

SPICE Modeling and Analysis for Metal Island Ternary QCA Logic Device

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Abstract The exploration of work ability of the new trend in quantum dot cellular automata (QCA)—the ternary QCA, is the major focus in this paper. Both physically and electrically, our tQCA approach is proving its excellence in comparison to the existing binary QCA (bQCA). We also propose a model description for tQCA that will help in determining its logic performances while operating it in the nano computing regime.

Keywords Metal island tQCA · bQCA · tQCA · Capacitive coupling · SPICE modeling

1 Introduction

The electronic charge state is being used in very intelligent way to represent the information. In digital electronics, the information representation with binary logic is an innovative idea by means of current switch. The binary ‘1’ and binary ‘0’ are

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represented by means of ‘on’ and ‘off’ respectively in Complementary Metal oxide Semiconductor (CMOS) technology. However, the device size reduction put it into the challenge. It has been noticed that in most promising CMOS technology, charge quantization, power dissipation etc. become problematic in nano scale device design. Now it is an established fact that Quantum dot Cellular Automata (QCA) can be an alternative of promising and empirical CMOS technology [1–8]. Researchers are mostly concentrating on binary logic representation in QCA while it may not be suitable to think all aspects by means of binary logic. Multi-valued logic especially ternary logic has potential advantages like greater data storage capability, faster arithmetic operations, better support for numerical analysis, non-deterministic and heuristic procedures, communication protocol and effective solution for non-binary problems [9–12].

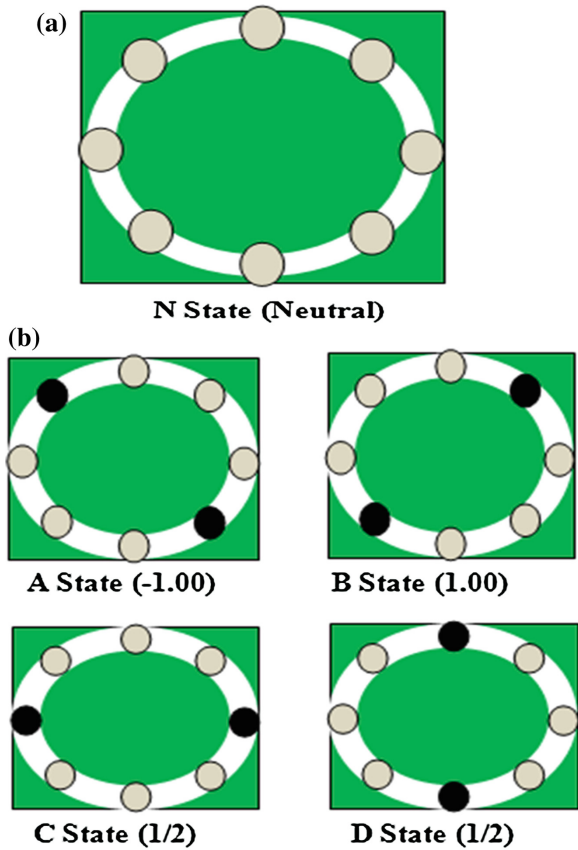
In binary QCA (bQCA), it is considered that a cell consist of four-quantum dot (QD) as charge container at four corner of square shaped cell and two extra electrons confined within the cell. Electrons can tunnel from one QD to another QD through tunnel junction. Now QCA platform is being utilized for ternary logic implementation [9–12]. It is really in infancy stage, several research works need to establish ternary logic implementation. Lebar Bajec et al. introduced ternary QCA (tQCA), reported in [9]. The tQCA can also be classified into semiconductor-based tQCA, Metal Island tQCA like bQCA. The tQCA cell is realized by means of eight QDs confined within a square shaped cell and two extra electrons confined within the cell. Eight QDs are arranged in a ring shape within square cell. The tQCA can have five state polarization ‘ -1.00 ’, ‘ $+1.00$ ’, ‘ $1/2$ ’ and No polarization represented by ‘A’, ‘B’, ‘C’, ‘D’ and ‘N’ and are shown in Fig. 1. Details of tQCA have been discussed in Sect. 2. To establish the tQCA as a logic design paradigm, the attention should be given on device performance.

In this paper, the physical properties of tQCA cell are discussed. Focus has been given on time independent steady state behavior of tQCA cell with the help of extended Hubbard model and the empirical studies on metal island tQCA cell with SPICE model is explored. Rest of the paper is organized in the following subsequence; Sect. 2 is dedicated to explore the basic ternary logic and tQCA. The time independent steady state behavior of tQCA cell is explored in Sect. 3. The SPICE model for the realization of metal island tQCA cell is described in Sect. 4. Finally, the conclusion of this work is presented in Sect. 5.

2 Ternary Logic and TQCA

The tQCA cell differs from bQCA cell in number of QD present in the cell and polarization state. There are eight numbers of QD arranged in a ring/circular shape within the square shaped cell, shown in Fig. 1. The diameter of ring must be equal to length of a square shape cell. As a result $R = L/2$, where ‘R’ is radius of ring/circle and ‘L’ is Length of square shape cell. Therefore, maximum distance of separation between two QD is L. The two extra electrons similar to bQCA are

Fig. 1 **a** Eight quantum dot arranged in ring within square shape tQCA cell, shows no polarization state and **b** tQCA cell with 'A' state (-1.00), 'B' state (+1.00), 'C' state or 'D' state (1/2)



confined within the cell. The internal circuitry of tQCA cell is made of tunnel junction and junction capacitance. Due to coulomb interaction, the tQCA cell is being polarized. Five possible state of polarity marked as 'A', 'B', 'C' or 'D' and 'N', are shown in Fig. 1.

3 Steady State Behavior of TQCA

The previous section has explored the different physical properties and cell architecture in quantum mechanics. Now in this section, an attempt is made to analyze the steady state behavior of tQCA with the help of extended Hubbard model. The Hamiltonian of eight-dot tQCA cell can be expressed as:

$$\begin{aligned}
H^{tQCA} = & \sum_{i,\sigma} (V_0 + V_i) \hat{n}_{i,\sigma} + \sum_{i>j,\sigma} t_{ij} \left(\hat{a}_{i,\sigma}^\dagger \hat{a}_{j,\sigma} + \hat{a}_{i,\sigma}^\dagger \hat{a}_{j,\sigma} \right) \\
& + \sum_i E_Q \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + \sum_{i>j,\sigma,\sigma'} V_Q \frac{\hat{n}_{i,\sigma} \hat{n}_{j,\sigma'}}{|R_i - R_j|}
\end{aligned} \tag{1}$$

Each quantum dot is considered as a site, so tQCA is simply an eight-site system as shown in Fig. 1. For tQCA system, it is considered that the number of extra electrons in the cell is fixed. The overall energy is constant and V_0 is set to zero. The first term V_i is potential energy of all electrons at site ‘i’ and $\hat{n}_{i,\sigma} = \hat{a}_{i,\sigma}^\dagger \hat{a}_{i,\sigma}$ is the number operator for the electron spin ‘ σ ’. σ is considered to have two states, up-spin ‘ \uparrow ’ and ‘down-spin ‘ \downarrow ’. The charging cost E_Q defined as

$$E_Q = \frac{V_Q}{(2R/3)} = \frac{3V_Q}{2R},$$

where ‘ R ’ is the radius of quantum dot and V_Q is the coulomb energy of two electrons separated by two third of the quantum dot’s radius.

For the steady state problem, Hamiltonian equation is solved with time independent Schrödinger equation

$$H^{tQCA} |\psi_i\rangle = E_i |\psi_i\rangle \tag{2}$$

where $|\psi_i\rangle$ is Eigen state of Hamiltonian and E_i is the corresponding Eigen value for $i = 0$, we have ground state of cell

$$|\psi_0\rangle = \sum_j \psi_j^0 |\phi_j\rangle$$

$|\phi_j\rangle$ is jth basis vector and ψ_j^0 is the coefficient of that basis vector. For eight-site tQCA system, we have 2^8 different basis vectors and these are

$$\begin{aligned}
|\phi_1\rangle &= \left| \begin{array}{c} 00000001 \\ 00000001 \end{array} \right\rangle \\
|\phi_2\rangle &= \left| \begin{array}{c} 01000000 \\ 01000000 \end{array} \right\rangle \\
&\dots\dots \\
&\dots\dots \\
|\phi_{256}\rangle &= \left| \begin{array}{c} 10000000 \\ 10000000 \end{array} \right\rangle
\end{aligned}$$

It means that site i and j are with up-spin electron state, others are with down-spin electron state. However, the polarization of tQCA cell can be expressed in

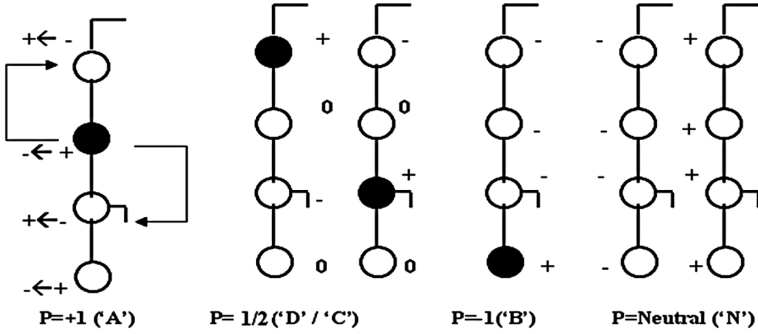


Fig. 2 Logic states in half metal island tQCA cell

terms of σ_i , the expectation value of number operator. The expectation value σ_i is proportional to radius of quantum dot of site 'i'. The polarization P can be expressed as

$$P \equiv \frac{(\sigma_1 + \sigma_5) - (\sigma_2 + \sigma_6) - (\sigma_3 + \sigma_7) - (\sigma_4 + \sigma_8)}{\sum_i \sigma_i} \tag{3}$$

tQCA cell can have four polarization state namely 'A', 'B', 'C' or 'D' and 'N' as shown in Fig. 2. We explore 'A' state ($p = -1.00$) as logic '0', 'B' state ($p = +1.00$) as logic '+1', 'C' and 'D' ($p = 1/2$) represent as logic '1/2' and finally 'N' state is denoted as Neutral state. The tQCA can be compared with existing bQCA counterpart and it has tri-stable or tri-state cell response. Due to its tri-state functionality, it is possible to store a single ternary bit in quantum state.

4 Metal Island tQCA Realization

The tQCA can also be manufactured with metal and small capacitance. The principle difference between metal island tQCA and semiconductor-based tQCA, is that in former one device is made with metal capacitive coupled rather than coulomb coupled quantum dot. In case of metal island tQCA, there are several electrons in conduction band, unlike the semiconductor quantum dot QCA device. The metal capacitive coupling cannot be represented with the help of Schrödinger equation. The capacitive model is to be adopted to realize the metal island tQCA. If two metal islands are connected with capacitor, the free charge can be removed and store into the capacitor. However, through the tunneling junction few electrons can tunnel in or out to metal island. The metal island tQCA half-cell architecture can be realized with SPICE model. The parameter considerations are similar with Dolan shadow-evaporation techniques. The schematic diagram of tQCA is shown in Fig. 3. Metal Island is connected with tunnel junction and capacitor, due to capacitive effect the

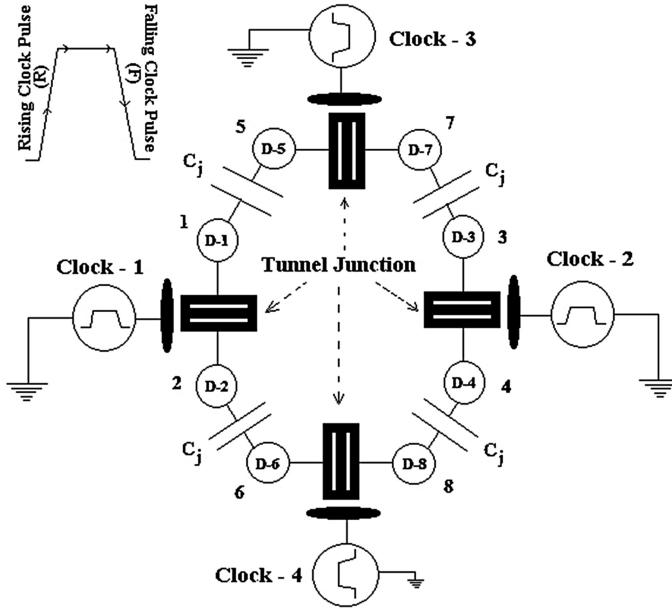


Fig. 3 Schematic of eight-dot tQCA

metal island become charge free. There are eight input bias voltage sources and four controls or clock input sources.

The charge configuration of system is composed of control electrode or clock and Metal Island, coupled by tunnel junctions and capacitance. The electrostatic energy of charge configuration system can be expressed in terms of voltage and charge as

$$E = \frac{1}{2} \begin{bmatrix} q \\ q' \end{bmatrix}^T C^{-1} \begin{bmatrix} q \\ q' \end{bmatrix} - v^T q' \quad (4)$$

where C is capacitance matrix, v is column vector of voltage source, q , q' are column vector of island charge and lead charge.

4.1 SPICE Model Description of tQCA

As stated in previous sections, the SPICE model, shown in Fig. 3 has four control knobs or clocks to operate the polarization within its cell. It has eight dot sites (i.e., D-1, D-2, D-3, D-4, D-5, D-6, D-7, D-8) where the free electrons can reside depending on the clock input. C_j is the junction capacitance between the dots. The tunnel junction provides the potential barrier to the electron transport and is an

Table 1 Logic configuration of tQCA

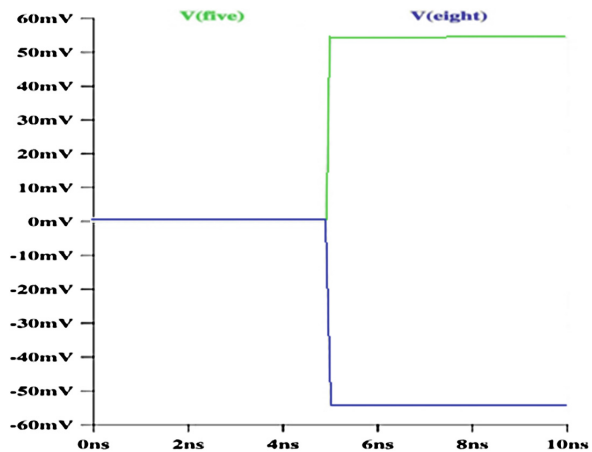
S. No.	Clock-1	Clock-2	Clock-3	Clock-4	Logic state
1.	(F)	(F)	(R)	(F)	“A”
2.	(F)	(F)	(F)	(F)	“Neutral”
3.	(F)	(F)	(F)	(R)	“B”
4.	(F)	(F)	(R)	(R)	“Neutral”
5.	(R)	(F)	(F)	(F)	“C or D”
6.	(R)	(R)	(F)	(F)	“Neutral”
7.	(F)	(R)	(F)	(F)	“D or C”

important criterion in tQCA. The position of the electrons on the dots determines the logic state of the cell. In our previous discussion, we have mentioned the possible logic states, shown in Fig. 2. We have altered the clock inputs (clock-1, clock-2, clock-3, clock-4) issuing them with possible rising pulse (R) and falling pulse (F) combination and obtained the logic states as stated in Fig. 2 and Table 1. The clock implemented triggers at 5 ns.

4.2 Determination of the Logic States

Logic ‘A’ states the polarization of the tQCA as ‘+1.00’ and is determined by calculating the potential differences $V(5 \sim \text{five})$ and $V(8 \sim \text{eight})$ with respect to the clock inputs. It is seen that $V(5 \sim \text{five}) > V(8 \sim \text{eight})$ stating the probability of the free electrons to be living at dot D-5 (see Fig. 4). When $V(5 \sim \text{five}) < V(8 \sim \text{eight})$, the electrons are likely to be residing at dot D-8, stating the possibility of logic ‘B’ which gives the polarization ‘-1.00’. We report of seeing two unknown logic states

Fig. 4 SPICE simulation curve stating logic ‘A’



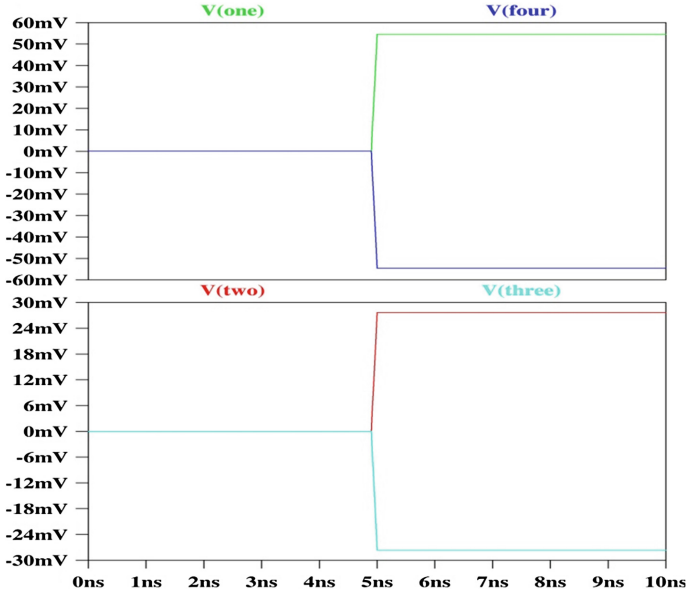


Fig. 5 Voltage curve stating logic ‘C’ or ‘D’

namely ‘C’ and ‘D’ on alternation of the clocks 1 and 2 as talked in Table 1. When clock-1 is high and clock-2 is low, simulation shows that $V(1 \sim \text{one})$ and $V(2 \sim \text{two})$ is greater than $V(3 \sim \text{three})$ and $V(4 \sim \text{four})$, depicting free electron can live either in D-1 or D-2 shown in Fig. 5 and again on changing the clock orientation, we see $V(1 \sim \text{one})$ and $V(2 \sim \text{two})$ less than $V(3 \sim \text{three})$ and $V(4 \sim \text{four})$, depicting free electron can live either in D-3 or D-4. The neutral state is determined on noticing same voltage level at every diagonal dot site (i.e., D-1, D-4, D-5, and D-8).

5 Conclusions

The proposed tQCA has proved its presence in this work. Our work has enabled the path to estimate the performance of tQCA at an extensive level. The approach of SPICE analysis shows that tQCA operates in the milli-volts range ($\sim 0\text{--}60$ mV), proving its possibility to be used for low power applications in near future. There is no power leakage in tQCA, as in tQCA electronic charge is confined in closed dimension and is also advantageous for static power dissipation since physical movement of electron is not seen here. The excellent steady state behavior of tQCA talks in favor of its ability to operate at room temperature. It will be feasible for fabrication and surely be cost effective in production.

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