Interval Goal Programming Approach to Multiobjective Programming Problems with Fuzzy Data Uncertainty

Shyamal Sen and Bijay Baran Pal

Abstract This paper presents interval goal programming approach for solving multiobjective programming problems with fuzzy parameter sets. In the proposed approach, the notion of interval approximation technique to fuzzy numbers is used to transform the objectives with interval parameter sets. In the model formulation, interval arithmetic is employed to convert the problem into the standard goal programming problem. In goal achievement function, both the modelling aspects in goal programming (GP), *minsum* GP and *minmax* GP are taken into account as a convex combination of them to minimize possible deviations from specified target intervals for goal achievements from optimistic point of view of decision maker (DM) in the decision situation. A numerical example is solved to illustrate the proposed approach.

Keywords Fuzzy number · Interval approximation · Interval arithmetic · Goal programming · Fuzzy goal programming

1 Introduction

The occurrence of inexact data is inherent to most of the real-world decision problems owing to imprecise nature of human judgments. Two prominent techniques, fuzzy programming (FP) and interval programming (IVP) are used to capture such uncertainty. The FP approaches [1], based on fuzzy set theory [2],

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concentrate on imprecise data (neither crisp nor random). Interval programming, based on interval arithmetic [3], emphasizes on intrinsic vagueness of estimated model parameters.

Now, in most of the practical decision situations, it is found that problems are with multiplicity of objectives, where objectives often conflict each other to achieve solutions by optimizing objectives in the decision environment. To resolve the crisis, GP approach [4], introduced by Charnes and Cooper to solve problems in crisp environment has been appeared as a robust tool for multiobjective decision analysis. In a fuzzy multiobjective decision environment, fuzzy goal programming (FGP) as an extension of conventional GP has been studied [5], and further extended by pal et al. [6]. Thereafter, FGP has been applied to wide range of real-life problems [7, 8] and widely circulated in the literature [9, 10].

Now, in the real-world decision situation it is to be observed that setting of the imprecise goal values and measuring of tolerance ranges for goal achievement in FGP approaches may not always be possible in a highly sensitive decision environment. To tackle such a situation, IVP [11, 12] has appeared as a flexible tool for solving decision problems with interval parameter sets.

IVP approach in the framework of GP called interval GP (IVGP) has been introduced by Inuiguchi and Kume [13] in 1991. The methodological development on IVP made in the past has been surveyed by Oliveira and Antunes [14] in 2007. However, methodological extension of interval programming is still at an early stage form the viewpoint of its use to different real-life problems.

However, in an uncertain environment, when all the model parameters are represented by sets of fuzzy numbers, the notion of α-cut in fuzzy sets is used for crisp equivalent formulation of a decision problem. Here, various defuzzification operators [15, 16] are employed to solve α-level multiobjective programming problem. But, the use of such a traditional approach involves a set of optimization problems, and which leads to involve huge computational load in the solution search process.

To overcome the above situation, the interval approximation approach for decision analysis has been studied [17] in the recent past. But, the constructive solution procure to solve fuzzily described multiobjective decision making (MODM) problems with crisp coefficient interval representations of model parameters is yet to be widely circulated in the literature.

In this paper, IVGP approach to fuzzily described MODM problems is introduced to make a reasonable decision for achievement of objectives within the intervals specified for optimizing them in the fuzzy environment. In the proposed approach, first the intervals within which fuzzy objective coefficients possibly take their values are determined by using the nearest interval approximation method [17] to fuzzy numbers in the decision situation. In IVGP model formulation, the structural constraint sets with fuzzy numbers are transformed into their crisp equivalents by employing the concept of magnitude of a fuzzy number [18] to solve the problem in the framework of IVGP method. In the solution process, minimization of deviational variables associated with objective goals of the formulated

IVGP model is taken into account to arrive at a decision for achievement of goals within the target interval specified for each of them in the decision environment.

Now, the basic definitions related to fuzzy set and fuzzy number are described in the next section.

2 Preliminaries on Fuzzy Sets

Fuzzy Sets: A fuzzy set $\tilde{A}$ of a universe X (relevance in particular context) is characterized by its membership function $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ and defined as:

$$\tilde{A} = \{(x,\ \mu_{\tilde{A}}(x)) | x \in X\}.$$

α-Cut of Fuzzy Sets: α-cut of a fuzzy set $\tilde{A}$ is a subset of $\tilde{A}$ and defined as $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha\}$, for all $\alpha \in [0, 1]$.

Fuzzy Number: A fuzzy set $\tilde{A}$ defined on $\Re$ (a set of real numbers) is said to be fuzzy number if the following conditions are satisfied

(i) $\tilde{A}$ is normal, that is height of $\tilde{A}$ is unity.
(ii) $\tilde{A}$ is a convex set.
(iii) the membership function $\mu_{\tilde{A}}(x), x \in \Re$, is at least piecewise continuous.

The membership function $\mu_{\tilde{A}}(x)$ of is a continuous mapping from $\Re$ to closed interval [0, 1] and presented as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } \ x \in (\propto, a_1] \\ f^L_{\tilde{A}}(x), & \text{if } x \in [a_1, a_2] \\ 1, & \text{if } \ x \in [a_2, a_3] \\ f^R_{\tilde{A}}(x), & \text{if } x \in [a_3, a_4] \\ 0, & \text{if } x \in [a_4, \propto) \end{cases}$$

where $f^L_{\tilde{A}}(x)$ is strictly increasing and $f^R_{\tilde{A}}(x)$ a is strictly decreasing function.

If $f^L_{\tilde{A}}(x)$ and $f^R_{\tilde{A}}(x)$ linear, then the fuzzy number is called trapezoidal fuzzy number and denoted by $\langle a_1,a_2,a_3,a_4\rangle$ and has α-cut $A_\alpha = [a_1 + (a_2 - a_1)\alpha,\ a_4 - (a_4 - a_3)\alpha]$. In particular, if $a_2 = a_3$, then trapezoidal fuzzy number is reduced to triangular fuzzy number and denoted by $< a_1, a_2, a_4 >$ and has α-cut $A_\alpha = [a_1 + (a_2 - a_1)\alpha,\ a_4 - (a_4 - a_2)\alpha]$.

Interval Arithmetic Operation: Let a closed interval A (named as interval number) is defined as $A = [a^L, a^U] = \{a{:}a^L \leq a \leq a^U\}$, where a^L, a^U left and right are limits, respectively of the interval A.

For a particular case, A = [a, a] represents only the real number a.

If ξ be a scalar then, the scalar multiplication of A is defined as: $\xi A = \begin{cases} [\xi \mathrm{a}^{\mathrm{L}}, \xi a^{\mathrm{U}}], & \xi \geq 0 \\ [\xi \mathrm{a}^{\mathrm{U}}, \xi a^{\mathrm{L}}], & \xi < 0 \end{cases}$

The two binary operations, on the two interval numbers A_1 and A_2 are defined as:

$$\mathrm{A}_1 + \mathrm{A}_2 = [\mathrm{a}_1^{\mathrm{L}} + \mathrm{a}_2^{\mathrm{L}},\ \mathrm{a}_1^{\mathrm{U}} + \mathrm{a}_2^{\mathrm{U}}]$$

$$\mathrm{A}_1 - \mathrm{A}_2 = [\mathrm{a}_1^{\mathrm{L}} - \mathrm{a}_2^{\mathrm{U}},\ \mathrm{a}_1^{\mathrm{U}} - \mathrm{a}_2^{\mathrm{L}}]$$

Fuzzy Arithmetic Operation: Let $\tilde{\mathrm{A}}$, $\tilde{\mathrm{B}}$ be two fuzzy numbers and $\tilde{\mathrm{C}}$ be another fuzzy number is defined as $\tilde{\mathrm{C}} = \tilde{\mathrm{A}} * \tilde{\mathrm{B}}$. Then α-cut of fuzzy number $\tilde{\mathrm{C}}$ can be expressed as:

$$\mathrm{C}_\alpha = \mathrm{A}_\alpha * \mathrm{B}_\alpha$$

where '$*$' represents a binary operation.

Interval Approximation of Fuzzy Number: The fuzzy number can be approximated to an interval number. Uncertainties related to the parameters can be effectively handled by using a suitable approximated interval. The idea of the nearest interval approximation based on the metric 'D' (distance function) has been introduced by Grezegorzwiski and defined as [17]:

$$\mathrm{N_D}(\tilde{\mathrm{A}}) = \left[\int_0^1 \mathrm{A}_\alpha^{\mathrm{L}} d\alpha, \int_0^1 \mathrm{A}_\alpha^{\mathrm{U}} d\alpha\right]$$

where $\mathrm{N_D}(\tilde{\mathrm{A}})$ represents the approximated interval of fuzzy number $\tilde{\mathrm{A}}$ based on the metric 'D'.

Ranking of Fuzzy Numbers: Fuzzy numbers are generally not rank ordered. To establish the order relation between two fuzzy numbers, the ranking function is to be defined. The ranking function $g : F(\Re) \to \Re$ where $F(\Re)$ the set of all fuzzy numbers on the real line is $\Re$, is defined as:

$$\begin{array}{lll} g(\tilde{\mathrm{A}}_i) < g(\tilde{\mathrm{A}}_j) & \text{implies } \tilde{\mathrm{A}}_i < \tilde{\mathrm{A}}_j, & i \neq j \\ g(\tilde{\mathrm{A}}_i) > g(\tilde{\mathrm{A}}_j) & \text{implies } \tilde{\mathrm{A}}_i < \tilde{\mathrm{A}}_j, & i \neq j \\ g(\tilde{\mathrm{A}}_i) = g(\tilde{\mathrm{A}}_j) & \text{implies } \tilde{\mathrm{A}}_i = \tilde{\mathrm{A}}_j & \text{for all} \quad i,\ j \in I \end{array}$$

where $\tilde{\mathrm{A}}_i, \tilde{\mathrm{A}}_j \in F(\Re)$ and I is the subset of natural number and it represents the index set.

Magnitude of a Trapezoidal fuzzy number: A special type of ranking function, the magnitude of the trapezoidal fuzzy number $< a_0, x_0, y_0, b_0 >$ and defined as [18]:

$$\mathbf{mag}(\tilde{A}) = \frac{1}{2}\left(\int_0^1 (A_\alpha^L + A_\alpha^U + x_0 + \ y_0)f(\alpha)d\alpha\right)$$

where $A_\alpha^L = a_0 + (x_0 - a_0)\alpha$, $A_\alpha^U = b_0 - (b_0 - y_0)\alpha$ and $f(\alpha)$ represents the weight function which is continuous and satisfies the following relations:

$$f(0) = 0, \quad f(1) = 1 \text{ and } \int_0^1 f(\alpha)d\alpha = \frac{1}{2}.$$

For the simplicity, $f(\alpha) = \alpha$ is taken as weight function to define the magnitude of the fuzzy number.

Now, the problem formulation is described in the Sect. 3.

3 Problem Formulations

The generic form of a multiobjective programming with fuzzy number involved in the objective function as well as in the constraints can be presented as:

Find X (x_1, x_2,…,x_n) so as to:

$$\text{Maximize} \sum_{j=1}^{n} \tilde{C}_{kj}x_j, \quad k = 1, 2, \ldots, K.$$

so as to satisfy

$$\begin{aligned} &\sum_{j=1}^{n} \widetilde{A}_{ij}x_j \leq \widetilde{B}_i, \ i = 1, 2, \ldots, m \\ &X \geq 0 \end{aligned} \tag{1}$$

where X is the vector of decision variables, and the parameters $\tilde{C}_{kj}$, $\tilde{A}_{ij}$ and $\tilde{B}_i$ (i = 1, 2,…, m; j = 1, 2,…, n) are fuzzy number.

In fuzzy linear programming, fuzzy coefficients are involved in the objective function. To optimize the objective, fuzzy objectives are to be defuzzified by using suitable defuzzification operator. Different defuzzification operators have been circulated in the literature. α-cut is the widely used operator for the conversion of the objective to its crisps equivalent form. Huge computational loads are involved to solve the α-level multiobjective programming problems. By using nearest interval approximation of fuzzy number [17], the objective function of the problem can be written as:

$$\begin{aligned}
\text{Maximize } Z_k(X) &= N_D\left(\sum_{j=1}^{n} \tilde{C}_{kj} x_j\right) \\
&= \sum_{j-1}^{n} N_D(\tilde{C}_{kj})\, x_j \\
&= \sum_{j=1}^{n} \left[\int_0^1 C^L_{kj\alpha} d\alpha, \int_0^1 C^U_{kj\alpha} d\alpha\right] x_j \\
&= \sum_{j=1}^{n} \left[c^L_{kj}, c^U_{kj}\right] x_j
\end{aligned}$$

where $\left[c^L_{kj}, c^U_{kj}\right]$ represents the nearest interval approximation of the fuzzy number $\tilde{C}_{kj}$ and the function $N_D(\tilde{C}_{kj})$ satisfies the linear property and where $c^L_{kj} = \int_0^1 C^L_{kj\alpha} d\alpha$, $c^U_{kj} = \int_0^1 C^U_{kj\alpha} d\alpha$.

Now, the parameters involved in the constraints are also fuzzy numbers. It is necessary to transform the constraints into deterministic equivalent according to some point of view defined on the basis of needs and desires of the decision maker (DM). To compare the constraints from both sides with respect to the inequality relation (the symbol '≤' only signifies the relation between the fuzzy numbers from both sides of the relation), a suitable ranking function is to be defined. Using the concept of magnitude of a fuzzy number defined in the Sect. 2, the equivalent crisp form of the constraints in (1) can be presented as [18]:

$$\mathbf{mag}\,(\sum_{j=1}^{n} \tilde{A}_{ij} x_j) \le \mathbf{mag}\,(\tilde{B}_i), \quad i = 1, 2, \ldots, m \tag{2}$$

where **mag**(.) represents the Magnitude of fuzzy number (.).

The interval programming problem formulation is described in the Sect. 3.1.

3.1 Interval Programming Problem Formulation

Using interval arithmetic defined in the Sect. 2, the crisp equivalent interval programming problem of fuzzy programming problem in (1) can be formulated as:

$$\begin{aligned}
\text{Maximize} \quad & Z_k(X) = \left[\sum_{j=1}^{n} c_{kj}^{L} x_j, \sum_{j=1}^{n} c_{kj}^{U} x_j\right] \\
& \quad = [Z_{kL}(X), Z_{kU}(X)] \text{ (say)} \\
\text{subject to} \quad & \sum_{j=1}^{n} M_{ij} x_j \le u_i, \quad i = 1, 2, \ldots, m \\
& X \ge 0
\end{aligned} \tag{3}$$

where **mag** $(\tilde{A}_{ij}) = M_{ij}$ and **mag** $(\tilde{B}_i) = u_i$.

In IVP problem, targets have to be set for achieving the objective values in the specified target intervals.

Determination of target intervals is presented in the Sect. 4.

4 Determination of Target Intervals

To determine the target intervals, the best and worst solutions of the defined interval valued objectives are to be obtained first.

Let, the individual best and worst solutions of the k-th objective be $(X_k^b; T_{kU}^*)$ and $(X_k^w; T_{kL}^*)$, respectively,

$$\text{where } T_{kU}^* = \underset{X \in S}{\text{Max}}\, Z_{kU}(X), \text{ and } T_{kL}^* = \underset{X \in S}{\text{Min}}\, Z_{kL}(X) \tag{4}$$

where S is the feasible region bounded by the set of constraints in (3).

Now, from the viewpoint of achieving the objective values within the best and worst decisions, the target intervals can be considered as $\left[t_k^L, t_k^U\right]$,

$$\text{where } T_{kL}^* \le t_k^L < t_k^U \le T_{kU}^*, \quad k = 1,2,\ldots,K \tag{5}$$

Then, incorporating the target intervals, interval valued objectives in (3) can be expressed as [13]:

$$Z_k(X) : [Z_{kL}(X), Z_{kU}(X)] = [t_k^L, t_k^U], \quad k = 1, 2, \ldots, k \tag{6}$$

The Goal programming formulation of the problem is described in the Sect. 5.

5 Goal Programming Formulation

The interval valued k-th objective with target interval can be restated as:

$$[Z_{kL}(X), Z_{kU}(X)] = [t_k^L, t_k^U], \quad k = 1, 2, \ldots, K \tag{7}$$

In the above formulation, it is to be mentioned that a possible solution of the problem exists if the relations

$$
\begin{aligned}
&Z_{kL}(X) \geq t_k^L \\
&\text{and } Z_{kU}(X) \leq t_k^U
\end{aligned} \tag{8}
$$

are satisfied simultaneously.

Now, introducing under- and over-deviational variables, the goal expressions in (8) can be presented as:

$$Z_{kL}(X) + \rho_{kL}^{-} - \rho_{kL}^{+} = t_k^L, \quad k = 1, 2, \ldots, K \tag{9}$$

$$Z_{kU}(X) + \rho_{kU}^{-} - \rho_{kU}^{+} = t_k^U, \quad k = 1, 2, \ldots, K \tag{10}$$

where $(\rho_{kL}^{-}, \rho_{kU}^{-}) \geq 0$ and $(\rho_{kL}^{+}, \rho_{kU}^{+}) \geq 0$ represent the under- and over-deviational variables, respectively.

Now, in a decision making situation, the aim of the DM is to achieve the goal values within the specified ranges by means of minimizing the possible regrets in terms of minimizing the deviational variables involved in the decision situation.

In the field of IVGP, both the aspects of GP, *minsum* GP [6] for minimizing the sum of the weighted unwanted deviational variables as well as *minmax* GP [8] for minimizing the maximum of the deviations, are simultaneously taken into account as a convex combination of them to reach a satisfactory solution within the specified target intervals of the goals.

Then, the regret function appears as:

$$\text{Minimize } Z = \lambda\left\{\sum_{k=1}^{k} w_k(\rho_{kL}^{-} + \rho_{kU}^{+})\right\} + (1-\lambda)\left\{\max_k(\rho_{kL}^{-} + \rho_{kU}^{+})\right\} \tag{11}$$

Taking $\max_k(\rho_{kL}^{-} + \rho_{kU}^{+})$ = V, the executable GP model of the problem can be restated as:

$$
\begin{aligned}
\text{Minimize} \quad & Z = \lambda \sum_{k=1}^{K} w_k(\rho_{kL}^{-} + \rho_{kU}^{+}) + (1-\lambda)V \\
\text{and satisfy} \quad & \\
& Z_{kL}(X) + \rho_{kL}^{-} - \rho_{kL}^{+} = t_k^L, \quad k = 1, 2, \ldots, K \\
& Z_{kU}(X) + \rho_{kU}^{-} - \rho_{kU}^{+} = t_k^U, \quad k = 1, 2, \ldots, K \\
\text{subject to} \quad & \sum_{j=1}^{n} M_{ij} x_j \leq u_i, \quad i = 1, 2, \ldots, m \\
& \rho_{kL}^{-} + \rho_{kU}^{+} \leq V, \quad k = 1, 2, \ldots, K \\
& X \geq 0
\end{aligned} \tag{12}
$$

where Z represents the regret function for goal achievement, and where $w_k(>0)$ with $\sum_{k=1}^{K} w_k = 1$ denote the numerical weights of importance of achieving the goals within the respective target intervals, and where $0<\lambda<1$.

6 Illustrative Example

To illustrate the proposed approach the following example is considered.

Find $X(x_{1,}x_2)$ so as to:

$$\begin{aligned} &\text{Maximize } Z_1(X) = \langle 1,\ 2,\ 3\rangle x_1 + \langle 2,\ 4,\ 6\rangle x_2 \\ &\text{Maximize } Z_2(X) = \langle 2,\ 3,\ 4\rangle x_1 + \langle 3.5,\ 5,\ 6.5\rangle x_2 \\ &\text{subject to } \langle 1,\ 2,\ 3,\ 4\rangle x_1 + \langle 1,\ 3,\ 4,\ 7\rangle x_2 \le \langle 10,\ 11,\ 12,\ 14\rangle \\ &\qquad \langle 6,\ 7,\ 8,\ 9\rangle x_1 + \langle 4,\ 6,\ 7,\ 8)\rangle x_2 \le \langle 20,\ 22,\ 23,\ 26\rangle \\ &\qquad x_1, x_2 \ge 0. \end{aligned} \tag{13}$$

Using the proposed procedure defined in the Sect. 2, the approximated interval of the triangular fuzzy numbers associated with the problem are obtained as: $N_D(<1,2,3>) = [1.5,\ 2.5]$, $N_D(<2,4,6>) = [3,\ 5.5]$, $N_D(<2,3,4>) = [2.5,\ 3.5]$ $N_D(<3.5,5,6.5>) = [4.25,\ 5.75]$;

Then, magnitudes of the trapezoidal fuzzy numbers of the parameters associated with the constraints in (13) are also obtained as:

$$(M_{11}, M_{12}, M_{21}, M_{22}) = (2.5, 3.417, 7.5, 6.583),\ \text{and}\ (u_1, u_2) = (10.917, 22.417)$$

Using the relation in (4) the best and least solution sets are obtained as:

$$(T_{1L}^*, T_{1U}^*) = (0,\ 17.57),\ \text{and}\ (T_{2L}^*, T_{2U}^*) = (0,\ 18.37)$$

Now, taking $[8,\ 17.57]$ and $[12, 18.37]$ as target interval for the objectives, respectively, the executable model can be presented as:

Find $X(x_1, x_2)$ so as to:

$$\text{Minimize}\,\vec{Z} = \lambda \sum_{k\vec{=}1}^{\vec{2}} \vec{w}_{\vec{k}} (\vec{\rho}_{\vec{kL}}^{\,\vec{-}} + \vec{\rho}_{k\vec{U}}^{\,\vec{+}})\vec{\ } + (1-\lambda)V$$

and satisfy

$$
\begin{aligned}
&1.5x_1 + 3x_2 + \rho_{1L}^- - \rho_{1L}^+ = 8 \\
&2.5x_1 + 5.5x_2 + \rho_{1U}^- - \rho_{1U}^+ = 17.57 \\
&2.5x_1 + 4.25x_2 + \rho_{2L}^- - \rho_{2L}^+ = 12 \\
&3.5x_1 + 5.75x_2 + \rho_{2U}^- - \rho_{2U}^+ = 18.37 \\
&2.5x_1 + 3.417x_2 \le 10.917 \\
&7.5x_1 + 6.583x_2 \le 22.417 \\
&\rho_{1L}^- + \rho_{1U}^+ \le V, \\
&\rho_{2L}^- + \rho_{2U}^+ \le V \\
&\quad 0 < \lambda < 1
\end{aligned} \tag{14}
$$

For simplicity and without loss of generality, taking $\lambda = 0.5$ and using the software (LINGO VER 6.0), solution is obtained as $x_1 = 0, \quad x_2 = 2.823$ with $Z_1 = [8.47.15.52], \quad Z_2 = [11.99, 16.23]$

Note. If the constraints in (13) are defuzzified by using the conventational notion of α-cut of fuzzy numbers, then the constraints are obtained as:

$$
\begin{aligned}
&[1+\alpha, 4-\alpha]x_1 + [1+2\alpha, 7-3\alpha]x_2 \le [10+\alpha, 14-2\alpha] \\
&[6+\alpha, 9-\alpha]x_1 + [4+2\alpha,\ 8-\alpha]x_2 \le [20+2\alpha,\ 26-3\alpha]
\end{aligned} \tag{15}
$$

Here, it may be mentioned that, for different values of $\alpha \in [0, 1]$, different results can be obtained in the solution search process. It is found that the optimal solution corresponds to $\alpha = 1$.

The optimal solution is: $x_1 = 0, \quad x_2 = 2.823$ with $Z_1 = [8.47.15.52]$, $Z_2 = [11.99, 16.23]$, which is the same as that obtained by using the proposed method.

Therefore, it may be said that the proposed approach is computationally more efficient than the conventional method to reach decision in a fuzzy decision environment.

7 Conclusion

In this paper, the potential use of IVGP approach to fuzzy MODM problems is presented. The main advantage of the approach presented here is that the computational load for performing sensitivity analysis with variation of level of satisfaction for degree of achievement of fuzzy objectives does not arise in the solution search process.

The proposed method can be extended to solve fractional programming problem as well as multilevel programming problem in hierarchical decision making situations, which may be the problems in future study. However, it is hoped that the proposed method can lead to open up new areas of research for solving real-life problems in uncertain decision environment.

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