Improving the Performance of the Optimization Technique Using Chaotic Algorithm

R. Arunkumar and V. Jothiprakash

Abstract Optimizing the operations of a multi-reservoir systems are complex because of their larger dimension and convexity of the problem. The advancement of soft computing techniques not only overcomes the drawbacks of conventional techniques but also solves the complex problems in a simple manner. However, if the problem is too complex with hardbound variables, the simple evolutionary algorithm results in slower convergence and sub-optimal solutions. In evolutionary algorithms, the search for global optimum starts from the randomly generated initial population. Thus, initializing the algorithm with a better initial population not only results in faster convergence but also results in global optimal solution. Hence in the present study, chaotic algorithm is used to generate the initial population and coupled with genetic algorithm (GA) to optimize the hydropower production from a multi-reservoir system in India. On comparing the results with simple GA, it is found that the chaotic genetic algorithm (CGA) has produced slightly more hydropower than simple GA in fewer generations and also converged quickly.

Keywords Optimization • Genetic algorithm • Chaotic algorithm **Multi-hydropower system**.

1 Introduction

The last decade had witnessed several optimization techniques from conventional linear programming to recently soft computing techniques. Among the soft computing techniques, the genetic algorithm (GA) had been widely used for optimization of

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reservoir operation [\[1](#page-6-0)[–3\]](#page-6-1). Application of simple GA technique to solve real-life water resources problems, especially optimizing multi-hydropower system is always cumbersome because of the complex nature, large number of variables and nonlinearity of the problem. Some recent studies reported that simple optimization techniques often succumbs to premature convergence and results in local optimal solution for complex water resources problems [\[4](#page-6-2)[–6](#page-6-3)]. Hence, still researches are emerging to introduce new techniques and also to improve the performance of the existing techniques to achieve global optimum and faster convergence.

The search for global optimal solution in evolutionary algorithms begins from the randomly generated initial population. Thus, a better initial population leads to faster convergence and not only saves substantial computational time but also results in global optimal solution. Hence, to generate a good initial population, chaotic algorithm is used, since chaos is highly sensitivity to the initial value, ergodic, and randomness in nature [\[7](#page-6-4)]. Yuan et al. [\[4\]](#page-6-2) proposed a hybrid chaotic genetic algorithm (HCGA) model to prevent premature convergence for a short-term hydropower scheduling problem. Cheng et al. [\[5](#page-6-5)] optimized the hydropower reservoir operation using HCGA and reported that the long-term average annual energy production was the best in HCGA and also it converges faster than the standard GA. It was also reported that the combination of chaotic characteristics along with general optimization algorithm would more likely result in global optimal solution. Han and Lu [\[6\]](#page-6-3) proposed a mutative scale chaos optimization algorithm (MSCOA) for the economic load dispatch problem. Huang et al. [\[8\]](#page-6-6) optimized the hydropower reservoir with ecological consideration using chaotic genetic algorithm (CGA) approach. These studies show that GA results better when coupled with chaotic algorithm and also escapes premature convergence. Hence in this study, in order to improve the performance of GA, chaotic algorithm is used to generate initial population and tested for a real-life hydropower system. The proposed algorithm is applied to maximize the hydropower production from a multi-reservoir system, namely Koyna hydroelectric project (KHEP).

The KHEP consists of two reservoirs, namely Koyna and Kolkewadi, which has four powerhouses to a total capacity of 1,960 MW [\[9\]](#page-7-0). Earlier, Jothiprakash and Arunkumar [\[10\]](#page-7-1) optimized the operation of Koyna reservoir alone considering it as a single reservoir system. However, in the present study, both the reservoirs are considered and optimized as multi-reservoir system. Apart from hydropower production, it also serves multiple purposes such as irrigation and flood control. In this system, the major powerhouses of the Koyna reservoir and Kolkewadi reservoir are in the western side and the irrigation releases are in the eastern side. The diversion of large quantity of water toward the western side for power production has resulted in disputes from the eastern side stakeholders, and hence, it was limited by Krishna water dispute tribunal [\[11\]](#page-7-2). This limiting constraint on the discharge for power production from the western side powerhouses made the system more complex by making the variables hard bound. This hard-binding constraint is considered in this study.

2 Chaotic Genetic Algorithm

GA is a search and optimization algorithm based on the principles of natural genetics [\[12\]](#page-7-3). In contrast to traditional optimization technique, GA searches the optimal solution from a randomly generated population within the upper and lower bounds of the variables. Each solution is represented through group of genes (sub-string) called chromosome (string) in the population space (search space). Each gene controls one or more features of the chromosomes. In the present study, the chaotic logistic mapping method is used to generate the initial population. Chaos often exists in nonlinear systems [\[13](#page-7-4)] and exhibits many good properties such as ergodicity, stochasticity, and irregularity [\[7](#page-6-4)]. May [\[7\]](#page-6-4) proposed a one-dimensional logistic mapping equation to generate a chaotic sequence. It is given as:

$$
Y_{j+1} = \lambda Y_j (1 - Y_j) \quad j = 1, 2, 3 \dots \tag{1}
$$

where λ is a control parameter and varies between $0 \leq \lambda \leq 4$. The chaotic sequence is produced when λ is equal to 4. The initial random variable (Y_1) is generated in the range between 0 and 1; however, it should not be equal to 0.25, 0.5, and 0.75, since it leads to a deterministic sequence [\[7\]](#page-6-4). Thus, the generated sequence using the logistic Eq. [\(1\)](#page-2-0) is highly irregular and possesses chaotic characteristics. Each variable in the sequence is dependent on initial variable, and a small change in initial value causes a large difference in its long-time behavior, which is the basic characteristic of chaos. This can be correlated with the releases in reservoir operation, such that the releases in the subsequent months depend on previous month releases. Hence in the present study, the initial population is generated as floating-point chaotic values within the upper (UB_i) and lower bounds (LB_i) of the variables using the equation:

$$
X_{i,j} = Y_{i,j} \times (\text{UB}_j - \text{LB}_j) + \text{LB}_j \quad i = 1, 2, \dots, N_p; \ j = 1, 2, \dots, N_v \tag{2}
$$

where $X_{i,j}$ is the '*j*th' sub-string of '*i*th' string, N_p is the population size, $Y_{i,j}$ is the chaotic variable, and N_v is the number of variables. Once the initial population is generated, the fitness of each string is evaluated using an appropriate fitness function. Based on the fitness value, the strings are selected for crossover and mutation to create a new population for the next generation. The tournament selection is used in the present study, since it provides selective pressure by holding a tournament among the selected individuals [\[12](#page-7-3)]. In the tournament selection, the fitness of the randomly selected strings from the population is compared with each other and the string having with higher fitness value will be copied to the mating pool. This process is repeated until the mating pool is filled with strings for generating new offspring for the next generation. The mating pool comprising the winners of the tournament will have higher average fitness value. Then, the strings in the mating pool are made to cross each other to create a new population. The simulated binary crossover (SBX) [\[14](#page-7-5)] is applied to create new population. The following steps were performed during SBX operation $[12]$. In the first step, a random number (u_i) between 0 and 1 is generated.

Then, the spread factor (β_{qi}) is computed using the equation

$$
\beta_{qi} = \begin{cases} (2u_i)^{\frac{1}{(\eta_c+1)}}, & \text{if } u_i < 0.5\\ \left(\frac{1}{2(1-u_i)}\right)^{\frac{1}{(\eta_c+1)}}, & \text{otherwise} \end{cases}
$$
(3)

where η_c is the distribution index for crossover. Deb [\[12\]](#page-7-3) reported that larger value of η_c produces '*near-parent*' offspring's and vice versa. Then, the off-springs $x_i^{1,t+1}$ and $x_i^{2,t+1}$ are computed from $x_i^{1,t}$ and $x_i^{2,t}$ using the equations,

$$
x_i^{1,t+1} = 0.5 \left[\left(1 + \beta_{qi} \right) x_i^{1,t} + \left(1 - \beta_{qi} \right) x_i^{2,t} \right]
$$
 (4)

$$
x_i^{2,t+1} = 0.5 \left[\left(1 - \beta_{qi} \right) x_i^{1,t} + \left(1 + \beta_{qi} \right) x_i^{2,t} \right]
$$
 (5)

where $x_i^{1,t}$ and $x_i^{2,t}$ are the parent string with *i*th sub-string in the *t*th generation. If the created offsprings are not within the upper and lower limits, the probability distribution needs to be adjusted accordingly. The new two offsprings are symmetric about the parent to avoid bias toward any particular parent solution in a single crossover operation. After crossover, the strings are subjected to mutation. The mutation operator introduces random changes into the characteristic of the offsprings. Mutation is generally applied at the sub-string level at a very small rate and depends on the length of the string. The mutation reintroduces the genetic diversity into the population and assists the search to escape from the local optima [\[12\]](#page-7-3). Then, the fitness of the newly created population is evaluated, and the procedure is continued until the termination criteria are reached.

3 Hydropower Model Development

The objective of the present study is to maximize the power production from all the four powerhouses of the KHEP and is expressed as:

$$
\text{Max } Z = \sum_{t=1}^{12} \sum_{n=1}^{4} \text{PH}_{n,t} \tag{6}
$$

where $PH_{n,t}$ is the power production from the powerhouse '*n*' during the time period '*t*' in terms of kWh. The hydropower production from the power plant [\[15\]](#page-7-6) is given by

$$
PH_{n,t} = K \times R_{n,t} \times HN_{n,t} \times \eta
$$
 (7)

where *K* is the constant for converting hydropower production in terms of kilo Watt hour (kWh), $R_{n,t}$ is discharge to the powerhouse '*n*' during the time period '*t*', $HN_{n,t}$

is the net head available for the powerhouse '*n*' during the time period '*t*', and η is the plant efficiency.

The above objective function is subjected to various constraints. They are:

$$
H_{n,t} \ge \text{MDDL}_{n,t} \qquad t = 1, 2, \dots 12; \ n = 1, 2, 3, 4 \tag{8}
$$

$$
PH_{n,t} \le P \max_{n,t} \qquad t = 1, 2...12; \ n = 1, 2, 3, 4 \tag{9}
$$

$$
R_{4,t} \geq ID_t \qquad t = 1, 2 \dots 12 \tag{10}
$$

$$
S_{x,\min} \le S_{x,t} \le S_{x,\max} \qquad t = 1, 2 \dots 12; \ \mathbf{x} = 1, 2 \tag{11}
$$

$$
S_{1,(t+1)} = S_{1,t} + I_{1,t} - \sum_{n=1}^{1,3,4} R_{n,t} - O_{1,t} - E_{1,t} \qquad t = 1, 2...12
$$
 (12)

$$
S_{2,(t+1)} = S_{2,t} + I_{2,t} + R_{1,t} + R_{3,t} - R_{2,t} - O_{2,t} - E_{2,t} \qquad t = 1, 2, \dots 12 \tag{13}
$$

$$
O_{x,t} = S_{x,(t+1)} - S_{x,\text{max}} \qquad t = 1, 2 \dots 12 \tag{14}
$$

$$
O_{x,t} \ge 0 \t t = 1, 2...12 \t (15)
$$

where $MDDL_{n,t}$ is the minimum drawdown level (m) for the powerhouse '*n*'; *Pmax_{nt}* is the maximum generation capacity (kWh) for the powerhouse '*n*'; R_4 _{*t*} is the irrigation release (10⁶ m³); ID_t is the monthly irrigation demand (10⁶ m³); $S_{x,\text{min}}$ is the minimum storage of the reservoir '*x*' (10⁶ m³); $S_{x,\text{max}}$ is the maximum storage of the reservoir '*x*' (10⁶ m³); $S_{x,t}$ is the storage in the reservoir '*x*' (10⁶ m³); $S_{x,(t+1)}$ is the final storage in the reservoir '*x*' (10⁶ m³); $I_{x,t}$ is the inflow into the reservoir '*x*' (10⁶ m³); $\ddot{O}_{x,t}$ is the overflow from the reservoir '*x*' (10⁶ m³); $E_{x,t}$ is the evaporation losses from the reservoir (10^6 m^3) ; *t* is the time period.

As already stated, the diversion of water to the western side powerhouses are limited by KWDT [\[11](#page-7-2)]. This constraint is given by:

$$
\sum_{t=1}^{12} \sum_{n=1,3} R_{n,t} \le R_{w,\text{max}} \tag{16}
$$

$$
\sum_{t=1}^{12} R_{4,t} \le \text{AID}_{\text{max}} \tag{17}
$$

where $R_{w, \text{max}}$ is the maximum water that can be diverted to the western side for power production, and AID_{max} is the water to be released annually for irrigation to the eastern side. *R*4*,^t* is the monthly irrigation release on the eastern side of the reservoir. These constraints make the system more complex by limiting the discharge to the powerhouses.

Fig. 1 Comparison of CGA and simple GA technique

4 Results and Discussion

The proposed CGA is applied to a complex multi-powerhouse system, namely KHEP to maximize its power production. The KHEP is one of the largest hydropower projects in India and consists of two reservoirs with various stages of development. The constraint on diverting large quantity of water for power production on the western side powerhouses makes the problem complex. This hard-binding constraint on discharge restricts the operation of the powerhouses on the western side to is full potential. Evaporation is one of the important components of reservoir operation studies, since a considerable amount of water is lost, especially from large reservoirs. Hence in the present study, the regression equation developed by Arunkumar and Jothiprakash [\[16\]](#page-7-7) for estimating reservoir evaporation is considered and directly incorporated in the continuity equation. Both the CGA and simple GA used in the study employs tournament selection, simulated binary crossover, and random mutation for comparison. The crossover probability of CGA and simple GA is varied from 0.5 to 0.95 with an increment of 0.05 and found that 0.80 resulted better. The mutation probability is fixed as the ratio of the number of variable (1/*n*) [\[12\]](#page-7-3). The elitism is also applied to preserve the best strings in the population. The algorithm is evaluated for 1,000 generation for a population size of 250. The constraints of the problems are handled by penalty function approach. Thus, heavy penalties are imposed on fitness function, if the constraints are violated. For each constraint different value of penalties are assumed. The total annual power production shows that CGA has resulted slightly higher power production of 3,225.71 \times 10⁶ kWh than simple GA (3,224.23 \times 10⁶ kWh). However, the convergence of these techniques over the generation varied largely and is given in Fig. [1.](#page-5-0) From the figure, it can be observed the convergence to optimal solution by CGA is faster than simple GA. It can also be noted that due to hard-binding constraints on releases, both the

techniques have resulted in sub-optimal solution for first few generations. Thus imposing heavy penalty on fitness function leads to negative fitness value and results in sub-optimal solution. However, the CGA has satisfied all the constraints and reaches the optimal solution in lesser generations than simple GA. This also shows that when hard-binding constraints are imposed strictly, the simple GA takes more generations for convergence. The time taken by CGA is 1,790.753 s where as simple GA took 2,418.966 s.

5 Conclusion

In the present study, the chaotic algorithm is combined with genetic algorithm to maximize the power production from a multi-powerhouse system. Based on the performances, it is found that the chaotic genetic algorithm resulted slightly higher power production than simple GA within fewer generations and also converged quickly. This shows that coupling the chaotic algorithm with evolutionary algorithm has enriched the global search of the optimization technique by having better initial population. Thus, it may be concluded that the chaotic algorithm with general optimizer converges quickly to global optimum in lesser generation compared to simple optimization technique.

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