in x, y, and z direction, respectively. The free space wave number is k_0. The exact solution of RDRA resonant frequency can be determined from combined solution of transcendental equation and characteristic equation. These equations have unique solution if RDRA boundary conditions are fixed. For example, top and bottom walls are PMC and rest of the four walls is PEC and vice versa, only two different transcendental equations will be developed.

To get this solution, H_z fields and derivative of H_z fields need to be solved. They are solved for continuous propagating fields conditions. The fields are assumed continuous at interface of RDRA. The RDRA along with eigen vectors is shown in Fig. 4.1a, b.

CASE#1 RDRA solution:
See Fig. 4.2.

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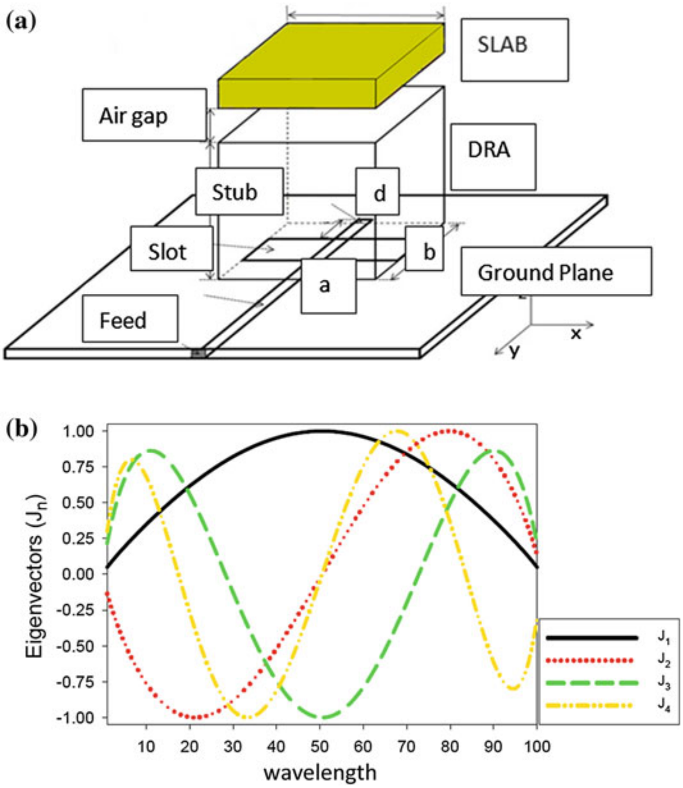


Fig. 4.1 **a** Rectangular DRA. **b** Eigen currents (current vectors) versus wavelength

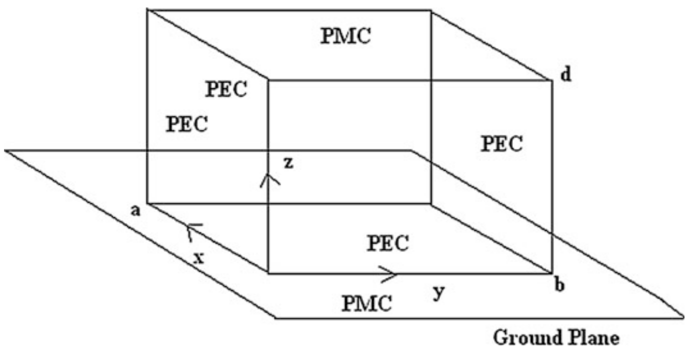


Fig. 4.2 RDRA under defined boundaries

To derive transcendental equation, the fields inside the resonator and outside the resonator are required.

$$\tan(k_z d) = \frac{k_z}{\sqrt{(\epsilon_r - 1){k_0}^2 - {k_z}^2}}(\text{transcendental equation}) \tag{4.1}$$

$$\epsilon_r {k_0}^2 = {k_x}^2 + {k_y}^2 + {k_z}^2(\text{wave equation}) \tag{4.2}$$

$$k_x = m\pi/a \tag{4.3a}$$

$$k_y = n\pi/b \tag{4.3b}$$

$$k_z = p\pi/d \tag{4.3c}$$

where a, b, and d are dimensions; m, n, and p are the indices.

$TE_{\delta 11}$, $TE_{1\delta 1}$, and $TE_{11\delta}$ are dominant modes.

The solution of resonant frequency can be had if solution of k_y propagation constant is obtained from characteristic equation, $\epsilon_r {k_0}^2 = {k_x}^2 + {k_y}^2 + {k_z}^2$, and then substituted in transcendental equation to compute resonant frequency $f_{0.}$

Boundary condition

Propagation constant, $\gamma_{mn}^2 = {k_0}^2 + {h_{mn}}^2$

$k = 2\pi/\lambda = \omega\sqrt{\mu\epsilon} = \omega/\mathrm{c};$

$$\int E^2 \mathrm{d}V = \int H^2 \mathrm{d}V$$

Time average electric energy = time average magnetic energy

$$\epsilon_r {k_0}^2 = {k_x}^2 + {k_y}^2 + {k_z}^2 \tag{4.4}$$

$$\epsilon_0 {k_0}^2 = {k_x}^2 + {k_y}^2 + {k_z}'^2 \tag{4.5}$$

$$k_z = p\pi/d$$

Subtracting Eq. (4.1) from Eq. (4.2), we get

$${k_z}'^2 - {k_z}^2 = \epsilon_0 {k_0}'^2 - \epsilon_r {k_0}^2$$

$${k_z}'^2 - {k_z}^2 = \epsilon_0 \mu_0 \omega^2 - \epsilon_r \mu_0 \omega^2$$

Taking value of $\epsilon_0 = 1$ and $\mu_0 = \mu$, we get

$${k_z}'^2 - {k_z}^2 = \omega^2 \mu (1 - \epsilon_r) \tag{4.6}$$

4.1 Case-1: Top and Bottom Walls as PMC and Rest of the Four Walls are PEC

See Fig. 4.3.

Assuming that the top and bottom surface plane be at $z = 0, d$ to be PMC

$$\therefore \quad n \times H = 0$$

And

$$n \cdot E = 0$$

or,

$$H_y = H_x = 0$$
$$E_z = 0$$

Rest of the other walls is PEC

$$\therefore \quad n \times E = 0$$

And

$$n \cdot H = 0$$

At

$$x = 0, a \quad E_y = E_z = 0$$
$$H_x = 0$$

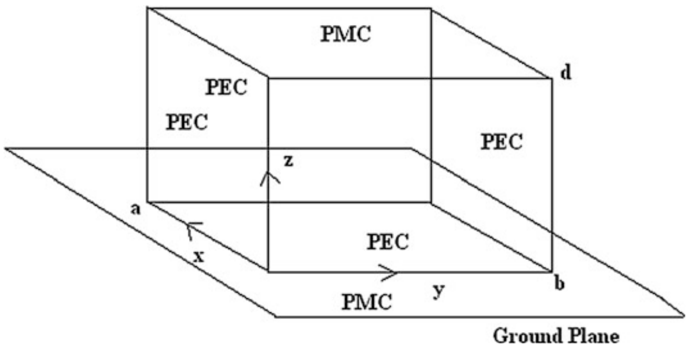


Fig. 4.3 RDRA with boundaries

At

$$y = 0, b \quad E_x = E_z = 0$$
$$H_y = 0$$

We also know

$$E_x = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial H_z}{\partial y} - \frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial x}\right] \tag{4.7a}$$

$$E_y = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[-\frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial y} - \frac{\partial H_z}{\partial x}\right] \tag{4.7b}$$

$$H_x = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial E_z}{\partial y} - \frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial x}\right] \tag{4.7c}$$

$$H_y = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial y} - \frac{\partial E_z}{\partial x}\right] \tag{4.7d}$$

Now, the solution of second-order differential equation is given as follows:

$$\psi_z = X(x)Y(y)Z(z)$$

where

$$X(x) = A_1 \sin k_x x + A_2 \cos k_x x \tag{4.8a}$$

$$Y(y) = A_3 \sin k_y y + A_4 \cos k_y y \tag{4.8b}$$

$$Z(z) = A_5 \sin k_z z + A_6 \cos k_z z \tag{4.8c}$$

For TE mode $(E_z = 0 \text{ and } H_z \neq 0)$

$$\psi_{H_z} = X(x)Y(y)Z(z)$$

After putting $E_z = 0$, we get

$$E_y = C'\left[-\frac{\partial H_z}{\partial x}\right]$$

or,

$$E_y = C'X'(x)Y(y)Z(z)$$

Now

$$X'(x) = A_1 \cos k_x x - A_2 \sin k_x x$$

But at

$$\begin{aligned} x = 0, a \quad E_y = 0 \\ \therefore \quad 0 = A_1 \cos k_x 0 - A_2 \sin k_x 0 \end{aligned}$$

or,

$$A_1 = 0 \text{ and } k_x = \frac{m\pi}{a}$$

Similarly,

$$E_x = C' \left[\frac{\partial H_z}{\partial y} \right]$$

or,

$$E_x = C'X(x)Y(y)Z(z)$$

Now

$$Y'(y) = A_3 \cos k_y y - A_4 \sin k_y y$$

But at

$$\begin{aligned} y = 0, b \quad E_x = 0 \\ \therefore \quad 0 = A_3 \cos k_y 0 - A_4 \sin k_y 0 \end{aligned}$$

or,

$$A_3 = 0 \text{ and } k_y = \frac{n\pi}{b}$$

from

$$H_x = C' \left[-\frac{1}{j\omega\epsilon} \frac{\partial^2 H_z}{\partial z \partial x} \right]$$

or,

$$H_x = C'X'(x)Y(y)Z'(z)$$

Now

$$Z'(z) = A_5 \cos k_z z - A_6 \sin k_z z$$

At

$$z = 0, d \quad H_x = 0$$

$$\therefore \quad A_5 \cos k_z 0 - A_6 \sin k_z 0 = 0$$
$$A_5 = 0 \text{ and } k_z = \frac{p\pi}{d}$$

Hence,

$$H_z = A_2 A_4 A_6 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{4.9}$$

Using Eqs. (4.1)–(4.4), and (4.8a)–(4.8c), we get

$$H_x = C'' A_2 A_4 A_6 \left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{4.10a}$$

$$H_y = C'' A_2 A_4 A_6 \left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{4.10b}$$

$$E_y = C'' A_2 A_4 A_6 \left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{4.10c}$$

$$E_x = C'' A_2 A_4 A_6 \left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{4.10d}$$

Now, evaluate E_x and H_z at the boundary walls of the dielectric waveguide.

As we know that at the PMC wall, the tangential component of magnetic field and normal component of electric field are equal to "zero" at the interface $z = 0$, d.

Hence,

$$H_x, H_y = 0$$

and

$$E_z = 0$$

Also, for propagation to be possible, we need two normal components of E and H.

Thus, we take E_x and H_y.

Now, the propagating wave is continuous at the interface, i.e., $E_x = E'_x$.

Therefore,

$$A\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\left(C_1e^{jk_zz}+C_2e^{-jk_zz}\right)=A\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)C_2'e^{-jk_z'z} \tag{4.11}$$

or,

$$C_1e^{jk_zz}+C_2e^{-jk_zz}=C_2'e^{-jk_z'z} \tag{4.12}$$

But at $z = 0$, only the inside waveform exists.

Therefore,

$$C_1e^{jk_zz}+C_2e^{-jk_zz}=0$$

Now substituting the value of $z = 0$, we get

$$\begin{aligned}&C_1+C_2=0\\ &\text{or,}\quad C_1=-C_2\end{aligned} \tag{4.13}$$

As H_z is continuous at the interface $z = d$,

$$H_z=H_z' \quad \text{and} \quad \frac{\partial H_z}{\partial x}=\frac{\partial H_z'}{\partial x}$$

From Eq. (4.9),

$$H_z=B\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(k_zz) \tag{4.14a}$$

and

$$H_z'=B\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(k_z'z) \tag{4.14b}$$

Equating Eqs. (4.14a) and (4.14b), we get

$$B\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\left(C_1e^{jk_zz}+C_2e^{-jk_zz}\right)=B\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\left(C_2'e^{-jk_z'z}\right)$$

or,

$$C_1e^{jk_zz}+C_2e^{-jk_zz}=C_2'e^{-jk_z'z} \tag{4.15}$$

From Eq. (4.15), i.e., $C_1 = -C_2$, we get, at $z = d$,

$$2jC_1 \sin(k_z d) = C_2' e^{-jk_z d} \tag{4.16}$$

Now, equating the derivative of H_z, we get

$$jk_z\left(C_1 e^{jk_z z} - C_2 e^{-jk_z z}\right) = -jk_z' C_2' e^{-jk_z' z} \tag{4.17}$$

or,

$$2k_z C_1 \cos(k_z d) = -k_z' C_2' e^{-jk_z' z}.$$

Dividing equation (4.16) by (4.17)

$$\frac{j \tan k_z d}{k_z} = \frac{-1}{k_z'} \tag{4.18a}$$

Squaring both sides and substituting the value of k_z^2 from Eq. (4.3c),

$$k_z'^2 = k_z^2 - \omega^2 \mu(\epsilon_r - 1)$$

and substituting $\mu = 1$. We get,

$$\tan(k_z d) = \frac{k_z}{\sqrt{k_0^2(\epsilon_r - 1) - k_z^2}} \tag{4.18b}$$

The above equation is the required transcendental equation.

4.2 Case-2

For transcendental equation, we need to compute the fields inside the resonator and outside it.

$$\tan(k_z d) = \frac{k_z}{\sqrt{(\epsilon_r - 1)k_0^2 - k_z^2}} \tag{4.19}$$

where $\epsilon_r k_0^2 = k_x^2 + k_y^2 + k_z^2$ (characteristic wave equation)

$$k_x = \frac{m\pi}{a} \tag{4.20a}$$

$$k_y = \frac{n\pi}{b} \tag{4.20b}$$

$$k_z = \frac{p\pi}{d} \tag{4.20c}$$

where a, b, and d are dimensions; m, n, and p are modes.

$TE_{\delta 11}$, $TE_{1\delta 1}$, and $TE_{11\delta}$ are dominant modes.

Boundary condition

Propagation constant, $\gamma_{mn}^2 = k_0^2 + hmn^2$ where $k = \frac{2\pi}{\lambda} = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}$.

From the energy conservation principle,

$$\int E^2 \mathrm{d}V = \int H^2 \mathrm{d}V.$$

i.e., time average electric energy = time average magnetic energy.

When top and bottom walls are PMC, rest of the other walls is PEC

Assuming that the top and bottom surface plane be at $z = 0, d$

$$\therefore \quad n \times H = 0$$

And

$$n \cdot E = 0$$

or,

$$H_y = H_x = 0$$
$$E_z = 0$$

Rest of the other walls is PEC

$$\therefore \quad n \times E = 0$$

And

$$n \cdot H = 0$$

At

$$x = 0, a \quad E_y = E_z = 0$$
$$H_x = 0$$

At

$$y = 0, b \quad E_x = E_z = 0$$
$$H_y = 0$$

We also know

$$E_x = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial H_z}{\partial y} - \frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial x}\right] \tag{4.21a}$$

$$E_y = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[-\frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial y} - \frac{\partial H_z}{\partial x}\right] \tag{4.21b}$$

$$H_x = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial E_z}{\partial y} - \frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial x}\right] \tag{4.21c}$$

$$H_y = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial y} - \frac{\partial E_z}{\partial x}\right] \tag{4.21d}$$

Now, the solution of second-order differential equation is given as follows:

$$\psi_z = X(x)Y(y)Z(z) \tag{4.22}$$

where

$$X(x) = A_1 \sin k_x x + A_2 \cos k_x x$$
$$Y(y) = A_3 \sin k_y y + A_4 \cos k_y y$$
$$Z(z) = A_5 \sin k_z z + A_6 \cos k_z z$$

For TE mode $(E_z = 0 \text{ and } H_z \neq 0)$

$$\psi_{H_z} = X(x)Y(y)Z(z)$$
$$E_z = 0$$

we get

$$E_y = C'\left[-\frac{\partial H_z}{\partial x}\right]$$

or,

$$E_y = C'X'(x)Y(y)Z(z)$$

Now

$$X'(x) = A_1 \cos k_x x - A_2 \sin k_x x$$

But at

$$x = 0, a \;\; E_y = 0$$
$$\therefore \quad 0 = A_1 \cos k_x 0 - A_2 \sin k_x 0$$

or,

$$A_1 = 0 \;\; \text{and} \;\; k_x = \frac{m\pi}{a}$$

Similarly,

$$E_x = C'\left[\frac{\partial H_z}{\partial y}\right]$$

or,

$$E_x = C'X(x)Y'(y)Z(z)$$

Now

$$Y'(y) = A_3 \cos k_y y - A_4 \sin k_y y$$

But at

$$y = 0, b \;\; E_x = 0$$
$$\therefore \quad 0 = A_3 \cos k_y 0 - A_4 \sin k_y 0$$

or,

$$A_3 = 0 \;\; \text{and} \;\; k_y = \frac{n\pi}{b}$$

$$H_x = C'\left[-\frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial x}\right]$$

or,

$$H_x = C'X'(x)Y(y)Z'(z)$$

Now

$$Z'(z) = A_5 \cos k_z z - A_6 \sin k_z z$$

At

$$\begin{aligned} & z = 0, d \;\; H_x = 0 \\ \therefore \quad & A_5 \cos k_z 0 - A_6 \sin k_z 0 = 0 \\ & A_5 = 0 \;\text{ and }\; k_z = \frac{p\pi}{d} \end{aligned}$$

Hence,

$$H_z = A_2 A_4 A_6 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{d}z\right) \tag{4.23}$$

Using Eqs. (4.1)–(4.4), and (4.8a)–(4.8c), we get

$$H_x = C'' A_2 A_4 A_6 \left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{p\pi}{d}z\right) \tag{4.24a}$$

$$H_y = C'' A_2 A_4 A_6 \left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \sin\left(\frac{p\pi}{d}z\right) \tag{4.24b}$$

$$E_y = C'' A_2 A_4 A_6 \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{d}z\right) \tag{4.24c}$$

$$E_x = C'' A_2 A_4 A_6 \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \cos\left(\frac{p\pi}{d}z\right) \tag{4.24d}$$

Above equations can also be written as follows:

$$\begin{aligned} H_x &= \frac{k_x k_z}{j\omega\mu_0} \sin(k_x x) \cos(k_y y) \sin(k_z z) \\ H_y &= \frac{k_y k_z}{j\omega\mu_0} \cos(k_x x) \sin(k_y y) \sin(k_z z) \\ E_y &= -k_x \sin(k_x x) \cos(k_y y) \cos(k_z z) \\ E_x &= k_y \cos(k_x x) \sin(k_y y) \cos(k_z z) \\ H_z &= \frac{k_x^2 + k_y^2}{j\omega\mu_0} \cos(k_x x) \cos(k_y y) \cos(k_z z) \end{aligned}$$

Since H_z is continuous, i.e., $\frac{\mathrm{d}H_z}{\mathrm{d}z} \neq 0$,

$$H'_z = \frac{k_x^2 + k_y^2}{j\omega\mu_0}\cos(k_x x)\cos(k_y y)\cos(k'_z z)$$

Now, H_y can be written as follows:

$$H_y = \frac{k_y k_z}{j\omega\mu_0}\cos(k_x x)\sin(k_y y)(C_1 e^{jk_z d} - C_2 e^{-jk_z d})$$

But

$$H_y = 0 \text{ at } d = 0$$

$$C_1 - C_2 = 0$$

or,

$$C_1 = C_2$$

$$\frac{\mathrm{d}H_y}{\mathrm{d}z} = A' jk_z \cos(k_x x)\sin(k_y y)(C_1 e^{jk_z d} + C_2 e^{-jk_z d})$$

or,

$$\frac{\mathrm{d}H_y}{\mathrm{d}z} = C_1 jk_z \cos(k_x x)\sin(k_y y)(e^{jk_z d} + e^{-jk_z d})$$

or,

$$\frac{\mathrm{d}H_y}{\mathrm{d}z} = C_1 2jk_z \cos(k_x x)\sin(k_y y)\cos(k_z d)$$

$$H'_y = C'_1 \cos(k_x x)\sin(k_y y)e^{-jk'_z d} \text{ outside the cavity}$$

For H_z to be continuous,

$$\frac{\mathrm{d}H_y}{\mathrm{d}z} = \frac{\mathrm{d}H'_y}{\mathrm{d}z}$$

or,

$$C_1 2jk_z \cos(k_x x)\sin(k_y y)\cos(k_z d) = -jk'_z C'_1 \cos(k_x x)\sin(k_y y)e^{-jk'_z d}$$

or,

$$2C_1 k_z \cos(k_z d) = -k'_z C'_1 e^{-jk'_z d} \tag{4.25}$$

From above equations, we have

$$E_x = k_y \cos(k_x x) \sin\left(k_y y\right)\left(C_1 e^{jk_z d} + C_2 e^{-jk_z d}\right)$$

At

$$d = 0,\ E_x = 0,$$

so,

$$C_1 + C_2 = 0$$

or,

$$\begin{aligned} & C_1 = -C_2 \\ \therefore \quad & E_x = k_y \cos(k_x x) \sin\left(k_y y\right) C_1 \left(e^{jk_z d} - e^{-jk_z d}\right) \end{aligned}$$

or,

$$E_x = 2jC_1 k_y \cos(k_x x) \sin\left(k_y y\right) \sin(k_z d)$$

Also

$$E'_x = k_y \cos(k_x x) \sin\left(k_y y\right) \cos\left(k'_z z\right)$$

or,

$$E'_x = C'_1 k_y \cos(k_x x) \sin\left(k_y y\right) e^{-jk_z d}$$

For H_z to be continuous,

$$E_x = E'_x$$

or,

$$jC_1 k_y \cos(k_x x) \sin(k_y y) \sin(k_z d) = C'_1 k_y \cos(k_x x) \sin(k_y y) e^{-jk_z d}$$

or,

$$2jC_1 \sin(k_z d) = C'_1 e^{-jk_z d} \tag{4.26}$$

Dividing Eq. (4.16) by Eq. (4.25), we get

$$\frac{2C_1 j\sin(k_z d)}{2C_1 k_z\cos(k_z d)} = \frac{-C_1' e^{-jk_z d}}{k_z' C_1' e^{-jk_z' d}}$$

or,

$$\frac{j\tan k_z d}{k_z} = \frac{-1}{k_z'}$$

or,

$$j\tan k_z d = -\frac{k_z}{k_z'}$$

On squaring and putting ${k_z'}^2 = k_z^2 + \omega^2\mu_0(1-\epsilon_r)$

$$\tan^2 k_z d = -\frac{k_z^2}{k_z^2 + \omega^2\mu_0(1-\epsilon_r)}$$

or,

$$\tan^2 k_z d = \frac{k_z^2}{\omega^2\mu_0(\epsilon_r - 1) - k_z^2}$$

$$\tan k_z d = \frac{k_z}{\sqrt{(\epsilon_r - 1)k_0^2 - k_z^2}} \tag{4.27}$$

With the help of transcendental equation, we can find the propagation factor. Also with the help of this equation, we can obtain resonant frequency.

CASE#3

For transcendental equation, we need to compute the fields inside the resonator and outside it.

$$k_z\tan(k_z d) = \sqrt{(\epsilon_r - 1)k_0^2 - k_z^2};$$
$$\epsilon_r k_0^2 = k_x^2 + k_y^2 + k_z^2;$$

and

$$k_x = m\pi/a \tag{4.28a}$$

$$k_y = n\pi/b \tag{4.28b}$$

$$k_z = p\pi/d \tag{4.28c}$$

where a, b, and d are dimensions; m, n, and p are the indices.

$TE_{\delta 11}$, $TE_{1\delta 1}$, $TE_{11\delta}$ are dominant modes.

Boundary conditions

Propagation constant, $\gamma_{mn}^2 = k_0^2 + h_{mn}^2$

$k = 2\pi/\lambda = \omega\sqrt[2]{\mu\epsilon} = \omega/c;$

$$\int E^2 \mathrm{d}V = \int H^2 \mathrm{d}V$$

Time average electric energy = time average magnetic energy

$$\epsilon_0 \epsilon_r k_0^2 = k_x^2 + k_y^2 + k_z^2 \tag{4.29a}$$

$$\epsilon_0 k_0^2 = k_x^2 + k_y^2 + k_z'^2 \tag{4.29b}$$

$$k_z' \neq p\pi/d$$

Subtracting Eq. (4.1) from Eq. (4.2), we get

$$k_z'^2 - k_z^2 = \epsilon_0 k_0'^2 - \epsilon_0 \epsilon_r k_0^2$$
$$k_z'^2 - k_z^2 = \epsilon_0 \mu_0 \omega^2 - \epsilon_0 \epsilon_r \mu_0 \omega^2$$

Taking the value of $\epsilon_0 = 1$ and $\mu_0 = \mu$, we get

$$k_z'^2 - k_z^2 = \omega^2 \mu \epsilon_0 \,(1 - \epsilon_r) \tag{4.30}$$

When top and bottom walls are PEC, rest of the other walls is PMC.
Now,
Assuming that the top and bottom surface plane be at $z = 0, d$

$$\therefore \quad n \times E = 0$$

and

$$n \cdot H = 0$$

or,

$$E_y = E_x = 0$$

$$H_z = 0$$

Rest of the other walls is PMC

$$\therefore \quad n \times H = 0$$

And

$$n \cdot E = 0$$

At

$$\begin{aligned} x = 0, a \quad & H_y = H_z = 0 \\ E_x = 0 & \end{aligned}$$

At

$$\begin{aligned} y = 0, b \quad & H_x = H_z = 0 \\ E_y = 0 & \end{aligned}$$

We also know

$$E_x = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial H_z}{\partial y} - \frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial x}\right] \tag{4.31a}$$

$$E_y = \frac{1}{j\omega\epsilon\left(1 + \frac{\gamma^2}{k^2}\right)}\left[-\frac{1}{j\omega\mu}\frac{\partial^2 E_z}{\partial z \partial y} - \frac{\partial H_z}{\partial x}\right] \tag{4.31b}$$

$$H_x = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{\partial E_z}{\partial y} - \frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial x}\right] \tag{4.31c}$$

$$H_y = \frac{-1}{j\omega\mu\left(1 + \frac{\gamma^2}{k^2}\right)}\left[\frac{1}{j\omega\epsilon}\frac{\partial^2 H_z}{\partial z \partial y} - \frac{\partial E_z}{\partial x}\right] \tag{4.31d}$$

Now, the solution of second-order differential equation is given as follows:

$$\psi_z = X(x)Y(y)Z(z)$$

where

$$X(x) = A_1 \sin k_x x + A_2 \cos k_x x$$
$$Y(y) = A_3 \sin k_y y + A_4 \cos k_y y$$
$$Z(z) = A_5 \sin k_z z + A_6 \cos k_z z$$

(i) For TE mode $(E_z = 0 \text{ and } H_z \neq 0)$

$$\psi_{H_z} = X(x)Y(y)Z(z)$$

At

$$x = 0, a \quad H_z = 0,$$

or,

$$A_1 \sin k_x 0 + A_2 \cos k_x 0 = 0$$
$$\therefore \quad A_2 = 0 \text{ and } k_x = \frac{m\pi}{a}$$

Also at

$$y = 0, b \quad H_z = 0$$

or,

$$A_3 \sin k_y 0 + A_4 \cos k_y 0 = 0$$
$$\therefore \quad A_4 = 0 \text{ and } k_y = \frac{n\pi}{b}$$

At

$$z = 0, d \quad H_z = 0$$

$$\therefore \quad A_5 \sin k_z 0 + A_6 \cos k_z 0 = 0$$
$$A_6 = 0 \quad \text{and} \quad k_z = \frac{p\pi}{d}$$

Hence,

$$H_z = A_1 A_3 A_5 \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \sin\left(\frac{p\pi}{d} z\right) \tag{4.32}$$

Using Eqs. (4.1)–(4.4), and (4.8a)–(4.8c), we get

$$H_x = C'' A_1 A_3 A_5 \left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{4.33a}$$

$$H_y = C'' A_1 A_3 A_5 \left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right) \tag{4.33b}$$

$$E_y = C'' A_1 A_3 A_5 \left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{4.33c}$$

$$E_x = C'' A_1 A_3 A_5 \left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right) \tag{4.33d}$$

Now, evaluate H_x and H_z at the boundary walls of the dielectric waveguide.

As we know that at the PEC wall, the tangential component of electric field and normal component of magnetic field is equal to "zero" at the interface $z = 0, d$.

Hence,

$$E_x, E_y = 0$$

and

$$H_z = 0$$

Also, for propagation to be possible, we need two normal components of E and H.

Thus we take E_y and H_x.

Now, the propagating wave is continuous at the interface, i.e., $H_x = H'_x$.

Therefore,

$$A\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\left(C_1 e^{jk_z z} + C_2 e^{-jk_z z}\right) = A\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)C'_2 e^{-jk'_z z}$$

or,

$$C_1 e^{jk_z z} + C_2 e^{-jk_z z} = C'_2 e^{-jk'_z z} \tag{4.34}$$

But at $z = 0$, only the inside waveform exists.

Therefore,

$$C_1 e^{jk_z z} + C_2 e^{-jk_z z} = 0$$

Now, substituting the value of $z = 0$, we get

$$C_1 + C_2 = 0$$

or,

$$C_1 = -C_2 \tag{4.35}$$

As H_z is continuous at the interface $z = d$.
Therefore,

$$H_z = H_z' \text{ and } \frac{\partial H_z}{\partial x} = \frac{\partial H_z'}{\partial x}$$

$$H_z = B \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(k_z z) \tag{4.36a}$$

and

$$H'_z = B \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \sin(k'_z z) \tag{4.36b}$$

Equating Eqs. (i) and (ii), we get

$$B \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)(C_1 e^{jk_z z} - C_2 e^{-jk_z z}) = B \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) C_2' e^{-jk_z' z}$$

or,

$$C_1 e^{jk_z z} - C_2 e^{-jk_z z} = C_2' e^{-jk_z' z}$$

From Eq. (1b), i.e., $C_1 = -C_2$, we get, at $z = d$,

$$2C_1 \cos(k_z d) = C_2' e^{-jk_z d} \tag{4.37}$$

Now, equating the derivative of H_z, we get

$$jk_z(C_1 e^{jk_z z} + C_2 e^{-jk_z z}) = -jk_z' C_2' e^{-jk_z' z}$$

or,

$$2jk_z C_1 \sin(k_z d) = -k_z' C_2' e^{-jk_z' z} \tag{4.38}$$

Dividing equation (iv) by (iii), we get

$$jk_z \tan k_z d = -k_z'$$

Squaring both sides and substituting the value of $k_z'^2$ from Eq. (4.3c), we get

$$k_z' = k_z^2 - \omega^2\mu(\epsilon_r - 1)$$

and substituting $\mu = 1$, we get isolated DRA case as:

$$k_z \tan(k_z d) = \sqrt{(\epsilon_r - 1)k_0^2 - k_z^2} \tag{4.39a}$$

DRA with ground plane case as:

$$k_z \tan(k_z d/2) = \sqrt{(\epsilon_r - 1)k_0^2 - k_z^2} \tag{4.39b}$$

Hence, the solution of transcendental equation is completely obtained.

4.3 MATLAB Simulation Results

The same can be seen if MATLAB simulation is obtained as given below:

```
clear
clear all
er=9.8;
c=3*10^8;
d=10*10^-3;
for p=1:1:10
  f=c*p*(sqrt(1+tan(p*pi/2).^2))/2*d*(sqrt(er-1));
 end
plot(p,f);
title('pvsf')
xlabel('p-------->>>');
ylabel('f-------->>');
grid on;
```

Relationship between delta distance and its impact on resonant frequency is shown in Fig. 4.4.

The resonant frequency is increasing as the delta length is increasing as shown in Fig. 4.4. Also, radiation lobe is increasing as the number of resonant mode is increasing as shown in Fig. 4.5.

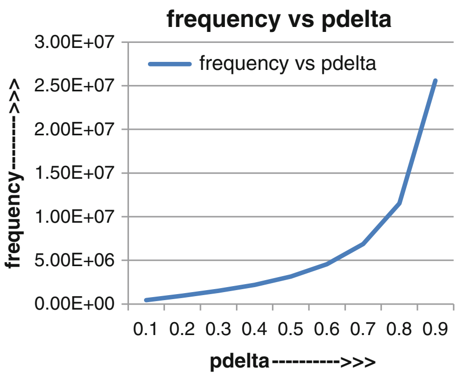


Fig. 4.4 Frequency versus delta distance

Radiation Lobes: RDRA dimensions are given to compute resonant modes using MATLAB.

Program for RDRA (a=length=10mm, b=width=5mm ,d=height=2mm)

```
m=3;
n=3;
a=10;
b=5;
x=linspace(-5,5,51);
y=linspace(-2.5,2.5,51);
[xi,yi] = meshgrid(x,y);
Ez= cos(m*pi*xi/a).*cos(n*pi*yi/b);
Ez= Ez.^2;
Ez= sqrt(Ez);
surf(xi,yi,Ez)
view([-45,60])
%%view([180,0])
drawnow
```

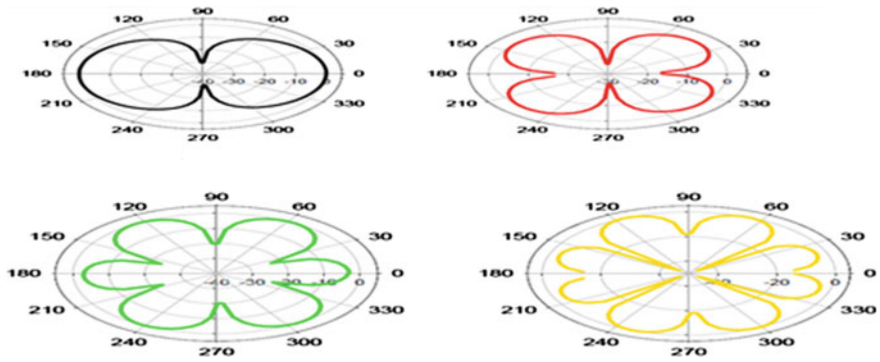


Fig. 4.5 Radiation lobes of radiation pattern in RDRA

MATLAB Program for Ez field

```
m=5;
n=4;
p=3;
a=10;
b=5;
c=2;
x=linspace(-5,5,51);
y=linspace(-2.5,2.5,51);
z=linspace(-1,1,51);
[xi,yi,zi] = meshgrid(x,y,z);
Ez= (cos(m*pi*xi/a).*cos(n*pi*yi/b)).*sin(p*pi*zi/c);
Ez= Ez.^2;
Ez= sqrt(Ez);
xslice = -4.5; yslice = -2.5; zslice =1;
slice(xi,yi,zi,Ez,xslice,yslice,zslice)
colormap hsv
```

MATLAB program for transcendental equation and resonant frequency of RDRA:

```
d=9;
w=6;
h=7.6;
c=3e8;
cons=9.8;
syms y real
kx=pi/d;
kz=pi/2/h;
ko=sqrt((kx^2+y^2+kz^2)/cons);
f=real(y*tan(y*w/2)-sqrt((cons-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/w)-0.01]);
fresonance = c/2/pi*sqrt((kx^2+ky^2+kz^2)/cons)/1e7;
```

The MATLAB-simulated resonant modes in Figs. 4.6, 4.7, 4.8, 4.9, 4.10, 4.11 and 4.12 have been drawn, and resonant frequency using transcendental equation is placed in table form.

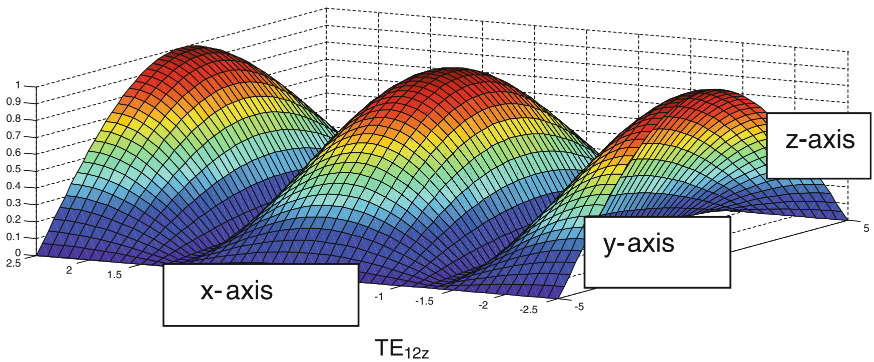


Fig. 4.6 Resonant modes in *xy* plane

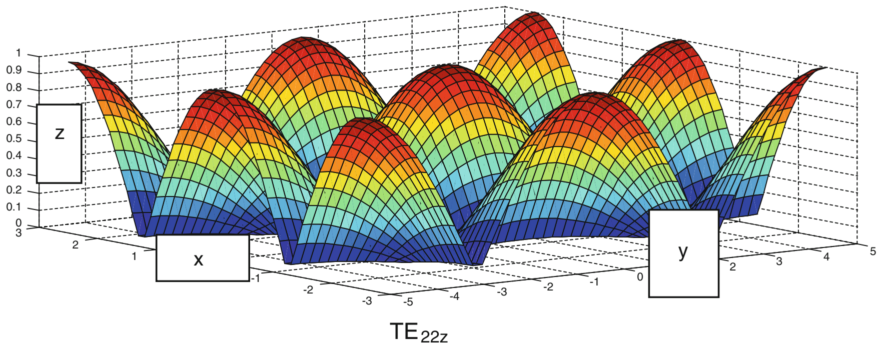


Fig. 4.7 Resonant modes in *xy* plane

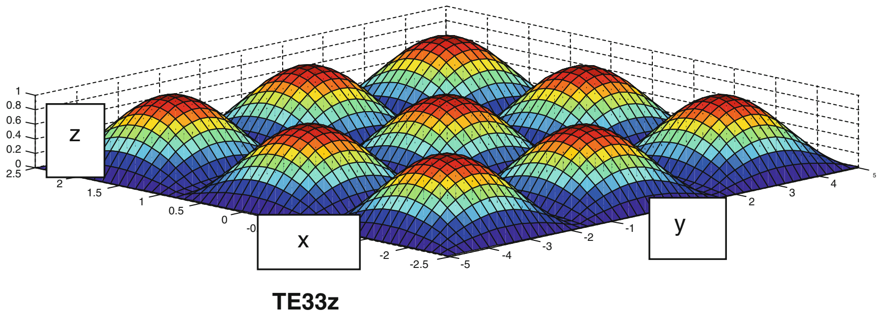


Fig. 4.8 Resonant modes in RDRA in *xy* plane

Solved examples of RDRA resonant frequency:

Example 1 Calculate the dimension of "*d*" in RDRA:
For TE_{111} mode when

$$\epsilon_r = 100$$
$$a = 10\,\text{mm}$$
$$b = 10\,\text{mm}$$
$$f_r = 7.97\,\text{GHz}$$

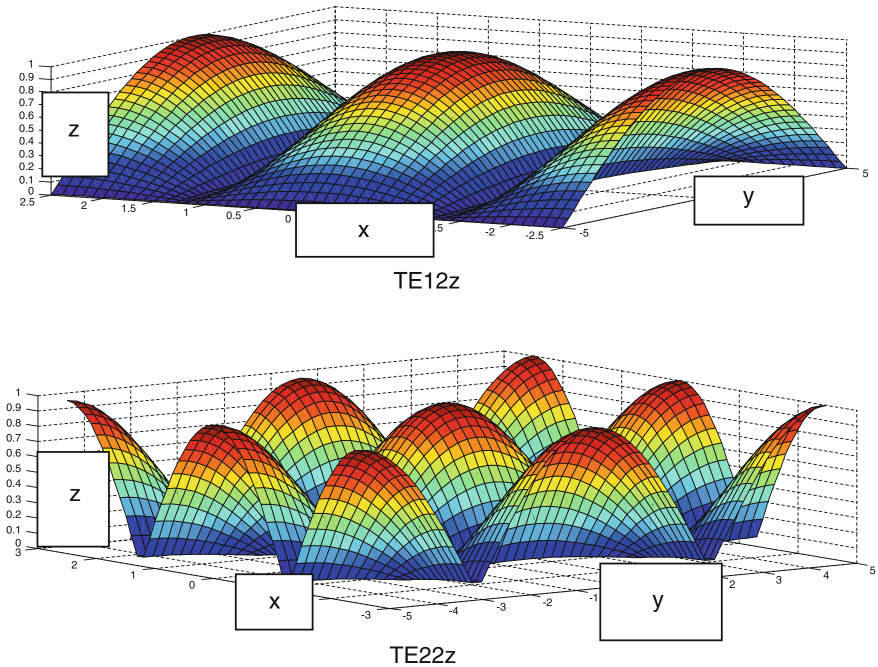


Fig. 4.9 Resonant modes in RDRA in *xy* plane

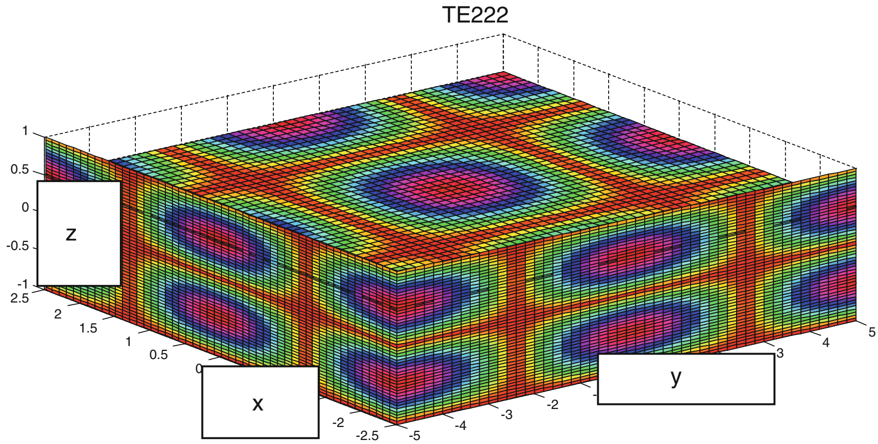


Fig. 4.10 Resonant modes 3D in RDRA in *xyz* plane

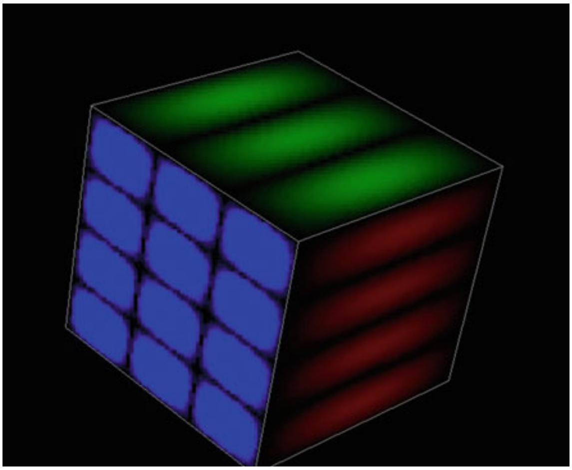


Fig. 4.11 TE 341 resonant modes

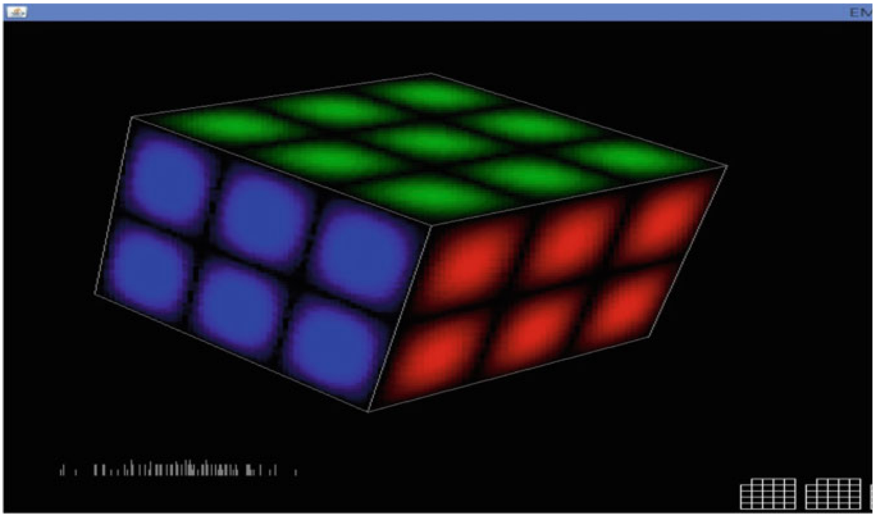


Fig. 4.12 TE 323 resonant modes

Solution Resonant frequency:

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m^2}{a} + \frac{n^2}{b} + \frac{p^2}{d}}$$

$$7.97 \times 10^9 = \frac{3 \times 10^8}{2\sqrt{100}}\sqrt{100^2 + 100^2 + \frac{1^2}{d}}$$

$$531.33 = \sqrt{20000 + \frac{1^2}{d}}$$

$$\frac{1}{d} = 512.167$$

$$d = 1.95\,\text{mm}$$

Example 2 RDRA with following data:

$$\epsilon_r = 35$$
$$a = 18\,\text{mm}$$
$$b = 18\,\text{mm}$$
$$f_r = 2.45\,\text{GHz}$$

Solution

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m^2}{a} + \frac{n^2}{b} + \frac{p^2}{d}}$$

$$2.45 \times 10^9 = \frac{3 \times 10^8}{2\sqrt{35}}\sqrt{\frac{1000^2}{18} + \frac{1000^2}{18} + \frac{1^2}{d}}$$

$$9337.222 = \sqrt{2\left(\frac{1000^2}{18}\right) + \frac{1^2}{d}}$$

$$\frac{1}{d} = 56.252$$

$$d = 17.77\,\text{mm}$$

Example 3 Calculate the resonant frequency for TE_{111} mode using the given data of RDRA:

$$\epsilon_r = 10$$
$$a = 14\,\text{mm}$$
$$b = 8\,\text{mm}$$
$$d = 8\,\text{mm}$$

Solution

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m^2}{a} + \frac{n^2}{b} + \frac{p^2}{d}}$$
$$f_r = \frac{3\times 10^8}{2\sqrt{10}}\sqrt{\frac{1000^2}{14} + \frac{1000^2}{8} + \frac{1000^2}{8}}$$
$$f_r = 9.04\,\text{GHz}$$

Example 4

$$\epsilon_r = 10$$
$$a = 14\,\text{mm}$$
$$b = 8\,\text{mm}$$
$$d = 16\,\text{mm}$$

Solution

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m^2}{a} + \frac{n^2}{b} + \frac{p^2}{d}}$$
$$f_r = \frac{3\times 10^8}{2\sqrt{10}}\sqrt{\frac{1000^2}{14} + \frac{1000^2}{8} + \frac{1000^2}{16}}$$
$$f_r = 7.44\,\text{GHz}$$

Example 5 Calculate the resonant frequency for the $TE_{11\delta}$ mode using the given data:

$$\epsilon_r = 10$$
$$a = 14\,\text{mm}$$
$$b = 8\,\text{mm}$$
$$d = 8\,\text{mm}$$

Solution

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m^2}{a} + \frac{n^2}{b} + \frac{\delta^2}{d}}$$
$$f_r = \frac{3\times 10^8}{2\sqrt{10}}\sqrt{\frac{1000^2}{14} + \frac{1000^2}{8} + 0}$$
$$f_r = 6.82\,\text{GHz}$$

Example 6

$$\begin{aligned}\epsilon_r &= 10\\ a &= 14\,\text{mm}\\ b &= 8\,\text{mm}\\ d &= 16\,\text{mm}\end{aligned}$$

Solution

$$f_r = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{m2}{a} + \frac{n2}{b} + \frac{p2}{d}}$$

$$f_r = \frac{3\times 10^8}{2\sqrt{10}}\sqrt{\frac{1000^2}{14} + \frac{1000^2}{8} + 0}$$

$$f_r = 6.82\,\text{GHz}$$

4.4 Resonant Frequency of RDRA for Experimentations

The RDRAs can be prototyped with various materials and sizes as per the requirements.

Table 4.1 consists of list of RDRA materials, permittivity, dimensions, and computed resonant frequency.

Example 7 Compute resonant frequency when RDRA dimensions are 10 × 10 × 10 mm^3 and dielectric constant of material used is 10.

$$(f_r)m,n,p = \frac{c}{2\pi\sqrt{\epsilon\mu}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

Resonant frequencies in isolated case are 49.7 and 25.8 GHz with ground plane (Table 4.2).

Table 4.1 RDRA materials, permittivity, dimensions, and computed resonant frequency

S. no.	Material	Permittivity	RDRA dimension ($a \times b \times h$) mm	Resonant frequency simulated by HFSS	Resonant frequency calculated
Countis Laboratories					
1.	$MgO–SiO_2–TiO_2$ (CD-9)	9.8	9 × 6 × 7.6	7.43	7.6757
2.	$MgO–SiO_2–TiO_2$ (CD-9)	9.8	14.3 × 25.4 × 26.1	3.5	3.7430
3.	$MgO–CaO–TiO_2$ (CD-20)	20.0	10.16 × 10.2 × 7.11	4.71	4.6215
4.	$MgO–CaO–TiO_2$ (CD-20)	20.0	10.16 × 7.11 × 10.2	4.55	4.5941
5.	$MgO–CaO–TiO_2$ (CD-20)	20.0	10.2 × 10.2 × 7.89	4.635	4.4833
6.	$MgO–CaO–TiO_2$ (CD-100)	100.0	10 × 10 × 2	4.57	4.2158
7.	$MgO–CaO–TiO_2$ (CD-100)	100.0	10 × 10 × 1	7.97	7.7587
8.	$MgO–CaO–TiO_2$ (CD-100)	100.0	12.7 × 12.7 × 1	7.72	7.6628
9.	$MgO–CaO–TiO_2$ (CD-100)	100.0	5 × 10 × 1	8.85	8.1828
10.	$MgO–CaO–TiO_2$ (CD-100)	100.0	10 × 5 × 1	8.5	8.0147
Emerson & Cuming Microwave Products N.V.					
11.	Magnesium titanate (ECCOSTOCK@)	10.0	14 × 8 × 8	5.5	5.6117
12.	Magnesium titanate (ECCOSTOCK@)	10.0	14.3 × 25.4 × 26.1	3.92	3.7055
13.	Zirconia (ECCOSTOCK@)	20.0	10.16 × 10.2 × 7.11	4.71	4.6215
14.	Zirconia (ECCOSTOCK@)	20.0	10.16 × 7.11 × 10.2	4.55	4.5941
15.	Zirconia (ECCOSTOCK@)	20.0	10.2 × 10.2 × 7.89	4.635	4.4833
16.	Strontium titanate (ECCOSTOCK@)	100.0	10 × 10 × 2	4.57	4.2158
17.	Strontium titanate (ECCOSTOCK@)	100.0	10 × 10 × 1	7.97	7.7587
18.	Strontium titanate (ECCOSTOCK@)	100.0	12.7 × 12.7 × 1	7.72	7.6628
19.	Strontium titanate (ECCOSTOCK@)	100.0	5 × 10 × 1	8.85	8.1828

(continued)

Table 4.1 (continued)

S. no.	Material	Permittivity	RDRA dimension ($a \times b \times h$) mm	Resonant frequency simulated by HFSS	Resonant frequency calculated
20.	Strontium titanate (ECCOSTOCK@)	100.0	10 × 5 × 1	8.5	8.0147
Morgan Advanced Materials					
21.	CaMgTi (Mg, Ca titanate) (D20)	20.0	10.16 × 10.2 × 7.11	4.71	4.6215
22.	CaMgTi (Mg, Ca titanate) (D20)	20.0	10.16 × 7.11 × 10.2	4.55	4.5941
23.	CaMgTi (Mg, Ca titanate) (D20)	20.0	10.2 × 10.2 × 7.89	4.635	4.4833
24.	ZrTiSn (Zr, Sn titanate) (D36)	37.0	18 × 18 × 9	2.45	2.1617
Temex Components & Temex Telecom, USA					
25.	Zr Sn Ti Oxide (E2000)	37.0	18 × 18 × 9	2.45	2.1617
Trans-Tech Skyworks Solutions, Inc.					
26.	BaZnCoNb (D-83)	35.0–36.5	18 × 18 × 6	2.532	2.7081
27.	BaZnCoNb (D-83)	35.0–36.5	18 × 6 × 18	2.835	2.3947
T-CERAM, RF & Microwave					
28.	E-11	10.8	15.2 × 7 × 2.6	11.6	10.379
29.	E-11	10.8	15 × 3 × 7.5	6.88	7.0937
30.	E-11	10.8	15.24 × 3.1 × 7.62	6.21	6.9440
31.	E-20	20.0	10.16 × 10.2 × 7.11	4.71	4.6215
32.	E-20	20.0	10.16 × 7.11 × 10.2	4.55	4.5941
33.	E-20	20.0	10.2 × 10.2 × 7.89	4.635	4.4833
34.	E-37	37.0	18 × 18 × 9	2.45	2.1617
TCI Ceramics, Inc.					
35.	DR-36	36.0	18 × 18 × 6	2.532	2.7081
36.	DR-36	36.0	18 × 6 × 18	2.835	2.3947

Table 4.2 Fringing effect along "*b*" dimensions increased effective along *y*-direction of RDRA

S. no.	Permittivity	Dimension (*a* (length) × *b* (width) × *d* (depth)) mm	Resonant frequency	Effective width (b')	Multiple factor	% change in width
37.	10.0	14.3 × 25.4 × 26.1	3.5	34.22	1.3474	34.7381
38.	10.0	14 × 8 × 8	5.5	14.13	1.7665	76.6535
39.	10.0	15.24 × 3.1 × 7.62	6.21	8.33	2.8872	168.7230
40.	20.0	10.2 × 10.2 × 7.89	4.635	15.31	1.5014	50.1419
41.	20.0	10.16 × 10.2 × 7.11	4.71	15.15	1.4858	48.5797
42.	35.0	18 × 18 × 6	2.532	24.12	1.34	33.9973
43.	35.0	18 × 18 × 9	2.45	25.64	1.4244	42.4423
44.	100.0	10 × 10 × 1	7.97	11.24	1.1242	12.4237

MATLAB program and simulation effective length due to fringing effect:

```
%%Dimensions of RDRA
%%length
d=[14.3,14.0,15.24,10.2,10.16,18,18,10];
%%width
w=[25.4,8,3.1,10.2,10.2,18,18,10];
%%height
h=[26.1,8,7.62,7.89,7.11,6,9,1];
%%Mode
m=1;
n=1;
p=1;
c=3e8;
cons=[10.0,10.0,10,20,20,35,35,100];
syms y real
for i=drange(1:8)
kx(i)=pi/d(i);
kz(i)=pi/2/h(i);
ko=sqrt((kx(i).^2+y.^2+kz(i).^2)/cons(i));
f=real(y.*tan(y*w(i)/2)-sqrt((cons(i)-1)*ko.^2-y.^2));
ky(i)=fzero(inline(f),[0,(pi/w(i))-0.01]);
%%Resonant frequency
fre(i)=c/2/pi*sqrt((kx(i).^2+ky(i).^2+kz(i).^2)/cons(i))*1e3;
Effwidth(i)=pi/ky(i);
factor(i)=Effwidth(i)./w(i);
perchangwidth(i)=((Effwidth(i)-w(i))/w(i))*100;
end
```

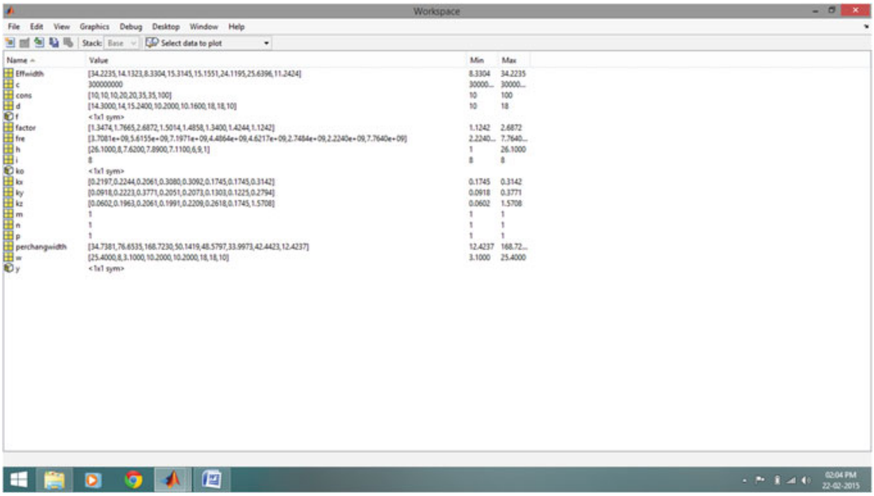
Effective increased length computations due to fringing effect:

Program 1

```
%%Dimensions of DRA
%%length
d=[14.3,14.0,15.24,10.2,10.16,18,18,10];
%%width
w=[25.4,8,3.1,10.2,10.2,18,18,10];
%%height
h=[26.1,8,7.62,7.89,7.11,6,9,1];
%%Mode
m=1;
n=1;
```

```
p=1;
c=3e8;
cons=[10.0,10.0,10,20,20,35,35,100];
syms y real
for i=drange(1:8)
kx(i)=pi/d(i);
kz(i)=pi/2/h(i);
ko=sqrt((kx(i).^2+y.^2+kz(i).^2)/cons(i));
f=real(y.*tan(y*w(i)/2)-sqrt((cons(i)-1)*ko.^2-y.^2));
ky(i)=fzero(inline(f),[0,(pi/w(i))-0.01]);
%%Resonant frequency
fre(i)=c/2/pi*sqrt((kx(i).^2+ky(i).^2+kz(i).^2)/cons(i))*1e3;
Effwidth(i)=pi/ky(i);
factor(i)=Effwidth(i)./w(i);
perchangwidth(i)=((Effwidth(i)-w(i))/w(i))*100;
end
```

Results:



Workspace

File Edit View Graphics Debug Desktop Window Help

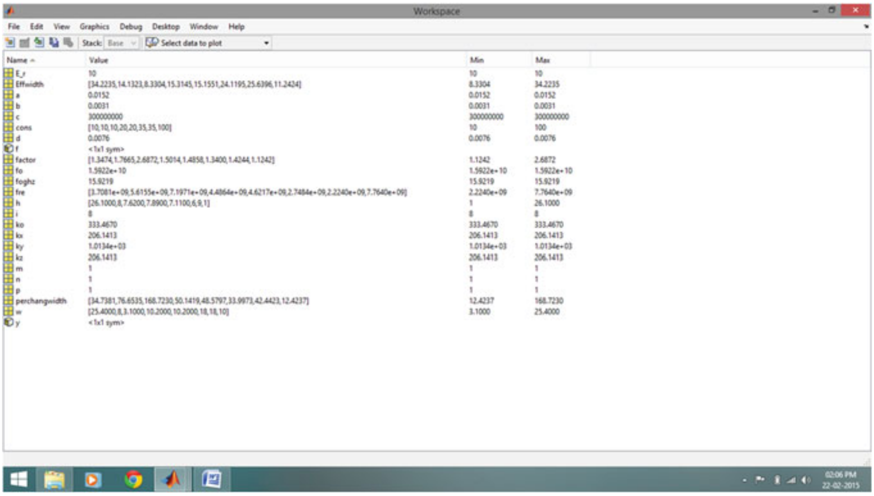
Stack: Base | Select data to plot

Name	Value	Min	Max
Effwidth	[34.2235,14.1323,8.3304,15.3145,15.1551,24.1195,25.6396,11.2424]	8.3304	34.2235
c	300000000	30000...	30000...
cons	[10,10,10,20,20,35,35,100]	10	100
d	[14.3000,14,15.2400,10.2000,10.1600,18,18,10]	10	18
f	<1x1 sym>		
factor	[1.3474,1.7665,2.6872,1.5014,1.4858,1.3400,1.4244,1.1242]	1.1242	2.6872
fre	[3.7081e+09,5.6155e+09,7.1971e+09,4.4864e+09,4.6217e+09,2.7484e+09,2.2240e+09,7.7640e+09]	2.2240...	7.7640...
h	[26.1000,8,7.6200,7.8900,7.1100,6,9,1]	1	26.1000
i	8	8	8
ko	<1x1 sym>		
kx	[0.2197,0.2244,0.2061,0.3080,0.3092,0.1745,0.1745,0.3142]	0.1745	0.3142
ky	[0.0918,0.2223,0.3771,0.2051,0.2073,0.1303,0.1225,0.2794]	0.0918	0.3771
kz	[0.0602,0.1963,0.2061,0.1991,0.2209,0.2618,0.1745,1.5708]	0.0602	1.5708
m	1	1	1
n	1	1	1
p	1	1	1
perchangwidth	[34.7381,76.6535,168.7230,50.1419,48.5797,33.9973,42.4423,12.4237]	12.4237	168.72...
w	[25.4000,8,3.1000,10.2000,10.2000,18,18,10]	3.1000	25.4000
y	<1x1 sym>		

Program 2

```
m=1;
n=1;
p=1;
E_r=10;
a=15.24e-03;
b=3.1e-03;
d=7.62e-03;
c=3e+08;
kx=m*pi/a;
ky=n*pi/b;
kz=p*(pi/d)/2;
ko=sqrt(kx^2+ky^2+kz^2)/sqrt(E_r);
fo=(c*ko/pi)/2;
foghz=fo/(1e+09);
```

Results:

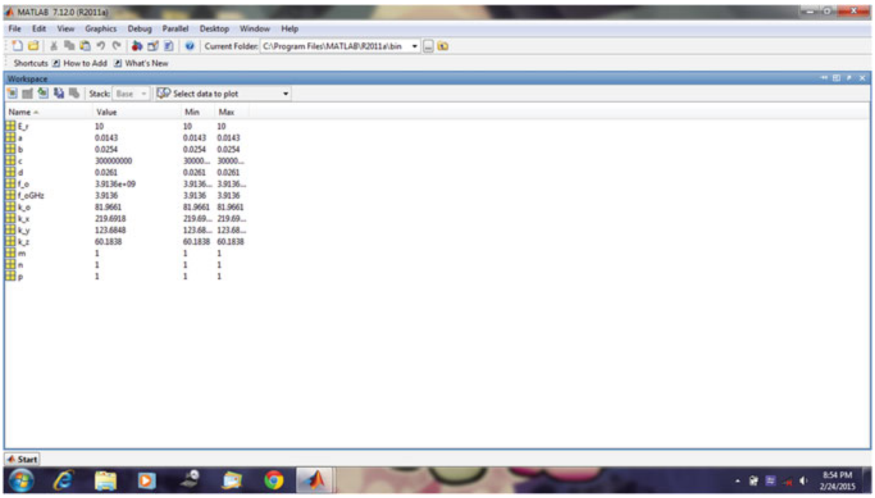


Name	Value	Min	Max
E_r	10	10	10
Effwidth	[34.2235,14.1323,8.3304,15.3145,15.1551,24.1195,25.6396,11.2424]	8.3304	34.2235
a	0.0152	0.0152	0.0152
b	0.0031	0.0031	0.0031
c	300000000	300000000	300000000
cons	[10,10,10,20,20,35,35,100]	10	100
d	0.0076	0.0076	0.0076
f	<1x1 sym>		
factor	[1.3474,1.7665,2.6872,1.5014,1.4858,1.3400,1.4244,1.1242]	1.1242	2.6872
fo	1.5922e+10	1.5922e+10	1.5922e+10
foghz	15.9219	15.9219	15.9219
fre	[3.7081e+09,5.6155e+09,7.1971e+09,4.4864e+09,4.6217e+09,2.7484e+09,2.2240e+09,7.7640e+09]	2.2240e+09	7.7640e+09
h	[26.1000,8,7.6200,7.8900,7.1100,6,9,1]	1	26.1000
i	8	8	8
ko	333.4670	333.4670	333.4670
kx	206.1413	206.1413	206.1413
ky	1.0134e+03	1.0134e+03	1.0134e+03
kz	206.1413	206.1413	206.1413
m	1	1	1
n	1	1	1
p	1	1	1
perchangwidth	[34.7381,76.6535,168.7230,50.1419,48.5797,33.9973,42.4423,12.4237]	12.4237	168.7230
w	[25.4000,8,3.1000,10.2000,10.2000,18,18,10]	3.1000	25.4000
y	<1x1 sym>		

Program 3

```
MATLAB programs taking parameters a, b, d same and comparing
frequency using:
Program 1: Characteristic equation
m=1
n=1
p=1
E_r=10
a=14.3e-03
b=25.4e-03
d=26.1e-03
c=3e+08
k_x=m*pi/a
k_y=n*pi/b
k_z=p*(pi/d)/2
k_o=sqrt(k_x^2+k_y^2+k_z^2)/sqrt(E_r)
f_o=(c*k_o/pi)/2
f_oGHz=f_o/1e+09
```

Output:

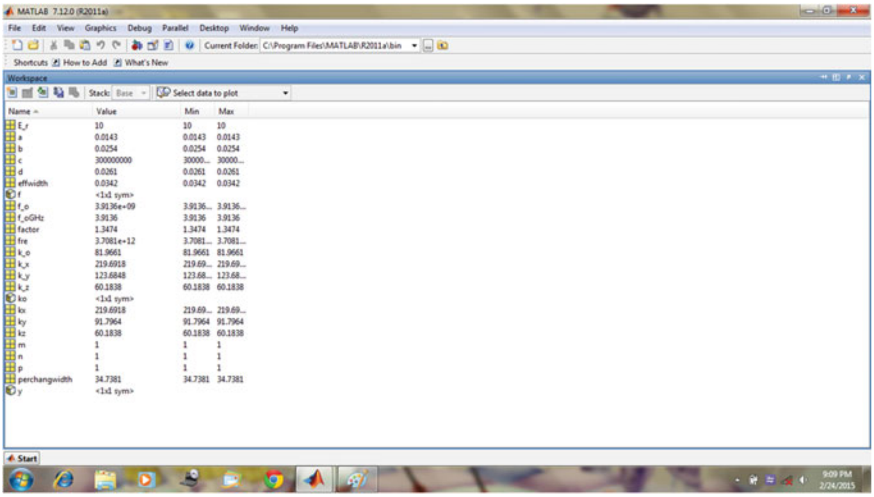


Program 4

Transcendental equation for same dimensions:

```
m=1;
n=1;
p=1;
E_r=10;
a=14.3e-03;
b=25.4e-03;
d=26.1e-03;
c=3e+08;
syms y real
kx=pi/a;
kz=pi/d/2;
ko=sqrt(kx^2+y^2+kz^2)/sqrt(E_r);
f=real(y*tan(y*b/2)-sqrt((E_r-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
fre=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;
```

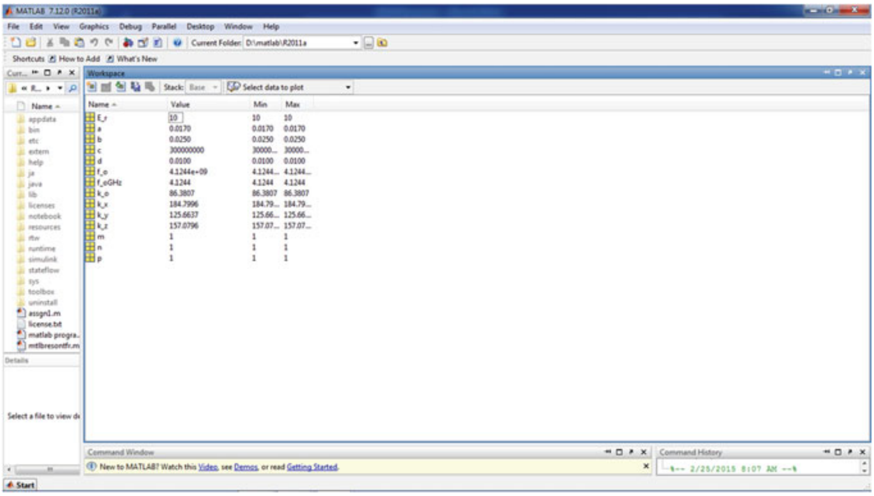
Output:



Program 5

```
MATLAB programs taking parameters a, b, d same and comparing
frequency using:
Characteristic equation
       Where  a=17mm
              b=25mm
              c=10mm
m=1;
n=1;
p=1;
E_r=10;
a=17e-03;
b=25e-03;
d=10e-03;
c=3e+08;
k_x=m*pi/a;
k_y=n*pi/b;
k_z=p*(pi/d)/2;
k_o=sqrt(k_x^2+k_y^2+k_z^2)/sqrt(E_r);
f_o=(c*k_o/pi)/2;
f_oGHz=f_o/1e+09;
```

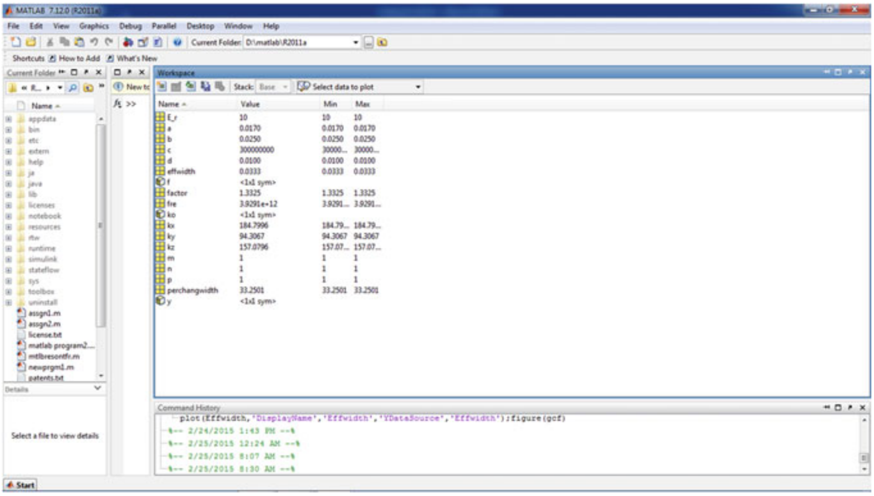
Output:



Program 6

Transcendental equation

```
m=1;
n=1;
p=1;
E_r=10;
a=17e-03;
b=25e-03;
d=10e-03;
c=3e+08;
syms y real
kx=pi/a;
kz=pi/d/2;
ko=sqrt(kx^2+y^2+kz^2)/sqrt(E_r);
f=real(y*tan(y*b/2)-sqrt((E_r-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
fre=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;
```

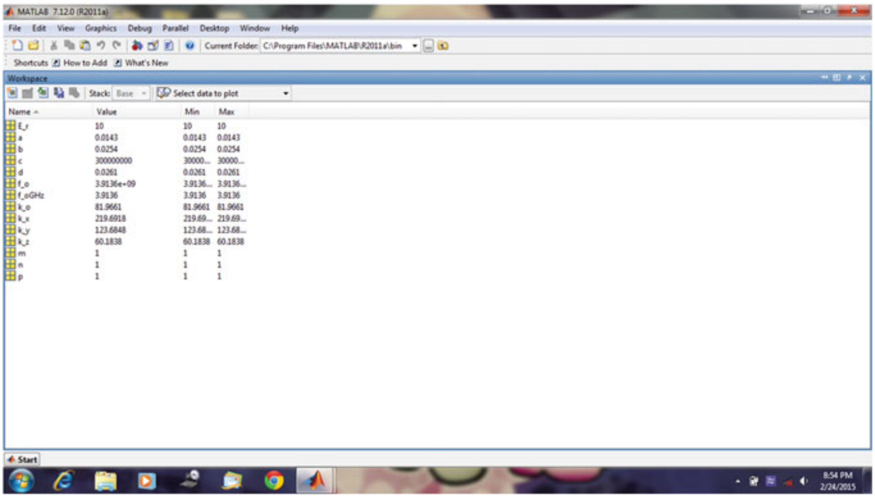


Program 7

```
MATLAB programs taking parameters a,b,d same and comparing
frequency using:
Characteristic equation
```

```
m=1
n=1
p=1
E_r=10
a=14.3e-03
b=25.4e-03
d=26.1e-03
c=3e+08
k_x=m*pi/a
k_y=n*pi/b
k_z=p*(pi/d)/2
k_o=sqrt(k_x^2+k_y^2+k_z^2)/sqrt(E_r)
f_o=(c*k_o/pi)/2
f_oGHz=f_o/1e+09
```

Output:

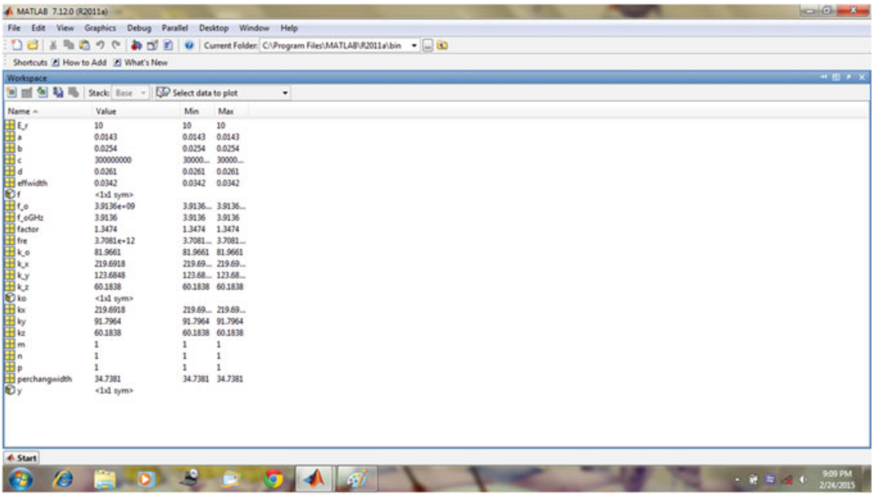


Program 8

Transcendental equation

```
m=1;
n=1;
p=1;
E_r=10;
a=14.3e-03;
b=25.4e-03;
d=26.1e-03;
c=3e+08;
syms y real
kx=pi/a;
kz=pi/d/2;
ko=sqrt(kx^2+y^2+kz^2)/sqrt(E_r);
f=real(y*tan(y*b/2)-sqrt((E_r-1)*ko^2-y^2));
ky=fzero(inline(f),[0,(pi/b)-0.01]);
fre=c/2/pi*sqrt((kx^2+ky^2+kz^2)/E_r)*1e3;
effwidth=pi/ky;
factor=effwidth/b;
perchangwidth=((effwidth-b)/b)*100;
```

Output:



Program 9

```
m=3;
n=3;
a=10;
b=5;
x=linspace(-5,5,51);
y=linspace(-2.5,2.5,51);
[xi,yi]=meshgrid(x,y);
Ez=cos(m*pi*xi/a).*cos(n*pi*yi/b);
Ez=Ez.^2;
Ez=sqrt(Ez);
surf(xi,yi,Ez)
view([-45,60])
%%view([180,0])
drawnow
```

Workspace

Stack: Base Select data to plot

Name	Value	Min	Max
Ez	<51x51 double>	3.37...	1
a	10	10	10
b	5	5	5
m	3	3	3
n	3	3	3
x	<1x51 double>	-5	5
xi	<51x51 double>	-5	5
y	<1x51 double>	-2.50...	2.50...
yi	<51x51 double>	-2.50...	2.50...

Program 10

```
d=[14.3,14.0,15.24,10.2,10.16,18,18,10];
w=[25.4,8,3.1,10.2,10.2,18,18,10];
h=[26.1,8,7.62,7.89,7.11,6,9,1];
m=1;
n=1;
p=1;
c=3e8;
cons=[10.0,10.0,10,20,20,35,35,100];
syms y real
for i=drange(1:8)
kx(i)=pi/d(i);
kz(i)=pi/2/h(i);
ko=sqrt((kx(i).^2+y.^2+kz(i).^2)/cons(i));
f=real(y.*tan(y*w(i)/2)-sqrt((cons(i)-1)*ko.^2-y.^2));
ky(i)=fzero(inline(f),[0,(pi/w(i))-0.01]);
fre(i)=c/2/pi*sqrt((kx(i).^2+ky(i).^2+kz(i).^2)/cons(i))*1e3;
Effwidth(i)=pi/ky(i);
factor(i)=Effwidth(i)./w(i);
perchangwidth(i)=((Effwidth(i)-w(i))/w(i))*100;
end
```

Workspace

Stack: Base | Select data to plot

Name	Value	Min	Max
Effwidth	[34.2235,14.1323,...	8.33...	34.2...
Ez	<51x51 double>	3.37...	1
a	10	10	10
b	5	5	5
c	300000000	3000...	3000...
cons	[10,10,10,20,20,3...	10	100
d	[14.3000,14,15.24...	10	18
f	<1x1 sym>		
factor	[1.3474,1.7665,2....	1.12...	2.68...
fre	[3.7081e+09,5.61...	2.22...	7.76...
h	[26.1000,8,7.6200...	1	26.1...
i	8	8	8
ko	<1x1 sym>		
kx	[0.2197,0.2244,0....	0.17...	0.31...
ky	[0.0918,0.2223,0....	0.09...	0.37...
kz	[0.0602,0.1963,0....	0.06...	1.57...
m	1	1	1
n	1	1	1
p	1	1	1
perchangwidth	[34.7381,76.6535,...	12.4...	168....
w	[25.4000,8,3.1000...	3.10...	25.4...
x	<1x51 double>	-5	5
xi	<51x51 double>	-5	5
y	<1x1 sym>		
yi	<51x51 double>	-2.50...	2.50...

Q.No. 1 Develop transcendental equation for moat-shaped RDRA.

Q.No. 2 Compute propagation constants in *x*-, *y*-, and *z*-directed propagated RDRAs, when feed probe is given. Compute its resonant frequency when RDRA dimensions are 5 × 5× 3 mm^3 and dielectric constant used is 20.