Engineering Optimization Using SOMGA

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Abstract Many real-life problems arising in science, business, engineering, etc. can be modeled as nonlinear constrained optimization problems. To solve these problems, population-based stochastic search methods have been frequently used in literature. In this paper, a population-based constraint-handling technique C-SOMGA is used to solve six engineering optimization problems. To show the efficiency of this algorithm, the results are compared with the previously quoted results.

Keywords SOMGA · C-SOMGA · Optimization

1 Introduction

Constraint handling is considered to be challenging and difficult task in optimization. Many real-life problems in engineering can be modeled as nonlinear constrained optimization problems. In view of their practical utility, there is a need to develop efficient and robust computational algorithms, which can numerically solve problems in different fields irrespective of their size. These days a number of probabilistic techniques are available for obtaining the global optimal solution of nonlinear optimization problems. Though GAs are very efficient at finding the global optimal solution of unconstrained or simply constrained (i.e., box constraints) optimization problems but encounter some difficulties in solving highly constraint nonlinear optimization problems, because the operators used in GAs are not very efficient in dealing with the constraints. Several methodologies have been developed to handle constraints when

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GAs are used to solve constrained optimization problems refer Kim and Myung [12], Michalewicz [13], Myung and Kim [14], Orvosh and Davis [15]. Deep and Dipti [8] proposed a penalty parameter free hybrid approach C-SOMGA for solving the nonlinear constrained optimization problems. It is not only easy to implement but also does not require any parameter to be fine-tuned for constraint handling. It works with a very low population size, hence uses low function evaluations where the term "function evaluations" represents the number of times an objective function is evaluated in the entire run. In this paper, six engineering optimization algorithms has been solved using C-SOMGA. The results obtained are compared with the previously quoted results. On the basis of the results, it is concluded that the C-SOMGA is efficient to solve these problems.

The paper is organized as follows: in Sect. 1, introduction is given; in Sect. 2, methodology of C-SOMGA is presented; in Sect. 3, mathematical models of the problems are given and results obtained using C-SOMGA are discussed and compared with the previously quoted results; and Sect. 4 summarizes the conclusions based on the present study.

2 Methodology of C-SOMGA

The algorithm C-SOMGA is an extension of SOMGA [7] for solving the constraint nonlinear optimization problems in which SOMGA is combined with constraint-handling tournament selection scheme, and as a result of this, C-SOMGA has been proposed. The methodology of C-SOMGA algorithm is as follows:

First, the individuals are generated randomly. These individuals compete with each other through constraint tournament selection method: Create new individuals via single-point crossover and bitwise mutation. Then, the best individual among them is considered as leader and all others are considered as active. For each active individual, a new population of size N is created, where N is the ratio of path length and step size. This population is nothing but the new positions of the active individual proceeds in the direction of the leader in n steps of the defined length. The movement of this individual is given by

$$x_{i,j}^{\text{MLnew}} = x_{i,j,\text{start}}^{\text{ML}} + \left(x_{L,j}^{\text{ML}} - x_{i,j,\text{start}}^{ML} \right) t \text{PRTVector}_j \tag{1}$$

where $t \in < 0, \quad \text{by Step to}, \quad \text{PathLength} >$,

ML	is actual migration loop.
$x_{i,j}^{\text{MLnew}}$	is the new positions of an individual.
$x_{i,j,\text{start}}^{\text{ML}}$	is the positions of active individual.
$x_{L,j}^{\text{ML}}$	is the positions of leader.

PRT vector is created before an individual proceeds toward leader. This parameter has the same effect as mutation in GA. It is defined in the range <0, 1>. Then, sort this population according to the fitness value in decreasing order. Starting from the best one of the new population, evaluate the constraint violation function described by Eq. 2.

$$\psi(x) = \sum_{m=1}^{M} [h_m(x)]^2 + \sum_{k=1}^{K} G_k \, [g_k(k)]^2 \tag{2}$$

where G_k is the Heaviside operator such that $G_k = 0$ for $g_k(x) \geq 0$ and $G_k = 1$ for $g_k(x) < 0$.

If $\psi(x) = 0$, replace the active individual with the current position and move to the next active individual and if $\psi(x) > 0$, then move to the next best position of the sorted new population. In this way, all the active individuals are replaced by the new updated feasible position. If no feasible solution is available, then active individual remains the same. At last, the best individuals (number equal to population size) from the previous and current generations are selected for the next generation. The computational steps of this approach are given below:

Step 1: Generate the initial population.
Step 2: Evaluate all individuals.
Step 3: Apply tournament selection for constrained optimization on all individuals to select the better individuals for the next generation.
Step 4: Apply crossover operator on all individuals with crossover probability P_c to produce new child individuals.
Step 5: Evaluate the new child individuals.
Step 6: Apply mutation operator on every bit of every individual of the population with mutation probability P_m.
Step 7: Evaluate the mutated individuals.
Step 8: Find leader (best fit individual) of the population and consider all others as active individuals of the population.
Step 9: For each active individual, a new population of size N is created. This population is nothing but the new positions of the active individual toward the leader in n steps of the defined length. The movement of this individual is given in Eq. (1).
Step 10: Sort new population with respect to fitness in decreasing order.
Step 11: For each individual in the sorted population, check feasibility criterion.
Step 12: If feasibility criterion is satisfied, replace the active individual with the new position, else move to next position in sort order, and go to Step 11.
Step 13: Select the best individuals (in fitness) of previous and current generation for the next generation via tournament selection.
Step 14: If termination criterion is satisfied go to 15 else go to Step 3.
Step 15: Report the best chromosome as the final optimal solution.

3 Mathematical Models of Engineering Optimization Problems

In this section, mathematical model of six engineering optimization problems has been given and the results obtained using C-SOMGA are compared with the available results. These models have been taken from the literature to see the performance of the C-SOMGA on constrained optimization problems. Many researchers used these models to demonstrate the performance of their techniques [2, 16–18]. The experimental setup for C-SOMGA is given in Table 1.

3.1 Gas Transmission Compressor Design

This problem is taken from Beightler and Phillips [2]. This is a real-life problem in which the values of design parameters P_1, x_1, x_2, x_3 are to be determined that will deliver 100 million cu. Ft. of gas per day with minimum cost for a gas pipe line transmission system. Here,

P_1 Compressor discharge pressure,
Q Flow rate,
x_1 Length between compressor stations (in miles),
x_2 Compressor ratio $= P_1/P_2$,
x_3 Pipe inside diameter (in inches).

The mathematical model of the problem is

$$\text{Minimize } g_0 = 8.61 \times 10^5 x_1^{1/2} x_2 x_3^{-2/3} x_4^{-1/2} + 3.69 \times 10^4 x_3 + 7.72 \times 10^8 \times 10^8 x_1^{-1} x_2^{0.219} - 765.43 \times 10^6 x_1^{-1}$$

subject to $x_4 x_2^{-2} + x_2^{-2} \leq 1$, where $x_1, x_2, x_3, x_4 > 0$.

Bounds on the variables are as follows:

$$20 \leq x_1 \leq 50, 1 \leq x_2 \leq 10, 20 \leq x_3 \leq 50, 0.1 \leq x_4 \leq 60$$

Table 1 Experimental setup

Population size	20
P_c	0.85
P_m	0.009
Step size	0.31
Path length	3
String length	20

Table 2 Optimal solution to the design of a gas transmission compressor

	Value of objective	Values of variables
Solution obtained by C-SOMGA	296.490×10^4	$x_1 = 49.9996$, $x_2 = 1.17834$, $x_3 = 24.5996$, $x_4 = 0.388482$
Solution given in Pant [16]	296.528×10^4	$x_1 = 50.000$, $x_2 = 1.183$, $x_3 = 24.347$, $x_4 = 0.339$
Solution given in Beightler and Phillips [2]	299×10^4	$x_1 = 28.760$, $x_2 = 1.109$, $x_3 = 25.030$, $x_4 = 0.230$

The problem turns out to be a constrained geometric programming problem. This problem is earlier solved by Beightler and Phillips [2], Verma [18], Thanh [17], and Pant [16]. The results obtained using C-SOMGA and those given in source are shown in Table 2. It is evident with the Table 2 that the cost obtained by C-SOMGA in deliver the gas per day that is 2964900 is lesser than the cost obtained by Pant, i.e., 2965280 and by Beightler and Phillips, i.e., 2990000. In other words, C-SOMGA provides far better results than previously quoted results.

3.2 Optimization of a Riser Design

This problem is taken from Gaindhar et al. [9]. The objective of this problem is to determine the optimal volume of the riser. Any metal will shrink in volume when it is allowed to cool and solidify from a molten state. A riser is a device by which the location of a shrinkage cavity is shifted from within the casting to the riser, which is an extraneous portion cast as an integral but distinct portion of the casting. After the casting is solidified, all extraneous parts are cut off leaving behind the desired casting free of any shrinkage cavity.

The basic requirement for the riser design is that the solidification time of the riser must not be less than the solidification time of the casting. From the practical point of view, it is considered advantageous to have top riser connected to the casting through a neck. The molding sand in the neck region gets up more heated as compared to the rest of the region surrounding the riser. This ensures molten metal in the region of the neck. This also facilitates cutting off of the riser from the casting after the casting has been solidified.

The mathematical modal of the problem, as given in Gaindhar et al. [9] is

$$\text{Minimize } f(x) = (1/4)\,\pi x_1 x_2^2 + (1/12)\,\pi x_4 \left(3 - 3x_4/x_3 + x_4^2/x_3^3\right) x_2$$

$$\text{subject to } 2E\left(5 + (x_4/x_3)\,(2 - x_4/x_3)\left(1 + x_3^2\right)^{1/2}\right) x + 4Ex_2^{-1}$$

$$- (x_4/3)\left(3 - 3x_4/x_3 + x_4^2/x_3^2\right) \le 1$$

$x_1, x_2, x_3, x_4 > 0.$
where

x_1 height of riser
x_2 diameter of the riser
E riser modulus constant ($E = 10 / 7$)
x_3 $\tan\theta$ and
x_4 height of the neck riser.

The variable bounds are as follows:

$$1 \leq x_1 \leq 8; 1 \leq x_2 \leq 10; 0 \leq x_3 \leq 1; 0 \leq x_4 \leq 1$$

This problem is earlier solved by Gaindhar et. al [9] and by Pant [16]. The numerical results obtained are compared with the available results and are presented in Table 3. It is evident with the Table 3 that the result obtained by C-SOMGA that is 290.78142 is better than the result obtained by Pant [16] i.e 290.8532 and by Gaindhar et al. [9] i.e. 290.8069.

3.3 Optimum Design of a Welded Beam

Optimum design of a welded beam problem is a well-known problem. The formulation of this problem is available in literature with two models. In model (a), the number of constraints is six and in model (b), it is seven. Both the models are described below:

Model (a):
This problem is taken from Beightler and Phillips [2]. In this problem, the assembly of the welded structure as is being considered for mass production. Outside considerations fix the material of the bar A as well as the design parameters F and L. Assuming that the design engineer has fixed the specifications, $F = 6{,}000$ lb, $L = 14$ in and bar $A = 1{,}010$ steel; the objective function is to find a feasible combination of x_1, x_2, x_3 and x$_4$ such that the total cost assembly construction is minimum.

Table 3 Optimal design of a riser

	Value of objective	Values of variables
Solution obtained by C-SOMGA	290.78142	$x_1 = 4.276, x_2 = 8.7510,$ $x_3 = 1, x_4 = 0.1001$
Solution given in Pant [16].	290.8532	$x_1 = 4.2233, x_2 = 8.6055,$ $x_3 = 1.0000, x_4 = 0.1000$
Solution given in Gaindhar et al. [9]	290.8069	$x_1 = 4.266, x_2 = 8.5710,$ $x_3 = 1.000, x_4 = 0.1000$

The mathematical model of the problem is
Minimize $g_0(X) = 1.1047x_1^2x_2 + 0.6735x_3x_4 + 0.04811x_2x_3x_4$
Subject to

$$g_1(X) = 16.8x_4^{-1}x_3^{-2} \leq 1,\ g_2(X) = x_1x_4^{-1} \leq 1,\ g_3(X) = 0.125x_1^{-1} \leq 1,$$
$$g_4(X) = 9.08x_3^{-3}x_4^{-1} \leq 1,\ g_5(X) = 0.09428x_3^{-1}x_4^{-3} + 0.02776x_3 \leq 1$$

$$g_6(X) = \left[\left(\begin{array}{l}\frac{F^2}{2}x_1^{-2}x_2^{-2} + \frac{F^2x_2^{-1}(L+x_2/12)}{2\left(\frac{x_2^2}{12}+\frac{(x_3+x_1)^2}{4}\right)^{-1}} \\ +\frac{F^2(L+x_2/2)^2\left(\frac{x_2^2+(x_3+x_1)^2}{4}\right)}{2x_1^2x_2^2\left(\frac{x_2^2}{12}+\frac{(x_3+x_1)^2}{4}\right)^2}\end{array}\right)\right]^{1/2} \leq 13,000$$

$(x_1, x_2, x_3, x_4) > 0.$
The variable bounds are as follows:

$$0.1 \leq x_1 \leq 1;\ 5 \leq x_2 \leq 7;\ 7 \leq x_3 \leq 9;\ 0.1 \leq x_4 \leq 1$$

Model (b):
This model is taken from Xiaohui et al. [19]. The objective is to minimize the cost of a welded beam subject to constraints on shear stress, bending stress in the beam, bucking load on the bar, end deflection of the beam, and side constraints. The problem can be stated as follows:

$$\text{Minimize} \quad f(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4\,(14.0 + x_2)$$

subject to

$$g_1(X) = \tau(X) - \tau_{\max} \leq 0$$
$$g_2(X) = \sigma(X) - \sigma_{\max} \leq 0$$
$$g_3(X) = x_1 - x_4 \leq 0$$
$$g_4(X) = 0.10471x_1^2 + 0.04811x_3x_4\,(14.0 + x_2) - 5.0 \leq 0$$
$$g_5(X) = 0.125 - x_1 \leq 0,$$
$$g_6(X) = \delta(X) - \delta_{\max} \leq 0$$
$$g_7(X) = P - P_c(X) \leq 0$$

where

$$\tau(X) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2}, \quad \tau'' = \frac{MR}{J}, \quad M = P\left(L + \frac{x_2}{2}\right), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}, \sigma(X) = \frac{6PL}{x_4x_3^2}, \quad \delta(X) = \frac{4PL^3}{Ex_3^3x_4}$$

$$P_c(X) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

$$P = 6{,}000\,lb, \quad L = 14\,in, \quad E = 30 \times 10^6\,psi, \quad G = 12 \times 10^6\,psi,$$
$$\pi_{\max} = 13{,}600\,psi, \quad \sigma_{\max} = 30{,}000\,psi, \quad \delta_{\max} = 0.25\,in$$

The following ranges of the variables were used:

$$0.1 \le x_1 \le 2, \quad 0.1 \le x_2 \le 10, \quad 0.1 \le x_3 \le 10, \quad 0.1 \le x_4 \le 2$$

Both the models are solved by C-SOMGA. The numerical results obtained and the results given in source are presented in Table 4 for model (a) and Table 5 for model (b).

In Table 4, although the results available in source are lesser than the results obtained by C-SOMGA, but the solutions are not satisfying the feasibility conditions. Hence, these solutions cannot be accepted. The result obtained by C-SOMGA is a feasible solution. Therefore, C-SOMGA is best in this problem.

In Table 5, the result attained by C-SOMGA is superior to Coello [5] and Deb [6] but slightly inferior at fifth place to Xiaohui et al. [19]. It shows that the results are comparable.

3.4 Optimal Capacity of Gas Production Facilities

This problem is taken from Beightler and Phillips [2]. This is the problem of determining the optimum capacity of production facilities that combine to make an oxygen

Table 4 Optimal design of a welded beam based on model (a)

	Value of objective	Value of variables	Feasibility
Solution obtained by C-SOMGA	2.45694	$x_1 = 0.244241, x_2 = 6.4712,$ $x_3 = 8.43726, x_4 = 0.244364$	Satisfied
Solution given in Pant [16]	1.9786	$x_1 = 0.1489, x_2 = 5.000,$ $x_3 = 8.2736, x_4 = 0.2454$	Not Satisfied
Solution given in Beightler and Phillips [2]	2.3860	$x_1 = 0.2455, x_2 = 6.1960,$ $x_3 = 8.2730, x_4 = 0.2455$	Not Satisfied

Table 5 Optimal design of a welded beam based on model (b)

	Value of objective	Value of variables
Solution obtained by C-SOMGA	1.72486	$x_1 = 0.205731$, $x_2 = 3.47048$, $x_3 = 9.03669$, $x_4 = 0.20573$
Solution given in Xiaohui [19]	1.72485084	$x_1 = 0.20573$, $x_2 = 3.47049$, $x_3 = 9.03662$, $x_4 = 0.20573$
Solution given in Coello [5]	1.74830941	$x_1 = 0.2088$, $x_2 = 3.4205$, $x_3 = 8.9975$, $x_4 = .2100$
Solution given in Deb [6]	2.43311600	$x_1 = 0.2489$, $x_2 = 6.1730$, $x_3 = 8.1739$, $x_4 = 0.2533$

Table 6 Optimal capacity of gas production facilities

	Value of objective	Value of variables
Solution obtained by C-SOMGA	169.844	$x_1 = 17.500$, $x_2 = 600.000$,
Solution given in Pant [16]	169.844	$x_1 = 17.500$, $x_2 = 600.000$,
Solution given in Beightler and Phillips [2]	173.760	$x_1 = 17.500$, $x_2 = 465.000$,

producing and storing system. Oxygen for basic oxygen furnace is produced at a steady-state level. The demand for oxygen is cyclic with a period of one hour, which is too short to allow an adjustment of level of production to the demand. Hence, the manager of the plant has two alternatives:

1. He can keep the production at the maximum demand level; excess production is lost in the atmosphere.
2. He can keep the production at lower level; excess production is compressed and stored for use during the high demand period. The mathematical model of the problem is

$$\text{Minimize } g_0(\mathrm{X}) = 61.8 + 5.72x_1 + .2623\left[(40 - x_1)\ln\frac{x_2}{200}\right]^{-0.85} + .087\,(40 - x_1)\ln\frac{x_2}{200} + 700.23x_2^{-.75}$$

subject to $x_1 \geq 17.5$, $x_2 \geq 200$, $x_1, x_2 > 0$.

The variable bounds are as follows:$17.5 \leq x_1 \leq 40$; $300 \leq x_2 \leq 600$

The numerical results obtained using C-SOMGA and the numerical results given in source are presented in Table 6. In this problem, C-SOMGA produced better results than Beightler and Philips [2] but similar results as obtained by Pant [16] using GRST.

Table 7 Minimization of the weight of a tension/compression Spring

	Value of objective	Value of variables
Solution obtained by C-SOMGA	.0126656	$x_1 = .0516216, x_2 = 0.355094, x_3 = 11.385$
Solution given in Xiaohui [19]	0.0126661409	$x_1 = 0.05147, x_2 = .35138394, x_3 = 11.60865920$
Solution given in Coello [5]	.0127047834	$x_1 = .051480, x_2 = .351661, x_3 = 11.632201$
Solution given in Arora [1]	.127302737	$x_1 = .053396, x_2 = .399180, x_3 = 9.185400.$

3.5 Minimization of the Weight of a Tension/Compression Spring

This problem was described by Arora [1] and Belegundu [3]. The problem consists of minimizing the weight of a tension/compression spring subject to constrains on minimum deflection, shear stress, surge frequency, limits on outside diameter and on design variables. The design variables are the mean coil diameter D, the wire diameter d, and the number of active coils N. The problem can be expressed as follows:

$$Minimize\, f(X) = (N+2)\, Dd^2$$

subject to

$$g_1(X) = 1 - \frac{D^3 N}{71785 d^4} \leq 0$$

$$g_2(X) = \frac{4D^2 - dD}{12566\left(Dd^3 - d^4\right)} + \frac{1}{5108 d^2} - 1 \leq 0$$

$$g_3(X) = 1 - \frac{140.45 d}{D^2 N} \leq 0$$

$$g_4(X) = \frac{D + d}{1.5} - 1 \leq 0.$$

The following ranges of the variables were used:

$$0.05 \leq x_1 \leq 2, \quad 0.25 \leq x_2 \leq 1.3, \quad 2.0 \leq x_3 \leq 15.$$

The numerical results of the solution obtained using C-SOMGA and the numerical results given in source are presented in Table 7. The result attained by C-SOMGA is superior to Coello and Mezura [4] and Arora [1] at the fourth place and at the sixth place to Xiaohui et al. [19]. Hence, the results are comparable.

3.6 Himmelblau's Nonlinear Optimization Problem

This problem has been taken from Xiaohui [19]. This problem was proposed by Himmelblau [10], and it has been used before as a benchmark for several evolutionary algorithm-based techniques. In this problem, there are five design variables, six nonlinear inequality constraints, and ten boundary conditions. The problem can be stated as follows:

$$\textit{Minimize}\ f(X) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.2932239x_1 - 40792.141$$

subject to

$$0 \le 85.334407 + .0056858x_2x_5 + .00026x_1x_4 - .0022053x_3x_5 \le 92$$
$$90 \le 80.51249 + 0.0071317x_2x_5 + 0.00026x_1x_2 + 0.0021813x_3^2 \le 110$$
$$20 \le 9.300961 + 0.0047026x_3x_5 + 0.0012547x_1x_3 + 0.0019085x_3x_4 \le 25$$
$$78 \le x_1 \le 102,\quad 33 \le x_2 \le 45,\quad 27 \le x_3 \le 45,\quad 27 \le x_4 \le 45,\quad 27 \le x_5 \le 45.$$

The results obtained by C-SOMGA and available from the other source are presented in Table 8. C-SOMGA gives better results than Coello and Mezura [4] and Homaifar et al [11] and results are comparable to Xiaohui et al. [19].

Table 8 Himmelblau's Nonlinear Optimization Problem

	Value of objective	Value of variables
Solution obtained by C-SOMGA	−31025.6	$x_1 = 78, x_2 = 33.0001,$ $x_3 = 27.071, x_4 = 45,$ $x_5 = 44.969$
Solution given in Xiaohui [19]	−31025.56142	$x_1 = 78.0, x_2 = 33.0,$ $x_3 = 27.070997, x_4 = 45,$ $x_5 = 44.96924255$
Solution given in Coello [5]	−31020.859	$x_1 = 78.0495, x_2 = 33.0070,$ $x_3 = 27.0810, x_4 = 45,$ $x_5 = 44.9400$
Solution given in Homaifar et al. [11]	−30665.609	$x_1 = 78.0000, x_2 = 33.0000,$ $x_3 = 29.9950, x_4 = 45,$ $x_5 = 36.7760.$

4 Conclusions

In this paper, six real-life constrained optimization problems arising in various fields of engineering have been solved. For solving these constrained optimization problems, a population-based hybridized algorithm C-SOMGA has been used. In four problems, C-SOMGA provides better results than the previously quoted results, and in two problems, results are comparable. The algorithm requires only 20 population size for solving these problems. It is therefore concluded that C-SOMGA is well suited for obtaining the global optimal solution of engineering optimization problems.

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