On Multigranular Approximate Rough Equivalence of Sets and Approximate Reasoning

B.K. Tripathy, Prateek Saraf and S.Ch. Parida

Abstract As the notion of equality in mathematics is too stringent and less applicable in real life situations, Novotny and Pawlak introduced approximate equalities through rough sets. Three more types of such equalities were introduced by Tripathy et al. as further generalisations of these equalities. As rough set introduced by Pawlak is unigranular from the granular computing point of view, two types of multigranulations rough sets called the optimistic and the pessimistic multigranular rough sets have been introduced. Three of the above approximate equalities were extended to the multigranular context by Tripathy et al. recently. In this paper, we extend the last but the most general of these approximate equalities to the multigranular context. We establish several direct and replacement properties of this type of approximate equalities. Also, we illustrate the properties as well as provide counter examples by taking a real life example.

Keywords Rough sets · Approximate equalities · Approximate equivalence · Optimistic multigranulation · Pessimistic multigranulation · Replacement properties

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1 Introduction

The notion of equality of sets used in mathematics is not of much use in real life situations because of its stringent definition, which can only be used when the two sets have the same elements. Moreover, it does not use the knowledge of the observer regarding the domain while considering the equality. In real life situations we use user knowledge as a supporting tool, which also determines the equality or otherwise of sets under consideration. In an attempt to achieve this Novotny and Pawlak [1–3] introduced three types of equalities of sets through rough sets, where the equivalence relation plays the deciding role. Since human knowledge according to Pawlak is determined through their classification capability, which is dependent upon classification of universes and in turn is equivalent to equivalence relations defined over the domain, this definition while being more general than the mathematical equality, takes care of the human knowledge, making it more natural. This early notion of approximate equality was not considered further until Tripathy et al. [4, 5] introduced another such notion called the rough equivalence, which was proved later to be the most general of this kind of equalities and also is free from the notion of mathematical equality. In fact, two more types of approximate equalities using rough sets were introduced by Tripathy [6] in 2011, by the way completing the four types of possible approximate equalities using rough sets. The notion of rough sets introduced by Pawlak [7, 8] is unigranular from the granular computing point of view in the sense that it considers only one equivalence relation at a time. Extending this notion the concept of optimistic multigranular rough sets was introduced by Qian and Liang [9] in 2006. Later on they defined another type of multigranular rough sets called the pessimistic multigranular rough sets in 2010 [10]. Three of the four types of approximate equalities were extended to the setting of multigranular rough sets by Tripathy and Mitra [5, 11, 12] very recently. In this paper we extend the last but the most important type of approximate equality that is we define multigranular rough equivalence of sets and establish their properties. Also, we prove the replacement properties. In this paper we provide two diagrams which provide a comparative analysis of the unigranular rough set notions of lower and upper approximation and the multigranular upper and lower approximations for both the optimistic and pessimistic multigranular rough sets. We use a real life database to illustrate the concepts of the paper and also provide counter examples wherever required using this example.

2 Definitions and Notations

In this section we provide some of the definitions and notations to be used. First we start with the basic rough sets in the next section.

2.1 Basic Rough Sets

Let *U* be a universe of discourse and *R* be an equivalence relation over *U*. By *U/R* we denote the family of all equivalence class of *R*, referred to as categories or concepts of *R* and the equivalence class of an element $x \in U$ is denoted by $[x]_R$. By a knowledge base, we understand a relational system *K* = (*U*, *R*), where *U* is as above and *R* is a family of equivalence relations over *U*. For any subset $P(\neq \phi) \subseteq R$, the intersection of all equivalence relations in *P* is denoted by IND (P) and is called the indiscernibility relation over *P*. Given any $X \subseteq U$ and $R \in \mathrm{IND}(K)$, we associate two subsets, $\underline{R}X = \cup\{Y \in U/R : Y \subseteq X\}$ and $\bar{R}X = \cup\{Y \in U/R : Y \cap X \neq \phi\}$, called the R-lower and R-upper approximations of X respectively.

The R-boundary of *X* is denoted by $BN_R(X)$ and is given by $BN_R(X) = \bar{R}X - \underline{R}X$. The elements of $\underline{R}X$ are those elements of *U*, which can certainly be classified as elements of X, and the elements of $\bar{R}X$ are those elements of U, which can possibly be classified as elements of X, employing knowledge of *R*. We say that *X* is rough with respect to *R* if and only if $\underline{R}X \neq \bar{R}X$, equivalently $BN_R(X) \neq \phi$. *X* is said to be R-definable otherwise.

2.2 Multigranular Rough Sets

We introduce the two types of multigranulations in this direction using the notations in recent papers by Tripathy et al. [13–15] followed by some properties of these multigranulations.

Definition 2.2.1 Let K = (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X \subseteq U$ and $R, S \in \mathbf{R}$. We define [9] the optimistic multi-granular lower approximation and optimistic multi-granular upper approximation of X with respect to R and S in U as

$$\underline{R+S}\,X = \{x \mid [x]_R \subseteq X \text{ or } [x]_S \subseteq X\} \tag{2.2.1}$$

$$\overline{R+S}\,X = \sim(\underline{R+S}(\sim X)). \tag{2.2.2}$$

Definition 2.2.2 Let K = (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X \subseteq U$ and $R, S \in \mathbf{R}$. We define [10] the pessimistic multi-granular lower approximation and pessimistic multi-granular upper approximation of X with respect to R and S in U as

$$\underline{R*S}\,X = \{x \mid [x]_R \subseteq X \text{ and } [x]_S \subseteq X\}, \tag{2.2.3}$$

$$\overline{R*S}\,X = \sim(\underline{R*S}(\sim X)). \tag{2.2.4}$$

2.2.1 Properties of Multigranular Approximations

We present some properties of multigranular rough sets, which shall be used in the proofs of the results of this paper [14, 15].

$$\underline{R+S}(X \cap Y) \subseteq \underline{R+S}(X) \cap \underline{R+S}(Y) \tag{2.2.5}$$

$$\underline{R+S}(X \cup Y) \supseteq \underline{R+S}(X) \cup \underline{R+S}(Y) \tag{2.2.6}$$

$$\overline{R+S}(X \cap Y) \subseteq \overline{R+S}(X) \cap \overline{R+S}(Y) \tag{2.2.7}$$

$$\overline{R+S}(X \cup Y) \supseteq \overline{R+S}(X) \cup \overline{R+S}(Y) \tag{2.2.8}$$

$$\underline{R*S}(X \cup Y) \supseteq \underline{R*S}(X) \cup \underline{R*S}(Y) \tag{2.2.9}$$

$$\overline{R*S}(X \cap Y) \subseteq \overline{R*S}(X) \cap \overline{R*S}(Y) \tag{2.2.10}$$

$$\underline{R*S}(X \cap Y) = \underline{R*S}(X) \cap \underline{R*S}(Y) \tag{2.2.11}$$

$$\overline{R*S}(X \cup Y) = \overline{R*S}(X) \cup \overline{R*S}(Y) \tag{2.2.12}$$

We would like to note some cases when equalities hold in (2.2.8) and (2.2.10). These results will be helpful to us in establishing some properties later.

Lemma 2.2.3.1 *If* $\overline{R+S}(X) = \overline{R+S}(Y)$ *then equality holds in* (2.2.8).

Proof It is easy to see from definition (2.2.2) that

$$\overline{R+S}(X) = \{x : [x]_R \cap X \neq \phi \text{ or } [\mathrm{x}]_S \cap X \neq \phi\}$$

and

$$\overline{R+S}(\mathrm{Y}) = \{x : [x]_R \cap Y \neq \phi \text{ or } [\mathrm{x}]_S \cap Y \neq \phi\}.$$

Also,

$$\overline{R+S}(\mathrm{X} \cup \mathrm{Y}) = \{x : [x]_R \cap (X \cup Y) \neq \phi \text{ or } [\mathrm{x}]_S \cap (X \cup Y) \neq \phi\}.$$

$$So,\ x \in \overline{R+S}(X \cup Y) \Rightarrow \{[x]_R \cap X \neq \phi \text{ or } [x]_R \cap Y \neq \phi \text{ or } [x]_S \cap X \neq \phi \text{ or} [x]_S \cap Y \neq \phi\}$$

By our assumption this implies that

$$\{[x]_R \cap X \neq \phi \text{ or } [x]_R \cap X \neq \phi \text{ or } [x]_S \cap X \neq \phi \text{ or } [x]_S \cap X \neq \phi\}$$

So, $x \in \overline{R+S}(X)$ and hence $x \in \overline{R+S}(\mathrm{Y})$. This completes the proof. □

Lemma 2.2.3.2 *If* $\overline{R*S}(X) = \overline{R*S}(Y)$ *then equality holds in* (2.2.10).

Proof It is easy to see from definition (2.2.4) that

$$\overline{R*S}(X) = \{x : [x]_R \cap X \neq \phi \text{ and } [\mathrm{x}]_S \cap X \neq \phi\}$$

$$\overline{R*S}(\mathrm{Y}) = \{x : [x]_R \cap Y \neq \phi \text{ and } [\mathrm{x}]_S \cap Y \neq \phi\}.$$

Also,

$$\overline{R*S}(\mathrm{X} \cap \mathrm{Y}) = \{x : [x]_R \cap (X \cap Y) \neq \phi \text{ and } [\mathrm{x}]_S \cap (X \cap Y) \neq \phi\}.$$

So, $x \in \overline{R*S}(X) \cap \overline{R*S}(\mathrm{Y}) \Rightarrow x \in \{y : [\mathrm{y}]_R \cap X \neq \phi \text{ and } [\mathrm{y}]_S \cap X \neq \phi\}$ *and* $\mathrm{x} \in \{\mathrm{y} : [\mathrm{y}]_R \cap Y \neq \phi \text{ and } [\mathrm{y}]_S \cap Y \neq \phi\}$

By our assumption this implies that

$$x \in \{y : [\mathrm{y}]_R \cap X \neq \phi \text{ and } [\mathrm{y}]_R \cap Y \neq \phi\} \text{ and}$$
$$\mathrm{x} \in \{\mathrm{y} : [\mathrm{y}]_R \cap X \neq \phi \text{ and } [\mathrm{y}]_S \cap Y \neq \phi\}$$

So, $[x]_R \cap (X \cap Y) \neq \phi$ *and* $[x]_S \cap (X \cap Y) \neq \phi$. Hence, $x \in \overline{R*S}(X \cap Y)$. This completes the proof. □

3 Approximate Multigranular Rough Equivalences

In this section we introduce the notions of approximate rough equivalences and study their properties. First, we define the two types of approximate multigranular rough equivalence below.

Definition 3.1 Let R and S be two equivalence relations on U and X, $Y \subseteq U$. Then

(3.1) X and Y are pessimistic bottom multigranular approximate rough equivalent to each other with respect to R and S (X b_R*S_aeqv Y) if *and* only if $\underline{R*S}X$ and $\underline{R*S}Y$ are ϕ or not ϕ together.

(3.2) X and Y are pessimistic top multigranular approximate rough equivalent to each other with respect to R and S (X t_R*S_aeqv Y) if and only if $\overline{R*S}X = \overline{R*S}Y$.

(3.3) X and Y are pessimistic multigranular approximate rough equivalent to each other with respect to R and S (X R*S_aeqv Y) if and only if $\underline{R*S}X$ *and* $\underline{R*S}Y$ are ϕ or not ϕ together and $\overline{R*S}X = \overline{R*S}Y$.

Definition 3.2 Let R and S be two equivalence relations on U and X, $Y \subseteq U$. Then

(3.4) X and Y are optimistic bottom multigranular approximate rough equivalent with respect to R and S (X b_R + S_aeqv Y) if and only if $\underline{R+S}X$ and $\underline{R+S}Y$ are ϕ or not ϕ together .

(3.5) X and Y are optimistic top multigranular approximate rough equivalent with respect to R and S (X t_R + S_aeqv Y) if and only if $\overline{R+S}X = \overline{R+S}Y$

(3.6) X and Y are optimistic multigranular approximate rough equivalent to each other with respect to R and S (X R + S_aeqv Y) if and only if $\underline{R+S}X$ and $\underline{R+S}Y$ are ϕ *or not* ϕ together and $\overline{R+S}X = \overline{R+S}Y$.

It may be noted here that we do not specify the optimistic or pessimistic case specifically as it is clear from the context. We will use Table 1 to prove the properties in sections to follow.

Table 1 Faculty database

S.no.	Name	Division	Grade	Top degree
1.	Sam	Network	Assistant professor	MCA
2.	Ram	Information system	Professor	PhD
3.	Shyam	Software engineering	Assistant professor (junior)	M.Sc.
4.	Peter	Artificial intelligence	Associate professor	PhD
5.	Roger	Embedded system	Professor	PhD
6.	Albert	Artificial intelligence	Assistant professor (Junior)	M.Sc.
7.	Mishra	Embedded system	Assistant professor (junior)	M.Sc.
8.	Hari	Information systems	Senior professor	PhD
9.	John	Software engineering	Assistant professor	MCA
10.	Smith	Network	Associate professor	PhD
11.	Linz	Artificial intelligence	Senior professor	PhD
12.	Keny	Software engineering	Professor	PhD
13.	Williams	Embedded systems	Associate professor	PhD
14.	Martin	Information systems	Assistant professor (junior)	M.Sc.
15.	Jacob	Network	Assistant professor (junior)	M.Sc.
16.	Lakman	Software engineering	Associate professor	PhD
17.	Sita	Artificial intelligence	Assistant professor	PhD
18.	Fatima	Embedded systems	Assistant professor	M.Tech
19.	Biswas	Information systems	Senior professor	M.Tech
20.	Pretha	Software engineering	Senior professor	PhD

3.1 Properties of Optimistic Multigranular Approximate Rough Equalities

In this section, we shall deal with the properties of optimistic multigranular approximate equivalence of rough sets. First, we establish some basic properties in the next subsection. Taking Table 1 into consideration, we have

> U = {Sam, Ram, Shyam … Pretha}. Also, the three attributes "Division", "Grade" and "Top Degree" induce three equivalences relations. The three equivalence classes are as given below.
> U/Division = {{Sam, Smith, Jacob}, {Shyam, John, Keny, Lakman, Pretha}, {Peter, Albert, Linz, Sita}, {Roger, Mishra, Williams, Fatima}, {Ram, Hari, Martin, Biswas}}.
> U/Grade = {{Shyam, Albert, Mishra, Martin, Jacob}, {Sam, John, Sita, Fatima}, {Peter, Smith, Williams, Lakman}, {Ram, Roger, Keny}, {Linz, Biswas, Pretha, Hari}}.
> U/Top Degree = {{Shyam, Albert, Mishra, Martin, Jacob}, {Sam John}, {Sita, Fatima}, {Ram, Peter, Roger, Hari, Smith, Keny, Linz, Williams, Lakman, Biswas, Pretha}}.

3.1.1 Basic Properties

3.1.1.1. X b_R + S_aeqv Y if $X \cap Y$ b_R + S_aeqv X and $X \cap Y$ b_R + S_aeqv Y. The converse may not be true.

Proof The first part follows directly from the definition of optimistic multigranular approximate bottom rough equivalence. The converse follows from the following example. We refer to Table 1, Let X = {Jacob, John, Peter, Albert, Linz, Sita} and Y = {Jacob, John, Sam, Sita, Fatima}. Then $X \cap Y$ = {Jacob, John, Sita}. Hence, $\underline{R+S}$(X) = {Peter, Albert, Linz, Sita} $\neq \phi$, $\underline{R+S}$(Y) = {Sam, John, Sita, Fatima} $\neq \phi$ and $\underline{R+S}$ $(X \cap Y) = \phi$. So, although X and Y are b_R + S_aeqv, none of them is b_R + S_aeqv to $X \cap Y$. □

3.1.1.2. X t_R + S_aeqv Y if $X \cup Y$ t_R + S_aeqv X and $X \cup Y$ t_R + S_aeqv Y. The converse is also true.

Proof If part follows directly from definition of optimistic multigranular approximate top rough equivalence. Conversely, X t_R + S_aeqv Y then $\overline{R+S}(\mathrm{X}) = \overline{R+S}(\mathrm{Y})$. Then by (2.2.8), $\overline{R+S}(X \cup Y) \supseteq \overline{R+S}(X) = \overline{R+S}(\mathrm{Y})$. The converse follows from Lemma 2.2.3.1. □

3.1.1.3. If X t_R + S_aeqv X' and Y t_R + S_aeqv Y' then we have $X \cup Y$ t_R + S_aeqv $X' \cup Y'$.

Proof The proof follows from Lemma 2.2.3.1. □

3.1.1.4. X b_R + S_aeqv X' and Y b_R + S_aeqv Y' may not imply that $X \cap Y$ b_R + S_aeqv $X' \cap Y'$.

Proof We provide one example to show this. Referring to Table 1, let us take X = {Ram, Hari, Martin, Biswas, Peter}, Y = {Peter, Smith, Williams, Lakman}, X' = {Peter, Albert, Linz, Sita, Biswas, Pretha, Hari} and Y' = {Linz, Biswas, Pretha, Hari}. Then $\underline{R+S}$ (X) = {Ram, Hari, Martin, Biswas} and $\underline{R+S}(X')$ = {Peter, Albert, Linz, Sita, Biswas, Pretha, Hari} are not ϕ together. So, X b_R + S_aeqv X'. Again, $\underline{R+S}$ (Y) = {Peter, Smith, Williams, Lakman} and $\underline{R+S}(Y')$ = {Linz, Biswas, Pretha, Hari} are not ϕ together. So, Y t_R + S_aeqvY', whereas $X \cap Y$ = {Peter}. So, $\underline{R+S}$ $(X \cap Y) = \phi$. Again, $X' \cap Y'$ = {Linz, Biswas, Pretha, Hari}. So, $\underline{R+S}$ $(X' \cap Y')$ = {Linz, Biswas, Pretha, Hari} $\neq \phi$. Hence $X \cap Y$ b_R + S_aeqv $X' \cap Y'$ is not true. □

3.1.1.5. If X t_R + S_aeqv Y then X ∪ ∼Y t_R + S_aeqv U.

Proof We have by hypothesis, $\overline{R+S}(X) = \overline{R+S}(Y)$. So, $\overline{R+S}(X \cup \sim Y) \supseteq \overline{R+S}(X) \cup \overline{R+S}(\sim Y) \supseteq \overline{R+S}(Y) \cup \overline{R+S}(\sim Y) = \overline{R+S}(Y) \cup (\sim \underline{R+S}(Y)) \supseteq \overline{R+S}(Y) \cup \sim BN_{R+S}(Y) \cup \sim \overline{R+S}(Y) \supseteq \overline{R+S}(Y) \cup \sim \overline{R+S}(Y) = U$. So, $\overline{R+S}$ (X ∪ ∼Y) = U. This completes the proof. □

3.1.1.6. if X b_R + S_aeqv Y then we may not have X ∩ ∼Y b_R + S_aeqv ϕ.

Proof An example can be provided as in the case of 3.1.1.4.

The proofs of the following two properties are obvious and hence omitted. □

3.1.1.7. If $X \subseteq Y$ and X t_R + S_aeqv U then Y t_R + S_aeqv U.
3.1.1.8. If $X \subseteq Y$ and Y t_R + S_aeqv ϕ then X t_R + S_aeqv ϕ.
3.1.1.9. If X t_R + S_aeqv Y then it is true that ∼X b_R + S_aeqv ∼Y

Proof By hypothesis, we have $\overline{R+S}(X) = \overline{R+S}(Y)$. So, $\sim\underline{R+S}(\sim X) = \sim\underline{R+S}(\sim Y)$ and hence $\underline{R+S}(\sim X) = \underline{R+S}(\sim Y)$. This implies that $\underline{R+S}(\sim X)$ and $\underline{R+S}(\sim Y)$ are ϕ or not ϕ together. This completes the proof. □

3.1.1.10. If X b_R + S_aeqv ϕ or Y b_R + S_aeqv ϕ then $X \cap Y$ b_R + S_aeqv ϕ.

Proof By hypothesis, we have $\underline{R+S}(X) = \phi$ or $\underline{R+S}(Y) = \phi$. In any case, $\underline{R+S}(X) \cap \underline{R+S}(Y) = \phi$. Thus by (2.2.5) $\underline{R+S}(X \cap Y) = \phi$. This completes the proof. □

3.1.1.11 If X t_R + S_aeqv U or Y t_R + S_aeqv U then $X \cup Y$ t_R + S_aeqv U.

Proof By hypothesis, $\overline{R+S}$(X) = U or $\overline{R+S}$(Y) = U. In any case, $\overline{R+S}(X) \cup \overline{R+S}(Y) = U$. Now, by (2.2.8), $\overline{R+S}$ (X ∪ Y) ⊇ $\overline{R+S}$ (X) ∪ $\overline{R+S}$ (Y) = U. Hence, the proof follows. □

3.1.2 Replacement Properties

We would like to note that in the properties below, we have avoided providing examples due to scarcity of space. These examples can be constructed as in the earlier cases.

3.1.2.1. X t_R + S_aeqv Y if $X \cap Y$ t_R + S_aeqv X and $X \cap Y$ t_R + S_aeqv Y. The converse may not be true.

Proof The first part follows directly from the definition of optimistic multigranular approximate top rough equivalence. To establish the second part we can provide an example. □

3.1.2.2. X b_R + S_aeqv Y if $X \cup Y$ b_R + S_aeqv X and $X \cup Y$ b_R + S_aeqv Y. The converse may not be true.

Proof The proof of the first part follows from the definition of bottom optimistic almost equivalence. For the second part we can provide an example. □

3.1.2.3. X b_R + S_aeqv X′ and Y b_R + S_aeqv Y′ may not imply that $X \cup Y$ b_R + S_aeqv X′ ∪ Y′.

Proof We can provide an example to establish our claim. □

3.1.2.4. X t_R + S_aeqv X' and Y t_R + S_aeqv Y' may not imply that $X \cap Y$ t_R + S_aeqv $X' \cap Y'$.

Proof An example can be constructed to establish our claim. □

3.1.2.5. If X b_R + S_aeqv Y then X ∪ ∼ Y may not be b_R + S_aeqv U.

Proof An example can be provided to establish our claim. □

3.1.2.6. If X t_R + S_aeqv Y then it may not be true that X ∩ ∼Y t_R + S_aeqv ϕ.

Proof We can provide an example in favour of our claim.

The next two properties follow directly from definition. □

3.1.2.7. If $X \subseteq Y$ and X b_R + S_aeqv U then Y b_R + S_aeqv U

3.1.2.8. If $X \subseteq Y$ and Y b_R + S_aeqv ϕ then X b_R + S_aeqv ϕ.

3.1.2.9. If X b_R + S_aeqv Y then it may not be true that ∼X t_R + S_aeqv ∼ Y.

Proof From hypothesis $\underline{R+S}$ (X) and $\underline{R+S}$ (Y) are ϕ or not ϕ together. So, $\sim\overline{R+S}$ ($\sim$X) and $\sim\overline{R+S}$ ($\sim$Y) are ϕ or not ϕ together. This implies that $\overline{R+S}$ ($\sim$X) and $\overline{R+S}$ ($\sim$Y) are U or not U together. When both are equal to U, there is no problem. They are equal. So the conclusion is true. This can be shown through an example. □

3.1.2.10. If X t_R + S_aeqv ϕ or Y t_R + S_aeqv ϕ then $X \cap Y$ t_R + S_aeqv ϕ.

Proof From the hypothesis, we have by definition, $\overline{R+S}$ (X) = ϕ or $\overline{R+S}$ (Y) = ϕ. Hence, $\overline{R+S}$ (X ∩ Y) ⊆ $\overline{R+S}$ (X) ∩ $\overline{R+S}$ (Y) ⊆ ϕ ∩ $\overline{R+S}$ (Y) or $\overline{R+S}$ (X) ∩ ϕ ⊆ ϕ. So, $\overline{R+S}$ (X ∩ Y) = ϕ. □

3.1.2.11. If X b_R + S_aeqv U or Y b_R + S_aeqv U then $X \cup Y$ may not be b_R + S_aeqv U.

Proof As $\underline{R+S}$ (U) = U, it follows from the hypothesis that $\underline{R+S}$ (X) and $\underline{R+S}$ (U) are not ϕ together or $\underline{R+S}$ (Y) and $\underline{R+S}$ (U) are not ϕ together. Now, $\underline{R+S}(X \cup Y) \supseteq \underline{R+S}X \cup \underline{R+S}Y = \phi$. This completes the proof. □

3.2 *Properties of Pessimistic Multigranular Approximate Rough Equivalences*

In this section, we shall deal with the properties of pessimistic multigranular approximate equivalence of rough sets. Due to shortage of space, we only state the properties below.

3.2.1 Basic Properties

3.2.1.1. X b_R*S_aeqv Y if $X \cap Y$ b_R*S_aeqv X and $X \cap Y$ b_R*S_aeqv Y. The converse may not be true.

3.2.1.2. X t_R*S_aeqv Y iff $X \cup Y$ t_R*S_aeqv Y and $X \cup Y$ t_R*S_aeqv Y.

3.2.1.3. If X t_R*S_aeqv X' and Y t_R*S_aeqv Y' then $X \cup Y$ t_R*S_aeqv$X' \cup Y'$.

3.2.1.4. X b_R*S_aeqv X' and Y b_R*S_aeqv Y' may not imply that $X \cap Y$ b_R*S_aeqv $X' \cap Y'$.

3.2.1.5. If X t_R*S_aeqv Y then X ∪ $\sim$ Y t_R*S_aeqv U.

3.2.1.6. If X b_R*S_aeqv Y then X ∩ $\sim$Y may not be b_R*S_aeqv ϕ.

3.2.1.7. If $X \subseteq Y$ and X t_R*S_aeqv U then Y t_R*S_aeqv U.

3.2.1.8. If $X \subseteq Y$ and Y t_R*S_aeqv ϕ then X t_R*S_aeqvϕ.

3.2.1.9. If X t_R*S_aeqv Y then it is true that $\sim$ X b_R*S_aeqv $\sim$ Y.

3.2.1.10. If X b_R*S_aeqv ϕ or Y b_R*S_aeqv ϕ then $X \cap Y$ b_R*S_aeqvϕ.

3.2.1.11. If X t_R*S_aeqv U or Y t_R*S_aeqv U then $X \cup Y$ t_R*S_aeqv U.

3.2.2 Replacement Properties

3.2.2.1. X t_R*S_aeqv Y if $X \cap Y$ t_R*S_aeqv X and $X \cap Y$ t_R*S_aeqv Y. The converse may not be true.

3.2.2.2. X b_R*S_aeqv Y if $X \cup Y$ b_R*S_aeqv X and $X \cup Y$ b_R*S_aeqv Y. The converse is also true.

3.2.2.3. X b_R*S_aeqv X' and Y b_R*S_aeqv Y' may not imply that $X \cup Y$ b_R*S_aeqv$X' \cup Y'$.

3.2.2.4. X t_R*S_aeqv X' and Y t_R*S_aeqv Y' may not imply that $X \cap Y$ t_R*S_aeqv $X' \cap Y'$.

3.2.2.5. If X b_R*S_aeqv Y then X ∪ $\sim$Y may not be b_R*S_aeqv U.

3.2.2.6. If X t_R*S_aeqv Y then X ∩ $\sim$Y may not be t_R*S_aeqv ϕ.

3.2.2.7. If $X \subseteq Y$ and X b_R*S_aeqv U then Y b_R*S_aeqv U

3.2.2.8. If $X \subseteq Y$ and Y b_R*S_aeqv ϕ then X b_R*S_aeqv ϕ.

3.2.2.9. If X b_R*S_aeqv Y then it may not be true that $\sim$X t_R*S_aeqv $\sim$ Y.

3.2.2.10. If X t_R*S_aeqv ϕ or Y t_R*S_aeqv ϕ then $X \cap Y$ t_R*S_aeqvϕ.

3.2.2.11. If X b_R*S_aeqv U or Y b_R*S_aeqv U then $X \cup Y$ may not be b_R*S_aeqv U.

4 Rough Equivalence Based Approximate Reasoning

As mentioned by Zadeh, approximate reasoning is viewed as a process of approximate solution of a system of relational assignment equations. We can consider the approximate equalities in this sense providing approximate reasoning. The usual practice is to generalize the modus ponens used in discrete mathematics for generation of rules. But here, we have used it in the first sense when we mention approximate reasoning.

5 Conclusions

In this paper the notion of multigranular rough equivalence of sets for both the optimistic and pessimistic multigranular rough sets are introduced. Several of their direct as well as replacement properties have been established. Two diagrams showing the comparison of the lower and upper approximations for unigranular rough sets and two types of multigranular rough sets have been presented. We have taken a real life database for the description of the concepts of this paper and also provided counter examples using this real life database.

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