Chapter 8
A Systems View of Pathological Tremors

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Abstract In this paper, we consider a specific case of movement disorders, i.e., resting tremors and attempt to formulate a simple mathematical description of these tremors. A novel aspect of the paper is that it is a first attempt at using standard tools from systems theory, such as state space and Lyapunov stability analysis, to model resting tremors. We formulate tremor control as a disturbance rejection problem, and derive conditions under which disturbance rejection is achievable.

Keywords Movement disorders · Tremors · State space model · Disturbance rejection

8.1 Introduction

Movement disorders often occur due to a defective central nervous system. One particular symptom of movement disorders is tremors, and there are 120 kinds of tremors categorized [1]. Tremors can have a debilitating affect on the normal life of subjects. A common reason for tremors is due to lack of sufficient dopamine produced in the substantia nigra, which affects the functioning of the motor cortex [2]. In this paper, we are interested in analyzing the behavior of one particular tremor, called the resting tremor. This is a tremor where the muscles are at rest and

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supported against gravity. Other types of tremor include kinetic tremor—which is caused due to goal-directed movements and has the same frequency as resting tremor.

The tremor waveform is roughly sinusoidal with characteristic frequency for each of the 120 kinds of tremors. Hence, frequency is important in tremor classification, and it is often used to characterize between different types of tremors, whereas the amplitude is not consistent between different tremors and is not used for characterization, e.g., the amplitude widely vary under controlled conditions since there are many factors (psychological, pathological, environmental, etc.) which can influence it [3].

Tremors are classified as physiological tremor, generally occurring in the 8–12 Hz frequency range and pathological tremor occurring in the 4–8 Hz frequency range; there is often a small overlap between the two frequency ranges. Physiological tremors are normal tremors which all humans have, and they do not interfere with motion. There are various conditions under which pathological tremors are generated. Often, underlying neurological (e.g., Parkinsonian) conditions can make a patient susceptible to tremors.

Currently, there exist few system theoretic formulations of tremors, especially from a control theoretic viewpoint. Notions of stability, disturbance rejection, reachability, and so on could find important use in tremor analysis and control. In this paper, we attempt to formulate resting tremors as a control problem, specifically in a disturbance rejection framework. Thus, we shall argue that subjects experiencing resting tremors have a poor disturbance rejection control mechanism.

8.2 Formal Definitions of Tremors

In this section, we will provide formal notions of resting tremors, where a tremor is essentially a function mapping an input signal space into an output signal space. Tremors are usually generated under the influence of certain events, and these event (trigger)-driven dynamics can be modeled from a system theoretic viewpoint. For this paper, let us consider a single body part—the arm for our study; without loss of generality. Consider two systems $\Sigma_{\text{phystremor}}$ and $\Sigma_{\text{pathtremor}}$. $\Sigma_{\text{phystremor}}$ is a system describing the dynamics[1] of an arm experiencing only physiological tremors but not resting tremors; $\Sigma_{\text{pathtremor}}$ is a system describing the dynamics of an arm experiencing pathological tremors (in our case: resting tremors).

The tremor signals themselves can be modeled as external disturbances acting on the arm, and we are essentially interested in studying the output behavior of the systems $\Sigma_{\text{phystremor}}$ and $\Sigma_{\text{pathtremor}}$ subject to the disturbances. Figure 8.1 shows a plant G—modeling the human arm—experiencing a disturbance d at the output, with a reference signal r. Such a system provides a simple starting model of resting

[1] We shall study these dynamics later.

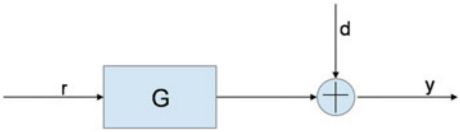

Fig. 8.1 Plant with external disturbance

tremors, where r can be considered to be zero since the arm (or system G) is at rest and supported against gravity. The disturbance d, in our case, is a sinusoidal signal and is defined as follows:

$$d := \{d_{\mathrm{lf}}, d_{\mathrm{hf}}\}, \text{ or any linear combination of } \{d_{\mathrm{lf}}, d_{\mathrm{hf}}\}$$

where d_{hf} is a high-frequency sinusoidal input with frequency in the range 8–12 Hz, and d_{lf} is a low-frequency sinusoidal input with frequency in the range 4–8 Hz.

The block diagram in Fig. 8.1 indicates that feedback is not present—though is not true for an actual human arm. Instead, we make a simplifying assumption that the plant G is modeled as an open-loop system with passive damping present, subject to external disturbances. A model of such a system is presented in the following section. Let us first formally define the physiological tremor system and the pathological tremor system.

Definition 1 (*Physiological Tremor System*) A physiological tremor system, denoted $\Sigma_{\mathrm{phystremor}}$, is formally defined by the maps

$$\Sigma_{\mathrm{phystremor}} : d_{\mathrm{hf}} \longrightarrow y_{\mathrm{hf}}, \quad \Sigma_{\mathrm{phystremor}} : d_{\mathrm{lf}} \longrightarrow 0$$

where y_{hf} is the output sinusoidal response corresponding to the sinusoidal disturbance signal d_{hf}; we define the output of $\Sigma_{\mathrm{phystremor}}$ to be zero when the input is d_{lf}. This definition simply says that the system $\Sigma_{\mathrm{phystremor}}$ is capable of rejecting the low-frequency disturbance signals and passes through the high-frequency disturbance signals.

Definition 2 (*Pathological Tremor System*) A pathological tremor system, denoted $\Sigma_{\mathrm{pathtremor}}$, is formally defined by the maps

$$\Sigma_{\mathrm{pathtremor}} : d_{\mathrm{hf}} \longrightarrow y_{\mathrm{hf}}, \quad \Sigma_{\mathrm{pathtremor}} : d_{\mathrm{lf}} \longrightarrow y_{\mathrm{lf}}$$

where y_{hf} is the output sinusoidal response corresponding to the sinusoidal disturbance signal d_{hf}; we define the output of $\Sigma_{\mathrm{phystremor}}$ to be another sinusoid y_{lf} when the input is d_{lf}. This definition says that the system $\Sigma_{\mathrm{pathtremor}}$ is unable to reject both the low- and high-frequency disturbance signals. The above two definitions provide a simple mathematical description of the two types of tremors. The following block diagram illustrates a simple systems approach to elucidate the above definitions. Figure 8.2 attempts to explain the onset of tremors under the influence of certain

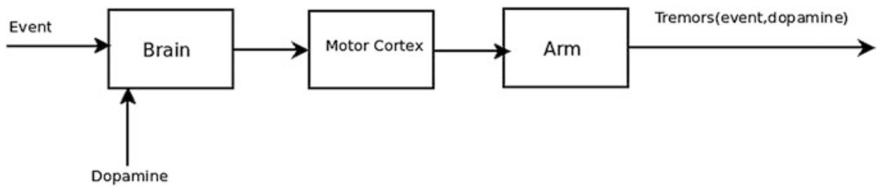

Fig. 8.2 An open-loop system representation of tremors on the human arm

events and the availability of dopamine in the brain. The figure is a massive simplification of a highly complex process but is sufficient for our purpose here. When certain events[2] occur (e.g., stress or sudden loss of dopamine producing neurons for various reasons) and if sufficient dopamine is not available in the brain[3] then abnormal synchronization activities, [4], occur in the brain and affect the functioning of the motor cortex and can induce pathological tremors, see [5, 6]. If sufficient dopamine is present, the motor cortex functions normal and pathological tremors are not observed. It is well known, [7], that Parkinson patients often rely on other signals, apart from proprioceptive feedback,[4] to control their motions during tremor. If we consider tremors in the arm, one possibility of control would be to essentially increase the damping of the arm (by stiffening the muscles) so as to reduce the tremors. In the absence of such sensory feedback, patients cannot control the tremors. Also, often the available sensory measurement is weak or faulty (e.g., in proprioception) and hence sufficient active damping is not produced to minimize the tremors.

8.3 Simple Dynamical Model of the Human Arm

In the previous section, we have provided formal definitions for systems modeling tremors—physiological and pathological. We defined tremor systems by functions mapping input spaces (signal spaces) into output spaces (signal spaces). In this section, we attempt to define simple models for these systems focusing on a specific part of the human anatomy (a single segment of the arm), and we use Lyapunov theory to show that pathological tremors essentially represent poor disturbance rejection. There exist many models to capture the behavior of the human arm [8, 9]. We adopt the simple mass spring damper (MSD) model [10, 11] for our study.

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[2] Note that events may be instantaneous or may build up overtime.

[3] Observe that we are not localizing the specific part of the brain here, e.g., the substantia nigra is affected in Parkinson's case. For the purposes of this paper, we will use the generic term *brain* instead of focussing on the specific part responsible.

[4] Proprioceptive feedback often degenerates during dopamine loss.

We also do not explicitly model the feedback for motion control of the arm; instead, we implicitly assume that the damping term in the MSD model is controlled appropriately[5] when sufficient dopamine is present and is poorly controlled in the absence of sufficient dopamine. Thus, we will argue that the available damping, for patients with loss of dopamine, is not sufficient to control the tremors.

The MSD system under the influence of an external force, u, is governed by the differential equation $m\ddot{q} + c\dot{q} + kq = u$. In the state space form, this can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u \tag{8.1}$$

where $x = [x_1\ x_2]^T; x_1 = q,\ x_2 = \dot{q}$; q is the measured displacement of the mass. Consider the unforced dynamics of this system, i.e., when $u = 0$. If we assume that $x = [0\ 0]^T$ is a stable equilibrium of the system (8.1), then there correspondingly exists a Lyapunov function, $V(x)$ which is positive definite and satisfying $\frac{\mathrm{d}V(x)}{\mathrm{d}t} \leq 0$. One of the standard choices of a Lyapunov function for the unforced ($u = 0$) MSD system is $V(x) = \frac{1}{2}kx_1^2 + \frac{1}{2}mx_2^2$. For the unforced MSD system, we then obtain $\dot{V}(x) = -cx_2^2$ which obviously satisfies $\dot{V}(x) \leq 0, \forall t$.

Remark 1 One could think of the unforced system as corresponding to the human arm, where there are no tremors generated (as sufficient dopamine is present in the brain) and thus we have the arm (modeled as a simple MSD system) at rest. In the absence of the tremors, we would expect the arm position to be stable; indeed, this is what we observe in the simple model used above.

8.3.1 Tremor Control as a Disturbance Rejection Problem

Pathological tremors are often classified as either resting or action [12]. Tremor is an involuntary and (often) rhythmic motion of a body part in a fixed plane and resting tremor occurs in a body part, [12], which is supported against gravity and is at rest. Thus, there is no *intentional* movement of the limb, and any observed dynamics is purely resting tremors. Resting tremors are most commonly caused by Parkinsonian disease and sometimes occur in severe essential tremors as well. We are specifically concerned with the tremors occurring in the 4–6 Hz range.

For the purpose of disturbance rejection, we can treat resting tremors as caused due to an external disturbance (a sinusoid) acting on the MSD system (8.1), also refer to Figs. 8.1 and 8.2. Thus, we have the following dynamics:

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[5] Through some complex feedback mechanism, not discussed here.

$$\Sigma := \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{A\sin(\omega t)}{m} \end{bmatrix} \tag{8.2}$$

Note here that the MSD system is completely supported against gravity, thus the only force acting on it is the external sinusoid (modeling the tremor signal). Our objective is to compute the lower bound on the active damping in the system (Σ) to be able to reject the (low frequency) disturbance.

Lemma 1 *The system, Σ, in (8.2) achieves disturbance rejection if* $A\sin(\omega t) \ll cx_2$.

Proof It is easily seen that if $A\sin(\omega t) \ll cx_2$ (strong damping force) then for any initial condition sufficiently close to the origin $x = [0\ 0]^T$, the resulting trajectory will also be sufficiently close to the origin (note that the chosen Lyapunov function has a global minima at the origin). However, since the output response of a linear system to an input sinusoid is again a sinusoid of varying amplitude and phase—the trajectory will be a sinusoid but will be in a neighborhood of the origin and has very small amplitude. □

Remark 2 Lemma 1 describes a system experiencing resting tremors if $A\sin(\omega t) > cx_2$. If $A\sin(\omega t) \ll cx_2$ then Lemma 1 describes a system where the resting tremors are negligible.

8.4 Conclusions

This paper attempts a system theoretic formulation of resting tremors, a type of movement disorder. We model the pathological tremors occurring in the 4–8 Hz range acts as a disturbance on the system, and this disturbance can be rejected provided sufficient active damping is present in the system. We derive bounds on the damping factor so as to reject this disturbance.

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