# A Partial Backlogging Inventory Model for Decaying Items: Considering Stock and Price Dependent Consumption Rate in Fuzzy Environment

**S. R. Singh and Swati Sharma**

**Abstract** In this article, an inventory model is developed to deal with the impreciseness present in the market demand and the various cost parameters. The presented model is developed in crisp and fuzzy environments. Signed distance method is used for defuzzification. In most of the classical models, constant demand rate is considered. But in practice purchasing deeds of the customers is affected by the selling price and inventory level. In this study, we have considered demand rate as a function of stock-level and selling price. Two parameters Weibull distribution deterioration is considered. It is assumed that shortages are allowed and are partially backordered with the time dependent backlogging rate. A numerical experiment is provided to illustrate the problem. Sensitivity analysis of the optimal solution with respect to the changes in the value of the system parameters is also discussed.

**Keywords** Inventory model · Triangular fuzzy numbers · Signed distance · Partial backlogging · Stock and price dependent demand rate

## 1 Introduction

In most of the inventory models, it is assumed that the inventory parameters like demand rate and ordering cost, etc., are precisely known. But in actual living situations, the nature of these parameters is imprecise, so it is important to consider them as fuzzy numbers. The concept of fuzzy set theory first introduced by Zadeh [1], after that Park [2] extended the classical EOQ model by introducing the fuzziness

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of ordering cost and holding cost. Recently, Chang [3] and Wang et al. [4] presented an inventory model under fuzzy demand.

Deterioration is a natural phenomenon in many real situations so it plays an important role in developing an inventory model. Generally, deterioration is defined as damage, spoilage, decay andobsolescence, vaporization, etc., that result in decrease of value of the original one. The first model for decaying items was presented by Ghare and Schrader [5]. It was extended by Covert and Philip [6] considering Weibull distribution deterioration. A complete survey for deteriorating inventory models was presented by Raafat [7]. Some other papers relevant to this topic are Teng et al. [8] and Bakker et al. [9].

Various researchers considered the situation in which shortages are either completely backlogged or completely lost which is not realistic. Many practical experiences disclose that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable supplies and the products with short life cycle. According to such phenomenon, backlogging rate should not be disregarded. Thus, it is necessary to consider the backlogging rate. Researchers, such as Park [10] and Wee [11] developed inventory models with partial backorders. Some recent work in this direction is done by Singh [12] and Hsieh et al. [13].

In many classical research articles, it is assumed that demand rate is constant during the sales period. In real life, the demand may be inspired if there is a large pile of goods displayed on shelf. Gupta and Vrat [14] developed the first models for stock-dependent consumption rate. Mandal and Phaujdar [15] then developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Other papers related to this research area are Ray et al. [16], Goyal and Giri [17], and Chang et al. [18].

All the above-cited papers reveal that many research articles are developed in which demand is considered as the function of stock level or selling price, shortages are allowed and partially backlogged, but there is no such research paper which is partially backlogged assuming demand rate as the function of selling price and inventory level in fuzzy environment. In lots of business practices, it is observed that several parameters in inventory system are imprecise. Therefore, it is necessary to consider them as fuzzy numbers while developing the inventory model.

In this study, we have developed a partial backlogging inventory model for deteriorating items considering stock and price sensitive demand rate in crisp and fuzzy surroundings. A numerical example to prove that the optimal solution exists and is unique is provided and the sensitivity analysis with respect to system parameters is discussed. The concavity is also shown through the figure (Fig. 1).

## 2 Assumptions and Notations

The notations and basic assumptions of the model are as follows:

(1) The demand rate is a function of stock and selling price considered as $f(t) = (a + bQ(t) - p)$ where $a > 0, 0 < b < 1, a > b$ and $p$ is selling price.

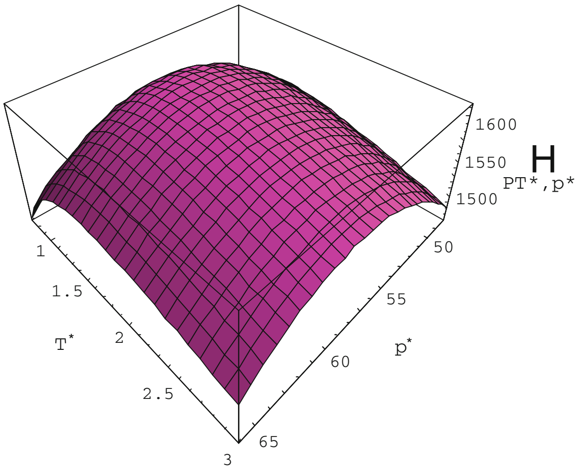


**Fig. 1** Concavity of the profit function

(2) Holding cost $h(t)$ per item per time-unit is time dependent and is assumed to be $h(t) = h + \delta t$ where $\delta > 0, h > 0$.
(3) Shortages are allowed and partially backlogged and rate is assumed to be $1/(1 + \eta t)$, which is a decreasing function of time, $\eta \geq 0$ .
(4) The deterioration rate is time-dependent.
(5) $T$ is the length of the cycle.
(6) Replenishment is instantaneous and lead time is zero.
(7) The order quantity in one cycle is $Q$.
(8) The selling price per unit item is $p$.
(9) $A$ is the ordering cost per order.
(10) $c_1$ is the purchasing cost per unit per unit time.
(11) $c_2$ is the backordering cost per unit per unit time.
(12) $c_3$ is the opportunity cost per unit.
(13) $c_4$ is the deterioration cost per unit.
(14) $P(T, t_1, p)$ the total profit per unit time.
(15) The deterioration of units follows the two parameter Weibull distribution (say) $\theta(t) = \alpha\beta t^{\beta-1}$ where $0 < \alpha < 1$ is the scale parameter and $\beta > 0$ is the shape parameter.
(16) During time $t_1$, inventory is depleted due to deterioration and demand of the item. At time $t_1$ the inventory becomes zero and shortages start going on (Fig. 2).

## 3 Mathematical Formulation and Solution

Let $Q(t)$ be the inventory level at time $t\,(0 \leq t \leq T)$. During the time interval $[0, t_1]$ inventory level decreases due to the combined effect of demand and deterioration both and at $t_1$ inventory level depletes up to zero. The differential equation to describe immediate state over $[0, t_1]$ is given by

$$Q'(t) + \alpha\beta t^{\beta-1} Q(t) = -(a + bQ(t) - p) \quad 0 \le t \le t \tag{1}$$

Again, during time interval $[t_1, T]$ shortages stars occurring and at $T$ there are maximum shortages, due to partial backordering some sales are lost. The differential equation to describe instant state over $[t_1, T]$ is given by

$$Q'(t) = -\frac{(a-p)}{[1+\eta(T-t)]} \quad t_1 \le t \le T \quad t_1 \le t \le T \tag{2}$$

With condition $Q(t_1) = 0$. Solving Eqs. (1) and (2) and neglecting higher powers of $\alpha$, we get

$$Q(t) = (a-p)\left[t_1 - t + \frac{b}{2}(t_1^2 - t^2) + \frac{\alpha}{(\beta+1)}(t_1^{\beta+1} - t^{\beta+1})\right] e^{-bt-\alpha t^{\beta}} 0 \le t \le t_1 \tag{3}$$

$$Q(t) = \frac{(a-p)}{\eta}[\log\ 1 + \eta(T-t) - \log\ 1 + \eta(T-t_1)] t_1 \le t \le T \tag{4}$$

At time $t = 0$ inventory level is $Q(0)$ and is given by

$$Q(0) = (a-p)\left(t_1 + \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta+1}}{\beta+1}\right)$$

At time $T$ maximum shortages $(Q_1)$ occurs and is given by

$$Q_1 = \frac{(a-p)}{\eta}[\log\{1 + \eta(T-t_1)\}]$$

The order quantity is $Q$ and is given by

$$Q = (a-p)\left(t_1 + \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{1}{\eta}\log(1+\eta(T-t_1))\right) \tag{5}$$

The purchasing cost is

$$\text{PC} = c_1(a-p)\left(t_1 + \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{1}{\eta}\log(1+\eta(T-t_1))\right) \tag{6}$$

Ordering cost is given by

$$\text{OC} = A \tag{7}$$

Holding cost during the period $[0, t_1]$ is given by

$$\begin{aligned}\text{IHC} &= \int_0^{t_1} (h+\delta t)Q(t)dt \\ &= \int_0^{t_1} (h+\delta t)(a-p)\left[t_1 - t + \frac{b}{2}(t_1^2 - t^2) + \frac{\alpha}{(\beta+1)}(t_1^{\beta+1} - t^{\beta+1})\right] \\ &\quad \times e^{-bt-\alpha t^{\beta}} dt \end{aligned} \tag{8}$$

Deterioration cost during the period $[0, t_1]$ is given by

$$\text{DC} = c_4 \left\{ Q(0) - \int_0^{t_1} (a + bQ(t) - p)dt \right\} \tag{9}$$

Shortage cost due to backordered is

$$\begin{aligned}\text{BC} &= c_2 \int_{t_1}^{T} [-Q(t)]\, dt \\ &= -c_2 \int_{t_1}^{T} \frac{(a-p)}{\eta} \left[\log\{1+\eta(T-t)\} - \log\{1+\eta(T-t_1)\}\right] dt \end{aligned} \tag{10}$$

Lost sales cost due to lost sales is

$$\text{LS} = c_3(a-p) \int_{t_1}^{T} \left[1 - \frac{1}{(1+\eta(T-t))}\right] dt \tag{11}$$

Sales revenue is given by

$$\text{SR} = p \int_0^{t_1} (a + bQ(t) - p)\, dt + p \int_{t_1}^{T} \frac{(a-p)}{[1+\eta(T-t)]} dt \tag{12}$$

From Eqs. (6), (7), (8), (9), (10), (11), and (12) total profit per unit time is given by

$$P(T, t_1, p) = \frac{1}{T}\text{SR} - \text{OC} - \text{PC} - \text{IHC} - \text{BC} - \text{LS} - \text{DC} \tag{13}$$

Let $t_1 = \gamma T$, $0 < \gamma < 1$. Hence we get the profit function is

$$P(T,p)=\frac{1}{T}\left[p(a-p)\gamma T+b(a-p)pK_1-b(a-p)pK_2+p(a-p)K_3-A\right.$$
$$\left.-c_1(a-p)K_4-\frac{c_2(a-p)K_5}{\eta}-c_3(a-p)K_5-c_4(a-p)K_6-(a-p)\,K_7\right] \tag{14}$$

Where,

$$K_1=\left(\gamma T+\frac{b\gamma^2T^2}{2}+\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}\right)\left(\gamma T-\frac{b\gamma^2T^2}{2}-\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}\right)$$

$$K_2=\left[\frac{\gamma^2T^2}{2}+\frac{b\gamma^3T^3}{6}+\frac{\alpha\gamma^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)}-b\left\{\frac{\gamma^3T^3}{3}+\frac{b\gamma^4T^4}{8}\right.\right.$$
$$\left.\left.+\frac{\alpha\gamma^{\beta+3}T^{\beta+3}}{(\beta+1)(\beta+3)}\right\}-\alpha\left\{\frac{\gamma^{\beta+2}T^{\beta+2}}{\beta+2}+\frac{b\gamma^{\beta+3}T^{\beta+3}}{2(\beta+3)}+\frac{\alpha\gamma^{2\beta+2}T^{2\beta+2}}{2(\beta+1)^2}\right\}\right]$$

$$K_3=\frac{\{\log\{1+\eta(T-\gamma T)\}\}}{\eta},\ \ K_4=\left(\gamma T+\frac{b\gamma^2T^2}{2}+\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}\right.$$
$$\left.+\frac{1}{\eta}\log\left(1+\eta(T-\gamma T)\right)\right),\ \ K_5=\frac{1}{\eta}\left\{\eta(T-\gamma T)-\log\left(1+\eta(T-\gamma T)\right)\right\},$$

$$K_6=\left(\frac{b\gamma^2T^2}{2}+\frac{\alpha\gamma^{\beta+1}T^{\beta+1}}{\beta+1}-b\left(\frac{\gamma^2T^2}{2}+\frac{b\gamma^3T^3}{6}-\frac{b^2\gamma^4T^4}{8}-\frac{2\alpha\gamma^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)}\right.\right.$$
$$\left.\left.-\frac{b\alpha\gamma^{\beta+3}T^{\beta+3}}{2\,(\beta+1)}-\frac{\alpha^2\gamma^{2\beta+2}T^{2\beta+2}}{2(\beta+1)^2}\right)\right)$$

$$K_7=\left\{\frac{h\gamma^2T^2}{2}+\frac{\delta\gamma^3T^3}{6}+\frac{bh\gamma^3T^3}{6}+\frac{b\delta\gamma^4T^4}{24}-\frac{b^2h\gamma^4T^4}{8}-\frac{b^2\delta\gamma^5T^5}{15}\right.$$
$$+\frac{\alpha h\gamma^{\beta+2}T^{\beta+2}}{\beta+2}-\frac{\alpha h\gamma^{\beta+2}T^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{3\alpha\delta\gamma^{\beta+3}T^{\beta+3}}{2(\beta+3)}-\frac{\alpha bh\gamma^{\beta+3}T^{\beta+3}}{2(\beta+1)}$$
$$-\frac{\alpha\delta\gamma^{\beta+3}T^{\beta+3}}{(\beta+2)}+\frac{\alpha\delta b\gamma^{\beta+4}T^{\beta+4}}{6(\beta+4)}-\frac{\alpha\delta b\gamma^{\beta+4}T^{\beta+4}}{2(\beta+2)}-\frac{\alpha^2h\gamma^{2\beta+2}T^{2\beta+2}}{2(\beta+1)^2}$$
$$\left.-\frac{\alpha^2\delta\gamma^{2\beta+3}T^{2\beta+3}}{(\beta+2)(2\beta+3)}\right\}$$

Our objective is to maximize the profit function $P(T,p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0\quad\text{and}\quad\frac{\partial P(T,p)}{\partial p}=0$$

Using the software Mathematica-8.0, from these two equations, we can determine the optimum values of $T^*$ and $p^*$ simultaneously and the optimal value $P^*(T^*, p^*)$ of the average net profit can be determined by (14) provided they satisfy the sufficiency conditions for maximizing $P^*(T^*, p^*)$ are

$$\frac{\partial^2 P(T,p)}{\partial T^2} < 0, \frac{\partial^2 P(T,p)}{\partial p^2} < 0 \quad \text{and} \quad \frac{\partial^2 P(T,p)}{\partial T^2}\frac{\partial^2 P(T,p)}{\partial p^2} - \left(\frac{\partial^2 P(T,p)}{\partial T \partial p}\right)^2 > 0$$

## 4 Numerical Example

To illustrate the theory of the model, we consider the following data on the basis of the previous study.

$A = 200, a = 100, b = 0.02, c_1 = 12, \gamma = 0.6, c_2 = 8, c_3 = 15, c_4 = 0.4, \alpha = 0.1, h = 0.6, \beta = 0.01, \eta = 0.5, \delta = 0.02$.

Based on these input data, the findings are as follows: $p^* = 57.2434, t_1^* = 0.896826, Q^* = 64.8475, T^* = 1.49471$ and $P^*(T^*, p^*) = 1612.65$.

## 5 Sensitivity Analysis

See Table 1.

## 6 Fuzzy Mathematical model

In this study, we consider $a, A, c_1, c_2, c_3$ and $c_4$ as fuzzy numbers, i.e., $\tilde{a}, \tilde{A}, \tilde{c}_1, \tilde{c}_2, \tilde{c}_3$ and $\tilde{c}_4$. Then $\tilde{P}(T, P)$ is regarded as the estimate of total profit per unit time in the

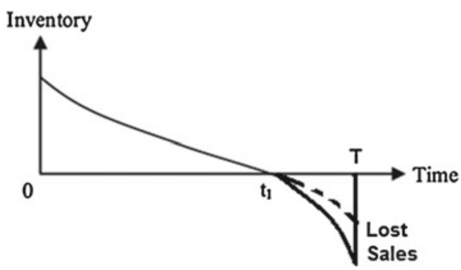


**Fig. 2** Graphical representation of the inventory system

**Table 1** Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 10 and 20 % and considering one parameter at a time, keeping the left over parameters at their original values

| Changing parameter | % change in system | Change in $T^*$ | Change in $p^*$ | Change in $t_1^*$ | Change in $Q^*$ | Change in $P^*(T^*, p^*)$ |
|---|---|---|---|---|---|---|
| $A$ | −20 | 1.30930 | 57.1406 | 0.785580 | 53.1966 | 1641.20 |
| | −10 | 1.40364 | 57.1932 | 0.842184 | 61.1014 | 1626.45 |
| | +10 | 1.58302 | 57.2914 | 0.949812 | 68.4592 | 1599.65 |
| | +20 | 1.66899 | 57.3376 | 1.001390 | 71.9558 | 1587.35 |
| $a$ | −20 | 1.89826 | 47.4582 | 1.138960 | 62.1062 | 896.308 |
| | −10 | 1.67136 | 52.3388 | 1.002820 | 63.6073 | 1229.32 |
| | +10 | 1.35289 | 62.1650 | 0.811734 | 65.8912 | 2046.22 |
| | +20 | 1.23631 | 67.0994 | 0.741786 | 66.7825 | 2529.98 |
| $b$ | −20 | 1.48084 | 57.2373 | 0.888504 | 64.2087 | 1610.57 |
| | −10 | 1.48771 | 57.2403 | 0.892626 | 64.5252 | 1611.61 |
| | +10 | 1.50184 | 57.2464 | 0.901104 | 65.1760 | 1613.70 |
| | +20 | 1.50910 | 57.2496 | 0.905460 | 65.5101 | 1614.76 |
| $c_1$ | −20 | 1.45761 | 55.9485 | 0.874566 | 65.2108 | 1718.40 |
| | −10 | 1.47591 | 56.5958 | 0.885546 | 65.0308 | 1665.12 |
| | +10 | 1.51403 | 57.8912 | 0.908418 | 64.6609 | 1561.00 |
| | +20 | 1.53389 | 58.5394 | 0.920334 | 64.4704 | 1510.15 |
| $c_2$ | −20 | 1.53379 | 57.1790 | 0.920274 | 66.5816 | 1619.58 |
| | −10 | 1.51388 | 57.2115 | 0.908328 | 65.6983 | 1616.09 |
| | +10 | 1.47623 | 57.2747 | 0.885738 | 64.0271 | 1609.25 |
| | +20 | 1.45842 | 57.3054 | 0.875052 | 63.2361 | 1605.88 |
| $c_3$ | −20 | 1.53126 | 57.1831 | 0.918756 | 66.4694 | 1619.14 |
| | −10 | 1.51266 | 57.2135 | 0.907596 | 65.6442 | 1615.88 |
| | +10 | 1.47737 | 57.2727 | 0.886422 | 64.0778 | 1609.46 |
| | +20 | 1.46061 | 57.3016 | 0.876366 | 63.3334 | 1606.30 |
| $\alpha$ | −20 | 1.49451 | 57.1739 | 0.896706 | 64.1849 | 1618.36 |
| | −10 | 1.49461 | 57.2086 | 0.896766 | 64.5166 | 1615.50 |
| | +10 | 1.49480 | 57.2781 | 0.896880 | 65.1776 | 1609.80 |
| | +20 | 1.49488 | 57.3128 | 0.896928 | 65.5067 | 1606.95 |
| $h$ | −20 | 1.50387 | 57.2314 | 0.902322 | 65.2491 | 1614.03 |
| | −10 | 1.49926 | 57.2374 | 0.899556 | 65.0318 | 1613.34 |
| | +10 | 1.49020 | 57.2493 | 0.894120 | 64.6499 | 1611.97 |
| | +20 | 1.48574 | 57.2552 | 0.891444 | 64.4543 | 1611.28 |

(continued)

fuzzy sense. In this study, we considered the signed distance method as proposed by Chang [3].

$\tilde{a} = (a - \Delta_1, a, a + \Delta_2)$ where $0 < \Delta_1 < a$ and $\Delta_1\Delta_2 > 0$, $\tilde{A} = A - \Delta_3, A, A + \Delta_4$ where $0 < \Delta_3 < A$ and $\Delta_3\Delta_4 > 0$.

$\tilde{c}_1 = c_1 - \Delta_5, c_1, c_1 + \Delta_6$ where $0 < \Delta_5 < c_1$ and $\Delta_5\Delta_6 > 0$, $\tilde{c}_2 = c_2 - \Delta_7, c_2, c_2 + \Delta_8$ where $0 < \Delta_7 < c_2$ and $\Delta_7\Delta_8 > 0$.

**Table 1** (continued)

| Changing parameter | % change in system | Change in $T^*$ | Change in $p^*$ | Change in $t_1^*$ | Change in $Q^*$ | Change in $P^*(T^*, p^*)$ |
|---|---|---|---|---|---|---|
| $\eta$ | −20 | 1.59373 | 57.2295 | 0.956238 | 69.6239 | 1634.32 |
| | −10 | 1.54094 | 57.2358 | 0.924564 | 67.0794 | 1623.19 |
| | +10 | 1.45381 | 57.2518 | 0.872286 | 62.8699 | 1602.62 |
| | +20 | 1.41731 | 57.2609 | 0.850386 | 61.1021 | 1593.04 |

$\tilde{c}_3 = c_3 - \Delta_9, c_3, c_3 + \Delta_{10}$ where $0 < \Delta_9 < c_3$ and $\Delta_9\Delta_{10} > 0, \tilde{c}_4 = c_4 - \Delta_{11}, c_4, c_4 + \Delta_{12}$ where $0 < \Delta_{11} < c_4$ and $\Delta_{11}\Delta_{12} > 0$.

And the signed distance of $\tilde{a}$ to $\tilde{0}$ is given by the relation $d(\tilde{a}, \tilde{0}) = a + 1/4(\Delta_2 - \Delta_1)$ where $d(\tilde{a}, \tilde{0}) > 0$ and $d(\tilde{a}, \tilde{0}) \in [a - \Delta_1, a + \Delta_2]$. Similarly, for other parameters signed distance can be defined as above.

Now, by the fuzzy triangular rule fuzzy total profit per unit is FP $(\tilde{a}, \tilde{A}, \tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \tilde{c}_4) = (F_1, F_2, F_3)$.

And $F_1$, $F_2$, $F_3$ are obtained as

$$F_1 = \frac{1}{T}[p(a - \Delta_1 - p)\gamma T + b(a - \Delta_1 - p)pK_1 - b(a + \Delta_2 - p)pK_2 + p(a - \Delta_1 - p)K_3 - (A + \Delta_4) - (c_1 + \Delta_6)(a + \Delta_2 - p)K_3 - \frac{(c_2 + \Delta_8)(a + \Delta_2 - p)K_5}{\eta} - (c_3 + \Delta_{10})(a + \Delta_2 - p)K_5 - (c_4 + \Delta_{12})(a + \Delta_2 - p)K_6 - (a + \Delta_2 - p)K_7] \quad (15)$$

$$F_2 = \frac{1}{T}[p(a - p)\gamma T + b(a - p)pK_1 - b(a - p)pK_2 + p(a - p)K_3 - A - c_1(a - p)K_4 - \frac{c_2(a - p)K_5}{\eta} - c_3(a - p)K_5 - c_4(a - p)K_6 - (a - p)K_7] \quad (16)$$

$$F_3 = \frac{1}{T}[p(a + \Delta_2 - p)\gamma T + b(a + \Delta_2 - p)pK_1 - b(a - \Delta_1 - p)pK_2 + p(a + \Delta_2 - p)K_3 - (A - \Delta_3) - (c_1 - \Delta_5)(a - \Delta_1 - p)K_4 - \frac{(c_2 - \Delta_7)(a - \Delta_1 - p)K_5}{\eta} - (c_3 - \Delta_9)(a - \Delta_1 - p)K_5 - (c_4 - \Delta_{11})(a - \Delta_1 - p)K_6 - (a - \Delta_1 - p)K_7] \quad (17)$$

Now, defuzzified average profit using the signed distance method is given by

$$\tilde{P}(T, p) = (F_1 + 2F_2 + F_3)/4 \quad (18)$$

Also, the defuzzified order quantity is Q and is given by

$$Q = \left( a + \frac{(\Delta_2 - \Delta_1)}{4} - p \right) \left( t_1 + \frac{bt_1^2}{2} + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{1}{\eta} \log\left(1 + \eta(T - t_1)\right) \right) \tag{19}$$

The necessary conditions for maximizing the average profit are $\partial \tilde{P}(T, p)/\partial T = 0$ and $\partial \tilde{P}(T, p)/\partial p = 0$.

Using the software Mathematica-8.0, from the above two equations, the optimum values of $\tilde{T}$ and $\tilde{p}$ can be determined simultaneously and the optimal value of the average net profit $(\tilde{P}(T, p))$ can be obtained by (18).

## 7 Numerical Example

$A = 200, \Delta_3 = 10, \Delta_4 = 20, a = 100, \Delta_1 = 5, \Delta_2 = 10, b = 0.02, c_1 = 12, \Delta_5 = 0.60, \Delta_6 = 1.2, c_2 = 8, \Delta_7 = 0.4, \Delta_8 = 0.8, c_3 = 15, \Delta_9 = 0.75, \Delta_{10} = 1.5, c_4 = 0.4, \Delta_{11} = 0.02, \Delta_{12} = 0.04, \alpha = 0.1, \beta = 0.01, \gamma = 0.6, h = 0.6, \eta = 0.5, \delta = 0.02$.

Based on these input data, the findings are as follows:
$p_f^* = 58.0034, T_f^* = 1.48131, t_{1f}^* = 0.888786, Q_f^* = 65.0234$ and $\tilde{P}(p_f^*, T_f^*) = 1646.53$.

## 8 Sensitivity Analysis

See Table 2.

### *8.1 Observations*

(1) When $\Delta_1 > \Delta_2$, i.e., demand parameter **a** decreases then the optimal profit $\tilde{P}(p_f^*, T_f^*)$ decreases and when $\Delta_1 < \Delta_2$ i.e., **a** increases then $\tilde{P}(p_f^*, T_f^*)$ increases.
(2) When $\Delta_3 > \Delta_4$, i.e., ordering cost **A** decreases then $\tilde{P}(p_f^*, T_f^*)$ slightly increases and when $\Delta_3 < \Delta_4$ i.e., **A** increases then profit decreases. Similarly, profit increases and decreases as $\Delta_5 > \Delta_6, \Delta_7 > \Delta_8, \Delta_9 > \Delta_{10}$ and $\Delta_5 < \Delta_6, \Delta_7 < \Delta_8, \Delta_9 < \Delta_{10}$ respectively.
(3) There is a small decline in profit $\tilde{P}(p_f^*, T_f^*)$, as $\Delta_{11} < \Delta_{12}$ i.e., deterioration cost $c_4$ increases and profit increases as $\Delta_{11} > \Delta_{12}$ i.e., $c_4$ decreases.

**Table 2** Sensitivity table with respect to system parameters

| Changing parameter | Change in parameter | Change in $T_f^*$ | Change in $p_f^*$ | Change in $t_{1f}^*$ | Change in $Q_f^*$ | Change in $\tilde{P}(p_f^*, T_f^*)$ |
|---|---|---|---|---|---|---|
| $\Delta_1, \Delta_2$ | (40, 20) | 1.57895 | 54.9325 | 0.947370 | 64.0666 | 1387.46 |
| | (10, 5) | 1.52093 | 56.7755 | 0.912558 | 64.7378 | 1545.21 |
| | (5, 10) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (20, 40) | 1.42152 | 59.8449 | 0.852912 | 65.2461 | 1792.81 |
| $\Delta_3, \Delta_4$ | (80, 40) | 1.42603 | 57.9725 | 0.855618 | 62.7247 | 1655.13 |
| | (20, 10) | 1.45933 | 57.9912 | 0.875598 | 64.1103 | 1649.93 |
| | (10, 20) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (40, 80) | 1.51398 | 58.0216 | 0.908388 | 66.3780 | 1641.53 |
| $\Delta_5, \Delta_6$ | (4.8, 2.4) | 1.47188 | 57.5999 | 0.883128 | 65.2270 | 1670.29 |
| | (1.2, 0.6) | 1.47660 | 57.8415 | 0.885960 | 65.0666 | 1660.11 |
| | (0.6, 1.2) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (2.4, 4.8) | 1.49085 | 58.2477 | 0.894510 | 65.0577 | 1615.99 |
| $\Delta_7, \Delta_8$ | (3.3, 1.6) | 1.49001 | 57.9809 | 0.894006 | 65.4258 | 1648.17 |
| | (0.8, 0.4) | 1.48605 | 57.9957 | 0.891630 | 65.2357 | 1647.42 |
| | (0.4, 0.8) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (1.6, 3.3) | 1.47002 | 58.0139 | 0.882012 | 64.5295 | 1644.42 |
| $\Delta_9, \Delta_{10}$ | (6, 3) | 1.48900 | 57.9833 | 0.893400 | 65.3794 | 1647.98 |
| | (1.5, 0.75) | 1.48576 | 57.9961 | 0.891456 | 65.2228 | 1647.36 |
| | (0.75, 1.5) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (3, 6) | 1.47135 | 58.0124 | 0.882810 | 64.5881 | 1644.67 |
| $\Delta_{11}, \Delta_{12}$ | (0.16, 0.08) | 1.48134 | 58.0018 | 0.888804 | 65.0271 | 1646.63 |
| | (0.04, 0.02) | 1.48133 | 58.0028 | 0.888798 | 65.0252 | 1646.59 |
| | (0.02, 0.04) | 1.48131 | 58.0034 | 0.888786 | 65.0234 | 1646.53 |
| | (0.08, 0.16) | 1.48128 | 58.0044 | 0.888768 | 65.0206 | 1646.41 |

# 9 Conclusion

In this study, the model is proposed in the following two senses: (1) crisp and (2) fuzzy. In fuzzy situation, demand rate, ordering cost, purchasing cost, deterioration cost, backordering cost, and opportunity cost are considered as triangular fuzzy numbers. It is assumed that the demand rate is a function of price and stock both, shortages are allowed and are partially backlogged. The proposed model is more practical as real life businesses are affected by the stock and price dependent demand. If there is a large pile of stock and a suitable selling price of the products is sustained in a business then it attracts the customers to buy more. In real life situations, all the shortages cannot be fully backlogged as some sales will be lost due to interruption of time, therefore partial backlogging is more realistic. In today's market due to the impreciseness of the inventory costs and demand, it is more useful to consider them as fuzzy numbers since the business strategy will be able to face the upper and down conditions of the market. Moreover, from sensitivity table, it is observed that the model is enough stable toward the changes in the system parameters.

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