

Research of Dynamics and Deploying Control Method on Tethered Satellite

Lei Gang, Xian Yong, Feng Jie and Wang Kui

Abstract Deployment of a tethered satellite system is the process of separating the two end bodies by spooling out the tether connecting them. In this paper, a 3D dynamic model of a tethered satellite was established. Three typical deploying parameters were chosen and discussed according to some correlative references, thus obtaining the changing law for the percentage of deployment. The numerical results show the process of deployment is much sensitive with the in-plane angle, and the stable points are prior to others in long-distance deployment considering energy needed. These results could provide ideas about systemic design and engineering practice for tethered satellite.

Keywords Tethered satellite · Dynamic model · Deploying parameter · Deployment

1 Introduction

The concept of tethered satellite system was first proposed by Tsiolkovsky [1]—the “Father of Space” of Russian—in 1895 in his book “Day-Dreams of Earth and Heaven”. It has great prospect in the momentum exchange, conductive tether, artificial gravity generation, cargo space transportation, deep space exploration, spacecraft rendezvous, and capture, etc., [2, 3].

Deploying and reclaiming have been based on the issues of tethered satellites’ dynamics. People has carried out a large research on the complex dynamics model, the programs of deploying and reclaiming control, rendezvous, precise capture, dynamic stability control, etc., [4–7]. In the release areas of the tethered satellite,

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Williams and Trivailo [7] studied the problems of rendezvous and capture in 3D tether system and got the rendezvous where both accordant to position coincidence and tether release percentage is zero in radial within 30 days. Bibliography [8] studied the problem about sub-star's quick release in the close distance, analyzed the motion characteristics of tether and sub-star attitude. To the question of tethered satellite parameters for selection, a system dynamics model is established in this paper. Considering relevant literature, the typical values of deploying parameters were analyzed. By numerical simulation, it analyzed the effect of the initial parameters on the length of the deployed tether. The conclusion shows that stable points are prior to others in long-distance deployment.

2 A System Dynamics Model

In Fig. 1, $EX_EY_EZ_E$, $CX_oY_oZ_o$ and $CX_sY_sZ_s$ separately represent the geocentric inertial coordinate system, orbit coordinate system, and satellite coordinates. Of which, E is the center of mass for the Earth; C is the tethered system mass; A and B are the main star and sub-star, respectively; θ is the in-plane angles (pitch angle); and φ is the out-of-angle (roll angle). Both ends of the tethered satellite are regarded as particle, ignoring tether's mass. Therefore, system dynamics equation is:

$$\frac{F_A}{m_A} - \frac{F_B}{m_B} = a_{A/E} - a_{B/E} \quad (1)$$

Assuming the main star and sub-star are only affected by the force of gravity and the tension of tether, Eq. (1) can be written as:

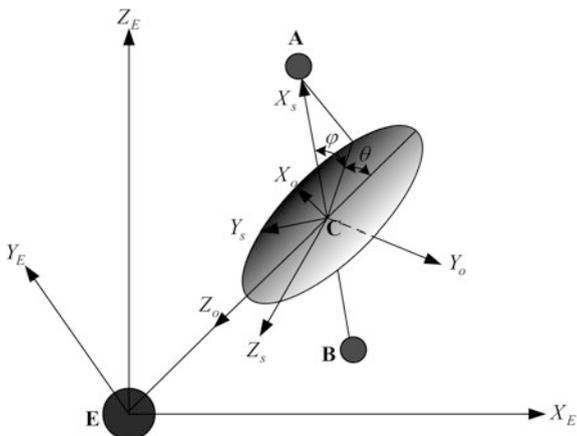
$$\begin{aligned} \frac{F_A}{m_A} - \frac{F_B}{m_B} = & \left(\frac{-GM}{|r_{A/E}|^3} r_{A/E} + \frac{T}{m_A} \frac{r_{C/A}}{|r_{C/A}|} \right) \\ & - \left(\frac{-GM}{|r_{B/E}|^3} r_{B/E} + \frac{T}{m_B} \frac{r_{C/B}}{|r_{C/B}|} \right). \end{aligned} \quad (2)$$

where $\frac{r_{C/A}}{|r_{C/A}|} = \frac{r_{C/B}}{|r_{C/B}|} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

In the satellite system:

$$\{r_{A/E}\}_s = \{r_{A/C}\}_s + \{r_{C/E}\}_s = \begin{bmatrix} L_a + R \cos \theta \cos \varphi \\ -R \sin \theta \\ -R \cos \theta \sin \varphi \end{bmatrix} \quad (3)$$

Fig. 1 The coordinate system of TSS



Then,

$$|r_{A/E}| = R \left(1 + \frac{L_a^2}{R^2} + \frac{2L_a}{R} \cos \theta \cos \varphi \right)^{1/2} \quad (4)$$

$L_a^2 \ll R^2$ when the tether length is much smaller than the orbital radius.

Ignoring higher-order items, second item launched of Eq. (4) to get:

$$|r_{A/E}|^{-3} \approx \frac{1}{R^3} \left(1 - 3 \frac{L_a}{R} \cos \theta \cos \varphi \right) \quad (5)$$

Similar to get:

$$\{r_{B/E}\}_s = \begin{bmatrix} -L_b + R \cos \theta \cos \varphi \\ -R \sin \theta \\ -R \cos \theta \sin \varphi \end{bmatrix} \quad (6)$$

$$|r_{B/E}|^{-3} \approx \frac{1}{R^3} \left(1 + 3 \frac{L_b}{R} \cos \theta \cos \varphi \right) \quad (7)$$

Setting $GM/R^3 = \Omega^2$, by (3)–(7), Eq. (2) can be written as:

$$\left\{ \frac{F_A}{m_A} - \frac{F_B}{m_B} \right\}_s = \begin{bmatrix} -\Omega^2 L + 3\Omega^2 L \cos^2 \theta \cos^2 \varphi - \frac{T}{m} \\ -3\Omega^2 L \cos \theta \sin \theta \cos \varphi \\ -3\Omega^2 L \cos^2 \theta \cos \varphi \sin \varphi \end{bmatrix} \quad (8)$$

m is the effective mass of the system, and $m = m_A m_B / (m_A + m_B)$.

Relatively inertial coordinate system, the angular percentage, and angular of acceleration satellite coordinate system is:

$$\omega_s^E = \begin{bmatrix} (\dot{\theta} + \Omega) \sin \varphi \\ -\dot{\varphi} \\ (\dot{\theta} + \Omega) \cos \varphi \end{bmatrix} \quad (9)$$

$$\dot{\omega}_s^E = \begin{bmatrix} \ddot{\theta} \cos \varphi + (\dot{\theta} + \Omega) \dot{\varphi} \cos \varphi \\ -\ddot{\varphi} \\ \ddot{\theta} \cos \varphi - (\dot{\theta} + \Omega) \dot{\varphi} \sin \varphi \end{bmatrix} \quad (10)$$

And the relationship of acceleration between inertial and satellite coordinate system is:

$$\begin{aligned} \{\ddot{r}_{A/B}\}_E &= \left\{ \ddot{r}_{A/B} \right\}_s + \dot{\omega}_s^E \times r_{A/B} \\ &+ 2\omega_s^E \times \{\dot{r}_{A/B}\}_s + \omega_s^E \times (\omega_s^E \times r_{A/B}) \end{aligned} \quad (11)$$

where

$$\{r_{A/B}\}_s = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}, \quad \{\dot{r}_{A/B}\}_s = \begin{bmatrix} \dot{L} \\ 0 \\ 0 \end{bmatrix}, \quad \{\ddot{r}_{A/B}\}_s = \begin{bmatrix} \ddot{L} \\ 0 \\ 0 \end{bmatrix}.$$

By (9)–(11), it can be simplified:

$$\{\ddot{r}_{A/B}\}_s = \begin{bmatrix} \ddot{L} - \dot{\varphi}^2 L - (\dot{\theta} + \Omega)^2 \cos^2 \varphi L \\ \ddot{\theta} \cos \varphi L - 2(\dot{\theta} + \Omega) \dot{\varphi} \sin \varphi + 2(\dot{\theta} + \Omega) \dot{L} \cos \varphi \\ \ddot{\varphi} L + 2\dot{\varphi} \dot{L} + (\dot{\theta} + \Omega)^2 \cos \varphi \sin \varphi L \end{bmatrix} \quad (12)$$

Contrast to Eq. (8), tethered system dynamics equation is:

$$\begin{cases} \ddot{L} = L[(\dot{\theta} + \Omega)^2 \cos^2 \varphi + \dot{\varphi}^2 + 3\Omega^2 \cos^2 \theta \cos^2 \varphi - \Omega^2] - \frac{T}{m} \\ \ddot{\theta} = 2(\dot{\theta} + \Omega) \dot{\varphi} \tan \varphi - 2\frac{\dot{L}}{L}(\dot{\theta} + \Omega) - 3\Omega^2 \cos \theta \sin \theta \\ \ddot{\varphi} = -2\frac{\dot{L}}{L} \dot{\varphi} - \left[\frac{(\dot{\theta} + \Omega)^2 + 3}{\Omega^2 \cos^2 \theta} \right] \cos \varphi \sin \varphi \end{cases} \quad (13)$$

3 Analyze the Values of System Parameters

This paper focuses on the released process of the sub-star in the different initial conditions. For this, it can select the typical system parameters as follows: the initial tethered tension T_0 , the system effective mass m , the initial separated velocity \dot{L}_0 , the initial tether angle θ_0 and φ_0 , track elevation H , and tether length

L_f . Among them, H and L_f may be regarded as objective parameters, the rest as the designed parameters. Values of the main parameters as follows:

1. The initial separated velocity \dot{L}_0

The initial velocity of tether can be gotten through ejection from the main star [8]. \dot{L}_0 is decided by the elasticity and compression degree of spring in the ejection institution. In 1993, SEDS-1 developed by NASA Marshall Space Flight Center in the USA, which was a small one-time-tethered expansion system, successfully carried out the long tether orbital flight tests, and the initial ejection speed was 1.64 m/s^9 . It is obvious that the capacity of catapult is limited by the device's narrow space. \dot{L}_0 was selected [3, 6] when simulated in Ref. [9].

2. The initial tension of tether T_0

By the release of the tethered reel device, as rope length is short, the gravity gradient force is negligible at the beginning. Therefore, the T_0 can be approximately equaled to the friction between the tether and reel device. The study about the reclaimed/deployed device showed that the friction is 50 mN approximately in room temperature [9]. Once deployed, the friction is 0, then the tension is related to the vibration cycle and the length of the tether. Taking into account the initial velocity of the tether is slow, focusing on the released process sensitivity to initial parameters, this article assumes that tension is constant in the process of entire release.

3. The effective mass of the system m

By Eq. (13), it shows that system dynamic model is directly related to the effective mass m . If the total mass of main star and the sub-star is constant, the m depends on the distribution of mass. This article assumes that total mass is 80 kg, the minimum mass of sub-star is 12 kg, and the m range 10.2–20 kg.

4 Simulation and Analysis

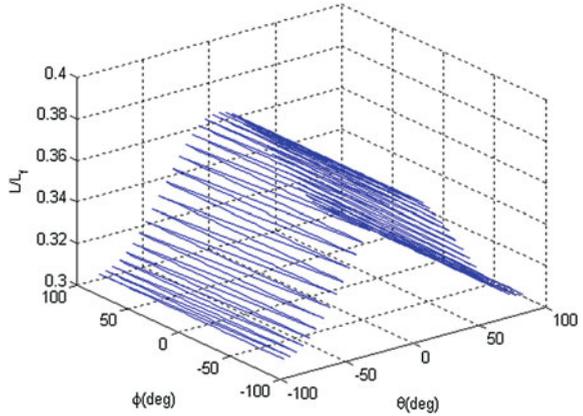
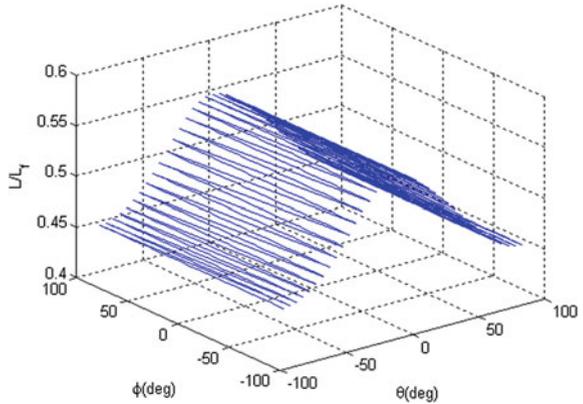
Simulation is based on assumptions that the tethered system in a circular orbit, with the conditions of $\dot{L} = 0$ or $L = L_f$, where L_f is scheduled deployed length of the tether. Table 1 shows the initial value of parameters.

The simulated results shown in Figs. 2 and 3.

Shown in Figs. 2 and 3, the extremum of tether's deployed percentage concentratedly present near the $\theta = 0$ resulted from the gravity gradient force, and the degree of increase gradually decreases with distance lengthen, which indicates that, compared to out-of-angle, the process of deployment is more sensitive to in-plane angles. When other conditions remain unchanged, the deployed percentage

Table 1 Initial value of parameters

T_0 (mN)	\dot{L}_0 (m/s)	m (kg)	θ_0 (deg)	φ_0 (deg)	H (km)	L_j (km)
50, 100	3, 4.5, 6	10.2, 15.1, 20	$-90 \sim 90$	$-90 \sim 90$	1,000	3

Fig. 2 Deployed percentage of tether when $T_0 = 50$ mN, $m = 10.2$ kg, $\dot{L}_0 = 3$ m/s**Fig. 3** Deployed percentage of tether when $T_0 = 100$ mN, $m = 10.2$ kg, $\dot{L}_0 = 3$ m/s

of tether is inversely proportional to T_0 and directly proportional to \dot{L}_0 and m . When sub-star is releasing, the stable equilibrium position is in radial direction. Therefore, the stable points are prior to others in long-distance deployment considering energy needed.

5 Conclusions

Based on the establishment of space tethered system dynamics model and analysis of typically released parameters, the relationship between the released parameters and the length of deployed tether was researched in this paper. The numerical results indicates that, compared to out-of-angle, the process of deployment is much sensitive to the in-plane angle and the stable points are prior to others in long-distance deployment considering energy needed. These results could provide ideas about systemic design and engineering practice for tethered satellite. With the in-depth study of tethered satellite complex models, deployment and recovery control, captured control, etc., the development prospects of tethered satellite systems will be broad.

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