

Shallow Foundation Response Variability due to Parameter Uncertainty

Prishati Raychowdhury and Sumit Jindal

Abstract Uncertainty in soil parameters may play a crucial role in response variation of foundations and the supporting structures and, consequently, may control several design decisions. It is, therefore, extremely important to identify and characterize the relevant parameters. Furthermore, it is also important to identify the sources and extent of uncertainty of soil and model input parameters, along with the effect of their uncertainty on the shallow foundation response. This chapter intends to investigate the effect of soil and model parameter uncertainty on the response of shallow foundation-structure systems resting on dry dense sand. In this study, the soil-foundation interface is modeled using Winkler-based concept, where the soil-foundation interface is assumed to be an assembly of discrete, nonlinear elements composed of springs, dashpots, and gap elements. The sensitivity of both soil and model input parameters on various force and displacement demands of the foundation-structure system is investigated using first-order second-moment analysis and Latin hypercube technique. It has been observed that the force and displacement demands of the foundation-structure system are highly sensitive to the soil and model parameters.

Keywords Soil-structure interaction • Winkler modeling • Parametric uncertainty • Nonlinear analysis • Sensitivity analysis

1 Introduction

Most soils are naturally formed in many different depositional environments; therefore, it shows variation in their physical properties from point to point. The soil properties exhibit variations even within an apparently homogeneous soil

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profile. Basic soil parameters that control the strength and stiffness of the soil-foundation system are cohesion, friction angle, unit weight, shear modulus, and Poisson's ratio of soil. These soil parameters can be delineated using deterministic or probabilistic models. Deterministic models use a single discrete descriptor for the parameter of interest, whereas probabilistic models define parameters by using discrete statistical descriptor or probability distribution (density) function. Uncertainty in soil properties can be formally grouped into aleatory and epistemic uncertainty [9]. Aleatory uncertainty represents the natural randomness of a property and is a function of spatial variability of the soil property. This type of uncertainty is inherent to the variable and cannot be reduced or eliminated by additional information. Epistemic uncertainty results from lack of information and shortcomings in measurements and calculations [9]. Epistemic uncertainty can usually be reduced by acquisition of more information or improvements in measuring methods.

In last few decades, significant research has been carried out for proper understanding of the behavior of structure due to uncertainty in soil parameters. In an early work, Lumb [10] showed that the soil parameters can be modeled as random variables conforming to the Gaussian distribution within the framework of probability theory. Ronold and Bjerager [11] observed that the model uncertainties are important in reliability analysis for prediction of stresses, capacities, deformation, etc., in structure and foundation systems. Chakraborty and Dey [2] studied the stochastic structural responses considering uncertainty in structural properties, soil properties, and loadings using Monte Carlo simulation technique. Lutes et al. [21] evaluated the response of a seismically excited structural system with uncertain soil and structural properties. Ray Chaudhuri and Gupta [16] investigated the variability in seismic response of secondary systems due to uncertain soil properties through a mode acceleration method. Foye et al. [4] described a thorough study for assessment of variable uncertainties by defining the probability density functions for uncertain design variables in load resistance factor design (LRFD). Na et al. [14] investigated the effect of uncertainties of geotechnical parameters on gravity-type quay wall in liquefiable condition using tornado diagram and first-order second-moment (FOSM) analysis. Raychowdhury [17] studied the effect of soil parameter uncertainty on seismic demand of low-rise steel building supported by shallow foundations on dense silty sand with considering a set of 20 ground motions. Raychowdhury and Hutchinson [18] carried out the sensitivity analysis of shallow foundation response to uncertain input parameters using simplified FOSM and tornado diagram methods.

This chapter focuses on studying the effect of uncertainty in soil and model parameters on the response of shallow foundation-supported [8] shear wall building. To incorporate the nonlinearity at the soil foundation interface, a beam-on-nonlinear-Winkler-foundation (BNWF) approach is adopted. The uncertainty analysis is carried out using simplified first-order second-moment method and Latin hypercube sampling technique.

2 Numerical Modeling

In this chapter, a beam-on-nonlinear-Winkler-foundation (BNWF) approach is used to model the nonlinear soil-structure interaction of shallow foundations subjected to lateral loads. The BNWF model includes a system of closely spaced independent, mechanistic, vertical, and lateral elements consisting of nonlinear springs, dashpots, and gap elements (Fig. 1). The vertical springs ($q - z$ elements) are intended to capture the axial and rotational behavior of the footing, whereas the lateral springs, $t - x$ element and $p - x$ element, are intended to capture the sliding and passive resistance, respectively. The material models were originally developed by Boulanger et al. [1] and modified by Raychowdhury and Hutchinson [15]. This model is capable of reasonably capturing the experimentally observed behavior for various shallow foundation conditions. For more details regarding the BNWF modeling, one can look into Raychowdhury and Hutchinson [15].

The backbone curves are thus characterized by a linear elastic region, then an increasingly growing nonlinear region (Fig. 2). For $q - z$ material, ultimate vertical capacity q_{ult} is calculated based on general bearing capacity equation given by Terzaghi [20]:

$$q_{ult} = cN_cF_{cs}F_{cd}F_{ci} + \gamma D_f N_q F_{qs}F_{qd}F_{qi} + 0.5\gamma BN\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \tag{1}$$

where q_{ult} is the ultimate vertical bearing capacity per unit area of footing, c the cohesion, γ the unit weight of soil, D_f is the depth of embedment, B the width of

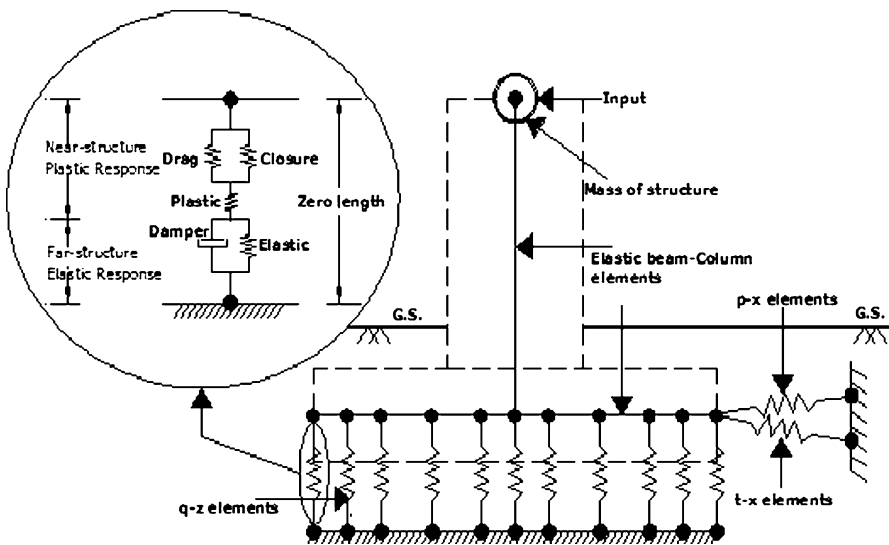


Fig. 1 Schematic diagram of structure with BNWF model for shallow foundations

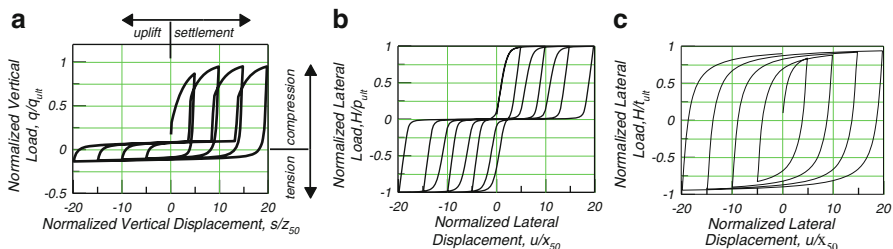


Fig. 2 Behavior of material models: (a) $q - z$ element, (b) $p - x$ element, and (c) $t - x$ element

footing, and N_c, N_q, N_γ are bearing capacity factors calculated after Mayerhof [13]. For the $p - x$ material, the passive resistance p_{ult} is calculated as

$$p_{ult} = 0.5\gamma K_p D_f^2 \tag{2}$$

where p_{ult} is the passive earth pressure per unit length of footing and K_p the passive earth pressure coefficient. For the $t - x$ material, the sliding resistance t_{ult} is determined using classical Mohr-Coulomb failure criteria

$$t_{ult} = W_g \tan\delta + cA_f \tag{3}$$

where t_{ult} is the frictional resistance per unit area of foundation, W_g the weight on the foundation from the structure, δ the angle of friction between foundation and soil, which typically varies from $1/3$ to $2/3 \phi$, and A_f the surface area of the foundation.

The vertical and lateral stiffness, K_v and K_h , are calculated using expressions given by Gazetas [5] as follows:

$$k_v = \frac{GL}{1 - \nu} \left[0.73 + 1.54 \left(\frac{B}{L} \right)^{0.75} \right] \tag{4}$$

$$k_h = \frac{GL}{2 - \nu} \left[2 + 2.5 \left(\frac{B}{L} \right)^{0.85} \right] \tag{5}$$

In order to investigate the effect of uncertain input parameters on the shallow foundation response, a single shear wall structure supported by strip footing resting on dense dry sand of relative density 80% is considered for this study. The dimensions of footing are 1.0 m wide and 0.25 m in height, and depth of embedment is 0.5 m. Shear wall dimensions are 0.5 m in length and 0.2 m in width, and the height of wall is 5.0 m. A monotonic loading is applied at the top of the structure, and responses are evaluated in terms of maximum absolute values of moment, shear, rotation, and settlement demands.

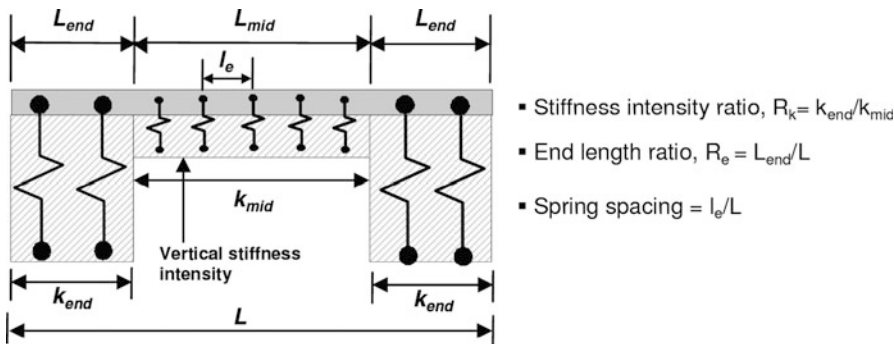


Fig. 3 Vertical spring distribution along the footing length

It is evident from the above discussion that the strength and stiffness of the spring elements of a particular size of footing are functions of basic soil properties such as friction angle, shear modulus, Poisson’s ratio, and unit weight. Therefore, these soil properties are expected to influence the response of the footing-structure system. However, in addition to these soil properties, few model parameters such as spring spacing and ratio of spring stiffness along the length (as shown in Fig. 3) may also alter the foundation responses.

3 Selection of Uncertain Parameters

Based on the discussion provided in the previous section, the parameters shown in Table 1 are chosen for the uncertainty analysis. The parameters are selected for dense dry sand of relative density 80%. These values are based on EPRI [3] and Harden et al. [6]. It is assumed that all uncertain input parameters are random variables with a Gaussian distribution, having no negative values. The upper and lower limits of the random variables are assumed to be in 95th and 5th percentile of its probability distribution. The corresponding mean (μ) and standard deviation (σ) can be calculated as

$$\mu = \frac{L_L + L_U}{2} \text{ and } \sigma = \frac{L_U - L_L}{2k}$$

where L_L and L_U are the lower and upper limits, respectively, and k depends on the probability level (e.g., $k = 1.645$ for a probability of exceedance = 5%). The assumed correlations among the uncertain parameters are provided in Table 2.

Table 1 Uncertain parameters considered in this study

Parameters	Symbol	Range	Mean(μ)	Coeff. of. variation (C_v) (%)
Friction angle (deg)	ϕ'	38–42	40	3
Poisson’s ratio	ν	0.3–0.5	0.4	16
Shear modulus (MPa)	G_s	12–20	16	15
End length ratio	R_e	1–17	9	54
Stiffness intensity ratio	R_k	1–9	5	48
Spring spacing (%)	S_s	1.0–3.0	2	30

Table 2 Correlation matrix of the parameters

	ϕ'	ν	G_s	R_e	R_k	S_s
ϕ'	1	0.1	0.6	0	0	0
ν		1	0.2	0	0	0
G_s			1	0	0	0
R_e				1	0.3	0.1
R_k					1	–0.1
S_s						1

4 Uncertainty Analysis

In order to perform uncertainty analysis, two different techniques are adopted: first-order second-moment (FOSM) method and Latin hypercube method. Below is a brief description of both methods.

4.1 First-Order Second-Moment (FOSM) Method

The FOSM method is used to perform simplified sensitivity analysis to evaluate the effect of variability of input variables on each response variable. This method uses a Taylor series expansion of the function to be evaluated, and expansion is truncated after the linear first-order term. It is assumed that the relationship between the response variables and the uncertain input parameters is assumed to be linear or low-to-moderately nonlinear.

The response of the foundation is considered as a random variable Q , which has been expressed as a function of the input random variables, P_i (for $i = 1, \dots, N$) denoting uncertain parameters and Q given by

$$Q = h(P_1, P_2, \dots, P_N) \tag{6}$$

P_i has been characterized by its mean μ_p and variance σ_p^2 . Now, Q can be expanded using a Taylor series as follows:

$$\begin{aligned}
 Q &= h(\mu_{P_1}, \mu_{P_2}, \dots, \mu_{P_N}) + \frac{1}{1!} \sum_{i=1}^N (P_i - \mu_{P_i}) \frac{\delta h}{\delta P_i} + \frac{1}{2!} \\
 &\times \sum_{j=1}^N \sum_{i=1}^N (P_i - \mu_{P_i})(P_j - \mu_{P_j}) \frac{\delta^2 h}{\delta P_i \delta P_j} + \dots
 \end{aligned}
 \tag{7}$$

Considering only the first-order terms of Eq. (7) and ignoring higher-order terms, Q can be approximated as taking expectation of both sides of Eq. (6), the mean of Q can be expressed as

$$Q \approx h(\mu_{P_1}, \mu_{P_2}, \dots, \mu_{P_N}) + \sum_{i=1}^N (P_i - \mu) \frac{\delta h}{\delta P_i}
 \tag{8}$$

$$\mu_Q = h(\mu_{P_1}, \mu_{P_2}, \mu_{P_N})
 \tag{9}$$

Utilizing the second-order moment of Q as expressed in Eq. (7), the variance of Q can be derived as

$$\begin{aligned}
 \sigma_Q^2 &\approx \sum_{i=1}^N \sum_{j=1}^N \text{covariance}(P_i, P_j) \frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_i} \frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_j} \\
 &\approx \sum_{i=1}^N \sigma_{P_i}^2 \left(\frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_i} \right)^2 \\
 &+ \sum_{i=1}^N \sum_{j=1}^N \rho_{P_i, P_j} \frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_i} \frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_j}
 \end{aligned}
 \tag{10}$$

where ρ_{P_i, P_j} denotes correlation coefficient for random variables P_i and P_j . The partial derivative of $h(P_1, P_2, \dots, P_N)$ with respect to P_i has been calculated numerically using the finite difference method (central) as follows:

$$\frac{\delta h(P_1, P_2, \dots, P_N)}{\delta P_i} = \frac{h(p_1, p_2, \dots, \mu_i + \Delta_{P_i}, p_N) - h(p_1, p_2, \dots, \mu_i - \Delta_{P_i}, p_N)}{2\Delta_{P_i}}
 \tag{11}$$

4.2 Latin Hypercube Method

For probabilistic analysis of engineering structures having uncertain input variables, Monte Carlo simulation (MCS) technique is considered as a reliable

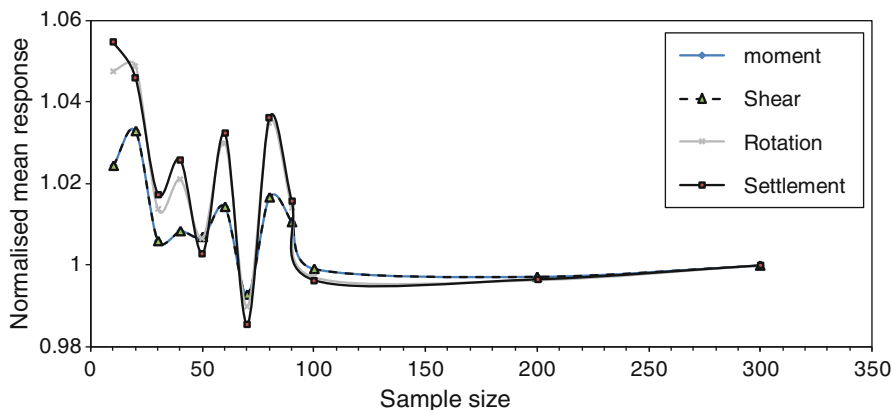


Fig. 4 Convergence test for Latin hypercube sampling method

and accurate method. However, this method requires a large number of equally likely random realizations and consequent computational effort. In order to decrease the number of realizations required to provide reliable results in MCS, Latin hypercube sampling (LHS) approach [12] is widely used in uncertainty analysis. LHS is a type of stratified MCS which provides a very efficient way of sampling variables from their multivariate distributions for estimating mean and standard deviations of response variables [7]. It follows a general idea of a Latin square, in which there is only one sample in each row and each column. In Latin hypercube sampling, to generate a sample of size K from N variables, the probability distribution of each variable is divided into segments with equal probability. The samples are then chosen randomly in such a way that each interval contains one sample. During the iteration process, the value of each parameter is combined with the other parameter in such a way that all possible combinations of the segments are sampled. Finally, there are M samples, where the samples cover N intervals for all variables. In this study, to evaluate the response variability due to the uncertainty in the input parameters, samples are generated using Stein's approach [19]. This method is based on the rank correlations among the input variables defined by Iman and Conove [7], which follows Cholesky decomposition of the covariance matrix.

In this study, the covariance matrix is calculated by using standard deviation (Table 1) and correlation coefficient (Table 2) between any two input parameters. The previously mentioned shear wall structure is considered for this analysis. The response of this soil-foundation structure system is dependent on the six independent input and normally distributed variables defined in Tables 1 and 2. In order to find out the correct sample size, pushover analysis is carried out using 10, 20, . . . , 100, 200, and 300 number of samples. Figure 4 shows the plot sample size versus the mean responses normalized by value corresponding to a sample size of 300. From Fig. 4, it can be observed that the mean of the responses tends to converge as the sample size increases. At the sample size of 100, the response of the system has almost converged. Therefore, for six independent and normally distributed variables, a sample size of 100 is used.

5 Results and Discussion

In order to evaluate the effect of soil and model parameter uncertainty on the response of the shallow foundation, four response parameters are chosen: absolute maximum moment $|M_{\max}|$, absolute maximum shear $|V_{\max}|$, absolute maximum rotation $|\theta_{\max}|$, and absolute maximum settlement $|S_{\max}|$. A monotonic loading is applied at the top of the structure, and responses and forces and displacements are obtained. The analysis is done using finite element software OpenSees (Open System for Earthquake Engineering Simulation). Figure 5 shows the comparison results for the centrifuge experiment conducted in the University of California, Davis. The results are for two extreme values of friction angle. The results include moment-rotation, settlement-rotation, and shear-rotation behaviors with the BNWF simulation shown in black and experimental results shown in gray scale. These comparisons indicate that the BNWF model is able to capture the hysteretic features such as shape of the loop, peaks, and unloading and reloading stiffness reasonably well. It can also be observed from Fig. 5 that with increasing the friction angle from 38° to 42° , peak moment and peak shear demands increase, whereas peak settlement demand decreases. It can also be noted that the variation of friction angle from 38° to 42° has the most significant effect on settlement prediction (more than 100%). However, the moment, shear, and rotation demands are moderately affected by this parameter. This indicates that the uncertainty in one parameter may have significantly different influence on the prediction of different responses, pointing out toward the importance of proper characterization of each parameter and conducting the sensitivity analysis.

Similarly, the analysis is carried out for varying each parameter at a time while keeping other parameters fixed at their mean values, and FOSM analysis is carried out to find out the sensitivity of each parameter on the responses. Figures 6, 7, 8, and 9 show the results of FOSM analysis for moment, shear, rotation, and settlement, respectively. It can be observed that for moment and shear, friction angle is the most important parameter (60% relative variance). Poisson's ratio and shear modulus are moderately important (about 23 and 16%, respectively), and model parameters have negligible effect (less than 5%). However, model parameter stiffness intensity ratio, R_k , seems to have great effect on the rotational demand (~67%). Settlement is observed to be affected by all three soil parameters (friction angle, shear modulus, and Poisson's ratio), almost equally (~30%) for each parameter. Model parameters do not affect this response much.

Table 3 shows the result obtained from Latin hypercube method. The response is presented in terms of the mean and coefficient of variation (C_v) of each demand parameters. It can be observed from this table that with 3, 16, 15, 54, 48, and 30% C_v of friction angle, Poisson's ratio, shear modulus, end length ratio, stiffness intensity ratio, and spring spacing input parameters, respectively, can result in moderate variation in demand parameters with C_v as 16, 16, 22, and 24% for the

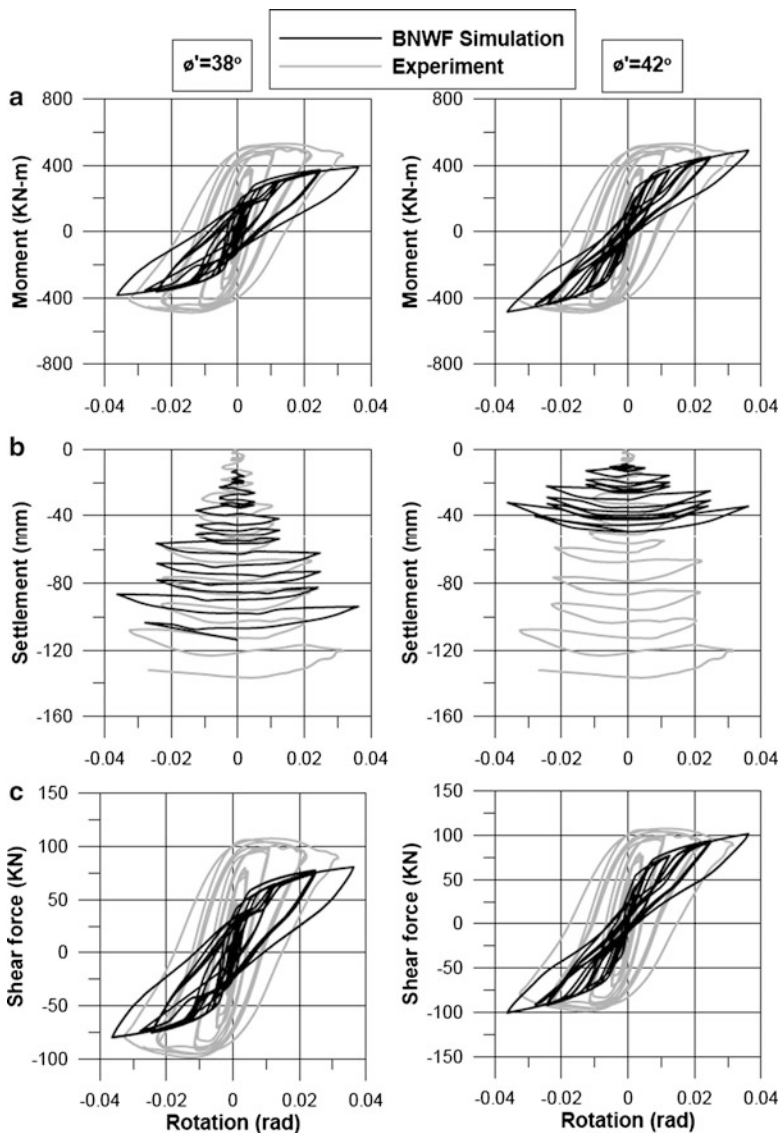


Fig. 5 Response of shear wall-footing system

absolute maximum moment, shear, rotation, and settlement demands, respectively. Note that all responses are more sensitive to the soil parameters than the model parameters. Friction angle is the most sensitive among all input parameter, as with a 3% C_v results in significant variation in the response variables.

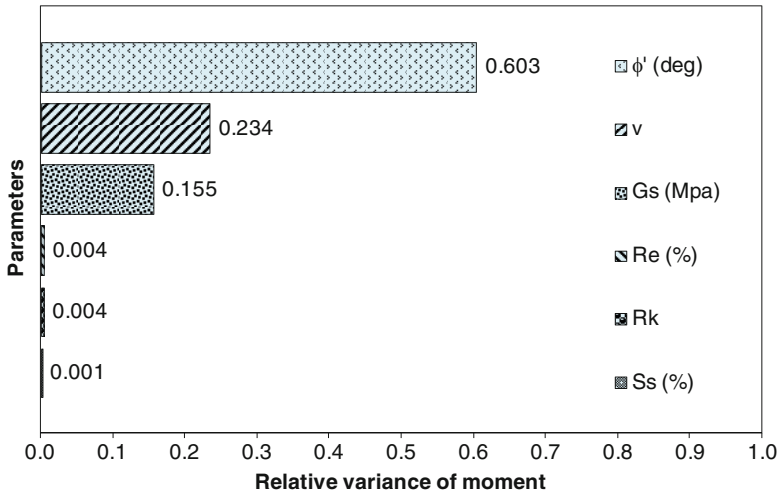


Fig. 6 Results of FOSM analysis: relative variance for moment

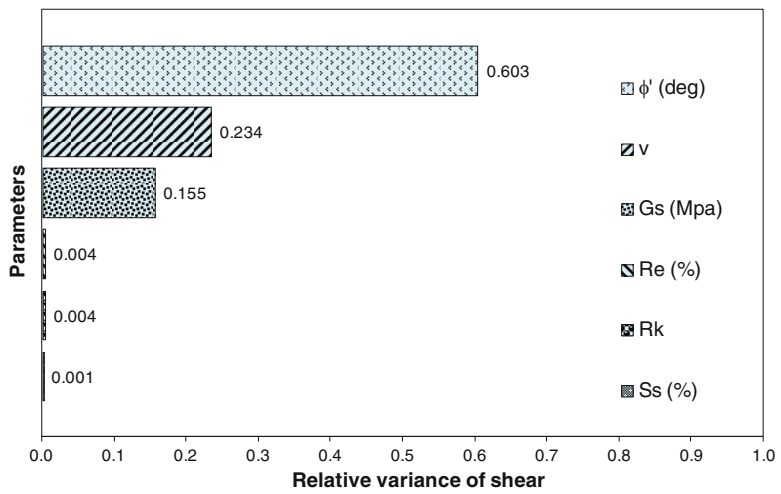


Fig. 7 Results of FOSM analysis: relative variance for shear

Table 3 Variability of response parameters using Latin hypercube method

	Moment $ M_{max} $	Shear $ V_{max} $	Rotation $ \theta_{max} $	Settlement $ S_{max} $
Mean (μ)	78.09 kN-m	15.62 kN	0.14 rad	39.11 mm
Coeff. of variation (C_v)	16%	16%	22%	24%

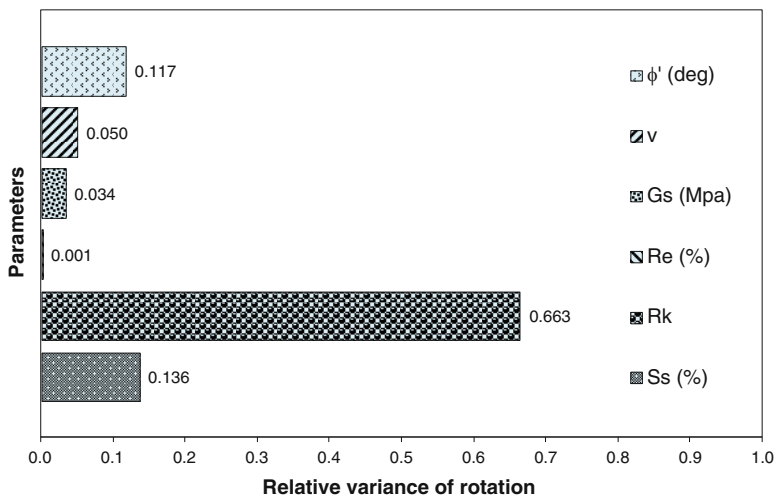


Fig. 8 Results of FOSM analysis: relative variance for rotation

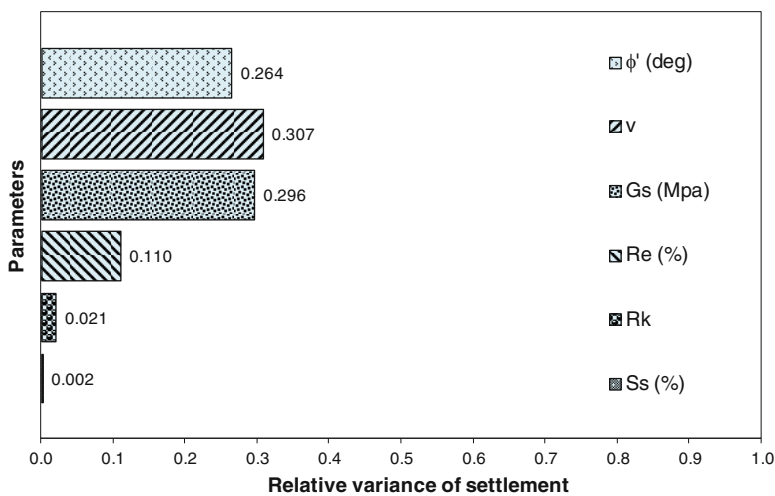


Fig. 9 Results of FOSM analysis: relative variance for settlement

6 Conclusions

The effect of uncertainty in soil and model parameters on the soil-foundation system response has been studied in this chapter. The soil-foundation system has been modeled using BNWF concept, and the uncertainty analyses are carried out using FOSM and Latin hypercube method. It has been observed that for moment and shear, friction angle is the most important parameter (60% relative variance),

Poisson's ratio and shear modulus are moderately important (about 23 and 16%, respectively), and model parameters have negligible effect (less than 5%). The rotational demand (~67%) is largely dependent on stiffness intensity ratio. The settlement demand is almost equally sensitive to friction angle, shear modulus, and Poisson's ratio (~30% variance for each parameter). The results from Latin hypercube method indicate that a coefficient of variation of 3% in friction angle results in 16, 16, 22, and 24% for the absolute maximum moment, shear, rotation, and settlement demands, respectively, indicating that these parameters have great effect on each response variables. It can finally be concluded that soil parameters such as friction angle, shear modulus, and Poisson's ratio may have significant effect on the response of foundation. Therefore, selection of these parameters should be considered critically when designing a structure with significant soil-structure interaction effect.

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