Facial Boundary Image Reconstruction Using Elliptical Approximation and Convex Hull

A. Bindu, C. N. Ravi Kumar, S. Harsha and N. Bhaskar

Abstract The Stretch of Biometrics' applications can only be limited by limiting ones' imagination! Biometrics is the Science and Technology of measuring and analyzing biological data for authentication purposes. In addition to verification the guiding force behind biometric verification has been convinience. Face enjoys a prime position in the realm of biometrics because it is very easily accessible when compared to the other biometrics. Efficient accomplishment of Face Recognition confronts innumerable hurdles in the form of variations in lighting conditions during image capture, Occlusions, damage in facial portions due to accidents etc. The application of Facial Image Inpainting also fails when the occlusions or the deformalities are present across the boundary of the object of interest(face), since the bounds for the application of the inpainting algorithm is not precisely defined. Hence recovery of the complete picture of a human face from partially occluded images is quite a challenge in Image Processing. The proposed FIREACH algorithm concentrates on the generation of a convex hull and a non linear elliptical approximation of the depleted and partially visible boundary of the human face, given different parameters to achieve an Efficient Boundary Recovery. The Boundary Recovery Algorithm is a pre-processing step which aids in setting up of a suitable platform for the proficient application of the Facial Image Inpainting.

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1 Introduction

Biometrics finds its application in Identity Authentication by means of analysis and measurement of the biological data. Face biometric enjoys a prime position for its ease of accessibility. Recognition of the face biometric is accomplished with the help of important features extracted from the subjects' face. But, efficient face recognition under some circumstances becomes very difficult viz., if facial region is severely disfigured during an accident [1], if the facial region is occluded with structural elements like beard, moustache, goggles [2–5] and other accessories, external occlusions like door, people etc. covering the prominent facial features at various degrees. Such hindrances are solved considerably by the application of Inpainting Algorithms [6–11] etc. Inpainting Algorithms aid in filling the missing image data in a visually credible manner such that the change is difficult to be percieved by a naïve unexpecting viewer [12]. But, there exist some extremities where it is totally impossible to get conducive results even after the application of Inpainting Algorithms. The proposed research work deals with such an extreme case, where the occlusion is present across the boundary of the region of interest. Under such a circumstance it becomes difficult to predict with precision the boundary of the region to be inpainted. Through the proposed work a modest effort has been initiated towards disoccluding the boundary of the region of interest thereby setting up a limiting condition for the application of the Inpainting Algorithm.

The proposed FIREACH Algorithm accomplishes its task of occluded face contour evolution with the help of 7 major steps, where the Step 1 involves detecting the face region by the application of the skin illumination compensation algorithm [13] which projects only the visible facial portions present in the image. Step 2 involves edge detection. Step 3 evolves the contour using Convex Hull for each of the visible face regions. Step 4 fits the ellipse around each of the visible face regions. Step 5 deals with the extension of the Convex Hull region into the depleted region until the boundary limits set by the ellipse fit. Step 6 involves finding the centroid followed by the Step 7 which evolves better contour of the facial regions, obtained by selectively choosing the contour points of the evolved face between the outputs obtained from Step 3 and Step 4.

2 Literature Review

The initiation in the work regarding manual face reconstruction (through surgery) dates back to a period unimaginable—"Reconstructive surgery techniques were being carried out in India around 800 BC by Sushruta, who is popularly known as the father of Surgery for his important contributions towards the field of plastic surgery and cataract surgery" [Wikipedia]. The research work defining the geographical boundaries for surgical reconstruction of marred face was proposed by Penn [1]. Nowadays, in the computer age the human kind is more reliant on getting the task automated by the machine. The research work initiated by Bertalmio [6] was a leading light towards opening a new avenue for image disocclusion [14] using the technique used by artists to retouch the damaged paintings. This new technique came to be widely known as Image Inpainting. In this technique the depleted region or the hole is filled by using the structure elements or isophotes (image details) present at the boundary of the hole (depleted image region). This inpainting technique was meddled by many scientists to achieve better results, but it was localised to the removal of small scratches in the image. Large depleted image regions could not be inpainted efficiently, for which [15] gave a solution in the form of Exemplar Based Inpainting. The Exemplar technique efficiently propogates both texture and structure information into the depleted region of the image. The evolution of the inpainting techniques was extrapolated to the 3D surfaces by Han and Zhu [16, Jin et al. [17]. The earliest work on line and edge extraction dates back to the early 60s. The process edge extraction for disocclusion using image segmentation and depth extraction was initiated by Nitzberg et al. [18] where the detection of T-junctions was accomplished and the connection was established amongst the related T-Junctions with the help of edge length and curvature minimizing energy function. The linking was accomplished by a new edge whose length and curvature are minimum. The drawback with this method was that it could be applied to highly segmented regions of interest with a few T Junctions available. Following the work proposed by Nitzberg et al. [18], Masnou and Morel [19] proposed a technique which involved evolving the deformities using level lines of larger objects in images relying on variational formulations and level sets. The method portrayed success without the necessity for T Junctions. The method by Masnou and Morel [19] had an overhead of finding the level lines of the region of interest and was efficient in removing minor scratches or deformities in the image. The contributory work in [6] came up with a technique which relied on manually selecting the region to be disoccluded using PDE which propogates the information in the direction of the isophotes and successfully removes minor scratches in the image. Bounded Variation based image model stated that the oscillation range of the image level lines are finitely constricted [20]. Following which the Total Variation tries to constrain and minimize the curve length of level lines over Bounded variation. But, it was later realized that a realistic inpainting output is achieved when the curvature is taken into account. Ballester et al. [21] stipulated an inpainting technique with a due consideration given to the curvature which was synonymous with Euler Elastica, in which prior models have a prime significance. The research contributions in [8] and [9] succeeding the previous inpainting contributions coined an inpainting approach with the nomenclature Curvature Driven Diffusion(CDD)—a computational scheme based on numerical PDE's, which is efficient of handling topologically complex Inpainting Domains. All the decisions realized in [8, 9, 11, 22–25] and [26] are primarily based on the finest guess or stated in a better way are based on Bayesian Inference (relies on the prior model of a specific class) of image objects. The major setback displayed by the prior models is their inefficiency in realizing the connectivity principle and also because of the fact that they create visible corners which is considerably eliminated by the usage of level sets. Level sets are efficient in segmenting the object of interest in the image, but the question arises what if the contour of the object visible in the image is not its actual one, but a pseudo contour of the object of interest remnant after occlusion? The problem was partially looked into in [27, 28] where the system using the level set approach requires the deformities to be regular! The question which lingers is ….. It is practically impossible to expect the deformities aprior in the case of realistic imagery with drastic illumination variations, erratic shapes of the objects occluding the object of interest etc.

With the backdrop of such complexities, disocclusion or invisible information recovery is to be accomplished. It is evident from the literature survey that the inpainting algorithms could be used. But what if the actual contour of the object of interest is unknown! Through the proposed FIREACH algorithm a humble effort has been put forth to tackle such a situation with face as the object of interest. Face is considered in our proposed work because of one major challenge it poses—occluded Face cannot be reconstructed from the information retrieved from the boundary of the hole. The Literature review related to Facial Inpainting are as follows: In the experimental results discussed by Hwang and Lee [29] and Mo et al. [30] rely on retrieving the missing portion of the face from a reference face image. The major drawback posed by these approaches is that the drastic variations in the illumination conditions in real life imagery and the various combinations of the reference facial images will not faithfully conform to the specific illumination and photographic conditions of the target image. The contribution in [31] proposed a facial inpainting technique which follows the principle of face recognition by classifying a query image into one or more categories in a database and extend the same principle for searching of faces with missing areas by using prior probabilistic distribution of facial structural information. The patch guided facial inpainting approach also has limitations. If the original face is partially covered by an occlusion the inpainted output does not yield a satisfactory outcome. The approach proposed in [31] assumes normal and uniform distribution of luminance across the face (which is not true in realistic imagery).

Literature Review of the related work to date percolates to the fact that the work done till date concentrates on occlusions of features like eyes and mouth, which are retrieved with reference to the database information. And the exuberant Survey of Literature makes it evident that contributions throwing light towards the evolution of the occluded face region contour has not been explored.

In a real life situation the occlusion will not always be oriented towards the eye or the mouth regions. For e.g.: if a person has met with an accident and his lower part of the face is completely lost! Under such circumstances it becomes necessary to evolve the contour of the face before the application of any reconstruction techniques. FIREACH is a humble step towards efficient accomplishment of Occluded Face Contour Evolution!

3 FIREACH Algorithm

Step 1: Face Region Localization

The Face Region Localization in the captured color image is accomplished by using the Skin Illumination Compensation Model proposed in [13]. The Model uses the concept of region signatures to separate out the skin regions corresponding to the face.

Step 2: Edge Detection and Sampling

Detected Visible Face Regions are Binarised to reduce the computational overhead. The Edge Maps of the detected Face regions are constructed by using the Canny Edge Detection Operator. The Edge points are periodically sampled to improvise the efficiency of the algorithm by reducing the computational time complexity

Step 3: Convex Hull

The Convex Hull [32] for the Face edge map is successfully evolved out in this module by using the Aki Toussaint Heuristic. The efficiency of the Chans' Algorithm [33] is exploited affirmatively. Initial assumption is that the 'h' points on the convex hull is known apriori. Chan's Algorithm starts by shattering the input points into 'n/h' arbitray subsets, each of size 'h', and computing the convex hull of each subset using the Graham's scan [34]. The algorithm requires O((n/h)hlogh) = O(nlogh) time for finding out the arbitrary subsets.

Following the computation of 'n/h' subhulls, the general outline of Jarvis's march is followed, which involves wrapping a string around the 'n/h' subhulls. Initiating with the leftmost input point l, starting with p = l (p is the boundary pixel of the Convex Hull) and successively evolving the convex hull vertices in counter-clockwise order until the traversal returns back to the original leftmost point again [35].

The output of the step 3 results in success only if the depleted regions are very small (within 10 % of the visible face region). As the percentage of depletion exceeds beyond 10 % of the visible face region the output is not satisfactory!

Step 4: Ellipse Fitting

Many unsuccessful attempts to make the fitting process computationally effective were succeeded efficiently by direct least squares based ellipse-specific method proposed by Fitzgibbon and Fischer [36], Fitzgibbon [37], Fitzgibbon et al. [38].

The Direct Least Square Fitting provides impressive results under noisy conditions with dependable computational efficiency. This method is stated to be a non-iterative ellipse-specific fitting.

Ellipse-a special case of a general conic, can be described by an implicit second order polynomial.

$$\mathrm{F}\ (\mathrm{a},\ \mathrm{x}) = \mathrm{a.\ x} = \mathrm{ax}^2 + \mathrm{bxy} + \mathrm{cy}^2 + \mathrm{dx} + \mathrm{ey} + \mathrm{f} = 0 \tag{1}$$

with an Ellipse specific constraint

$$\left(\mathrm{b}^2 - 4\mathrm{ac}\right) < 0 \tag{2}$$

where a, b, c, d, e, f are coefficients of the ellipse which fits the detected face and (x, y) are the coordinates of points lying on the detected face. The polynomial F(x, y) is the algebraic distance of point (x, y) to the given conic.

By introducing vectors $\mathrm{a}' = [\mathrm{a\ b\ c\ d\ e\ f}]^{\mathbf{T}}$

$$\mathrm{x}' = \left[x^2\,\mathrm{xy}\,\mathrm{y}^2\,\mathrm{x}\,\mathrm{y}\,1\right] \tag{3}$$

it can be rewritten to the vector form

$$\mathrm{F_a(x)} = \mathrm{x.\ a} = 0 \tag{4}$$

The fitting of ellipse to the set of points $(\mathrm{x_i}, \mathrm{y_i})$, i = 1,…,N., extracted from the boundary of the partially occluded face is accomplished by minimizing the sum of squared algebraic distances of the points to the ellipse fit which is represented by the coefficients a′:

$$\min_{\mathrm{a}} \sum_{i=1}^{N} F(x_i, y_i)^2 = \min_{\mathrm{a}} \sum_{i=1}^{N} (F_a(\mathrm{x_i}))^2 = \min_{\mathrm{a}} \sum_{i=1}^{N} (\mathrm{x_i} \cdot \mathrm{a})^2 \tag{5}$$

To ensure ellipse-specificity of the solution, constraint in Eq. (2)is taken into account.

Step 5: Convex Hull Extension

In the case of facial regions with occlusions: after the evolution of the ellipse fit for the facial region with occlusions the convex hull region is extended into the depleted region until the bounds of the contour created by the ellipse fit.

Step 6: Centroid Estimation

The Convex Hull for the detected face region is evolved in Step 3, and is succeeded by applying the Direct Least Squares Ellipse fit on the edge recognized detected face region in Step 4. The output of Step 3 and Step 4 are overlapped on their respective face regions and the Centroid for the ellipse is detected by using Eq. (6)

$$\mathrm{Centroid(EllipseDirectFit)} = \mathrm{mean(XY)} \tag{6}$$

where, X = Major Axis of the Ellipse and Y = Minor Axis of the Ellipse

Step 7: Boundary Estimation

The output of the Step 6 results in the evolved contour of the Face Region, along with the Convex Hull (B1) of the visible face region (in the occluded case) and the Ellipse fit of the region (B2) with the Centroid. It is evident that both the Convex Hull and the Ellipse Fit result in extraneous outliers which are to be eliminated. To accomplish this following steps are followed:

1. Move outwards from the centroid, until either B1 or B2 is confronted.
2. Once any one of the Boundaries is confronted while moving outwards from the centroid, set the corresponding value of the pixel (save the pixel coordinates in a vector) as the actual contour of the face region.
3. Continue this process starting from the Centroid in all possible orientations ranging from 0 to 360°.
4. Once the entire cycle from 0 to 360(degrees) is completed, the set of pixels whose corresponding coordinate values are saved in the vector depict the newly evolved Contour of the Disoccluded Face Region!

The Steps 1–7 are repeated iteratively on all the detected face regions in the image. Finally, the evolved contour is multiplied with the original image to get the exact face region.

4 Data Set

The prime goal of the proposed algorithm is to evolve the damaged/occluded contours of face biometric. The Face Database is suitably compiled to suit the application under consideration considering different degrees of occlusion across the face. The database consists of 2 sections, the first one comprises of images personally collected and the second section comprises of images collected from the World Wide Web. The first section includes 10 different variants per subject, with reference to the degree of occlusions and variations in the lighting conditions. Similar types of variants are collected for 100 different subjects with a uniform background, totaling the content of section 1–1,000 samples. For each of the 100 subjects, one of the sample collected is an ideal one without any occlusion and is considered as the ground truth for the respective subject. The second section in the dataset involves 50 random images collected from the World Wide Web (WWW). The face contours of the faces in the collected web images are considered as the ground truth and the occlusion is artificially induced at different degrees (10 different degrees of occlusions are induced for each of the facial components present in the web images).

The samples in the first section are captured in standard conditions with a uniform background and with the distance of the subject from the camera being maintained constant for all the samples. The ground truth samples are not taken into consideration for training or for testing.

5 Experiments and Results

The FIREACH Algorithm is designed to automatically select and assign the dataset samples randomly as training and testing categories in every iteration (for e.g.: 10 % training and 90 % testing, 60 % training and 40 % testing and the like) (Figs. 1, 2, 3, 4, 5, 6).

The Dataset totally includes section 1:1,000 samples captured individually by using a standard 12 Mega Pixel Nikon Digital Camera plus section 2:50 random image samples collected from the World Wide Web (WWW) for which the occlusions are manually induced resulting in 9 different degrees of occlusions for

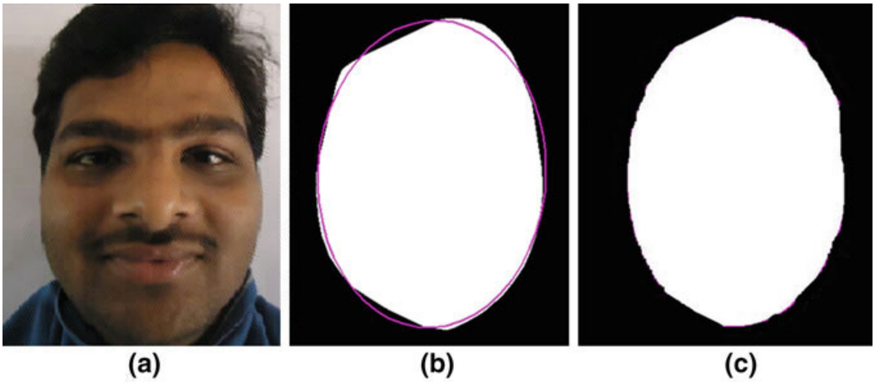

Fig. 1 Case 1: FIREACH output for frontal face with no occlusion (used as the **ground truth** to evaluate the success of FIREACH for various degrees of occlusions present across the face samples). **a** Represents the original image, **b** represents the contours of the detected face region resulting after the application of step 3 and step 4 in the FIREACH Algorithm, **c** represents the evolved face contour output resulting from step 7 in FIREACH Algorithm

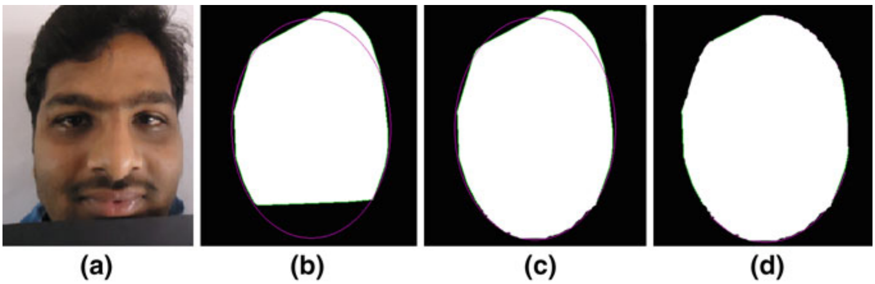

Fig. 2 Case 2: FIREACH output when 10 % of the face occluded. **a** pertains to the sample collected with 10% occlusion in the sample; **b** the output after the application of Convex Hull and Ellipse Fitting with evident occlusions; **c** The output after extending the Convex hull into the depleted/occluded face regions until the bounds set by ellipse fit; **d** Signifies the output after the application of Step 7 in the FIREACH Algorithm

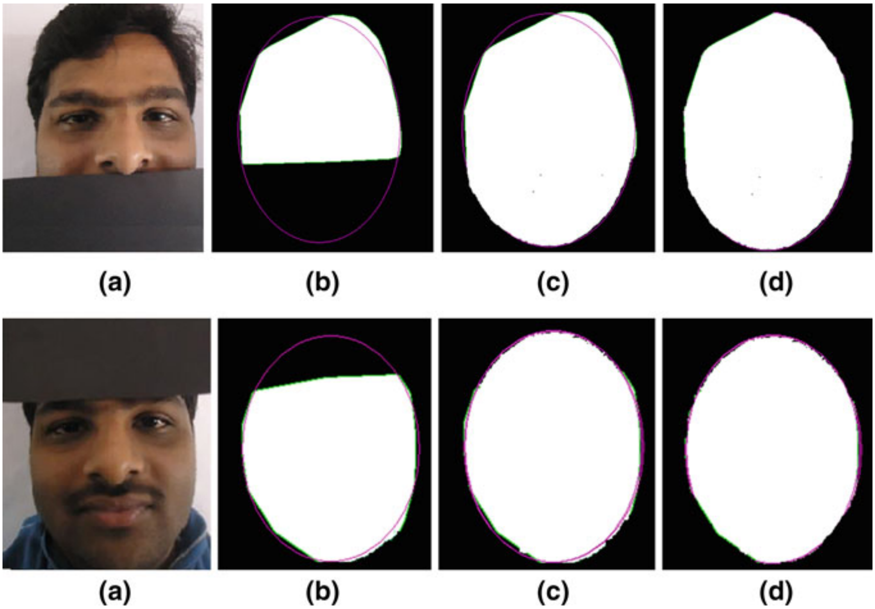

Fig. 3 Case 3: FIREACH output when (30–40) % of the face occluded. **a** pertains to the sample collected with (30-40)% occlusion in the sample; **b** the output after the application of Convex Hull and Ellipse Fitting with evident occlusions; **c** The output after extending the Convex hull into the depleted/occluded face regions until the bounds set by ellipse fit; **d** Signifies the output after the application of Step 7 in the FIREACH Algorithm

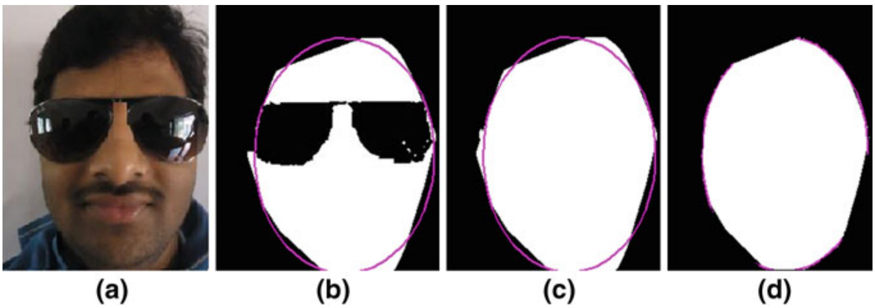

Fig. 4 Case 4: FIREACH output for a face with glasses/goggles. **a** Pertains to the sample collected with occlusion due to goggles; **b** the output after the application of Convex Hull and Ellipse Fitting with evident occlusions; **c** The output after extending the Convex Hull into the depleted/occluded face regions until the bounds set by ellipse fit; **d** Signifies the output after the application of Step 7 in the FIREACH Algorithm

each of the 50 random image samples (from WWW) inclusive of the ground truth totaling to 500 samples. The Compiled Dataset is composed of a total of 1500 samples (inclusive of the ground truth which is not included in either the Training

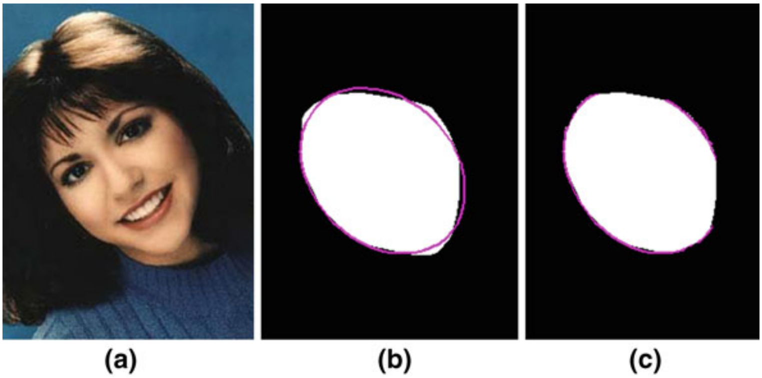

Fig. 5 Case 5: **a** indicates the image from the WWW; **b** indicates the evolved boundary of the image sample in ’a’ after the application of the Convex Hull and Ellipse Fitting; **c** indicates the face contour evolved after the application of Step 7 in the FIREACH Algorithm and is considered as the Ground Truth for all the corresponding occlusions induced on the image sample mentioned in ’a’

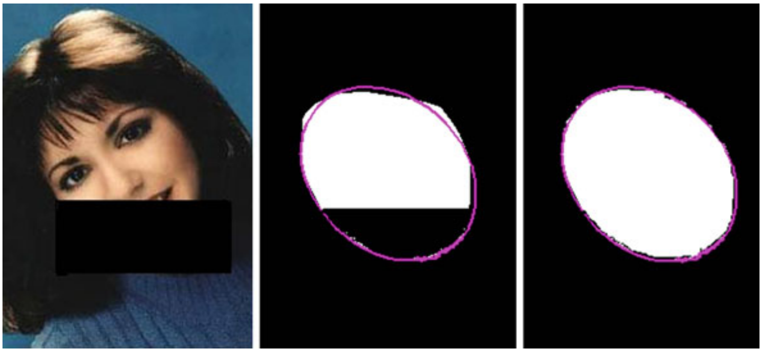

Fig. 6 Case 6: **a** indicates the image sample from the Web with artificially induced occlusion; **b** indicates the Occluded Face Contour after the application of Step 3 and Step 4 in the FIREACH Algorithm; **c** indicates the output after the application of Step 7 in the FIREACH Algorithm

or the Testing Set). The Tabular representation in the Table 1 depicts the fact that the performance of evolution of the occluded face contour with reference to the closeness of the evolved contour to the corresponding ground truth increases as the number of training images increase.

The results displayed in the Table 1 are the values obtained with reference to the closeness of the evolved face contour of the occluded face regions in comparison with the corresponding evolved ground truth face contour. The comparison of the evolved face contour with that of the ground truth is established with compatibility/acceptable variation of evolved contour with that of the reference contour being $\pm$ 3 %. It is evident that the performance of the algorithm demonstrates a commendable improvement as the number of training samples increases.

Table 1 The accuracy FIREACH algorithm in evolving occluded face contour

Degree of occlusion induced	Training data: testing data (wrt., percentage of samples in the data set)								
	10:90 %	20:80	30:70	40:60	50:50	60:40	70:30	80:20	90:10
10 % Occlusion (lower part of the face)	0.90	0.90	0.90	0.93	0.93	0.96	0.97	0.97	0.97
10 % Occlusion (upper part of the face)	0.87	0.89	0.90	0.90	0.92	0.92	0.92	0.93	0.93
(20–30) % Occlusion (lower part of the face)	0.80	0.80	0.86	0.86	0.86	0.87	0.88	0.90	0.90
(20–30) % Occlusion (upper part of the face)	0.79	0.79	0.79	0.80	0.80	0.80	0.80	0.80	0.81
(30–40) % Occlusion (lower part of the face)	0.74	0.75	0.75	0.77	0.76	0.77	0.77	0.79	0.80
(30–40) % Occlusion (upper part of the face)	0.70	0.72	0.72	0.72	0.75	0.75	0.75	0.74	0.76
(40–50) % Occlusion (around 50 %) (upper part/lower part/side parts of the face)	0.60	0.60	0.57	0.56	0.56	0.57	0.60	0.63	0.62

It is obvious from some of the tabular entries that the performance drops even with the increase in the training sample number. The reason being that when the system operates by randomly selecting the samples for testing and training during every iteration, if in a particular iteration the presence of web selected face images exceed the manually selected ones the efficiency is found to depreciate as the web images capturing conditions are not very ideal. But, still the depreciation is not very prominent as is evident from Table 1.

The system gives good results with occlusions present on the lower part of the face images when compared to the occlusion present in the upper part of the facial image, which is because of the variations in the hair style (because of which it becomes difficult to match with the exact face contour).

6 Conclusion

FIREACH achieves a commendable initiative towards evolution of occluded face region contours which would predominantly aid as a preliminary phase in achieving efficient and reliable face reconstruction for Face Recognition Systems. FIREACH algorithm works well with occlusions varying around 50 %. The Future Enhancement would be directed towards solving the contour evolution lacunas present in FIREACH with respect to occlusions covering the face regions beyond 50 %.

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