

Job Block Scheduling with Dual Criteria and Sequence-Dependent Setup Time Involving Transportation Times

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Abstract A two-machine flowshop scheduling with sequence-dependent setup time (SDST), job block, and transportation time is considered with the objective of minimizing makespan and the rental cost of machines taken on rent under a specified rental policy. The processing time of attributes on these machines is associated with probabilities. To find the optimal or near-optimal solution for these objectives, a heuristic algorithm is developed. To test the efficiency of the proposed heuristic algorithm, a numerical illustration is given.

Keywords Scheduling · Makespan · Job block · Sequence-dependent setup time · Processing time · Transportation time · Rental cost · Utilization time

1 Introduction

Scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over a given time periods, and its goal is to optimize one or more objectives [19]. Many of the heuristics with objective as makespan given by [5, 14, 17, 18], assumes that the setup times are negligibly small or are included in the processing time, but in some applications, setup time has a major impact on the performance

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measure considered for scheduling problem. The term “sequence dependent” implies that the setup time depends on the sequence in which the jobs are processed on the machines, i.e., setup times depend on the type of job just completed as well as on the job to be processed [4]. A typical example is the manufacturing of different colors of paint, in which a cleaning operation time is needed, and is related to sequence of colors to be processed. Also, a setup changeover from black to white in painting industry takes longer duration than from white to black or dark gray. Scheduling with sequence-dependent setup time (SDST) has received significant attention in recent years. Corwin and Esogbue [6] minimized makespan by considering SDST. Gupta [12] proposed a branch-and-bound algorithm to minimize setup cost in n jobs and m machines flowshop with SDST. Rajendran and Ziegler [21] gave a heuristic for scheduling to minimize the sum of weighted flowtime of jobs in a flowshop with SDST of jobs. Rios and Bard [22] solved permutation flowshops with sequence-dependent setup times using branch and bound. Rabadi et al. [20] extended branch-and-bound algorithm for the early/tardy machine scheduling problem with a common due date and SDST. Tan and Narasimhan [23] minimized tardiness on a single processor with SDST using simulated annealing approach. A review of literature on scheduling research with various setup considerations has been presented by Allahverdi et al. [1]. Also, Allahverdi et al. [2] extended this survey up to the year 2006. Gajpal et al. [8] described an ant colony algorithm for scheduling in flowshop with SDST of jobs. Gupta and Smith [13] proposed two heuristics: a greedy randomized adaptive search procedure (GRASP) and a problem space-based local search heuristic. They showed that the space-based local search heuristic performs equally well when compared to ACO of Gagne et al. [7] while taking much less computational time. Gupta and Smith [13] also showed that GRASP gives much better solutions than ACO, while it takes much more computation time than ACO. The bicriteria scheduling problems are motivated by the fact that they are more meaningful from practical point of view. Some of the noteworthy heuristic approaches are due to Bagga and Bambani [3], Gupta et al. [9–11], Van Wassenhove and Gelders [24]. The idea of transportation has a practical significance when the material from one place of processing is to be carried to another place for further processing. Maggu and Das [15] introduced the concept of job block in flowshop scheduling. With job block, the importance lies in the fact that how to create a balance between a cost of providing priority in service and cost of providing service with non-priority customers, i.e., how much is to be charged extra from the priority customer(s) as compared to non-priority customer(s). Renting of machines is an affordable and quick solution for an industrial setup to complete assignment when one does not have resources. Renting enables saving working capital, gives option for having the equipment, and allows up gradation to new technology. The two criteria of minimizing the maximum makespan, and the utilization of machines or rental cost is the concern in this paper with the technological constraints as posed with job block and transportation times.

2 Assumptions, Notations, and Rental Policy

The proposed algorithm is based on the assumptions as discussed for flow shop scheduling in [4]. The following notations will be used all the way through this present paper:

S : Sequence of jobs 1, 2, 3... n ; S_p : Sequence obtained by applying Johnson’s procedure; M_k : Machine k , $k = 1, 2, \dots$ $a_{i,k}$: Processing time of i th attribute on machine M_k ; $p_{i,k}$: Probability associated with the processing time $a_{i,k}$; $A_{i,k}$: Expected processing time of i th attribute on machine M_k ; J_i : i th job, $i = 1, 2, 3, \dots n$; $S_{ij,k}$: Setup time if job i is processed immediately after job j on k th machine; $L_k(S_p)$: The latest time when machine M_k is taken on rent for sequence S_p ; $t_{ij,k}(S_p)$: Completion time of i th job processed immediately after j th job for sequence S_p on machine M_k ; $I_{i,k}(S_p)$: Idle time of machine M_k for job i in the sequence S_p ; $R(S_p)$: Total rental cost for the sequence S_p of the machines; C_i : Rental cost of i th machine; $t'_{ij,k}(S_p)$: Completion time of i th job processed immediately after j th job for sequence S_p on machine M_k when machine M_k starts processing jobs at time $L_k(S_p)$; $U_k(S_p)$: Utilization time for which machine M_k is required, when M_k starts processing jobs at time $L_k(S_p)$; β : Equivalent jobs for job block; $T_{i,k \rightarrow l}$: Transportation time required for i th job from machine k to machine l .

The machines will be taken on rent as and when they are required and are returned back as and when they are no longer required, i.e., the first machine will be taken on rent in the starting of the processing the jobs, second machine will be taken on rent at time when first job is completed on first machine, transported to second machine, and is in ready mode for processing on second machine.

3 Problem Formulation

Completion time of i th job processed immediately after j th job for sequence S on machine M_k is defined as $t_{ij,k} = \max(t_{(i-1)j,k}, t_{ij,k-1}) + A_{i,k} + S_{ij,k} + T_{i,(k-1) \rightarrow k}$; $k \geq 2$

Also, completion time if i th job processed immediately after j th job on machine M_k at latest time L_k is defined as

$$\begin{aligned}
 t'_{ij,k} &= L_k + \sum_{q=1}^i A_{q,k} + \sum_{r=1}^{i-1} S_{rj,k} = \sum_{q=1}^i I_{q,k} + \sum_{q=1}^i A_{q,k} + \sum_{r=1}^{i-1} S_{rj,k} \\
 &= \max(t_{(i-1)j,k}, t'_{ij,k-1}) + A_{i,k} + S_{ij,k},
 \end{aligned}$$

Let some job J_i ($i = 1, 2, \dots n$) are to be processed on two machines M_k ($k = 1, 2$) under the specified rental policy. Let there are n attributes of jobs on machine M_1 and m attributes of jobs on machine M_2 . The mathematical model of the problem in matrix form can be stated as (Table 1).

Table 1 Attributes of jobs

	Machine M_2						
	1	2	3	–	j	–	m
Machine M_1							
1	J_1	–	J_2		J_3	–	–
2	–	J_4	–	–	–	–	J_5
3	–	–	J_6	–	–	–	–
–	–	–	–	–	–	–	–
i	–	–	–	–	J_i	–	–
–	–	–	–	–	–	–	–
n	J_{n-1}	–	–	–	–	–	J_n

Each job is characterized by its first attribute (row) on the first machine and second attribute (column) on the second machine

The processing times for various attributes on machine M_1 and M_2 are shown in Table 2.

The setup times for various attributes on machine M_k ($k = 1, 2$) are shown in Table 3.

Mathematically, the problem can be stated as minimize $t_{nj,2}(S)$ and minimize $U_k(S)$ or $R(S_i) = t_{n,1} \times C_1 + U_k(S_i) \times C_2$, subject to constraint: Rental Policy (P).

Table 2 Processing time with probability

Attributes	Machine M_1		Machine M_2	
1	$a_{1,1}$	$p_{1,1}$	$a_{1,2}$	$p_{1,2}$
2	$a_{2,1}$	$p_{2,1}$	$a_{2,2}$	$p_{2,2}$
3	$a_{3,1}$	$p_{3,1}$	$a_{3,2}$	$p_{3,2}$
–	–	–	–	–
m	$a_{m,1}$	$p_{m,1}$	$a_{m,2}$	$p_{m,2}$
–	–	–	–	–
n	$a_{n,1}$	$p_{n,1}$	–	–

Table 3 Setup time on machine M_k

Attributes	1	2	3	–	j	–	n
1	–	$S_{12,k}$	$S_{13,k}$	–	$S_{1j,k}$	–	$S_{1n,k}$
2	$S_{21,k}$	–	$S_{23,k}$	–	$S_{2j,k}$	–	$S_{2n,k}$
3	$S_{31,k}$	$S_{32,k}$	–	–	$S_{3j,k}$	–	$S_{3n,k}$
–	–	–	–	–	–	–	–
i	$S_{i1,k}$	$S_{i2,k}$	$S_{i3,k}$	–	–	–	$S_{in,k}$
–	–	–	–	–	–	–	–
n	$S_{n1,k}$	$S_{n2,k}$	–	–	$S_{nj,k}$	–	–

The attribute in row i is processed immediately after the attribute in column j

4 Theorems

The following theorems support the finding of optimal sequence of jobs processing.

Theorem 4.1 Consider a flowshop problem consisting of two machines M_1 and M_2 and a set of n -jobs to be processed on these machines. Let $A_{i,1}$ and $A_{i,2}$ be the given processing time for each job i ($1 \leq i \leq n$) on machine M_1 and M_2 , respectively. Each machine can handle at most one job at a time and the processing of each job must be finished on machine M_1 before it can be processed on machine M_2 . It has been assumed that the order of treatments in the process M_1 and M_2 are same. Let $T_{i,1 \rightarrow 2}$ denote the transportation time of job i from machine M_1 to M_2 . In the transportation process, several jobs can be handled simultaneously. Let β be an equivalent job for a given ordered set of jobs, then processing times $A_{\beta,1}$ and $A_{\beta,2}$ on machines M_1 and M_2 are given by

$$A_{\beta,1} = (A_{k,1} + T_{k,1 \rightarrow 2}) + (A_{k+1,1} + T_{k+1,1 \rightarrow 2}) - \min\{A_{k,2} + T_{k,1 \rightarrow 2}, A_{k+1,1} + T_{k+1,1 \rightarrow 2}\}$$

$$A_{\beta,2} = (A_{k,2} + T_{k,1 \rightarrow 2}) + (A_{k+1,2} + T_{k+1,1 \rightarrow 2}) - \min\{A_{k,2} + T_{k,1 \rightarrow 2}, A_{k+1,1} + T_{k+1,1 \rightarrow 2}\}$$

and the transportation time of β from machines M_1 to M_2 is given by $T_\beta = 0$ as given by Maggu and Das [16].

Theorem 4.2 The processing of jobs on M_2 at time $L_2 = \sum_{i=1}^n I_{i,2}$ keeps $t_{nj,2}$ unaltered as given by [10].

5 Algorithm

The following proposed algorithm of bicriteria here can be referred to as $F_2/S_{sd}/R(S), C_{max}$.

- Step 1: Calculate the expected processing times of the given attributes .
- Step 2: Introduce two fictitious machines G and H with processing times G_i and H_i , respectively, defined as $G_i = A_{i,1} + T_{i,1 \rightarrow 2}$ and $H_i = A_{i,2} + T_{i,1 \rightarrow 2}$
- Step 3: Take equivalent job $\beta(k_1, m_1)$ and calculate the processing time $A_{\beta,1}$ and $A_{\beta,2}$ on the guide lines of Maggu and Das [15].
- Step 4: Define a new reduced problem with the processing times $A_{i,k}$ as defined in step 1 and jobs (k_j, m_j) are replaced by single equivalent job β with processing time $A_{\beta,k}(k = 1, 2)$ as defined in step 3.
- Step 5: Using Johnson’s technique [14], obtain the sequences S_p having minimum total elapsed time. Let these sequences be S_1, S_2, \dots .
- Step 6: Compute total elapsed time $t_{n,2}(S_p), p = 1, 2, 3, \dots$ of second machine by preparing in-out tables for sequence S_p .
- Step 7: Compute $L_2(S_p) = t_{n,2} - \sum_{i=1}^n A_{i,2} - \sum_{i=1}^{n-1} S_{ij,2}$ for each sequence S_p .

Step 8: Find utilization time of 2nd machine $U_2(S_p) = t_{n,2}(S_p) - L_2(S_p)$.

Step 9: Find minimum of $\{(U_2(S_p))\}; p = 1, 2, 3, \dots$

Let it be the sequence S_γ . Then, S_γ is the optimal sequence, and minimum rental cost for the sequence S_γ is $R(S_\gamma) = t_{n,1}(S_\gamma) \times C_1 + U_2(S_\gamma) \times C_2$.

6 Numerical Illustration

Consider a two-stage furniture production system where each stage represents a machine. Seven jobs are to be processed on each machine. At stage one, sheets as raw materials (having six attributes) are cut and subsequently painted in the second stage (having four attributes) according to the market demand. A setup changeover is needed in cutting department when the thickness of two successive jobs differs substantially. In the painting department, a setup is required when the color of two successive jobs changes. The setup times are sequence dependent. Further, the machines M_1 and M_2 are taken on rent under rental policy P. The attributes, processing times as well as setup times on the first and second machines are shown in Tables 4, 5, 6, and 7, respectively.

Let the jobs 3 and 5 are processed as a job block $\beta = (3, 5)$. The rental cost per unit for the machines M_1 and M_2 be 8 and 10 units, respectively. Our objective is to find the sequence of jobs processing with minimum possible rental cost, when the machines are taken on rent under the specified rental policy.

Solution: As per steps 1, the expected processing times of the jobs on two machines for the possible attributes with transportation time $T_{i,1 \rightarrow 2}$ is given in Table 8.

Using Johnson’s technique [14] as per the algorithm, the sequence S_γ having minimum total elapsed time is

$$S_\gamma = J_1 - J_\beta - J_2 - J_4 - J_6 - J_7 = J_1 - J_3 - J_5 - J_2 - J_4 - J_6 - J_7.$$

The in-out flow table of jobs for the sequence S_γ gives the total elapsed time $t_{n,2}(S_\gamma) = 37.5$ units. Hence, the utilization time of machine M_2 is $U_2(S_\gamma) = t_{n,2}(S_\gamma) - L_2(S_\gamma) = 37.5 - 2.8 = 34.7$ units.

Total Minimum Rental Cost = $R(S_\gamma) = t_{n,1}(S_\gamma) \times C_1 + U_2(S_\gamma) \times C_2 = 587.8$ units.

Now, the latest time at which machine M_2 should be taken on rent

$$L_2(S_\gamma) = t_{n,2}(S_\gamma) - \sum_{q=1}^n A_{q,2}(S_\gamma) - \sum_{j=1}^{n-1} S_{ij,2}(S_\gamma) = 37.5 - 20 - 9 = 8.5 \text{ units.}$$

The bi-objective in-out flow table for the sequence S_γ is given in Table 9.

Table 4 Attributes of jobs

	Machine M_2			
	1	2	3	4
Machine M_1				
1	-	J_3	-	-
2	J_1	-	J_6	-
3	-	J_5	-	-
4	J_2	-	-	-
5	-	-	J_4	-
6	-	-	-	J_7

Table 5 Processing times of attributes with probability

Attributes	Machine M_1		Machine M_2	
1	15	0.2	11	0.2
2	9	0.2	8	0.4
3	20	0.1	10	0.3
4	7	0.2	24	0.1
5	13	0.2		
6	25	0.1		

Table 6 Setup time on M_1

Attributes	Attributes					
	1	2	3	4	5	6
1	-	3	1	1	1	2
2	2	-	2	3	3	6
3	3	4	-	2	1	2
4	2	2	3	-	2	6
5	3	1	2	1	-	4
6	7	2	8	6	5	-

The attribute in row i is processed immediately after the attribute in column j

Table 7 Setup time on M_2

Attributes	Attributes			
	1	2	3	4
1	-	1	2	1
2	2	-	1	2
3	4	2	-	3
4	3	4	2	-

The attribute in row i is processed immediately after the attribute in column j

Table 8 Expected processing time

Jobs	J_1	J_2	J_3	J_4	J_5	J_6	J_7
M_1	1.8	1.4	3.0	2.6	2.0	1.8	2.5
$T_{i,1 \rightarrow 2}$	1	2	3	1	2	4	5
M_2	2.2	2.2	3.6	3.0	3.6	3.0	2.4

Table 9 Bi-objective in-out table

Jobs	Machine M_1 In-out	$T_{i,1 \rightarrow 2}$	Machine M_2 In-out
J_1	0.0–1.8	1	8.5–10.7
J_3	4.8–7.8	3	12.7–16.3
J_5	10.8–12.8	2	16.3–19.9
J_2	15.8–17.2	2	20.9–23.1
J_4	18.2–20.8	1	27.1–30.1
J_6	23.8–25.6	4	30.1–33.1
J_7	27.6–30.1	5	35.1–37.5

Therefore, the utilization time of machine M_2 is
 $U_2(S_\gamma) = t'_{n,2}(S_\gamma) - L_2(S_\gamma) = 37.5 - 8.5 = 29$ units.

Total Minimum Rental Cost = $R(S_\gamma) = t_{n,1}(S_\gamma) \times C_1 + U_2(S_\gamma) \times C_2$.

Hence, effective decrease in the rental cost of machines = $587.8 - 530.8 = 57.0$ units.

7 Conclusion

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S_r) = t_{n,2}(S_r) - \sum_{i=1}^n A_{i,2}(S_r) - \sum_{i=1}^{n-1} S_{ij,2}(S_r)$ on M_2 will reduce its utilization time. Therefore, total rental cost of M_2 will be minimum. Also, rental cost of M_1 will always be minimum as idle time of M_1 is minimum always due to our rental policy. The study may further be extending by introducing the concept of weightage of jobs, breakdown interval, etc.

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