A Fuzzified Production Model with Time Varying Demand Under Shortages and Inflation

Shalini Jain and S. R. Singh

Abstract We develop an inventory model with time-dependent demand rate and deterioration, allowing shortages. The production rate is assumed to be finite and proportional to the demand rate. The shortages are partially backlogged with time-dependent rate. Inflation is also taken in this model. Inflation plays a very significant role in inventory policy. We developed the model in both fuzzy and crisp sense. The model is solved logically to obtain the optimal solution of the problem. It is then illustrated with the help of numerical examples. Sensitivity of the optimal solution with respect to changes in the values of the system parameters is also studied.

Keywords Time-dependent demand · Shortages · Deterioration · Fuzzy

1 Introduction

Inventory control involves human capability to deal with uncertainty of future demand of stock items. Hence, the application of fuzzy reasoning models in inventory control systems is quite important as fuzzy inferring procedures are becoming essential in managing uncertainties. In the past, a great deal of research has been done in the areas of inventory control systems. But, only few of the researchers have contributed in the applications of fuzzy logic. [29] first developed a inventory model in fuzzy sense. [24] fuzzified the ordering cost into trapezoidal fuzzy number in the total cost of an inventory without backordering and obtained the fuzzy total cost. Later, they used the centroid method and gained the total cost in the fuzzy sense. [7] fuzzified the

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B. V. Babu et al. (eds.), *Proceedings of the Second International Conference on Soft Computing for Problem Solving (SocProS 2012), December 28–30, 2012*, Advances in Intelligent Systems and Computing 236, DOI: 10.1007/978-81-322-1602-5_43, © Springer India 2014

ordering cost, inventory cost, and backordering cost into trapezoidal fuzzy numbers and used the functional principle to obtain the estimate of the total cost in the fuzzy sense. [14] proposed an inventory model without shortages by fuzzifying the order quantity into a triangular fuzzy number. [28] generalized an inventory model without any backlogging for fuzzy order quantity and fuzzy total demand quantity. [5] considered the fuzzy problems for the mixture of backorders and lost sales in inventory model. The total expected annual cost is obtained in the fuzzy sense. [6] considered the mixture inventory model involving a fuzzy variable and obtained the total cost in the fuzzy sense. [22] consider inflation and apply discounted cash flow in a inventory model and formulated the total cost of the system using genetic algorithm. [18] presented solution changed model to a crisp multipurpose problem using defuzzification of fuzzy constraints and fuzzy chance-constrained programming methods.

Production is a process whereby raw material is converted into semifinished products and then converted into finished products. The main purpose of production function is to produce the goods and services demanded by the customers in the most efficient and economical way. So efficient management of the production function is of utmost importance in order to achieve this objective. An optimal production quantity model for a deteriorating item was developed by [15]. [25] proposed economic production quantity (EPQ) deteriorating inventory with partial backordering. [11] introduced an EPQ model with marketing policies and a deteriorating item. [23] developed a model with price and stock-dependent demand considering a production model for deteriorating items. [20] introduced a model that generates an economic run quantity solution and the total production solution simultaneously. Economic run quantity is an extended model of production model. The objective of production planning, and control, like that of all other manufacturing controls, is to contribute to the profits of the industry. As with inventory management and control, this is accomplished by keeping the customers satisfied during the meeting of delivery schedules.

Usually, it is assumed that lifetime of an item is immeasurable when it is in storage. However, there are abundant types of items such as food grains, highly volatile substances, radioactive materials, films, drugs, blood, fashion goods, electronic components, and high-tech products in which there is gradual loss of potential or value with a passage of time. Therefore, the effect of deterioration cannot be ignored in inventory models. [10] were the first to consider deterioration of inventory with constant demand. As time passed, several researchers developed inventory models by assuming either instantaneous or finite production with different assumptions on the patterns of deterioration. In this connection, researchers may refer to work by [4, 8, 9, 13, 15]. Also, some researchers [26, 27] have studied the chances and effect of integration and co-operation between the buyer and the producer of deteriorating items. Interested readers may review the articles by [19] and [12]. Lin and Gong assumed varying production rate and deteriorating inventory.

Inflation plays a very significant role in inventory models. Inflation refers to the movement in the general level of prices. It does not refer to changes in one price relative to other prices. Rather than measure inflation by using the actual rate at which prices are rising, some economists prefer a measure of inflation that reflects primarily only the systematic factors that act to raise prices. [3] developed the first

EOQ model taking inflation into the model. [1] developed the inventory decisions under inflationary condition. [2] proposed an economic order quantity under variable rate of inflation and mark-up prices. [17] criticized a net present value. [16] studied the inflation effects on inventory system. Yang et al. presented a deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. [21] presented a fuzzy inventory model under inflation.

In the present paper, we assume the production model with time-dependent demand and deterioration. As a result, the finite production rate is also time dependent. Shortages are allowed and are partially backlogged. The model developed in both fuzzy and crisp sense. An analytical solution of the model is discussed and is illustrated with the help of numerical examples. Sensitivity of the optimal solution with respect to changes in different parameter values is also examined.

2 Assumptions and Notations

The following assumptions and notations have been used throughout the paper:

1. The demand rate is deterministic and is a function of time.
2. The rate of production is finite.
3. Production rate depends on the demand rate.
4. Inflation and time value of money are considered.
5. Shortages are allowed.
6. Holding cost is taken to be variable in nature

The following notations are used in our study:

D(t) Demand rate (units/unit time), $D(t) = a+bt$, a and b are positive constants, $a > b$.
P(t) Production rate (units/unit time), k > 1, P(t) = kD(t) for any t.
$\theta(t)$ Rate of deterioration where $\theta(t) = \theta t$, θ is a positive constant.
$I_i(t)$ Inventory level at any time t.
s Per unit selling price of the item.
B Backlogging rate, $B = e^{-\delta t}$, δ is a positive constant.
r Constant representing the difference between the discount rate and inflation rate.
c Production cost per unit item.
c_3 Set up cost per unit item.
c_2 Shortage cost per unit item.
$c_1 + \alpha t$ Inventory holding cost per unit item per unit time, $\alpha > 0$.

3 Model Illustration

The problem has been formulated in two steps. In the first step, we formulate a crisp model and then in the next step, we extend the model into a fuzzy sense. The crisp formulation of the model has been presented here:

$$I_1'(t) + \theta t I_1(t) = k(a+bt) - (a+bt), 0 \le t \le T_1 \tag{1}$$

$$I_2'(t) + \theta t I_2(t) = -(a+bt), T_1 \le t \le T_2 \tag{2}$$

$$I_3'(t) = -e^{-\delta t}(a+bt), T_2 \le t \le T_3 \tag{3}$$

$$I_4'(t) = k(a+bt) - (a+bt), T_3 \le t \le T_4 \tag{4}$$

With the boundary conditions:

$$I_1(0) = 0, I_2(T_2) = 0, I_3(T_2) = 0, I_4(T_4) = 0 \tag{5}$$

Solutions of (1)–(4) are

$$I_1(t) = (k-1)\left[\left(at + \frac{bt^2}{2} + \frac{a\theta t^3}{6} + \frac{b\theta t^4}{8}\right)\right] e^{-\frac{\theta t^2}{2}} \tag{6}$$

$$I_2(t) = \left[a(T_2 - t) + \frac{b}{2}\left(T_2^2 - t^2\right) + \frac{a\theta}{6}\left(T_2^3 - t^3\right) + \frac{b\theta}{8}\left(T_2^4 - t^4\right)\right] e^{-\frac{\theta t^2}{2}} \tag{7}$$

$$I_3(t) = \left[a(T_2 - t) + \frac{b}{2}\left(T_2^2 - t^2\right)\right] \tag{8}$$

$$I_4(t) = (k-1)\left[a(t - T_4) + \frac{b}{2}\left(t^2 - T_4^2\right)\right] \tag{9}$$

At $t = T_1$, $I_1(T_1) = I_2(T_1)$. From Eqs. (6) and (7), we get

$$T_1 = f(T_2) \tag{10}$$

At $t = T_3$, $I_3(T_3) = I_4(T_3)$. From Eqs. (8) and (9), we get

$$T_3 = f(T_2, T_4) \tag{11}$$

The present worth of holding cost for the period under consideration

$$HC = \int_{0}^{T_1} (c_1 + \alpha t) I_1(t) e^{-rt} dt + c_1 \int_{T_1}^{T_2} (c_1 + \alpha t) I_2(t) e^{-rt} dt \tag{12}$$

Production has been taking place in the period $[0, T_1]$ and $[T_3, T_4]$, hence, the present worth of the production cost is:

$$PC = \left(\int_{0}^{T_1} k(a + bt) e^{-rt} dt + \int_{T_3}^{T_4} k(a + bt) e^{-rt} dt \right) \tag{13}$$

Before the start of a production run, the fixed cost to be borne by the producer is:

$$SPC = c_3 + c_3 e^{-rT_3} \tag{14}$$

Shortages are accumulated in the system during $[T_2, T_4]$. The maximum level of shortages are present at $t = T_4$. The total present worth of shortages during this time is

$$SC = \left(\int_{T_2}^{T_3} -I_3(t) e^{-rt} dt + \int_{T_3}^{T_4} -I_4(t) e^{-rt} dt \right) \tag{15}$$

Inventory is present and sold during $[0, T_4]$. The present worth of the generated revenue is given by

$$SP = s \left(\int_{0}^{T_1} D(t) e^{-rt} dt + \int_{T_1}^{T_2} D(t) e^{-rt} dt + \int_{T_2}^{T_3} D(t) e^{-\delta t} e^{-rt} dt + \int_{T_3}^{T_4} D(t) e^{-rt} dt \right) \tag{16}$$

So, the net profit of the system is represented by:

$$NP = \frac{s.SP - c_1.HC - c.PC - c_3 - c_2.SC}{T_4} \tag{17}$$

4 Fuzzy Modeling

In order to develop the model in a fuzzy environment, we consider the Production cost per inventory unit per unit time $\tilde{c}^{\vee}$ and Shortage cost per inventory unit per unit time $c_2^{\vee}$ as the triangular fuzzy numbers $\tilde{c} = (c - \Delta_1, c, c + \Delta_2)$ and $\tilde{c}_2 = (c_2 - \Delta_3, c_2, c_2 + \Delta_4)$ such that that $0 < \Delta_1 < c, 0 < \Delta_2, 0 < \Delta_3 < c_2, 0 < \Delta_4$

and where Δ_1, Δ_2, Δ_3, Δ_4 are determined by the decision maker based on the uncertainty of the problem. Thus, the Production cost c and the shortage cost c_2 are considered as the fuzzy numbers $\tilde{c}$ and $\tilde{c}_2$ with membership functions.

$$\mathrm{NP} = \frac{\mathrm{s.SP} - \mathrm{c_1.HC} - \hat{\mathrm{c}}.\mathrm{PC} - \mathrm{c_3.SPC} - \hat{\mathrm{c}}_2.\mathrm{SC}}{T_4} \tag{18}$$

$$\tilde{c} = [c - \Delta_1, c, c + \Delta_2] \tag{19}$$

$$\tilde{c}_2 = [c_2 - \Delta_3, c_2, c_2 + \Delta_4] \tag{20}$$

By Centroid Method, we get

$$\tilde{c} = c + \frac{1}{3}(\Delta_2 - \Delta_1) \tag{21}$$

$$\tilde{c}_2 = c_2 + \frac{1}{3}(\Delta_4 - \Delta_3) \tag{22}$$

$$\mathrm{F_1} = \frac{\mathrm{s.SP} - \mathrm{c_1.HC} - (\mathrm{c} + \Delta_2).\mathrm{PC} - \mathrm{c_3.SPC} - (\mathrm{c_2} + \Delta_4).\mathrm{SC}}{\mathrm{T_4}} \tag{23}$$

$$\mathrm{F_2} = \frac{\mathrm{s.SP} - \mathrm{c_1.HC} - \mathrm{c.PC} - \mathrm{c_3.SPC} - \mathrm{c_2.SC}}{\mathrm{T_4}} \tag{24}$$

$$\mathrm{F_3} = \frac{\mathrm{s.SP} - \mathrm{c_1.HC} - (\mathrm{c} - \Delta_1).\mathrm{PC} - \mathrm{c_3.SPC} - (\mathrm{c_2} - \Delta_3).\mathrm{SC}}{\mathrm{T_4}} \tag{25}$$

By the method of Defuzzification,

$$\mathrm{NP} = \frac{F_1 + 2F_2 + F_3}{4} \tag{26}$$

we find out Net profit in fuzzy sense.

5 Solution Procedure

To maximize total average profit per unit time (NP), the optimal values of t_2 and t_4 can be obtained by solving the following equations simultaneously

$$\frac{\partial NP}{\partial T_2} = 0 \tag{27}$$

and

$$\frac{\partial NP}{\partial T_4} = 0 \tag{28}$$

Provided, they satisfy the following conditions

$$\frac{\partial^2 NP}{\partial T_2^2} < 0, \frac{\partial^2 NP}{\partial T_4^2} < 0 \text{ and} \left(\frac{\partial^2 NP}{\partial T_2^2}\right)\left(\frac{\partial^2 NP}{\partial T_4^2}\right) - \left(\frac{\partial^2 NP}{\partial T_2 \partial T_4}\right)^2 > 0$$

Equation (16) is our objective function which needs to be maximized. For this, we use the classical optimization techniques. The Eqs. (27) and (28) obtained thereafter are highly nonlinear in the continuous variable T_2, T_4 and the discreet variables T_1, T_3. However, if we give particular values to the discreet variable T_1, T_3, our objective function becomes the function of two variables T_2 and T_4. We have used the mathematical software MATHEMATICA 8.0 to arrive at the solution of the system in consideration. We can obtain the optimal values of different values of the time with the help of software. With the use of these optimal values, Eq. (17) provides maximum total average profit per unit time of the system in consideration.

6 Numerical Example

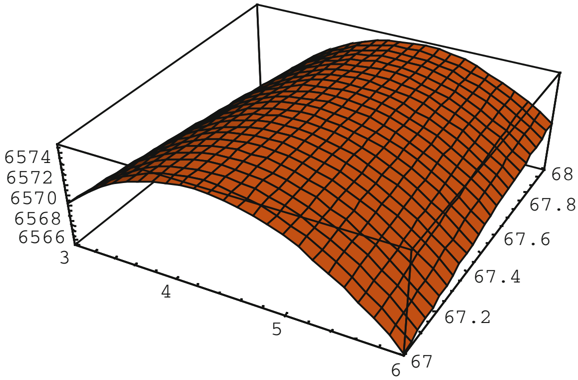
To illustrate the results, let us apply the proposed method to efficiently solve the following numerical example. For convenience, the values of the parameters are selected randomly.

$k = 1.5, a = 80, b = 0.2, \alpha = 0.02, c = 1.6, c_1 = 0.9, c_2 = 10, c_3 = 40, s = 22,$
$r = 0.03, \theta = 0.05, T_1 = 2, T_3 = 26.5472$

Optimal Solution of the Proposed Model

T_2	T_4	Total profit (NP)
4.49286	67.5452	6575.58

The graph shows the variation of the system cost with T_2 and T_4. From the figure, it is very clear that the total profit function is concave with respect to the two variables.

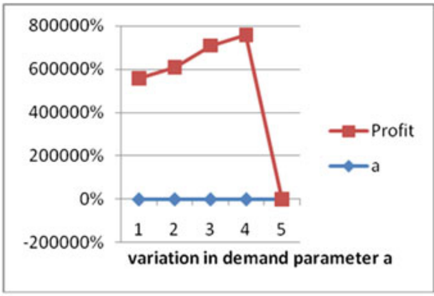


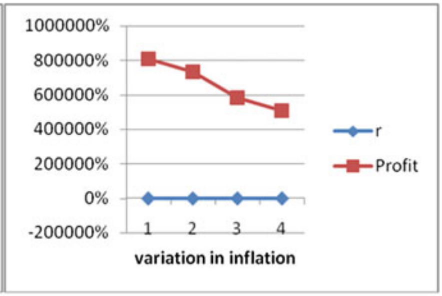
7 Sensitivity Analysis

We now study the effects of changes in the values of the system parameters $a,\ b,\ k,\ \alpha,\ c,\ c_1,\ c_2,\ c_3,\ s,\ r,\ \theta$ on the total system cost in consideration. The sensitivity analysis is performed by changing each of the parameters by -10–20 %, taking one parameter at a time and keeping the remaining parameters unchanged. The analysis is based on the familiar results obtained from Example.

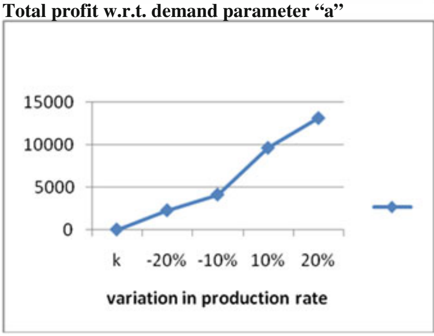
Parameter	Change in parameter			
	−20 %	−10 %	10 %	20 %
a	5579.61	6077.65	7073.41	7571.17
b	6255.86	6415.76	6735.3	6894.94
k	2257.36	4105.75	9606.43	13106.3
α	6575.99	6575.99	6575.98	6575.97
c	6583.38	6579.44	6571.67	6571.67
c_1	6578.17	6576.87	6574.28	6584.65
c_2	5409.32	5992.45	7158.71	7741.83
c_3	6575.75	6575.66	6576.49	6575.4
s	6416.06	6495.82	6655.33	6735.09
r	8082.49	7325.72	5831.22	5091.93
θ	6575.37	6575.47	6575.68	6575.78

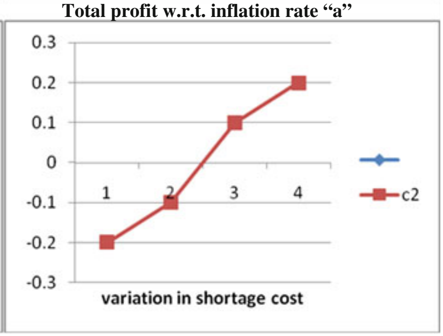
Graphical representation of the sensitivity results with respect to different system parameters have been plotted in figures shown below.





Total profit w.r.t. demand parameter "a" **Total profit w.r.t. inflation rate "a"**





Total profit w.r.t. production rate "k" **Total profit w.r.t. shortage cost "c**

8 Conclusion

The model has been developed for time-dependent demand with time-dependent deterioration in inventory. In this model, production rate is dependent on the demand rate. The average net profit function is the objective function in the case of crisp model. The same function extends to give the fuzzy model of the situation. This model is later defuzzified to a crisp model. Both the crisp as well as the fuzzy models have been solved numerically. In this study, shortages in inventory are allowed and the backlogging rate is taken as time dependent. The proposed model can be extended in numerous ways. For example, we may extend the inflation-dependent demand to inflation. Also, we could extend the model to incorporate some more features, such as quantity discount, and permissible delay in payment.

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