Slope Reliability Analysis Using the First-Order Reliability Method

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Abstract This chapter pertains to a study on the reliability evaluation of earth slopes under a probabilistic framework. This study is concerned in the first phase with the determination of reliability index and the corresponding probability of failure associated with a given slip surface and then in the second phase with the determination of the critical probabilistic slip surface and the associated minimum reliability index and the corresponding probability of failure. The geomechanical parameters of the slope system have been treated as random variables for which different probability distributions have been assumed. The reliability analyses have been carried out using two methods, namely, the approximate yet simple mean-value first-order second-moment (MVFOSM) method and the rigorous first-order reliability method (FORM). Based on a benchmark illustrative example of a simple slope in homogeneous soil with uncertain strength parameters along a slip circle, an effort has been made to numerically demonstrate the nature and level of errors introduced by adopting the MVFOSM method for reliability analysis of earth slopes still widely used in the geotechnical engineering practice, vis-à-vis a more accurate method such as the FORM.

Keywords Slope stability • Slip surface • Uncertainty • Random variable • Probability distribution • Reliability analysis

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1 Introduction

In recent years, there has been a growing appreciation among the researchers in the field of geotechnical engineering of the fact that geotechnical parameters, especially the strength parameters including pore water pressure, are highly uncertain or random. Conventional deterministic approach is, therefore, being increasingly replaced with probabilistic approach or reliability analysis within a probabilistic framework. Slope stability analysis is one of the important areas where the recent trend is to determine the probability of failure of slopes instead of, or complementary to, the conventional factor of safety.

During the last decade, quite a few studies on reliability evaluation of earth slopes have been reported in the literature. Most of these studies used the simple yet approximate reliability analysis method known as the mean-value first-order second-moment (MVFOSM) method based on a Taylor series expansion of the factor of safety. However, this method suffers from serious shortcomings such as the following: (1) The method does not use the distribution information about the variables when it is available. (2) The performance function is linearized at the mean values of the basic variables. When the performance function is nonlinear, significant errors may be introduced by neglecting higher order terms, for the reason that the corresponding ratio of mean of performance function to its standard deviation which is evaluated at the mean values may not be the distance to the nonlinear failure surface from the origin of the reduced variables. (3) Furthermore, first-order approximations evaluated at the mean values of the basic variates will give rise to the problem of invariance for mechanically equivalent limit states; that is, the result will depend on how a given limit-state event is defined.

The first-order reliability method (FORM), on the other hand, does not suffer from the above shortcomings and is, therefore, widely considered to be an accurate method. The method has been finding increasing use especially in structural engineering applications for more than a decade. More recently in the geotechnical engineering field also, there have been quite a few attempts at reliability analysis of earth slopes using the FORM method [2–4].

In this chapter, an attempt has been made to develop computational procedures for slope reliability analysis based on the first-order reliability method (FORM). Computer programs have been developed to demonstrate the application of FORM in the determination of (1) the reliability index for a given slip surface and, more importantly, in the determination of (2) the probabilistic critical slip surface and the associated minimum reliability index. Different probability distributions have been considered for the basic random variables. In determining the probabilistic critical slip surface, the basic methodology suggested by Bhattacharya et al. [1] has been adopted. The above reliability analyses have also been carried out using the approximate MVFOSM method, and the results obtained have been compared with those obtained by using the FORM to bring out the difference clearly and demonstrate numerically the shortcomings of the MVFOSM method.

2 Formulation

2.1 Deterministic Analysis

The conventional slope stability analysis follows a deterministic approach wherein out of a number of candidate potential slip surfaces, the one with the least value of factor of safety is searched out and is termed the critical slip surface. It has now been widely appreciated that the slope stability analysis is essentially a problem of optimization wherein the coordinates defining the shape and location of the slip surface are the design variables and the factor of safety functional expressed as a function of the design variables is the objective function to be minimized subject to the constraints that the obtained critical slip surface should be kinematically admissible and physically acceptable. In practice, analysis is often done based on the assumption that the slip surface is an arc of a circle, as it greatly simplifies the problem. The ordinary method of slices (OMS) [5] is the simplest and the earliest method of slices that assumes a circular slip surface geometry.

The factor of safety functional (FS) for the ordinary method of slices (OMS) is given by the following expression [Eq. (1)], where the notations have their usual meaning. Specifically, $c^{'}$ and $\phi^{'}$ denote the effective cohesion and effective angle of shearing resistance, respectively; W_i and u_i are the weight and the pore water pressure at the base of the ith slice, respectively; θ_i is the base inclination of the ith slice; and Δl_i and $\widehat{L}$ are the base length of the ith slice and the total arc length of the slip circle, respectively:

$$\mathrm{FS} = \frac{c' \ \widehat{L} + \tan\ \phi' \ \sum_{i=1}^{i=n} (W_i \ \cos \ \theta_i - u_i \Delta l_i)}{\sum_{i=1}^{i=n} (W_i \ \sin \ \theta_i)} \tag{1}$$

Substituting $W_i = \gamma b h_i$, and $u_i = r_u \gamma h_i$, where γ and b are the unit weight of soil and the common width of slice, respectively, h_i is the mean height of the ith slice, and r_u is the pore pressure ratio, Eq. (1) reduces to

$$\mathrm{FS} = \frac{c' \ \widehat{L} + \tan\ \phi' \ \sum_{i=1}^{i=n} (\gamma b h_i \ \cos \ \theta_i - r_u \gamma h_i \Delta l_i)}{\sum_{i=1}^{i=n} (\gamma b h_i \ \sin \ \theta_i)} \tag{2}$$

2.2 Reliability Index β Based on the MVFOSM Method

Taking the performance function as the expression for FS in a limit equilibrium method of slices such as Eq. (1) or (2) for analyzing slope stability and the corresponding limit-state equation as $\mathrm{FS}-1 = 0$, the reliability index β based on the MVFOSM method is given by

$$\beta = \frac{E[\mathrm{FS}] - 1}{\sigma\ [\mathrm{FS}]}$$
$$= \frac{\mathrm{FS}\left(\mu_{x_i}\right) - 1}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial \mathrm{FS}}{\partial X_i}\right)^2 \sigma^2 [X_i] + 2 \sum_{i,j=1}^{n} \left(\frac{\partial \mathrm{FS}}{\partial X_i}\right) \left(\frac{\partial \mathrm{FS}}{\partial X_j}\right) \rho\ \sigma[X_i]\ \sigma[X_j]}} \tag{3}$$

where n is the number of soil strength parameters (c', tan ϕ', r_u, γ, etc.) taken as random variables; E[FS], the expected value of FS; σ[FS], the standard deviation of FS; μ_{xi}, the mean value of random variable X_i; $\sigma[X_i]$, the standard deviation of X_i; and ρ, correlation coefficient between X_i and X_j.

2.2.1 Mechanically Equivalent Limit State

When using MVFOSM method, it is of interest to study how the results of reliability analysis differ when other mechanically equivalent limit states are adopted. A limit state equivalent to FS−1 = 0 mentioned above is given by ln (FS) = 0. For such a limit state, the reliability index is given by

$$\beta = \frac{E[ln\ \mathrm{FS}]}{\sigma_{ln\,\mathrm{FS}}} \tag{4a}$$

where

$$E[ln\ \mathrm{FS}] = ln(E[\mathrm{FS}]) - \frac{\sigma^2_{ln\,\mathrm{FS}}}{2} \tag{4b}$$

and

$$\sigma_{ln\,\mathrm{FS}} = \sqrt{ln\left(1 + \left(\frac{\sigma_{\mathrm{FS}}}{E[\mathrm{FS}]}\right)^2\right)} \tag{4c}$$

2.3 Reliability Index β Based on the FORM Method

In this method, the reliability index (β) is defined as the minimum distance (D_{min}) from the failure surface $[g(X') = 0]$ to the origin of the reduced variates, as originally proposed by Hasofer and Lind [7]. For general nonlinear limit states, the computation of the minimum distance ($D_{\min}$) becomes an optimization problem as stated below:

Minimize $D = \sqrt{X'^{t}X'}$
Subject to the constraint $g(X') = 0$

where X' represents the coordinates of the checking point on the limit-state equation in the reduced coordinates system.

Two optimization algorithms are commonly used to solve the above minimization problem to obtain the design point on the failure surface and the corresponding reliability index β [6]. In the first method [10] referred to as FORM method I by Haldar and Mahadevan [6], it is required to solve the limit-state equation during the iteration. The second method [11] referred to as FORM method II by Haldar and Mahadevan [6] does not require solution of the limit-state equation. It uses a Newton-type recursive formula to find the design point. The FORM method II is particularly useful when the performance function is implicit, that is, when it cannot be written as a closed-form expression in terms of the random variables. The FORM method, however, is applicable only for normal random variables. For non-normal variables, it is necessary to transform them into equivalent normal variables. This is usually done following the well-known Rackwitz–Fiessler method [6].

2.4 Probability of Failure

Once the value of the reliability index β is determined by any of the methods discussed above, the probability of failure p_{F} is then obtained as

$$p_{\mathrm{F}} = \Phi(-\beta) \tag{5}$$

where $\Phi(.)$ is the standard normal cumulative probability distribution function, values of which are tabulated in standard texts.

2.5 Determination of Probabilistic Critical Slip Surface

Bhattacharya et al. [1] proposed a procedure for locating the surface of minimum reliability index, $\beta_{\min}$, for earth slopes. The procedure is based on a formulation similar to that used to search for the surface of minimum factor of safety, $\mathrm{FS}_{\min}$, in a conventional slope stability analysis. The advantage of such a formulation lies in enabling a direct search for the critical probabilistic surface by utilizing an existing deterministic slope stability algorithm or software with the addition of a simple module for the calculation of the reliability index β. This is definitely an improvement over the indirect search procedure proposed earlier by Hassan and Wolff [8].

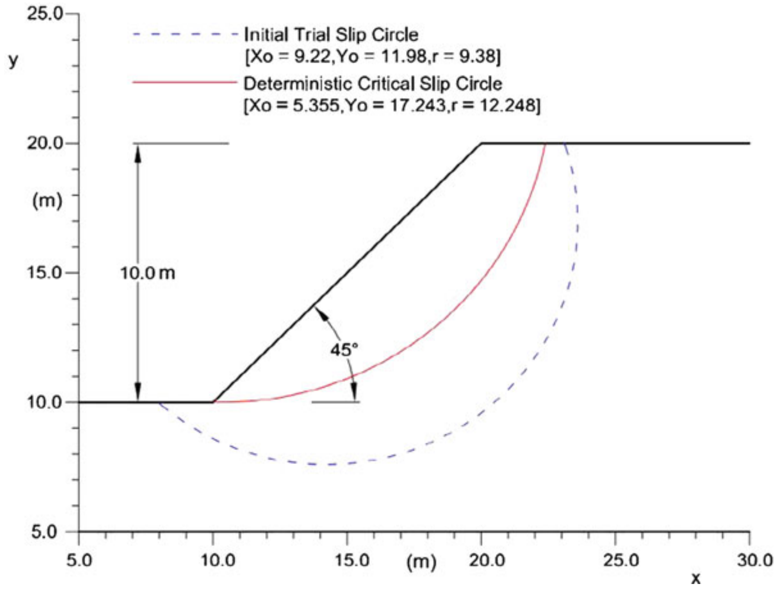

Fig. 1 Slope section and the deterministic critical slip circle in the illustrative example

Table 1 Statistical properties of soil parameters

Parameter (1)	Mean (2)	Standard deviation (3)	Coefficient of variation (4)
c'	18.0 kN/m^2	3.6 kN/m^2	0.20
$\tan \phi'$	tan 30°	0.0577	0.10
γ	18.0 kN/m^3	0.9 kN/m^3	0.05
r_u	0.2	0.02	0.10

3 Illustrative Example

Figure 1 shows a section of a simple slope of inclination 45° and height 10 m in a homogeneous c- ϕ soil. Previous reliability analyses of this slope under a probabilistic framework include those reported by Li and Lumb [9], Hassan and Wolff [8], and Bhattacharya et al. [1] using different methods of analysis. Thus, this example can well be regarded as a benchmark example problem. In all the previous investigations, all four geotechnical parameters, namely, the effective cohesion c', the effective angle of shearing resistance ϕ', the pore pressure ratio r_u, and the unit weight γ, were treated as random variables, and their statistical properties (mean, standard deviation, and coefficient of variation) are as in Table 1.

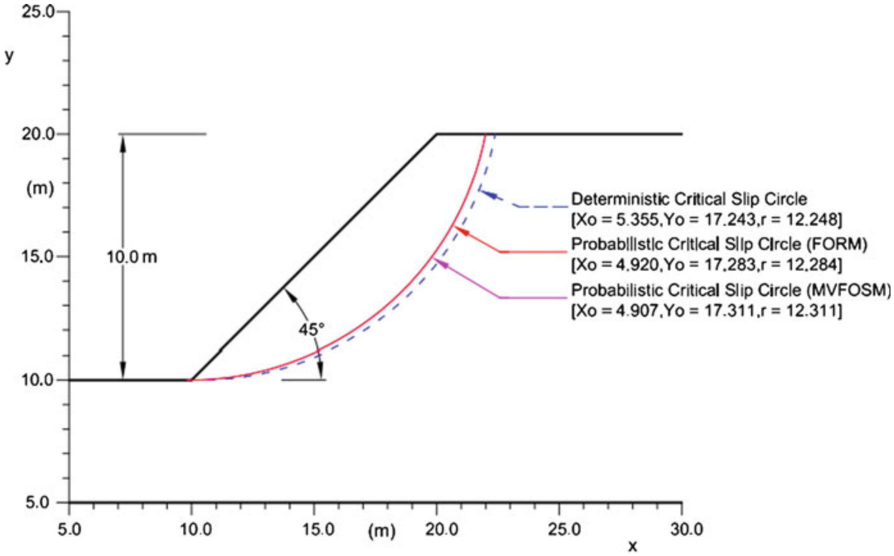

Fig. 2 Probabilistic and deterministic critical slip surfaces

4 Results and Discussion

4.1 Deterministic Analysis

For the purpose of determination of the critical slip circle, a trial slip circle ($x_o = 9.22$ m, $y_o = 11.98$ m, $r = 9.38$ m with reference to the axis system shown in Fig. 1) has been arbitrarily selected. Using Eq. (1) or (2) for the ordinary method of slices, its factor of safety (FS) is obtained as 1.70, when the parameters c', tan ϕ', γ, and r_u are assumed constant at their mean values (Table 1). With this slip circle as the initial slip surface, the developed computer program based on the sequential unconstrained minimization technique (SUMT) of nonlinear optimization coupled with the ordinary method of slices (OMS) yields a critical slip circle ($x_c = 5.355$ m, $y_c = 17.243$ m, $r_c = 12.248$ m) which passes through the toe, as shown in Fig. 2. The associated minimum factor of safety (F_{min}) is obtained as 1.26.

4.2 Reliability Analysis

Reliability analysis of this slope was attempted using two methods, namely, the mean-value first-order second-moment (MVFOSM) method and the first-order reliability method (FORM), with a view to compare the two sets of results. All four parameters c', tan ϕ', γ, and r_u are assumed to be normally distributed and uncorrelated. However, reliability analyses were carried out in three phases: In phase I only two parameters, namely, the cohesion c' and the effective angle of shearing resistance in the form of tan ϕ', were treated as random variables, while the other two parameters γ and r_u were assumed as constants at their mean values. In phase II, three parameters, namely, the cohesion c', the effective angle of shearing resistance tan ϕ', and r_u, were treated as random variables, while the parameter γ was assumed as constant at its mean value, while in phase III all four parameters were assumed as random variables.

4.3 Reliability Analysis for a Given Slip Surface Using MVFOSM

For the two slip surfaces shown in Fig. 1, namely, (1) the initial slip circle ($x_o = 9.22$, $y_o = 11.98$, $r = 9.38$) and (2) the deterministic critical slip circle ($x_o = 5.355$, $y_o = 17.243$, $r = 12.248$), the reliability indices were determined by MVFOSM method by taking two mechanically equivalent limit states: FS$-1 = 0$ and ln (FS) $= 0$ using Eqs. (3) and (4a, 4b, 4c), respectively, for phase I, phase II, and phase III described above. The reliability index values for the different cases are summarized in Table 2.

4.4 Reliability Analysis for a Given Slip Surface Using FORM

Reliability index values have also been determined for the above mentioned slip surfaces using FORM. In particular, the algorithm for FORM method I [6] has been used in this case. All three phases mentioned above have been analyzed. For the sake of comparison as well as numerical demonstration, both the limit states considered in the analyses by MVFOSM have also been used here. The results are summarized in Table 2 again, alongside those obtained by using MVFOSM.

From Table 2, the following observations are made:

1. For the same slip surface and the same set of random variables (in phases I, II, and III), values of reliability index obtained for different mechanically equivalent limit states are markedly different when MVFOSM is used as the method of reliability analysis, whereas these values are identical when analyzed by the FORM method. This observation clearly demonstrates that unlike the FORM method, the MVFOSM method suffers from the "problem of invariance," that is, the result depends on how a given limit-state event is defined. In this respect, another observation from Table 2 is that when the limit state is taken as ln (FS) $= 0$, the reliability index values are higher in all three phases of analysis.
2. It may be noted from Eqs. (1) and (2) that the performance function FS is linear when only c' and tan ϕ' are treated as random variables as in phase I of reliability analysis. However, when c', tan ϕ', and r_u are treated as random variables as in

Table 2 Summary of results of reliability analyses for given slip surfaces

Method of reliability analysis	Slip surface	Values of reliability index					
		Limit state: FS − 1 = 0			Limit state: ln (FS) = 0		
		Phase I	Phase II	Phase III	Phase I	Phase II	Phase III
		(c', tan ϕ')	(c', tan ϕ', r_u)	(c', tan ϕ', r_u, γ)	(c', tan ϕ')	(c', tan ϕ', r_u)	(c', tan ϕ', r_u, γ)
MVFOSM	Initial slip circle	3.955 (3.83×10^{-5})	3.812 (6.89×10^{-5})	3.736 (9.35×10^{-5})	5.059 (2.10×10^{-7})	4.874 (5.46×10^{-7})	4.775 (8.98×10^{-7})
	Deterministic critical slip circle	1.671 (4.73×10^{-2})	1.643 (5.02×10^{-2})	1.601 (5.47×10^{-2})	1.815 (3.47×10^{-2})	1.783 (3.73×10^{-2})	1.734 (4.14×10^{-2})
FORM (all random variables are normally distributed)	Initial slip circle	3.955 (3.83×10^{-5})	3.862 (5.63×10^{-5})	3.851 (5.88×10^{-5})	3.955 (3.83×10^{-5})	3.862 (5.63×10^{-5})	3.851 (5.88×10^{-5})
	Deterministic critical slip circle	1.671 (4.73×10^{-2})	1.646 (4.99×10^{-2})	1.625 (5.21×10^{-2})	1.671 (4.73×10^{-2})	1.646 (4.99×10^{-2})	1.625 (5.21×10^{-2})

Note: Figures in the parentheses indicate the values of the probability of failure for the respective slip surface phase II or when c', tan ϕ', r_u, and γ are treated as random variables as in phase III, the performance function FS becomes nonlinear, and the degree of nonlinearity increases from phase II to phase III. Now, from Table 2, it is seen that for phase I, the values of reliability index yielded by MVFOSM and FORM are exactly the same, whereas they are different for phases II and III. Further, this difference in case of phase III is more than in case of phase II. This observation clearly demonstrates that in those situations where the performance function is linear and all the variables are normally distributed and statistically independent, the values of reliability index by MVFOSM method agree with those given by the FORM, and the error associated with MVFOSM method increases as the degree of nonlinearity of the performance function (or limit-state equation) increases.

3. Another important observation from Table 2 is that when the number of random variables increases, the value of reliability index (β) decreases and probability of failure increases.

4.5 Reliability Analysis for Given Slip Surfaces: Effect of Probability Distributions of the Basic Variates

As already stated, the MVFOSM method does not use the information on probability distribution of the basic random variables. In the FORM method, on the other hand, this information can be incorporated in the analysis. In the present analysis, the effect of variation of probability distributions has been studied using FORM method I. Only two distributions have been considered, namely, the normal distribution and the lognormal distribution. Results have been obtained for phase I only, that is, when only two parameters c′. and tan ϕ' are treated as random variables. Table 3 summarizes the results. It can be observed that there are substantial differences in the values of the reliability index obtained by using FORM when different combinations of probability distributions for the random variates c'. and tan ϕ' are considered. It is further observed that the β values from MVFOSM agree with those from FORM only when both the random variables are assumed to be normally distributed. Thus, it can be said that the MVFOSM method, though does not make use of any such knowledge regarding distribution of variates, implicitly assumes that all variables are normally distributed.

4.6 Probabilistic Critical Slip Surface and the Associated β_{min}

The probabilistic critical slip surface (surface of minimum β) has been determined following the same computational procedure as used for the determination of the deterministic critical slip surface, simply by replacing the objective function FS with

Table 3 Variation of reliability index with different probability distributions for the basic variates

			Values of reliability index for			
	Probability distribution		Initial trial slip circle		Deterministic critical slip circle	
Limit state surface	c'	$\tan \phi'$	MVFOSM	FORM	MVFOSM	FORM
$FS-1 = 0$	Lognormal	Normal	3.955	4.806	1.671	1.859
	Normal	Lognormal		4.038		1.661
	Lognormal	Lognormal		5.318		1.854
	Normal	Normal		3.955		1.671
$\ln FS = 0$	Lognormal	Normal	5.059	4.806	1.815	1.859
	Normal	Lognormal		4.038		1.661
	Lognormal	Lognormal		5.318		1.854
	Normal	Normal		3.955		1.671

Note: These results correspond to the phase I analysis, that is, when only c'. and $\tan \phi'$ are treated as random variables

β [1]. A computer program was developed based on the sequential unconstrained minimization technique (SUMT) of nonlinear optimization coupled with a method of reliability analysis, MVFOSM or FORM, as the case may be.

For this search, the deterministic critical slip surface shown in Fig. 1 has been used as the initial slip surface. Several such probabilistic critical slip surfaces have been determined, and the associated minimum reliability index ($\beta_{\min}$) values are summarized in Table 4. For the sake of clarity, only two of these critical surfaces are plotted in Fig. 2: the probabilistic critical slip surface for phase III analysis using MVFOSM ($x_c = 4.907$ m, $y_c = 17.311$ m, $r_c = 12.311$ m) and the probabilistic critical slip surface for phase III analysis using FORM with all four random variables normally distributed ($x_c = 4.920$ m, $y_c = 17.283$ m, $r_c = 12.284$ m). For the sake of comparison, the deterministic critical slip surface ($x_c = 5.355$ m, $y_c = 17.243$ m, $r_c = 12.248$ m) has also been plotted in Fig. 2.

From Fig. 2, as well as from the magnitudes of the coordinates of centers and radii, it is seen that the two probabilistic critical slip surfaces are located very close to each other while the deterministic critical slip circle is somewhat apart. The closeness of the deterministic and the probabilistic critical slip surfaces for the case of simple homogeneous slopes is in agreement with those reported by earlier investigators. The detailed results presented in Table 4 generally corroborate the observations made earlier from Table 2 with reference to the reliability analyses of the given slip surfaces.

5 Summary and Conclusions

In view of the growing appreciation of the uncertainty associated with the geotechnical parameters, especially, the strength parameters including the pore water pressure, the conventional deterministic approach of analysis is increasingly

Table 4 Summary of results of the minimum reliability analyses associated with the probabilistic critical slip surface

	Values of minimum reliability index					
	Limit state: FS−1 = 0			Limit state: ln (FS) = 0		
	Phase I	Phase II	Phase III	Phase I	Phase II	Phase III
Method of reliability analysis	(c', tan ϕ')	(c', tan ϕ', r_u)	(c', tan ϕ', r_u, γ)	(c', tan ϕ')	(c', tan ϕ', r_u)	(c', tan ϕ', r_u, γ)
MVFOSM	1.643 (1.671)	1.618 (1.643)	1.576 (1.601)	1.785 (1.815)	1.756 (1.783)	1.707 (1.734)
FORM (all random variables are normally distributed)	1.643 (1.671)	1.620 (1.646)	1.599 (1.625)	1.643 (1.671)	1.620 (1.646)	1.599 (1.625)

Note: Figures in the parentheses indicate the values of reliability index for the deterministic critical slip surface

being replaced by probabilistic approach of analysis or reliability analysis under a probabilistic framework. The mean-value first-order second-moment (MVFOSM) method based on a Taylor series expansion is rather widely used by the practitioners in the geotechnical engineering field mainly due to the simplicity and early origin of the method. However, in other fields of engineering, for example, in the structural engineering field, it is an established fact for quite some time that the MVFOSM method suffers from serious shortcomings such as the problem of invariance, as mentioned in an earlier section of this chapter.

This chapter concerns a study on the reliability analysis of earth slopes with uncertain soil strength parameters under a probabilistic framework. Reliability analyses have been carried out using a rigorous method, namely, the first-order reliability method (FORM) in conjunction with a simple slope stability model, namely, the ordinary method of slices (OMS). For the sake of comparison, results in the form of the reliability index and probability of failure have also been obtained using the MVFOSM method. Computer programs have been developed for the determination of reliability index based on both FORM and MVFOSM method for a given slip surface and then for the optimization-based determination of the probabilistic critical slip surface and the associated minimum reliability index. The developed programs have been applied to a benchmark example problem concerning a simple slope in homogeneous soil in which the geotechnical parameters are treated as random variables with given values of statistical moments. The differences between the two sets of results have been brought out for the cases of an arbitrarily selected given slip surface, the deterministic critical slip surface, and also the probabilistic critical slip surface. The study has been successfully used to demonstrate numerically all the major shortcomings of the approximate MVFOSM method and the error involved vis-à-vis the more accurate FORM method.

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