Multi-Objective Optimization of PID Controller for Coupled-Tank Liquid-Level Control System Using Genetic Algorithm

Sanjay Kr. Singh, Nitish Katal and S. G. Modani

Abstract The main aim of this chapter is to obtain optimal gains for a PID controller using multi-objective genetic algorithm used in a coupled-tank liquid-level control system. Liquid level control system is a nonlinear system and finds a wide application in petrochemical, food processing, and water treatment industries, and the quality of control directly affects the quality of products and safety. This chapter employs the use of multi-objective genetic algorithm for the optimization of the PID gains for better plant operations in contrast to conventional tuning methods and GA. The simulations indicate that better performance is obtained in case of multi-objective genetic algorithm-optimized PID controller.

Keywords PID controller · Multi-objective genetic algorithm · PID optimization · Liquid level control

1 Introduction

Coupled-tank liquid-level control is the center to many diverse industrial applications ranging from petrochemical, food processing to nuclear power generation [1]. The main objective of this system is to control the flow of liquid between tanks so that optimum levels are maintained in both the tanks [2].

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In this chapter, coupled-tank liquid-level system has been considered, and the PID controller is implemented for either maintaining the liquid level at a desired set point, disturbance rejection or to be used for moving the liquid set point. For designing the PID controller, classical method of Ziegler Nichols has been used, followed by the optimization using multi-objective genetic algorithm. The gain parameters have been tuned with respect to the objective function, stated as “Sum of integral of the squared error and the sum of integral of absolute error”. According to the results obtained, considerably better results have been obtained in case of multi-objective genetic algorithm-optimized PID controllers when compared to Ziegler-Nichols method in their respective step response on the system.

2 Mathematical Modeling of Coupled-Tank Liquid-Level System

Considering the coupled-tank system, is in Fig. 1. The dynamic equations of the system, by considering the flow balances for each tank, the equations for rate of change of fluid volume in tanks are as [3, 4]:

$$\text{For Tank 1}: Q_i - Q_1 = A\frac{\mathrm{d}H_1}{\mathrm{d}t} \tag{1}$$

$$\text{For Tank 2}: Q_1 - Q_0 = A\frac{\mathrm{d}H_2}{\mathrm{d}t} \tag{2}$$

where

H_1, H_2	Height of tank 1 and 2
A	Cross sectional area of tank 1 and 2
Q_1, Q_2	Flow rate of the fluid
Q_i	Pump flow rate

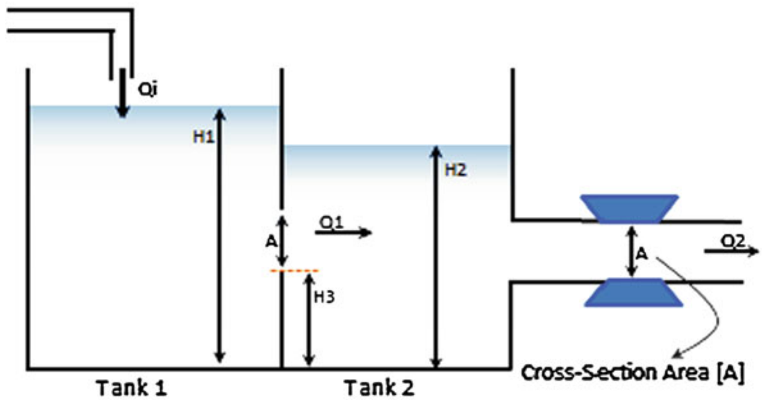

Fig. 1 Schematic representation of the coupled-tank system

The steady-state representation of the coupled-tank system can be given as follows:

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{pmatrix} -k_1/A & k_1/A \\ k_1/A & -\frac{(k_1+k_2)}{A} \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1/A \\ 0 \end{bmatrix} q_i \tag{3}$$

Taking the Laplace transformation of Eq. 3, the transfer function is obtained in Eq. 4.

$$G(s) = \frac{1/k_2}{\left(\frac{A^2}{k_1 k_2}\right).s^2 + \left(\frac{A(2k_1+k_2)}{k_1 k_2}\right).s + 1} = \frac{1/k_2}{(sT_1+1)(sT_2+1)}$$

where

$$\begin{aligned} T_1 T_2 &= A^2/k_1 k_2 \\ T_1 + T_2 &= \frac{A(2k_1+k_2)}{k_1 k_2} \\ k_1 &= \frac{\alpha}{2\sqrt{H_1 - H_2}} \\ \text{and } k_2 &= \frac{\alpha}{2\sqrt{H_2 - H_3}} \end{aligned}$$

Using; $H_1 = 18$ cm, $H_2 =$14 cm, $H_3 = 6$ cm, $\alpha = 9.5$ (constant for coefficient of discharge), $H = 32$; the transfer function can be obtained in Eq. 4.

$$G(s) = \frac{0.002318}{s^2 + 0.201.s + 0.00389} \tag{4}$$

3 Designing and Optimization of PID Controllers

PID controllers are the most widely used controllers in the industrial control processes [5], and 90 % of the controllers today used in industry are alone PIDs. The general equation for a PID controller can be given by Eq. 5.

$$C(s) = K_p.R(s) + K_i \int R(s)\mathrm{d}t + K_d \frac{\mathrm{d}R(s)}{\mathrm{d}t} \tag{5}$$

where K_p, K_i and K_d are the controller gains, $C(s)$ is output signal, and $R(s)$ is the difference between the desired output and output obtained [6].

3.1 PID Tuning Using Ziegler Nichols

Ziegler Nichols is the most operative method for tuning the PID controllers. But, this method is limited for application till ratio of 4:1 for the first two peaks in closed-loop response, leading to an oscillatory response [7]. Initially, unit-step response is derived (Fig. 2) followed by the computation of the PID gains as suggested by Ziegler-Nichols as in Table 1.

3.2 PID Optimization Using Genetic Algorithm

Genetic algorithms have vanguard advantage of wider adaptability to any constraints and hence are considered as one of the most robust optimization algorithms [8]. Optimization of the PID controllers with genetic algorithms focuses on obtaining the best possible solution for the three PID gains $[K_p, K_i, K_d]$ by minimizing the objective function. For the optimal tuning of the controller, the minimization of the integral square error (ISE) has been carried out.

$$ISE = \int_0^{T_s} e^2(t)\,\mathrm{d}t$$

The optimization has been carried out using Global Optimization Toolbox and Simulink [9] with a population size of 20, scattered crossover, both-side migration and roulette-wheel-based selection. The PID gains obtained by optimal tuning using GA are represented in Table 2, and Fig. 3 shows the closed-loop response of the GA-optimized controllers. Figure 4 represents the plot for best and mean fitness

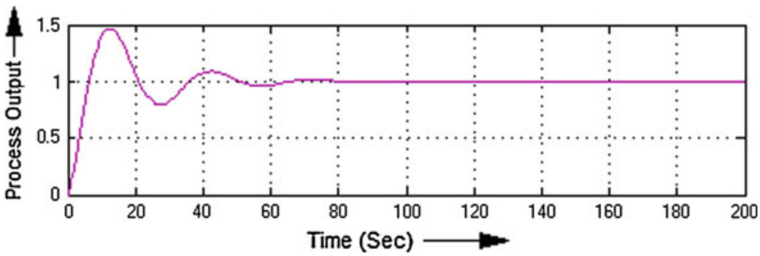

Fig. 2 Closed-loop step response of the system with ZN-PID controller

Table 1 PID parameters estimated by Ziegler-Nichols

PID gains	Value
K_p	28.214
K_i	4.155
K_d	47.89

Table 2 PID parameters estimated by genetic algorithm

PID gains	Value
K_p	79.9820
K_i	1.2042
K_d	83.4625

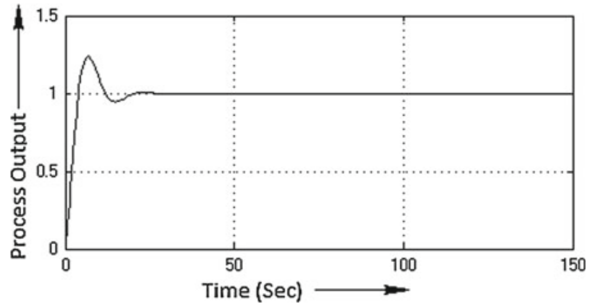

Fig. 3 Closed-loop step response of the system with GA-PID controller

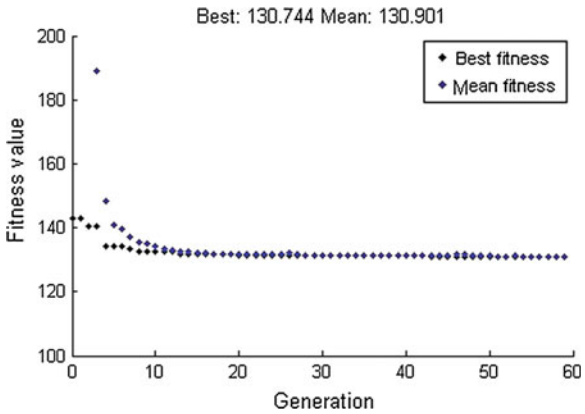

Fig. 4 Plot for the best and average fitness values of the genetic algorithm optimization

values across various generations obtained while optimizing the PID controller using Genetic Algorithm.

3.3 PID Optimization Using Multi-Objective Genetic Algorithm

Since the Ziegler-Nichols tuned PID controllers give an oscillatory response, they are not optimum for implementation for plant. PID optimization using multi-objective genetic algorithm aims at obtaining an optimal Pareto solution, simultaneously

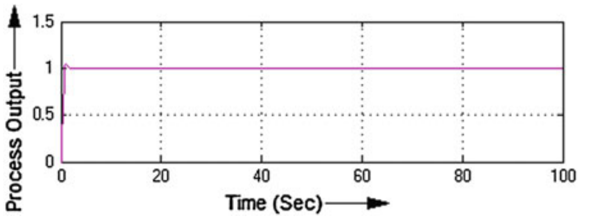

Fig. 5 Closed-loop response using Mobj-GA-optimized PID controllers

Table 3 PID parameters estimated by multi-objective genetic algorithm

PID gains	Value
K_p	255.1
K_i	5.5
K_d	1249.96

improving the objective function of both the objectives O_1 and $O_{2,}$ given as follows:

First objective function is integral square error (ISE) which discards the large amplitudes, and second objective function is integral absolute error (IAE) which gives the measure of the systems performance. ISE tends to suppress the larger errors, while ISE tends to suppress the smaller errors [10]. The algorithm used here is NSGA-II, which using the controlled elitist genetic algorithm boosts obtaining the better fitness value of the individuals; and if the value is less, it still favors increasing the diversity of the population [11, 12]. Diversity of the populations/gains is controlled by the elite members of the population, while elitism is controlled by Pareto fraction and at Pareto Front also bound the number of individuals.

$$ISE = O_1 = \int_0^{T_s} e^2(t)\,\mathrm{d}t \quad and \quad IAE = \int_0^{T_s} |u(t)|\,\mathrm{d}t$$

The system implementation and optimization have been carried out in MATLAB and Simulink [9] environment using Global Optimization Toolbox. The population

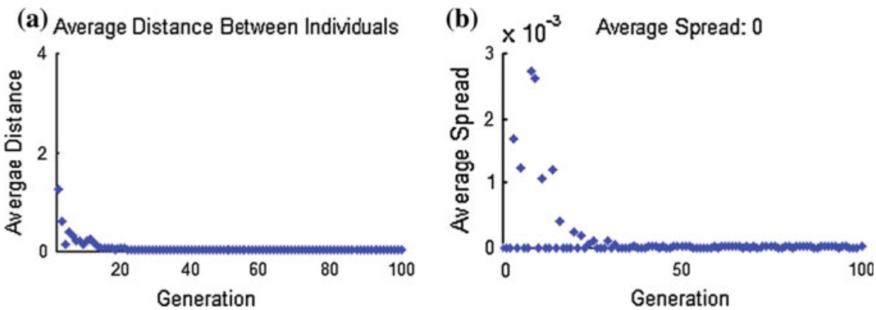

Fig. 6 Plot for the **a** Average distance **b** Average spread between individuals

size of 45 has been considered with adaptive feasible mutation function, heuristic crossover, and the selection of individuals on the basis of tournament with a tournament size of 2. A hybrid function of Fitness Goal Attain (*fgoalattain*) is used, which further minimizes the function after GA terminates. Figure 5 shows the closed-loop response, and the optimized PID parameters are shown in Table 3. In Fig. 6a, distance between members of each generation is shown, and Fig. 6b gives the plot for average Pareto spread, which is the change in distance measure with respect to the previous generations.

4 Results and Discussion

In this chapter, the implementation and simulations of the system has been carried out in Simulink. Initially, the gains of the PID have been estimated using Ziegler Nichols rules [13] which give an oscillatory response, followed by the optimization by genetic algorithm and multi-objective genetic algorithm. The computed parameters are implemented for obtaining the closed-loop response of the system. Figure 7 shows the compared closed-loop step response graph, clearly indicating that better results are obtained in case of multi-objective genetic algorithm-optimized PID controller with decreased overshoot percentage and rise and settling time values. Table 4 represents the numerical data of the results obtained.

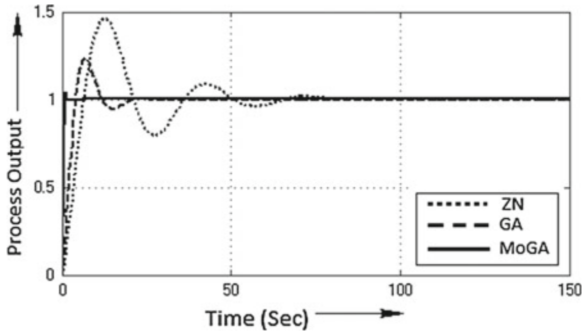

Fig. 7 Comparative closed-loop response of the ZN, GA, and MoGA-optimized PID controllers

Table 4 Comparison of the results

Method of design	Overshoot percentage	Rise time (s)	Settling time (s)
Ziegler-Nichols	46.4	4.83	62.4
Genetic algorithm	23.7	2.93	18.5
Multi-objective GA	4.47	0.504	1.41

5 Conclusion

The use of multi-objective genetic algorithm for the optimization of PID controller offers better results in terms of decreased overshoot percentage and rise and settling times as compared to Ziegler Nichols and genetic algorithm-tuned PIDs, thus offering better operation for the coupled-tank liquid-level control and better plant safety and performance.

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