

# Multi-objective Design Optimization of Three-Phase Induction Motor Using NSGA-II Algorithm

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**Abstract** The modeling of electrical machine is approached as a system optimization, more than a simple machine sizing. Hence wide variety of designs are available and the task of comparing the different options can be very difficult. A number of parameters are involved in the design optimization of the induction motor and the performance relationship between the parameters also is implicit. In this paper, a multi-objective problem is considered in which three phase squirrel cage induction motor (SCIM) has been designed subject to the efficiency and power density as objectives. The former is maximized where the latter is minimized simultaneously considering various constraints. Three single objective methods such as Tabu Search (TS), Simulated Annealing (SA) and Genetic Algorithm (GA) is used for comparing the Pareto solutions. Performance comparison of techniques is done by performing different numerical experiments. The result shows that NSGA-II outperforms other three for the considered test cases.

**Keywords** Multi-objective optimization • Induction motors • Multi-objective evolutionary algorithms • Single objective evolutionary algorithm

## 1 Introduction

Three-phase induction motors have been widely used in industrial applications. Over the past decade, there have been clear areas in motor utilization that demand higher power density and increased energy efficiency. In many industrial

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applications, motor size and inertia are critical. Motors with high power density can offer a performance advantage in applications such as paper machines. However, high-power density cannot compromise reliability and efficiency. In such multi-objective optimization (MO), it is impossible to obtain the solution with maximizing or minimizing all objectives simultaneously because of the trade off relation between the objectives. When the MO is applied to the practical design process, it is difficult to achieve an effective and robust optimal solution within an acceptable computation time. The solutions obtained are known as *Pareto-optimal* solutions or *non-dominated solutions*. The rest is called dominated solutions. There are several methods to solve MO problems and one method of them, Pareto optimal solutions are generally used for the balanced solutions between objectives.

Appelbaum proposed the method of “boundary search along active constrains” in 1987 [1]. Madescu proposed the nonlinear analytical iterative field-circuit model (AIM) in 1996 by Madescu et al. [2]. However, these techniques have many shortcomings to provide fast and accurate solution, particularly when the optimal solution to a problem has many variables and constraints. Thus, to deal with such difficulties efficient optimization strategies are required. This can be overcome by multi-objective optimization (MO) technique [3–7].

This paper aims at MO which incorporates NSGA-II algorithm for minimization of power density and maximization of efficiency of three phases SCIM using different nonlinear constrained optimization techniques [8–10]. The Pareto-optimization technique is used in order to solve the multi-objective optimization problem of electric motor drive in a parametric fashion. It results in a set of optimal solutions from which an appropriate compromise design can be chosen based on the preference of the designer. In addition to that various SOEA techniques such as Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithm (GA) is applied to compare among Pareto-optimal solutions [11]. Their performance has been evaluated by the metrics such as Delta, Convergence (C) and Spacing (S) through simulation studies.

## 2 Multi-objective Optimization Design

The general formulation of MOPs as [12]

Maximize/Minimize

$$f(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}), \dots, f_M(\vec{x})) \quad (1)$$

Subjected to constraints:

$$g_j(\vec{x}) \geq 0, \quad j = 1, 2, 3, \dots, J \quad (2)$$

$$h_k(\vec{x}) = 0, \quad k = 1, 2, 3, \dots, K \quad (3)$$

where  $\vec{x}$  represents a vector of decision variables  $\vec{x} = \{x_1, x_2, \dots, x_N\}^T$ .

The search space is limited by

$$x_i^L \leq x_i \leq x_i^U, \quad i = 1, 2, 3, \dots, N \quad (4)$$

$x_i^L$  and  $x_i^U$  represent the lower and upper acceptable values respectively for the variable  $x_i$ .  $N$  represents the number of decision variables and  $M$  represents the number of objective functions. Any solution vector  $\vec{u} = \{u_1, u_2, \dots, u_K\}^T$  is said to dominate over  $\vec{v} = \{v_1, v_2, \dots, v_k\}^T$  if and only if

$$\left. \begin{aligned} f_i(\vec{u}) &\leq f_i(\vec{v}) \quad \forall i \in \{1, 2, \dots, M\} \\ f_i(\vec{u}) &< f_i(\vec{v}) \quad \exists i \in \{1, 2, \dots, M\} \end{aligned} \right\} \quad (5)$$

Those solutions which are not dominated by other solutions for a given set are considered non-dominated solutions are called Pareto optimal solution.

The practical application of genetic algorithm to multi-objective optimization problem (MOP) involves various problems out of which NSGA-II [13, 14] algorithm has been implemented to find the Pareto-optimal solution between power density and efficiency.

### 3 Multi-objective Evolutionary Algorithm Frameworks

A majority of MOEAs in both the research and the application areas are Pareto-dominance based which are mostly the same frameworks as that of NSGA-II. In these algorithms a selection operator based on Pareto-domination and a reproduction operator is used. The operator of the MOEAs guides the population iteratively towards non-dominated regions by preserving the diversity to get the Pareto-optimal set. The evaluate operator leads to population convergence towards the efficient frontier and helps preserve the diversity of solutions along the efficient frontier. Both goals are achieved by assigning a rank and a density value to each solution. The MOEAs provide first priority to non-dominance and second priority to diversity. However, the methods by which they achieve these two fundamental goals differ. The main difference between the algorithms lies in their fitness assignment techniques. Coello et al. Classifies the constraints handling methods into five categories: (1) penalty functions (2) special representations and operators (3) repair algorithms (4) separate objective and constraints and (5) hybrid methods [15, 16].

### 4 Design Optimization of Induction Motor

In this paper the design of induction motor is formulated by MOEAs based on non-dominated sorting, NSGA-II which does not combine the two objectives to obtain the Pareto-optimal solution set. Here, the two objectives are taken individually and

an attempt is made to optimize both simultaneously. The main objective is to maximize efficiency ( $\eta$ ) and minimize power density ( $\zeta$ ). The proposed NSGA-II is suitably oriented in such a way as to optimize the two objectives. To express both the objectives in maximization form, the first objective  $\zeta$  is expressed as  $-\zeta$ . In addition to these objectives, different practical constraints mentioned are also considered. In order to design, the problem is expressed as Maximize  $\eta$  and  $-\zeta$  simultaneously considering all constraints [17, 18].

The sizing equation of an induction machine is

$$P_R(IM) = \frac{\sqrt{2}\pi^2}{2(1 + K_\phi)} K_\omega \eta \cos \phi_r B_g A \frac{f}{p} \lambda_0^2 D_0^2 L_e \quad (6)$$

In terms of efficiency ( $\eta$ ) can be written as

$$\eta = \frac{P_R(IM)2(1 + K_\phi)}{\sqrt{2}\pi^2 K_\omega \cos \phi_r B_g A \frac{f}{p} \lambda_0^2 D_0^2 L_e} \quad (7)$$

The power density of the induction machine is given by

$$\zeta(IM) = \frac{\sqrt{2}\pi^2}{2(1 + K_\phi)} K_\omega \eta \cos \phi_r B_g A \frac{f}{p} \lambda_0^2 \frac{L_g}{L_r} \quad (8)$$

where  $\cos \phi_r$  is the power factor which is related to the rated power  $P_R(IM)$ , the pole pairs  $p$  of the machine, and the converter frequency  $f$ . The design variables for induction motor are chosen as consisting of four flux densities at the teeth and yokes for the stator and rotor, one current density in stator winding and three geometric variables. Three geometric variables are the depth of stator slot, the ratio of the rotor slot bottom width of rotor tooth width and the ratio of rotor slot top radius of the rotor slot bottom radius [19, 20].

## 5 Performance Measure for Comparison

The final Pareto-optimal front obtained from different MOEAs techniques is compared using performance metrics such as Spacing (S), Diversity metric ( $\Delta$ ), Convergence metric (C) [17]. These performance metrics set the benchmark to compare the results and select the best outcomes.

## 6 Simulation Results

The 5 kW, 4-pole, three-phase squirrel-cage induction motor is chosen as a sample design. The rated frequency is 50 Hz and voltage is 170 V. Also, the ratio of maximum torque to nominal torque is set 2.5 as a constraint. Lower limit of

**Table 1** The performance evaluation metrics

Algorithm		SA	TS	GA	NSGA-II
S	Avg.	2.67E-5	9.73E-6	5.74E-6	4.21E-6
	Std.	6.72E-6	1.44E-6	1.43E-6	1.05E-6
Δ	Avg.	8.03E-1	7.86E-1	5.96E-1	5.53E-1
	Std.	2.08E-1	1.93E-1	1.48E-1	1.47E-1

**Table 2** The result obtained for C metric

	SA	TS	GA	NSGA-II
SA	–	0.3422	0.2192	0.1932
TS	0.4443	–	0.2653	
GA	0.5653	0.4922	–	0.25373
NSGA-II	0.5988	0.5911	0.3262	–

**Table 3** Comparison of CPU time (s)

Algorithms	NSGA-II		GA		TS		SA	
	Mean	Var	Mean	Var	Mean	Var	Mean	Var
CPU time	150	25	680	35	720	40	690	45

efficiency is 90 % and that of power density is 0.3 kW/kg. The population size is set to be 100. The algorithms stop after 20,000 function evaluations. Initial populations are generated by uniformly randomly sampling from the feasible search space. The uniform Crossover rate is taken 0.8. The mutation rate is 0.10 where it is taken as 1/n, i.e. n is 10, the number of decision variables.

Table 1 shows the S metric and Δ metric obtained using all four algorithms. Table 1 shows that the S and Δ metric value for NSGA-II is less than other three algorithms and hence its performance is better among all.

Table 2 shows the result obtained for Convergence (C) metrics. The values 0.5988 in the fourth row, first column means almost all solutions from final populations obtained by NSGA-II dominates the solutions obtained by SA. The values 0 in the first row, first column mean that no solution of the non-dominated population obtained by TS, GA and NSGA-II is dominated by solutions from final populations obtained by SA. From the result, it clear that the performance of NSGA-II significantly outperforms the competing algorithms in the considered optimal design of induction motor.

The comparison time computed by the CPU is shown in Table 3. The mean time and the variance (var) of time for NSGA-II algorithm is less than other algorithms. The Simulation statistics generated by the four algorithms NSGA-II, GA, TS, SA respectively are illustrated from Figs. 1, 2, 3 and 4. It is shown in Fig. 5 that NSGA-II results in wide areas of convergence and is diversified.

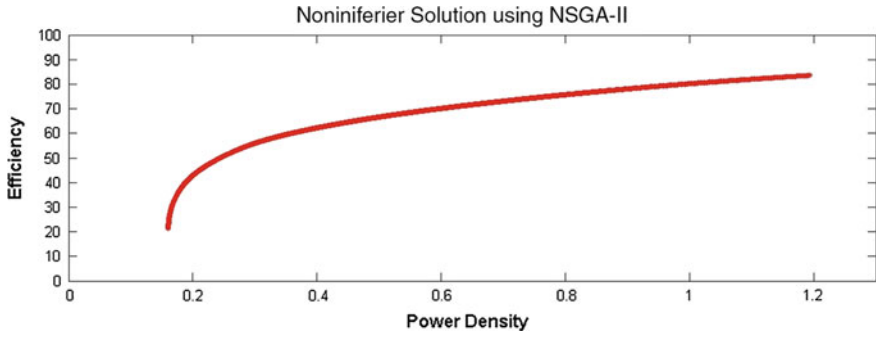


Fig. 1 Plots of Pareto fronts achieved by NSGA II

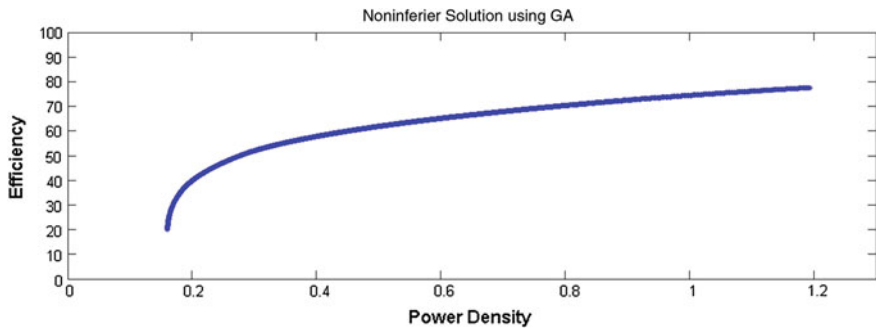


Fig. 2 Plots of Pareto fronts achieved by GA

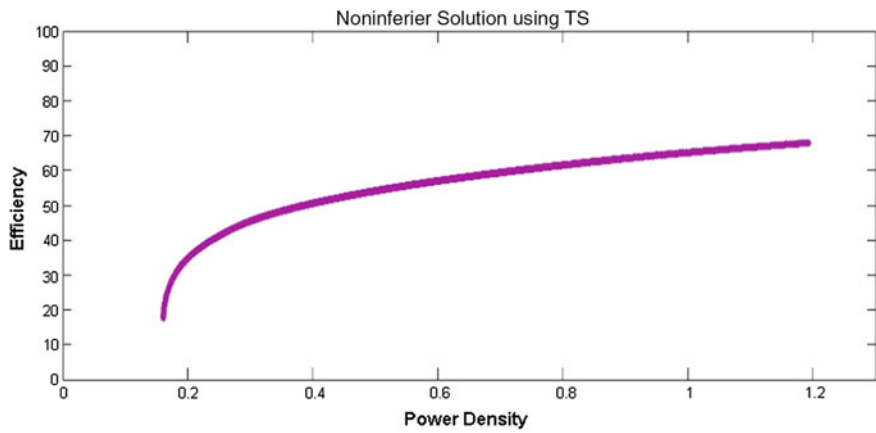


Fig. 3 Plots of Pareto fronts achieved by TS

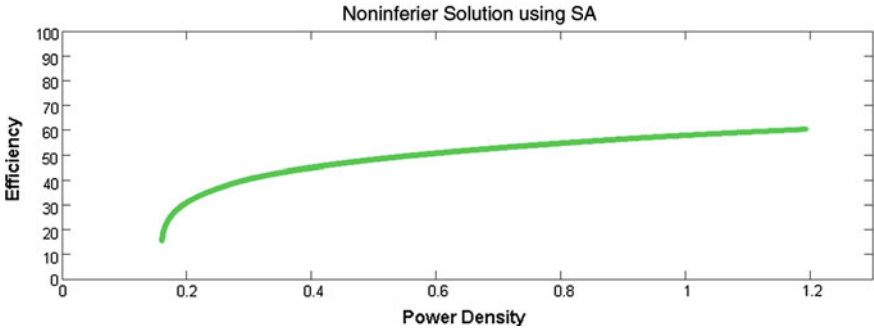


Fig. 4 Plots of Pareto fronts achieved by SA

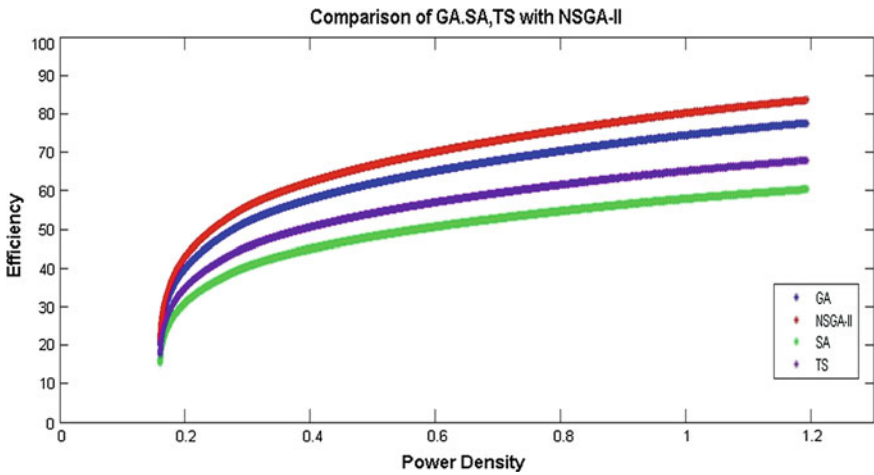


Fig. 5 Pareto front at different cardinality

## 7 Conclusion

In this paper, the multi-objective design optimization based on NSGA-II and size equations are applied for the three phase induction motors. In order to effectively obtain a set of Pareto optimal solutions, ranking method is applied. From the results, we can select the balanced optimal solution between the power density and efficiency. In case of optimized model, the efficiency increases at 80 % and the power density is also increased 12 kW/kg, compared to the SA, TS and GA result of the initial model. The performance metrics of NSGA-II results in best possible Pareto solutions. The proposed method can be efficiently and effectively used to multi-objectives design optimization of the machine cost and efficiency of electric machines.

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