

# Thermo Hydrodynamic Lubrication Characteristics of Power Law Fluids in Rolling/Sliding Line Contact

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**Abstract** Hydrodynamic lubrication of rollers having the same dimension and moving with different velocities is studied assuming the consistency of the non-Newtonian incompressible power law lubricants to vary with pressure and the mean film temperature. The equations of motion, continuity, and momentum energy are solved first analytically and then numerically by Runge–Kutta Fehlberg method. Some important bearing characteristics are analyzed and displayed in the form of some graphs to study their behaviors.

## Nomenclature

H	Film thickness at $x = -x_1$
h	Lubricant film thickness
$h_o$	Minimum film thickness
$\bar{h}$	$h/h_o$ etc.
m	Lubricant consistency
$m_o$	Initial consistency temperature
n	Consistency index of the power law lubricant
p	Hydrodynamic pressure
R	Radius of the equivalent cylinder
T	Lubricant temperature
$T_{11}$	Film temperature for $y \geq \delta$ in region-I etc.
$T_m$	Mean film temperature
$T_0$	Ambient temperature
$\bar{T}_{Fh+}$	Traction force ( $= - (2\alpha T_{Fh}/h_o)$ ) etc.
$U_1, U_2$	Cylinders velocities at $y = -h$ and $y = h$ respectively
u	Velocity of the lubricant in x-direction

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$u_m$	The mean velocity of the lubricant
$v$	Velocity of the lubricant in $y$ -direction
$W$	Load in $y$ -direction
$\overline{W}$	Dimensionless load ( $= \alpha W / (Rh_o)^{1/2}$ )
$\bar{x}$	$x / (2Rh_o)^{1/2}$ etc.
$x_1$	Point of maximum pressure
$x_2$	Cavitation point
$\varphi$	$\frac{\rho c u_m}{k} \left( \frac{dT_m}{dx} \right)$
$\alpha, \beta$	Pressure and temperature coefficients

## 1 Introduction

Hydrodynamic bearings play vital roles in lubrication mechanism as successful performance of mechanical components such as gears, roller bearings, came etc. When these bearings run at heavy loads and high speeds, the lubricant material properties vary with pressure [1]. Further, high pressure generation causes viscous heating, so the temperature of the lubricant increases significantly along with pressure [2]. Hence the effect of temperature on the lubricant cannot be neglected [3].

Furthermore, lubrication of the line contact of two cylindrical rollers is assumed to be symmetric. More research work has not been done by ignoring the symmetric condition except a few where closed form or semi analytical solutions were found. Savage [4] gave a new direction to anti symmetric line contact problem and obtained a semi analytical solution for isoviscous fluid. Later, Prasad et al. [5] analyzed this problem while adding thermal effects and obtained a semi analytical solution. Tsong-Rong Lin [6] studied this problem in detail with thermal compressibility including EHD effects and obtained fully numerical solution of the problem.

As such the classical hydrodynamic lubrication of heavily loaded cylindrical line contact problems were studied by various researchers. One of the most famous contributors among them is Dowson and his associates. Dowson and Higginson [7] studied this problem where the viscosity and density of the lubricant were taken to be pressure and temperature dependent. Cheng [8] studied the problem with thermal effects and developed a refined solution for the problem. These results, particularly frictional forces, were in good agreement with the experimental results (Fig. 1).

In most of the problems, lubricant is assumed to be Newtonian. Since, the lubricant is subject to high pressure and heavy load, the Newtonian fluid characteristics ceases to exist [9] i.e. the lubricant becomes non-Newtonian. The non-Newtonian characteristics have been invariably served in various lubrication problems and this is due to the shear rate and the high pressure gradient, or due to

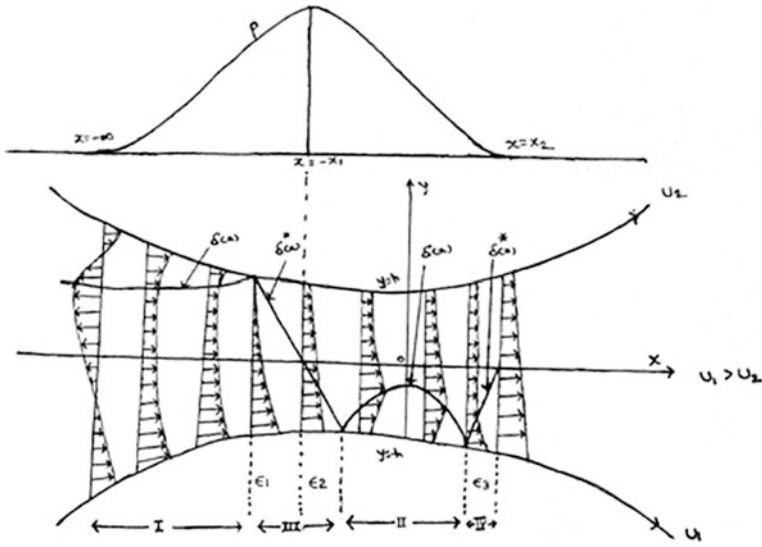


Fig. 1 Lubrication of asymmetric cylinders

the additives [10]. Hence the effect of non-Newtonian characteristic of the lubricant is also to be incorporated along with the effects of pressure and temperature. Hsiao and Hamrock [11] developed a numerical computational algorithm for non-Newtonian fluids and derived the temperature profile for this model lubricant throughout the region for pure rolling, rolling/sliding and pure sliding conditions. Punit and Khonsari [12] applied Carreau viscosity model for high viscosity oil to study the combined effects of shear thinning and temperature rise on the EHL behavior of rolling/sliding line contact problem. Later, Punit and Khonsari [13] exhibited their views on the role of the lubricant rheology and Piezo-viscous properties in EHL line/point contact where a comprehensive study of number of non-Newtonian fluid models have been proposed and applied for analyzing shear thinning and limiting shear stress behaviors.

The Power law non-Newtonian fluid model has got attention in the recent years because of their simplicity and potential to describe many lubricants like polymer solutions, silicon fluids etc. In fact, it characterizes two different types of non-Newtonian fluids i.e. viscoelastic and dilatants plus Newtonian as well when index  $n$  of the power law model is unity [14]. Recently, Punit et al. [15] dealt with a similar problem on isothermal EHL behavior of line contact using Newtonian fluid and power law fluid as base oil and additive oil respectively for the effect of polymeric fluid additives. Very recently, Hasim Khan et al. [16] described a computational algorithm for the solution of thermal EHL problem of infinite line contact and observed that the EHL of rough surfaces is significantly influenced by the consideration of thermal effects, and this scheme can also be extended to the analysis of two-dimensional thermal EHL problems such as finite line/point

contact problems. Dong Zhu [17] presented a latest review on history of EHL research, outlined the importance of mixed EHL and expressed some views about its further development as a gate way to interfacial mechanics.

### 1.1 Mathematical Formulation

The fluid flow governing equations of the hydrodynamic lubrication with some usual assumptions are [5]:

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \quad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

where

$$m = m_0 e^{\alpha p - \beta(T_m - T_0)} \quad (3)$$

$$T_m = \frac{1}{2h} \int_{-h}^h T \, dy \quad (4)$$

$$h = h_0 + x^2/2R. \quad (5)$$

$$\tau = m \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y}, \quad (6)$$

The boundary conditions for the problem are:

$$u = U_1 \text{ at } y = -h \quad (7)$$

$$u = U_2 \text{ at } y = h \quad (8)$$

$$p = 0 \text{ at } x = -\infty \quad (9)$$

$$p = 0 ; \quad \frac{dp}{dx} = 0 \text{ at } x = x_2 \quad (10)$$

The velocity gradient conditions for the geometry under consideration are:

$$\frac{\partial u_1}{\partial y} \geq 0, \quad \delta \leq y \leq h; \quad (11)$$

$$\frac{\partial u_2}{\partial y} \leq 0, \quad -h \leq y \leq \delta; \quad I: -\infty < x < -x_1; \quad (12)$$

$$\frac{\partial u_3}{\partial y} \geq 0, \quad -h \leq y \leq \delta \quad (13)$$

$$\frac{\partial u_4}{\partial y} \leq 0, \quad \delta \leq y \leq h; \quad \text{II} : -x_1 < x < x_2 \quad (14)$$

Now the volume flux Q for the region:

$$\begin{aligned} & -\infty < x < -x_1 \\ & Q = \int_{-h}^h u dy \\ & = U_1(h + \delta) + U_2(h - \delta) \\ & - \left( \frac{n}{2n+1} \right) \left( \frac{1}{m_1} \frac{dp_1}{dx} \right)^{1/n} \left[ (h + \delta)^{\frac{2n+1}{n}} + (h - \delta)^{\frac{2n+1}{n}} \right] \end{aligned} \quad (15)$$

The flux Q through the point  $x = -x_1$  can be evaluated directly from Eq. (1) yielding:

$$Q(-x_1) = (U_1 + U_2)H \quad (17)$$

where H is film thickness at  $x = -x_1$ .

Reynolds equation:

Equating the flux Eq. (16) and (17), and simplifying them, it can be obtained as

$$\frac{dp_1}{dx} = m_1 \left( \frac{2n+1}{n} \right)^n \left[ \frac{(U_1+U_2)(h-H) + (U_1-U_2)\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}} \right]^n, \quad (18)$$

$$-\infty < x \leq -x_1$$

$$\frac{dp_2}{dx} = -m_2 \left( \frac{2n+1}{n} \right)^n \left[ \frac{(U_1+U_2)(H-h) + (U_1-U_2)\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}} \right]^n, \quad (19)$$

$$-x_1 \leq x \leq x_2$$

Using  $u_1 = u_2$  and  $u_3 = u_4$  at  $y = \delta$  one can get

$$(U_1 - U_2) + \left( \frac{n}{n+1} \right) \left( \frac{1}{m_1} \frac{dp_1}{dx} \right)^{1/n} \left[ (h - \delta)^{\frac{n+1}{n}} - (h + \delta)^{\frac{n+1}{n}} \right] = 0, \quad (20)$$

$$-\infty < x \leq -x_1 - \epsilon_1$$

$$(U_1 - U_2) + \left( \frac{n}{n+1} \right) \left( \frac{1}{m_2} \frac{dp_2}{dx} \right)^{1/n} \left[ (h - \delta)^{\frac{n+1}{n}} - (h + \delta)^{\frac{n+1}{n}} \right] = 0, \quad (21)$$

$$-x_1 + \epsilon_2 \leq x \leq -x_2 - \epsilon_3$$

Eliminating  $\frac{dp_1}{dx}$  and  $\frac{dp_2}{dx}$  from Eq. (20) and (21) and using Reynolds Eq. (18) and (19), one can obtain a single relationship as:

$$\begin{aligned} & (U_1 - U_2) \\ & + \left( \frac{2n+1}{n+1} \right) \left[ \frac{(U_1 + U_2)(h-H) + (U_1 - U_2)\delta}{(h+\delta)^{\frac{2n+1}{n}} + (h-\delta)^{\frac{2n+1}{n}}} \delta \right] \left[ (h-\delta)^{\frac{n+1}{n}} - (h+\delta)^{\frac{n+1}{n}} \right] \\ & = 0 \end{aligned} \tag{22}$$

## 1.2 Heat Energy Equation

The heat energy equation with usual assumptions is considered to be [12, 18]:

$$\frac{\rho c}{k} + \left( u_m \frac{dT_m}{dx} \right) = \frac{\partial^2 T}{\partial y^2} \left( \frac{m}{k} \right) \left| \frac{\partial u}{\partial y} \right|^{n-1} \left( \frac{\partial u}{\partial y} \right)^2 \tag{23}$$

The boundary conditions for temperature:

$$T = T_{-h} \text{ at } y = -h \text{ and } T = T_h \text{ at } y = h \tag{24}$$

Now the lubricant film temperatures  $T_{11}$  and  $T_{12}$ , separated by  $\delta$ , are calculated for the region  $-\infty < x \leq -x_1$  as:

$$T_{11} = \varphi(x)y^2/2 + S_1(y-\delta)^{\frac{3n+1}{n}} + c_1y + d_1, \quad \delta \leq y \leq h \tag{25}$$

$$T_{12} = \varphi(x)y^2/2 + S_1(\delta-y)^{\frac{3n+1}{n}} + c_2y + d_2, \quad -h \leq y \leq \delta \tag{26}$$

Using the temperature matching condition Eliminating  $\frac{dp_1}{dx}$  and  $\frac{dp_2}{dx}$  from Eq. (20) and (21) and using Reynolds Eq. (18) and (19), one can obtain a single relationship as:

$T_{11} = T_{12}$  at  $y = \delta$  and the matching heat flux condition  $k \frac{\partial T_{11}}{\partial y} = k \frac{\partial T_{12}}{\partial y}$  at  $y = \delta$ , one may get

$c_1 = c_2 = c$  (say) and  $d_1 = d_2 = d$  (say). The boundary conditions (24) in (25) and (26) gives

$$c = \frac{T_h - T_{-h}}{2h} - \left( \frac{S_1}{2h} \right) \left[ (h-\delta)^{\frac{3n+1}{n}} - (h+\delta)^{\frac{3n+1}{n}} \right] \tag{27}$$

$$d = \frac{T_h + T_{-h}}{2} - \left( \frac{S_1}{2} \right) \left[ (h-\delta)^{\frac{3n+1}{n}} + (h+\delta)^{\frac{3n+1}{n}} \right] - \frac{h^2}{2} \varphi(x) \tag{28}$$

Similarly  $-x_1 \leq x \leq x_2$  in the region one can get

$$T_{21} = \varphi(x)y^2/2 + S_2 (\delta - y)^{\frac{3n+1}{n}} + ay + b \quad (29)$$

$$T_{22} = \varphi(x)y^2/2 + S_2 (y - \delta)^{\frac{3n+1}{n}} + ay + b \quad (30)$$

where

$$a = \frac{T_h - T_{-h}}{2h} - \left(\frac{S_2}{2h}\right) \left[ (h - \delta)^{\frac{3n+1}{n}} - (h + \delta)^{\frac{3n+1}{n}} \right] \quad (31)$$

$$b = \frac{T_h + T_{-h}}{2} - \left(\frac{S_2}{2}\right) \left[ (h - \delta)^{\frac{3n+1}{n}} + (h + \delta)^{\frac{3n+1}{n}} \right] - \frac{h^2}{2} \varphi(x) \quad (32)$$

Thus,  $T_{11}$ ,  $T_{12}$ ,  $T_{21}$ , and  $T_{22}$  are explicitly known functions of  $x$  and  $y$  analytically. Finally, the mean temperature  $T_{m1}$ , and  $T_{m2}$  as defined in Eq. (4), can be written as :

$$T_m = \frac{1}{2h} \int_{-h}^h T dy$$

or

$$T_{m1} = \varphi(x) \frac{h^2}{6} + \frac{nS_1}{2h(4n+1)} \left[ (h - \delta)^{\frac{4n+1}{n}} + (h + \delta)^{\frac{4n+1}{n}} \right] + d, \quad (33)$$

$-\infty < x \leq -x_1$

$$T_{m2} = \varphi(x) \frac{h^2}{6} + \frac{nS_2}{2h(4n+1)} \left[ (h - \delta)^{\frac{4n+1}{n}} + (h + \delta)^{\frac{4n+1}{n}} \right] + b, \quad (34)$$

$-x_1 \leq x \leq x_2$

where  $S_1 = \left(\frac{m_1}{k}\right) \left(\frac{1}{m_1} \frac{dp_1}{dx}\right)^{\frac{n+1}{n}} \left(\frac{-n^2}{(2n+1)(3n+1)}\right)$

$$S_2 = \left(\frac{m_2}{k}\right) \left(-\frac{1}{m_2} \frac{dp_2}{dx}\right)^{\frac{n+1}{n}} \left(\frac{-n^2}{(2n+1)(3n+1)}\right)$$

Now, using the following dimensionless scheme:

$$\bar{x} = \frac{x}{\sqrt{2Rh_0}}, \quad \bar{p} = \alpha p, \quad \bar{m} = 2mc_n \alpha,$$

$$c_n = \frac{1}{2} \left(\frac{2n+1}{n}\right)^n \left(\frac{U}{h_0}\right)^n \sqrt{\frac{2R}{h_0}}, \quad \bar{h} = 1 + \bar{x}^2 \quad (35)$$

$$\bar{T}_m = \beta T_m, \quad \bar{\delta} = \delta/h_0, \quad \bar{U} = U_1/U_2, \quad \bar{h} = h/h_0$$

Eq. (18), (19), (22), (25), (26), (29), (30), (33) and (34) can be rewritten as

$$\frac{d\bar{p}_1}{d\bar{x}} = \bar{m}_1 \overline{(f_x)^n} \quad (36)$$

$$\frac{d\bar{\phi}_2}{d\bar{x}} = -\bar{m}_2 \overline{(-f_x)^n} \quad (37)$$

$$(\bar{U} - 1) + \left( \frac{2n+1}{n+1} \right) \left[ \overline{(h-\delta)^{\frac{n+1}{n}}} - \overline{(h+\delta)^{\frac{n+1}{n}}} \right] \bar{f}_x = 0 \quad (38)$$

$$\bar{T}_{11} = \bar{\lambda} + \bar{\psi}_{1x} - \frac{\bar{\gamma}\bar{m}_1 n}{(3n+1)} \overline{(f_x)^{n+1}} \bar{g}_{1x} \quad (39)$$

$$\bar{T}_{12} = \bar{\lambda} + \bar{\psi}_{1x} - \frac{\bar{\gamma}\bar{m}_1 n}{(3n+1)} \overline{(f_x)^{n+1}} \bar{g}_{2x} \quad (40)$$

$$\bar{T}_{21} = \bar{\lambda} + \bar{\psi}_{2x} - \frac{\bar{\gamma}\bar{m}_2 n}{(3n+1)} \overline{(-f_x)^{n+1}} \bar{g}_{2x} \quad (41)$$

$$\bar{T}_{22} = \bar{\lambda} + \bar{\psi}_{2x} - \frac{\bar{\gamma}\bar{m}_2 n}{(3n+1)} \overline{(-f_x)^{n+1}} \bar{g}_{1x} \quad (42)$$

$$\begin{aligned} \frac{p_e \bar{h}^2}{3} \left( \bar{u}_m \frac{d\bar{T}_{m1}}{d\bar{x}} \right) + \bar{T}_{m1} &= \frac{1}{2} (\bar{T}_h + \bar{T}_{-h}) \\ &- \frac{\bar{\gamma}\bar{m}_1 n}{3n+1} \sqrt{\overline{(f_x)^{n+1}}} \bar{g}_x \end{aligned} \quad (43)$$

$$\frac{p_e \bar{h}^2}{3} \left( \bar{u}_m \frac{d\bar{T}_{m2}}{d\bar{x}} \right) + \bar{T}_{m2} = \frac{1}{2} (\bar{T}_h + \bar{T}_{-h}) - \frac{\bar{\gamma}\bar{m}_2 n}{3n+1} \overline{(-f_x)^{n+1}} \bar{g}_x \quad (44)$$

$$\bar{m} = \bar{m}_0 e^{(\bar{p}_1 - \bar{T}_{m1} + \bar{T}_0)} \text{ etc.} \quad (45)$$

$$\bar{\gamma} = \left( \frac{U_2 h_0 \beta}{2k\alpha} \right) \sqrt{\frac{h_0}{2R}} \quad (46)$$

$$\bar{f}_x = \frac{(\bar{U} + 1) \overline{(h-H)} + (\bar{U} - 1) \bar{\delta}}{\overline{(h+\delta)^{\frac{2n+1}{n}}} + \overline{(h-\delta)^{\frac{2n+1}{n}}}} \quad (47)$$

$$\bar{\lambda} = \left( \frac{\bar{T}_h + \bar{T}_{-h}}{2} \right) + \frac{\bar{T}_h - \bar{T}_{-h}}{2} \left( \frac{\bar{y}}{\bar{h}} \right) \quad (48)$$

$$\bar{\psi}_{1x} = \frac{\bar{p}_e}{2} \bar{u}_m \frac{d\bar{T}_{m1}}{d\bar{x}} \left( \bar{y}^2 - \bar{h}^2 \right) \quad (49)$$

$$\bar{\psi}_{2x} = \frac{\bar{p}_e}{2} \bar{u}_m \frac{d\bar{T}_{m2}}{d\bar{x}} \left( \bar{y}^2 - \bar{h}^2 \right) \quad (50)$$



where  $\bar{p}_e = \frac{\rho c U_2 h_0}{k} \sqrt{\frac{h_0}{2R}}$

$$\bar{g}_x = \frac{n}{(4n+1)\bar{h}} \left[ (\bar{h} + \bar{\delta})^{\frac{4n+1}{n}} + (\bar{h} - \bar{\delta})^{\frac{4n+1}{n}} \right] - \left[ (\bar{h} - \bar{\delta})^{\frac{3n+1}{n}} + (\bar{h} + \bar{\delta})^{\frac{3n+1}{n}} \right] \quad (51)$$

$$\bar{g}_{1x} = 2(\bar{y} - \bar{\delta})^{\frac{3n+1}{n}} - \left[ (\bar{h} + \bar{\delta})^{\frac{3n+1}{n}} + (\bar{h} - \bar{\delta})^{\frac{3n+1}{n}} \right] + (\bar{y}/\bar{h}) \left[ (\bar{h} + \bar{\delta})^{\frac{3n+1}{n}} - (\bar{h} - \bar{\delta})^{\frac{3n+1}{n}} \right] \quad (52)$$

$$\bar{g}_{2x} = 2(\bar{\delta} - \bar{y})^{\frac{3n+1}{n}} - \left[ (\bar{h} + \bar{\delta})^{\frac{3n+1}{n}} + (\bar{h} - \bar{\delta})^{\frac{3n+1}{n}} \right] + (\bar{y}/\bar{h}) \left[ (\bar{h} + \bar{\delta})^{\frac{3n+1}{n}} - (\bar{h} - \bar{\delta})^{\frac{3n+1}{n}} \right] \quad (53)$$

**Load and Traction:** The dimensionless load

$$\bar{W} = \frac{w\alpha}{\sqrt{2Rh_0}} \quad (54)$$

$$\bar{W} = \int_{-\infty}^{\bar{x}_2} \bar{p} d\bar{x} = - \int_{-\infty}^{\bar{x}_2} \bar{x} \frac{d\bar{p}}{d\bar{x}} d\bar{x} \quad (55)$$

Dimensionless tractions is

$$\bar{T}_{Fh-} \left( = \alpha \frac{T_{F_0}}{h_0} \right) = - \int_{-\infty}^{\bar{x}_2} \bar{\tau}_{\bar{y}=-\bar{h}} d\bar{x} \quad (56)$$

$$\bar{T}_{Fh+} = - \int_{-\infty}^{\bar{x}_2} \bar{\tau}_{\bar{y}=-\bar{h}} d\bar{x} \quad (57)$$

### 1.3 Result and Discussion:

A semi analytical solution of the Reynolds equations (36, 37) and the energy Eqs. (39-42, 43, 44) are obtained for the following representative values:  $U_2 = 400$  cm/s,  $h_0 = 4 \times 10^{-4}$  cm,  $\alpha = 1.6 \times 10^{-9}$  dyne $^{-1}$  cm $^2$ ,  $R = 3$  cm,  $\bar{y} = 1000.5$ ,  $\bar{T}_h + \bar{T}_{-h} = 3.2$ .

The Reynolds and the energy Eq. (36, 37, 43 and 44) coupled through  $\bar{m}$  contain two unknowns  $\bar{\delta}$  and  $\bar{x}_1$ . These unknowns are also present in Eq. (38). In order to solve Eq. (38), first of all, an initial value of  $\bar{x}$  is replaced by a large but a finite negative value i.e.  $\bar{x} = -5$ . For solution details, see Ref. [5]. Runge–Kutta Fehlberg method is used to solve the Eq. (38). As noted earlier [5],  $\bar{\delta}$  does not exist in the neighborhood of  $\bar{x} = -\bar{x}_1$  and lying in the interval:  $-\bar{h} \leq \bar{\delta} \leq \bar{h}$ . Hence in the neighborhood of  $\bar{x} = -\bar{x}_1$   $\bar{\delta}$  ( $= \bar{\delta}^*$ ) say has to be determined solely on the basis of physical considerations. The  $\in -$  neighborhood ( $-\bar{x}_1 - \epsilon_1 \leq \bar{x} \leq -\bar{x}_1 + \epsilon_2$ ) of  $\bar{x} = -\bar{x}_1$  is to be determined as the region where there does not exist any  $\bar{\delta}$  lying in the interval  $-\bar{h} \leq \bar{\delta} \leq \bar{h}$  and satisfying Eq. (38). To ease the mathematical complexity, a linear profile for  $\bar{\delta}^*$  chosen as below:

$$\bar{\delta}^* = -\bar{h}_{b1} (\bar{x} + \bar{x}_1) / \epsilon_1, \quad -\bar{x}_1 - \epsilon_1 \leq \bar{x} \leq -\bar{x}_1 \quad (58)$$

$$\bar{\delta}^* = -\bar{h}_{b2} (\bar{x} + \bar{x}_1) / \epsilon_2, \quad -\bar{x}_1 \leq \bar{x} \leq -\bar{x}_1 + \epsilon_2 \quad (59)$$

$\bar{h}_{b2} = 1 + (-\bar{x}_1 - \epsilon_1)^2$   $\bar{h}_{b2} = 1 + (-\bar{x}_1 - \epsilon_1)^2$  is assumed. Having determined  $\bar{\delta} = \bar{\delta}^*$  using Eq. (58) and (59) in the neighborhood of  $\bar{x} = -\bar{x}_1$ , the same procedure is followed in [5] with

$$\bar{\delta}^* = \bar{h}_{b3} (\bar{x} - \bar{x}_2) / \epsilon_3 \quad (60)$$

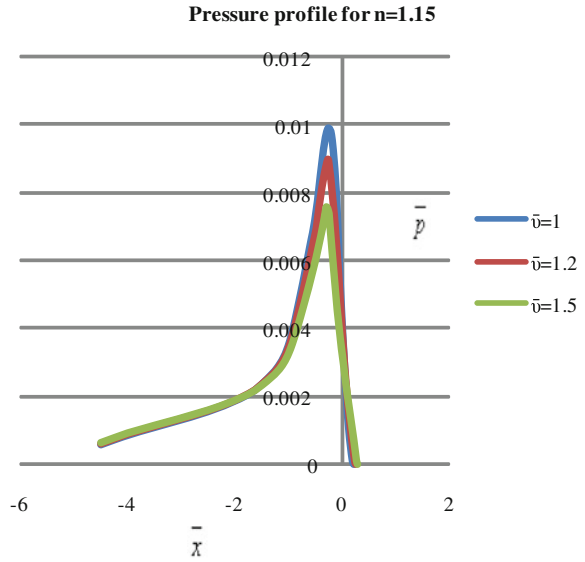
where  $\bar{h}_{b3} = 1 + (\bar{x}_2 - \epsilon_3)^2$   $\bar{\delta}^*$ , in the region  $\bar{x}_2 - \epsilon_3 \leq \bar{x} \leq \bar{x}_2$ , is calculated using Eq.(60) together with  $\bar{p}_2$  and  $\bar{T}_{m2}$ .

The lubricant pressure  $\bar{p}$  is presented in Figs. (2) and (3) i.e.  $\bar{p}$  increases with  $\bar{x}$  and then decreases till last. A similar type of profile was also obtained by Wang et al. [19]. The qualitative behavior of  $\bar{p}$  for different values of  $\bar{U}$  (for fixed n) is identical (see Fig. 2). Further,  $\bar{p}$  increases as  $\bar{U}$  decreases. This is somewhat contrary to the results of Prasad et al. [5], Jang and Khonsari [20], and Rong-Tsong and Hamrock [21], For fixed  $\bar{U}$ , pressure increases as n increases [22, 23].

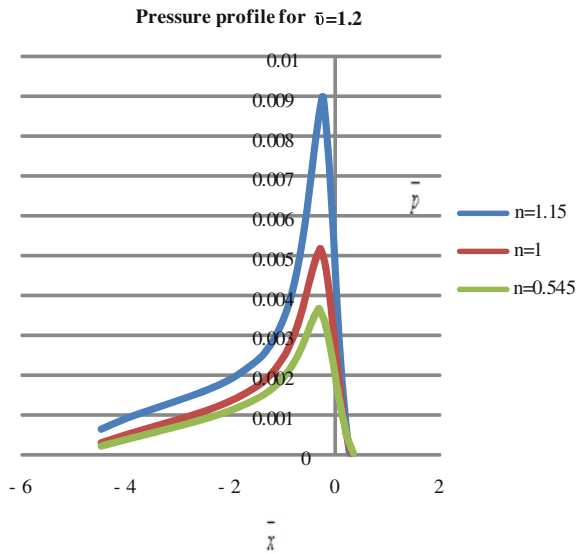
The mean temperature  $\bar{T}_m$  is shown in Fig. (4) and Fig. (5), as a function of n, and  $\bar{U}$  respectively. It may be observed that the qualitative behavior of  $\bar{T}_m$  versus  $\bar{x}$  is very similar to the temperature profile obtained in Prasad et al. [5], Saini et al. [24] and Liu et al. [25]. Further, it may be noted from Fig. (4) that for fixed values of  $\bar{U}$  and  $\bar{P}_e$ ,  $\bar{T}_m$  increases with n showing that the temperature for dilatants fluid is higher than that of Newtonian and pseudo plastic fluids both. For fixed values of n and  $\bar{P}_e$ ,  $\bar{T}_m$  increases with  $\bar{U}$ , indicates that the sliding temperature is higher than that of pure rolling [26].

The calculated values of the normal load carrying capacity  $\bar{W}$ , the traction force  $\bar{T}_{Fh+}$  are calculated and presented in Figs. 6 and 7 respectively. Results are in agreement with the previous findings [6]. All these have same characteristics, i.e., both increase with n and decreases as  $\bar{U}$  increases. The trend of the traction forces with n is quite similar to Punit and Khonsari [13],

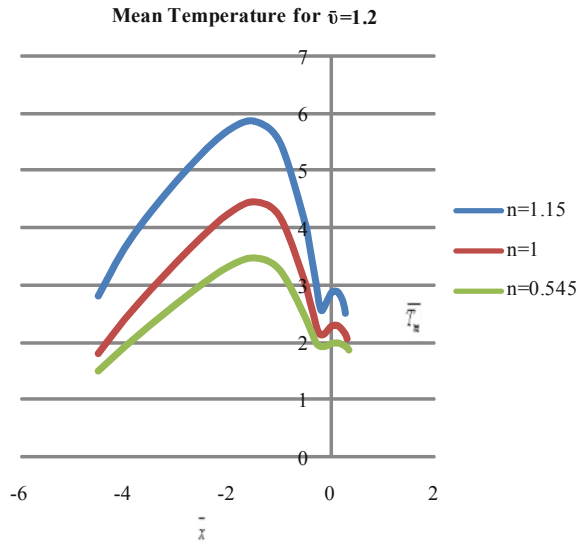
**Fig. 2** Pressure  $\bar{p}$  verses  $\bar{X}$  var rining  $\bar{U}$



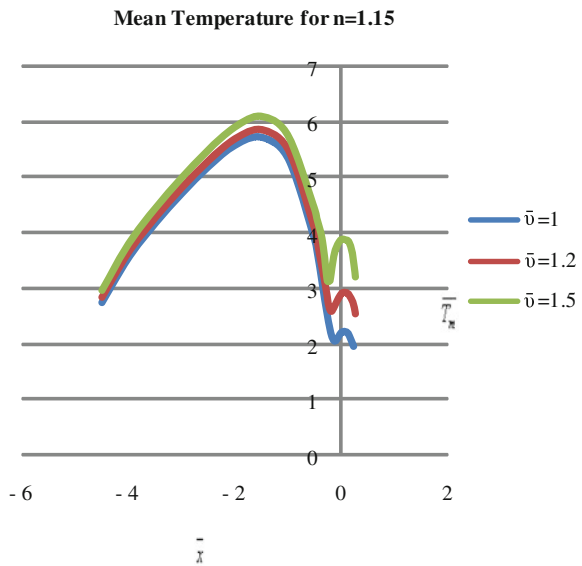
**Fig. 3** Pressure  $\bar{p}$  verses  $\bar{X}$  var rining n



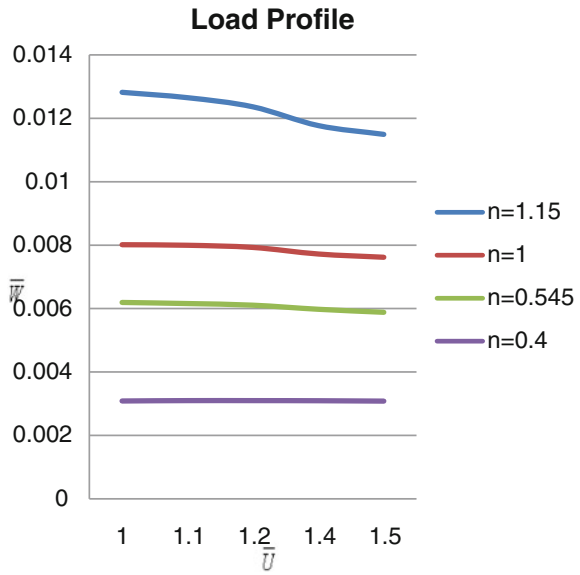
**Fig. 4** Temperature  $\bar{T}_m$  verses  $\bar{X}$  var rining n



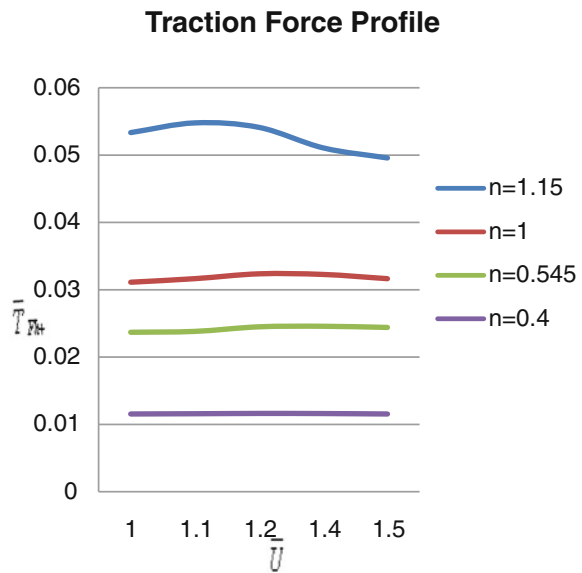
**Fig. 5** Temperature  $\bar{T}_m$  verses  $\bar{X}$  var rining



**Fig. 6** Load  $\bar{W}$  verses  $\bar{U}$  var rining n



**Fig. 7** Traction  $\bar{T}_{F+}$  verses  $\bar{U}$  var rining n



## 2 Conclusion

(i) There is a significant increase in pressure, hence the load and the traction, with the flow index  $n$  for a fixed value of  $\bar{U}$ . (ii) There is also a significant increase in the mean film temperature with  $n$  and  $\bar{U}$  both. Hence it is justifiable to treat  $\bar{m}$  as a function of  $\bar{p}$  and temperature.

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