Fuzzy VEISV Epidemic Propagation Modeling for Network Worm Attack

Muthukrishnan Senthil Kumar and C. Veeramani

Abstract An epidemic vulnerable—exposed—infectious—secured—vulnerable (VEISV) model for the fuzzy propagation of worms in computer network is formulated. In this paper, the comparison between classical basic reproduction number and fuzzy basic reproduction number is analyzed. Epidemic control strategies of worms in the computer network—low, medium, and high—are analyzed. Numerical illustration is provided to simulate and solve the set of equations.

Keywords Epidemic threshold · Reproduction number · Fuzzy logic · Worm propagation

1 Introduction

Recent myriad research contributions in Internet technology are considered to secure the worm attacks and to analyze the dynamics of worms in network. To study the dynamics of worms in network, a mathematical model is to be developed to analyze the propagation of worms. Worms behave like infectious diseases and are epidemic in nature. The propagation of worms throughout a network can be studied by using epidemiological models for disease propagation [1–4]. Using Kermack and McKendrick SIR classical epidemic model [5–7], dynamical models for malicious object propagation were proposed, providing estimations for temporal evolution of nodes depending on network parameters considering topological

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aspects of the network [1–4]. The similarity between the spread of a biological virus and malicious worm propagation motivates the researchers to adopt an epidemic model to the network environment [8]. Recent research in epidemic models such as SIR [8–10], SIS [8], SEIR [1, 11–14], SIRS [15, 16], SEIQV [17], and vulnerable—exposed—infectious—secured—vulnerable (VEISV) [18] is proposed to study the worm propagation by developing different transaction states based on the behavior of the virus or the worm.

In particular, epidemic systems in computer networks have strong nonlinearity and should be treated in a different way. The nonlinearity is due to the force of epidemic of an infectious agent. This intrinsically includes the fuzzy logic analysis. Fuzzy epidemic modeling for human infectious diseases has been studied in many research contributions [19–22]. Recently, Mishra and Pandey [23] proposed fuzzy epidemic model for the transmission of worms in computer network. This motivated us to consider the fuzzy epidemic model for VEISV propagation for network worm attack. Our model generalizes Mishra and Pandey [23] work.

2 Classical VEISV Epidemic Model

Toutonji et al. [18] proposed VEISV epidemic model for security countermeasures that have been used to prevent and defend against worm attacks. Thus, they used the state name *vulnerable* → *exposed* → *infectious* → *secured* → *vulnerable*. The parameters and notations used in this model are given with explanation in Table 1. The schematic representation of this model is shown in Fig. 1. The vulnerable state includes all hosts which are vulnerable to worm attack. Exposed state includes all hosts which are exposed to attack but not infectious due to the latent time requirement. Infectious state includes all hosts which were attacked and actively scanning and targeting new victims. Secured state includes all hosts which gained one or more security countermeasures, providing the host with a temporary or permanent immunity against the malicious worm. The following assumptions are considered:

1. The total number of hosts N is fixed and defined by

$$N = V(t) + E(t) + I(t) + S(t). \tag{1}$$

2. Initially, all hosts are vulnerable to attack. The total number of quarantined hosts, without considering the quarantine time, will move to the secure state after installing the required security patches or updates.
3. The number of replaced hosts is equal to the number of dysfunctional hosts, and the model is closed network defined as

Table 1 Notation and parameters for VEISV model

Notation	Explanation
$V(t)$	Number of vulnerable hosts at time t
$E(t)$	Number of exposed hosts at time t
$I(t)$	Number of infectious hosts at time t
$S(t)$	Number of secured hosts at time t
β	Contact rate
α	State transition rate from E to I
ψ_1	State transition rate from V to S
ψ_2	State transition rate from E to S
γ	State transition rate from I to S
ϕ	State transition rate from S to V
N	Total number of hosts
θ	Dysfunctional rate
μ_1	Replacement rate

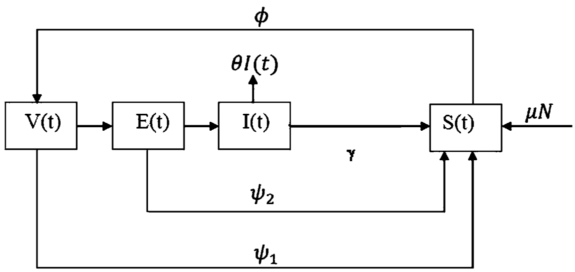

Fig. 1 VEISV model

$$\Gamma = \{(V, E, I, S) \in R_+^4 | V + E + I + S = N\}. \tag{2}$$

Since the number of hosts is large, we defined the incident of infection as $\beta \frac{V(t)}{N} I(t)$. β represents the number of incidents occurring in a unit of time. The transition of hosts from V state to E state in terms of Δt is

$$\Delta VE = \beta \frac{V(t)}{N} I(t) \Delta t. \tag{3}$$

Since $\alpha E(t)$ is the number of transitioning vulnerable hosts from time t to $(t + \Delta t)$ by the following equation:

$$V(t + \Delta t) - V(t) = -fE(t)V(t)\Delta t - \psi_1 V(t)\Delta t + \phi S(t). \tag{4}$$

We followed the mathematical approach [18] to derive the theoretical part. This set of differential equations governs the VEISV model:

$$
\begin{aligned}
\frac{\mathrm{d}V}{\mathrm{d}t} &= -fEV - \psi_1 V + \phi S, \\
\frac{\mathrm{d}E}{\mathrm{d}t} &= fEV - (\alpha + \psi_2)E, \\
\frac{\mathrm{d}I}{\mathrm{d}t} &= \alpha E - (\gamma + \theta)I, \\
\frac{\mathrm{d}S}{\mathrm{d}t} &= \mu_1 N + \psi_1 V + \psi_2 E + \gamma I - \phi S.
\end{aligned} \tag{5}
$$

3 Fuzzy VEISV Epidemic Model

During a worm attack, dysfunction occurred in infectious state; thereby, the hosts are taken over by a worm and are not capable of performing properly. The higher the worm load, the higher will be the chance of worm transmission. Let $\beta = \beta(x)$ measure the chance of a transmission to occur in a meeting between a vulnerable node and an exposed node with a large number of worms x. To obtain the membership function $\beta(x)$, we assume that the number of worms in a node is relatively low, that the chance of transmission is negligible, and that there are a minimum number of worms $x_{\min}$ needed to cause transmission. For certain number of worms x_M, the chance of transmission is maximum and equal to 1. Further, the number of worms in a node is always limited to $x_{\max}$. So the membership function of β (refer Fig. 2a) is defined as follows:

$$
\beta(x) = \begin{bmatrix} 0 & \text{if } x < x_{\min} \\ \frac{x - x_{\min}}{x_M - x_{\min}} & \text{if } x_{\min} < x < x_M \\ 1 & \text{if } x_M < x < x_{\max}. \end{bmatrix} \tag{6}
$$

To obtain the membership function of α, in latent period we assume that the number of worms exposed in a node is relatively low, that the chance of transmission is negligible, and that there are a minimum number of worms $x_{\min}$ needed

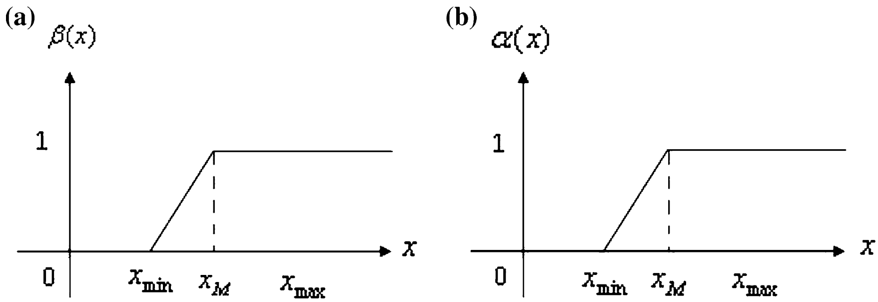

Fig. 2 **a** Membership function of β and **b** membership function of α

to cause transmission. For certain number of worms x_M, the chance of transmission is maximum and equal to 1. So the membership function of α (refer Fig. 2b) is defined as follows:

$$\alpha(x) = \begin{bmatrix} 0 & \text{if } x < x_{\min} \\ \frac{x - x_{\min}}{x_M - x_{\min}} & \text{if } x_{\min} < x < x_M \\ 1 & \text{if } x_M < x < x_{\max}. \end{bmatrix} \quad (7)$$

Now, the vulnerable node's recovery rate $\psi_1 = \psi_1(x)$ is also a function of worm load. The higher the worm load, the longer it will take to recover from infection; i.e., ψ_1 should be decreasing function of x. The membership function of ψ_1 (refer Fig. 3a) is as follows:

$$\psi_1(x) = \frac{\psi_{10} - 1}{x_{\max}} x + 1 \quad (8)$$

where ψ_{10} is the lowest recovery rate from vulnerable state to secured state. Also, the exposed node's recovery rate $\psi_2 = \psi_2(x)$ is also a function of worm load. The higher the worm load, the longer it will take to recover from infection; i.e., ψ_2 should be decreasing function of x. The membership function of ψ_2 (refer Fig. 3b) is as follows:

$$\psi_2(x) = \frac{\psi_{20} - 1}{x_{\max}} x + 1 \quad (9)$$

where ψ_{20} is the lowest recovery rate from exposed state to secured state. Furthermore, the infected node's recovery rate $\gamma = \gamma(x)$ is also a function of worm load. The higher the worm load, the longer it will take to recover from infection; i.e., γ should be decreasing function of x. The membership function of γ (refer Fig. 4a) is as follows:

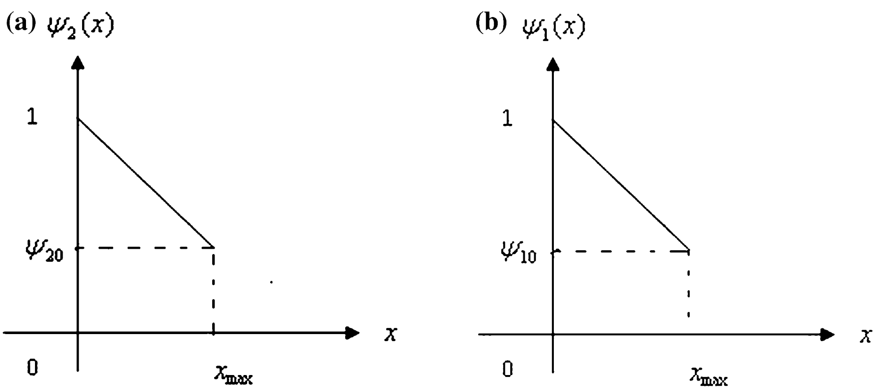

Fig. 3 **a** Membership function of ψ_1 and **b** membership function of ψ_2

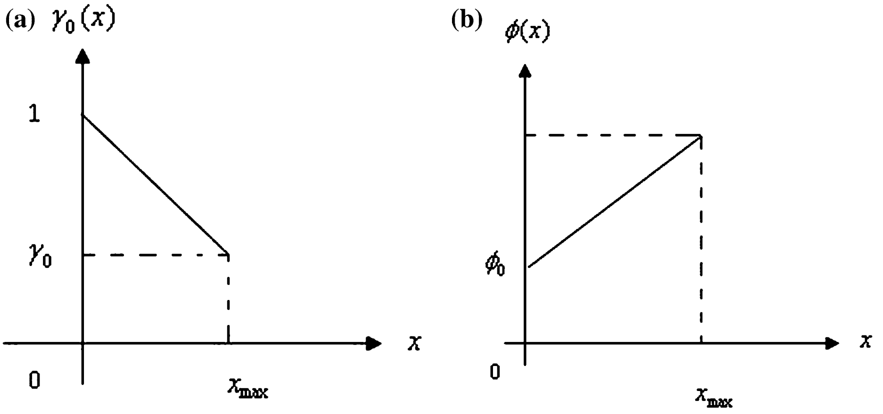

Fig. 4 **a** Membership function of γ and **b** membership function of ϕ

$$\gamma(x) = \frac{\gamma_0 - 1}{x_{\max}} x + 1 \tag{10}$$

where γ_0 is the lowest recovery rate from infected state to secured state. Here, ϕ is the rate of vulnerable after recovery, that is, the secured nodes may be vulnerable again. The higher we use the secondary devices and/or Internet services, the higher it will be vulnerable after recovery. The membership function of ϕ (refer Fig. 4b) will be increasing function of x. It is defined as follows:

$$\phi(x) = \frac{1 - \phi_0}{x_{\max}} x \tag{11}$$

where $\phi_0 > 0$ and (<1) is the lowest vulnerability after recovery. The number of worms differs in different nodes of the computer network. So we assume that x can be seen as fuzzy number, and hence, the membership function is defined as follows:

$$\rho(x) = \begin{bmatrix} 1 - \frac{|x - \overline{x}|}{\delta} & \text{if } x \in [\overline{x} - \delta, \overline{x} + \delta] \\ 0 & \text{otherwise} \end{bmatrix} \tag{12}$$

where $\overline{x}$ is a central value and δ gives the dispersion of each node of the fuzzy sets assumed by x. For a fixed $\overline{x}, \rho(x)$ can have a linguistic meaning such as low, medium, and high.

4 Stability Analysis of Fuzzy VEISV Model

Since $S(t) = N - V(t) - E(t) - I(t)$, we can use the reduction method by considering only the first three Eqs. of (5) to analyze the model

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \phi N - fEV - (\psi_1 + \phi)V - \phi(E + I),\\ \frac{\mathrm{d}E}{\mathrm{d}t} &= fEV - (\alpha + \psi_2)E,\\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \alpha E - (\gamma + \theta)I.\end{aligned} \tag{13}$$

Then, for the equilibrium points, we take $\frac{\mathrm{d}V}{\mathrm{d}t} = 0, \frac{\mathrm{d}E}{\mathrm{d}t} = 0,$ and $\frac{\mathrm{d}I}{\mathrm{d}t} = 0$. For $\frac{\mathrm{d}E}{\mathrm{d}t} = 0$, the equilibrium occurs at

$$E^* = 0 \text{ or } E^* > 0 \text{ and } v^* = \frac{\alpha + \psi_2}{\beta\alpha}N. \tag{14}$$

For $E^* = 0$, the worm-free equilibrium occurs at

$$\Pi_{\mathrm{wf}} = (v_1^*,\ E_1^*,\ I_1^*) = \left(\frac{\phi}{\psi_1 + \phi}N,\ 0,\ 0\right). \tag{15}$$

For $E^* > 0$, the worm-epidemic equilibrium is

$$\begin{aligned}\Pi_{\mathrm{we}} &= (v_2^*,\ E_2^*,\ I_2^*)\\ &= \left(\frac{\alpha + \psi_2}{\beta\alpha}N,\ \frac{\phi - \frac{(\alpha+\psi_2)}{\beta\alpha}(\psi_1 - \phi)}{\alpha + \psi_2 + \phi\left(1 + \frac{\alpha}{\gamma+\theta}\right)}N,\ \frac{\alpha}{\gamma + \theta}E_2^*\right).\end{aligned} \tag{16}$$

Now, taking into account worm load, we have

$$\Pi_{\mathrm{we}} = \left(\frac{\alpha(x) + \psi_2(x)}{\beta(x)\alpha(x)}N,\ \frac{\phi(x) - \frac{(\alpha(x)+\psi_2(x))}{\beta(x)\alpha(x)}(\psi_1(x) - \phi(x))}{\alpha(x) + \psi_2(x) + \phi(x)\left(1 + \frac{\alpha(x)}{\gamma(x)+\theta}\right)}N,\ \frac{\alpha(x)}{\gamma(x) + \theta}E_2^*\right). \tag{17}$$

As $\frac{f_1(x)N}{f_2(x)} < 1$, where $f_1(x) = \alpha(x)\phi(x) - \frac{\alpha(x)+\psi_2(x)}{\beta(x)}(\psi_1(x) - \phi(x))$ and $f_2(x) = (\gamma(x) + \theta)\left(\alpha(x) + \psi_2(x) + \phi(x)\left(1 + \frac{\alpha(x)}{\gamma(x)+\theta}\right)\right)$, so a value of bifurcation for x is x^*, the solution of the equation $f_1(x)N = f_2(x)$.

5 Basic Reproduction Number

The basic reproduction number (R_0) is obtained through the analysis of the stability of the trivial equilibrium point. For the classical VEISV propagation model, $R_0 = \frac{\alpha\beta\phi}{(\psi_1+\phi)(\alpha+\psi_2)}$. Based on the definition of R_0, the worm-free equilibrium is locally asymptotically stable when $R_0 \leq 1$ and unstable when $R_0 \geq 1$. As in this case, we taken $\alpha = \alpha(x)$, $\beta = \beta(x)$, $\phi = \phi(x)$, $\psi_1 = \psi_1(x)$, and $\psi_2 = \psi_2(x)$, then we write, $R_0(x) = \frac{\alpha(x)\beta(x)\phi(x)}{(\psi_1(x)+\phi(x))(\alpha(x)+\psi_2(x))}$.

We consider $\max R_0(x) < 1$ to control the worm transmission. But we take an average value of $R_0(x)$ because it can be an extreme attitude. For this, we consider the distribution of the worm load as given by a triangular fuzzy number $\rho(x)$. Then, fuzzy basic reproduction number is defined as follows:

$$R_0^f = \frac{1}{\gamma_0}\text{FEV}[\gamma_0 R_0(x)] \tag{18}$$

where FEV is fuzzy expected value. Suppose that $R_0(x) > 1$, but $\gamma_0 R_0(x) \leq 1$, so that the value of R_0^f is well defined. This is defined as the average number of secondary cases by just one infected node introduced into entirely susceptible nodes.

To obtain $\text{FEV}[\gamma_0 R_0(x)]$, we define a fuzzy measure μ and use the possibility measure:

$$\mu(A) = \sup_{x\in A}\rho(x), \quad A \subset R.$$

This measure shows that the infectivity of a group is the one presented by the node belonging to the group with the maximal infectivity. R_0^f is estimated by assuming that the number of worms classified as low, medium, and high. The fuzzy set is given by the membership function $\rho(x)$ for different cases:

(i) low, if $\bar{x} + \delta < x_{\min}$,
(ii) medium, if $\bar{x} - \delta > x_{\min}$ and $\bar{x} + \delta < x_M$ and
(iii) high, if $\bar{x} - \delta > x_M$.

Case (i): It is noted that $R_0^f < 1$ if x is low.

Case (ii): Since $R_0(x) = \frac{\alpha(x)\beta(x)\phi(x)}{(\psi_1(x)+\phi(x))(\alpha(x)+\psi_2(x))}$ is an increasing function of x, $H(\lambda) = [x', x_{\max}] = \sup_{x' \leq x \leq x_{\max}}\rho(x)$. Here $(\lambda) = \mu\{I(x,t) \geq \lambda\}$, $\text{FEV}[I(x,t)]$ is the fixed point of $H(\lambda)$ and x' is the solution of the equation $\gamma_0 \frac{\alpha(x)\beta(x)\phi(x)}{(\psi_1(x)+\phi(x))(\alpha(x)+\psi_2(x))} = \lambda$. Since the fixed point of $H(\lambda)$ is same as $\text{FEV}[\gamma_0 R_0^f(x)]$.
Hence,

$$H(\lambda) = \begin{cases} 0 & \text{if } 0 \leq \lambda \leq \gamma_0 \frac{\alpha(\bar{x})\beta(\bar{x})\phi(\bar{x})}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))} \\ \rho(x') & \text{if } \gamma_0 \frac{\alpha(\bar{x})\beta(\bar{x})\phi(\bar{x})}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))} \leq \lambda \\ & \leq \gamma_0 \frac{\alpha(\bar{x}+\delta)\beta(\bar{x}+\delta)\phi(\bar{x}+\delta)}{(\psi_1(\bar{x}+\delta)+\phi(\bar{x}+\delta))(\alpha(\bar{x}+\delta)+\psi_2(\bar{x}+\delta))} \\ 0 & \text{if } \gamma_0 \frac{\alpha(\bar{x}+\delta)\beta(\bar{x}+\delta)\phi(\bar{x}+\delta)}{(\psi_1(\bar{x}+\delta)+\phi(\bar{x}+\delta))(\alpha(\bar{x}+\delta)+\psi_2(\bar{x}+\delta))} < \lambda \leq 1. \end{cases} \tag{19}$$

For $\delta > 0$, H is a continuous and decreasing function, and in this case, $\mathrm{FEV}[\gamma_0 R_0(x)]$ is equal to fixed point of H. By direct manipulation, we have

$$\frac{\alpha(\bar{x})\beta(\bar{x})\phi(\bar{x})}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))} < \frac{\mathrm{FEV}[\gamma_0 R_0(x)]}{\gamma_0} < \frac{\alpha(\bar{x}+\delta)\beta(\bar{x}+\delta)\phi(\bar{x}+\delta)}{(\psi_1(\bar{x}+\delta)+\phi(\bar{x}+\delta))(\alpha(\bar{x}+\delta)+\psi_2(\bar{x}+\delta))}$$

So, $R_0(\bar{x}) < R_0^f < R_0(\bar{x}+\delta)$.
Case (iii): From the previous case, we have

$$\frac{1}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))} < R_0^f < \frac{1}{(\psi_1(\bar{x}+\delta)+\phi(\bar{x}+\delta))(\alpha(\bar{x}+\delta)+\psi_2(\bar{x}+\delta))}$$

It guarantees that the worms invade since $R_0^f > 1$.

6 Comparison Between R_0 and R_0^f

Here, we have analyzed the three cases discussed in the previous section related to the three classifications for the number of infections: low, medium, and high worm load. In any of the three cases, we have

$$\frac{\alpha(\bar{x})\beta(\bar{x})\phi(\bar{x})}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))} < \frac{\mathrm{FEV}[\gamma_0 R_0(x)]}{\gamma_0} < \frac{\alpha(\bar{x}+\delta)\beta(\bar{x}+\delta)\phi(\bar{x}+\delta)}{(\psi_1(\bar{x}+\delta)+\phi(\bar{x}+\delta))(\alpha(\bar{x}+\delta)+\psi_2(\bar{x}+\delta))}$$

i.e., $R_0(\bar{x}) < R_0^f < R_0(\bar{x}+\delta)$. Since the function $R_0 = \frac{\alpha(\bar{x})\beta(\bar{x})\phi(\bar{x})}{(\psi_1(\bar{x})+\phi(\bar{x}))(\alpha(\bar{x})+\psi_2(\bar{x}))}$ is continuous, by intermediate mean value theorem we have $\tilde{x}$ such that $\bar{x} < \tilde{x} < \bar{x}+\delta$. So,

$$R_0^f = R_0(\tilde{x}) > R_0(\bar{(x)}) \tag{20}$$

It means that R_0^f (fuzzy) and R_0 (classical) coincide if the number of infection is $\tilde{x}$.

7 Concluding Remarks

In summary, this paper describes an epidemic VEISV model for the fuzzy propagation of worms in computer network. The comparison between R_0 and R_0^f notices the importance of fuzzy logic in worm propagation. In future, fuzzy-based analysis of epidemic processes on large networks paves the way to more realistic models.

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