# Design of Fuzzy PD Controller for Inverted Pendulum in Real Time

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**Abstract.** This paper presents the control of 1-stage non-linear model of Inverted Pendulum (IP) in real time using conventional PID and Fuzzy PD controller. Fuzzy controller is non-linear controller in which it is cumbersome to tune the parameters of fuzzy membership function to acquire the desired results. To overcome this problem, linear fuzzy PD controller is used. Firstly controllers are designed in MATLAB Simulink environment and after that both are implemented in real time for controlling the IP. The simulation results reveals that the performance of Fuzzy PD controller is better, efficient and improved one compared to conventional PID controller.

**Keywords:** Inverted Pendulum, PID controller, Fuzzy controller.

## 1 Introduction

The Inverted Pendulum has the property of unstable, higher order, multivariable and highly coupled, which can be treated as a typical non-linear control problem [3]. The IP system provides an excellent experimental platform to test various control theories and techniques. Inverted pendulum can vividly simulate the flight control of rockets and the stabling control in walking robots etc. In conventional control theory, most of control problems are generally solved by mathematical tools based on system models. In practical world, it is not possible to derive exact mathematical model of complex system because of certain uncertainties. Various control techniques are present for which exact mathematical model of the system is not necessary such as fuzzy system, neural system, genetic algorithm etc.

In this paper, fuzzy PD control system is used to control the non-linear model of IP [5] in a better way. Fuzzy system [6] used linguistic variables to approximate the system. Two controllers are used to stabilize cart position and pendulum angle, one for cart position and another for pendulum angle. Firstly, PID controller has been developed by tuning the various gains of PID for stable the IP. Fuzzy controller is a non-linear controller; it is difficult to tune the parameters of membership function which gives better result. To reduce the difficulty, it is convenient to use the linear fuzzy controller [6]. Here, fuzzy PD controller is used which is one of the type of linear fuzzy controller.

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## 2 Mathematical Modeling of Inverted Pendulum

The 1-stage inverted pendulum is made up of cart onto which pendulum is hinged. The 1-stage inverted pendulum is shown in fig.1 [3]. The cart is constrained to move only in the horizontal x direction, while the pendulum can rotate in the x-y plane. The single inverted pendulum system has two degrees of freedom and can therefore be fully represented using two generalized coordinates: horizontal displacement of the cart, and rotational displacement of pendulum. The physical properties of the system are fixed and are shown in table 1.

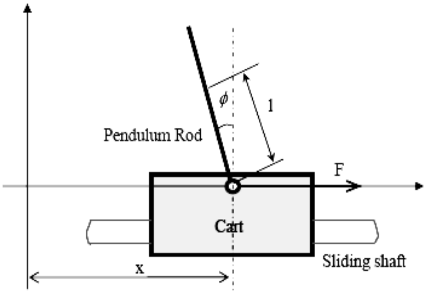


**Fig. 1.** 1- Stage Inverted Pendulum

In this section, the mathematical model of 1-stage inverted pendulum and dynamical equations will be given. After ignoring the air resistance and other frictions, 1-stage IP can be simplified as a system of cart and rod, as shown in Fig. 1. The two dynamic equations of Inverted Pendulum are:

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F \tag{1}$$

$$(I + ml^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta \tag{2}$$

Non linear model of Inverted Pendulum is made with the help of above two equations [5].

**Table 1.** Physical parameters of Inverted Pendulum

| Symbol | Definition | Value |
|---|---|---|
| M | Mass of the cart | 1.096 Kg |
| m | Mass of the rod | 0.1096 Kg |
| b | Friction coefficient of the cart | 0.1 N/m/sec |
| I | Rod inertia | 0.0034 Kg*m*m |
| l | Distance from the rod axis rotation center to the rod mass center | 0.25 m |
| F | Force acting on the cart | |
| x | Cart position | |
| θ | Angle between the rod and the vertically downward direction | |

## 3 Implementation of Controller

Controllers are used to stabilize the unstable system and make it robust to disturbances. The framework of the Inverted Pendulum-cart system controller is shown in fig.2. As in fig.2, Inverted Pendulum-cart system is controlled by two separate controller, pendulum angle controller and cart position controller. From the dynamic equations of this system, it is found that there are two dynamic objects in the inverted pendulum-cart system. One is the pendulum and the other is the cart. However, there is only one control action is allowed for the inverted pendulum–cart system.

Therefore, the control action Fp for the pendulum subsystem and the control action Fc for the cart subsystem need to be combined into one control action F for the inverted pendulum-cart system. It can be seen that to provide a control action to push the cart toward left-hand side will move the pendulum to the right-hand side. This instinctive knowledge indicates that the control actions to move the cart and pendulum to the same direction have opposite sign. Since the main purpose for the position control of the inverted pendulum-cart system is to balance the pendulum at the straight upward direction, the combination of Fp and Fc is defined as $F=F_p - F_c$ [1].

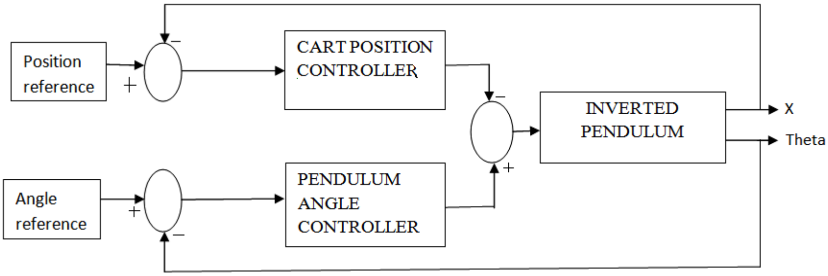


**Fig. 2.** Block diagram of inverted pendulum-cart controller system

### 3.1 Conventional PID Controller

Conventional PID controller is widely used in control applications for its simple structure, easy tuning and independence from system model. Inverted pendulum is an unstable system, and the main objective is to stable the pendulum rod in upright position with moving cart in particular position. For this two controllers are used, one for pendulum angle and another for cart position. For both PID controllers, the standard structure is as follows:

$$u = K_p e + K_I \int e dt + K_d \frac{de}{dt} \tag{3}$$

Where u is control action, e is error between desired output and the actual plant output, i.e. $e = y_{ref} - y_{output}$ . PID controller trying to reduce the error between the desired output and the actual output by varying the gains $K_p, K_I, K_d$. $K_p, K_I, K_d$ are the Proportional, Integral and Derivative gain respectively. In this work, Zeigler-Nicholas tuning is used.

## 3.2 Fuzzy PD Controller

A fuzzy controller is an automatic, non-linear controller, a self-acting or self-regulating mechanism that controls an object in accordance with the desired behaviour. A fuzzy controller acts or regulates by means of natural language or linguistic language, with the distinguishing feature, fuzzy logic. The basic block diagram of fuzzy controller is shown in fig.3.The fuzzy controller is in between pre-processing block and post-processing block. The fuzzification block converts the crisp input into fuzzy sets. Rule base is in the if-then format, and the variables are error and change in error. This gives control output in fuzzy form. For applying to the plant, it is necessary to convert it into crisp output. To obtain the crisp output, defuzzification block is used.

Conventional fuzzy controller is a non-linear controller, thus it is very difficult to tune the parameters of fuzzy controller for obtain the desired behaviour. To overcome this problem, linear fuzzy controller is used. Linear fuzzy controller is similar to PID controller. In this paper, fuzzy PD controller is used [6], block diagram is shown in fig.4.

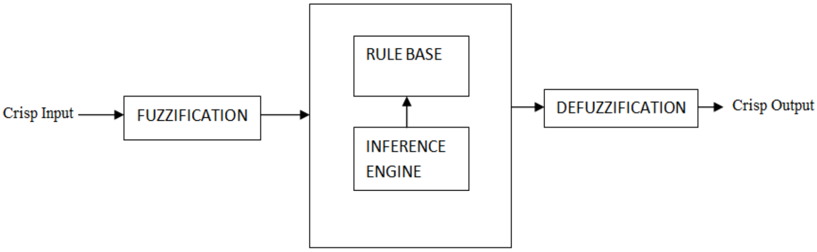


**Fig. 3.** Basic building block of fuzzy controller

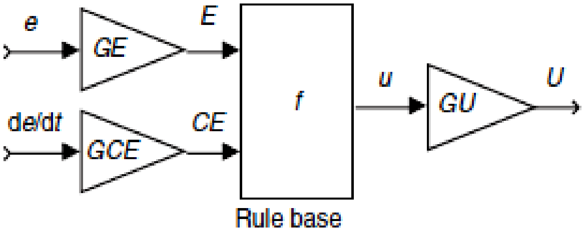


**Fig. 4.** Fuzzy PD controller

To control the whole inverted pendulum, two controllers are used. One is pendulum angle controller and cart position controller. Inputs to both the controllers are error and change in error. The fuzzy variables used for input variable error(e) , change in error(edot) and control output(u) for cart position controller are three. Therefore, total rules in rule base are 9 for position controller. The fuzzy variables used for input variable error(e), change in error(edot) and control output(u) for pendulum angle controller are five. The total rules in rule base for angle controller are 25.The triangular membership is used for all fuzzy variables and crossover point between membership function is at membership grade value 0.5, for linearize the fuzzy controller. At the time of real time simulation, large number of rules creates a problem; system will hang or shut down. Thus, it is necessary for real time simulation to reduce the rules. Reduction of rules takes place by the elimination of redundant rules, i.e rules which are not fire ever at any time of instant.

## 4 Simulation Results and Analysis

Inverted pendulum system has been real time simulated with two PID and two Fuzzy PD controllers. The reference values of cart position and pendulum angle are 0.2 and 0 respectively. Various gains of PID controllers are tuned by Zeigler- Nicholas criteria manually. It is necessary to choose the correct values of gains, otherwise system will unstable.

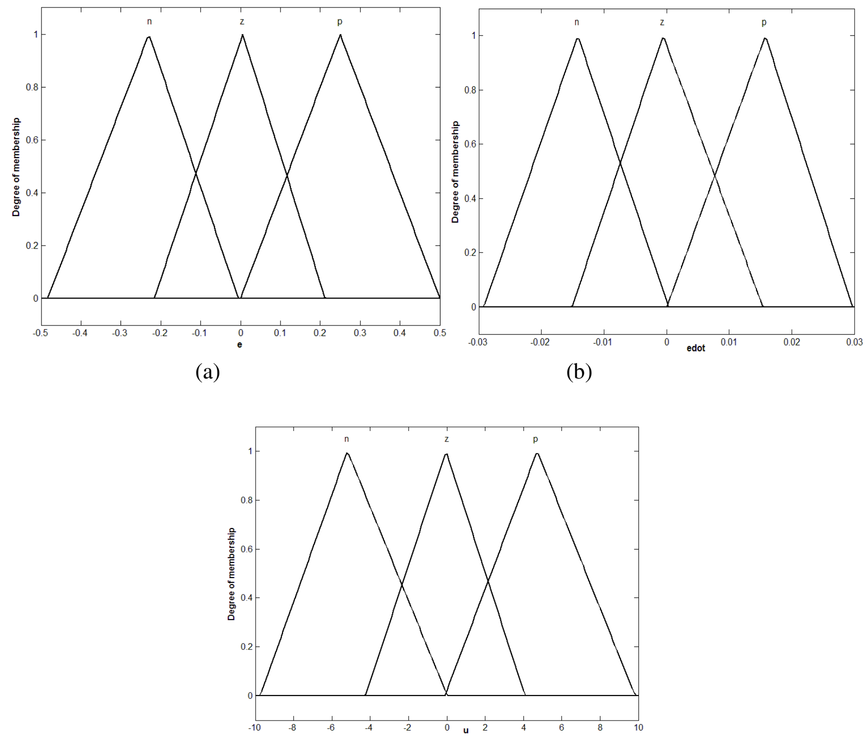
PID parameters of Cart position controller are:

$$K_p = 2.55, K_I = 0.005, K_d = 0.005$$

PID parameters of Pendulum angle controller are:

$$K_p = 50, K_I = 35, K_d = 8$$

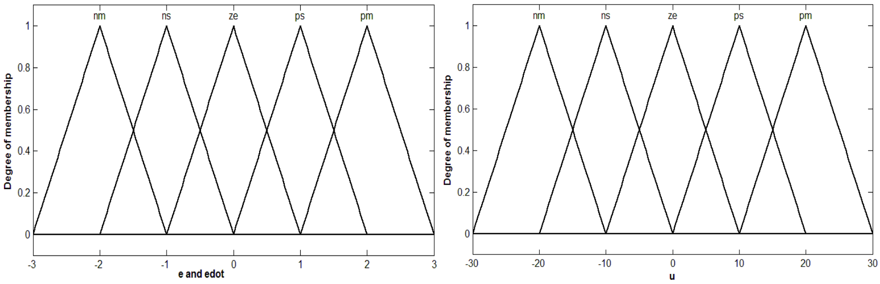
Two fuzzy PD controllers are used. Triangular membership function is used for all fuzzy variables for both controllers. The membership function of inputs error, errordot and control output for Cart position controller are in the range: [-0.5 0.5], [-0.03 0.03] and [-10 10] respectively as shown in fig.5. The membership function of inputs error, errordot and control output for pendulum angle are in the range: [-3 3], [-3 3], and [-30 30] respectively as shown in fig.6. Gain of angle fuzzy PD controller: GE=4, GCE=0.6, GU=1. Gain of cart position fuzzy PD controller: GE=2, GCE=0.05, GU=1. The results are shown in fig.7 and fig.8.



(a) (b)

(c)

**Fig. 5.** Membership function of (a) error (b) errordot (c) control output of Cart position controller



(a) (b)

**Fig. 6.** Membership Function of (a) error and erroedot (b) control output of Angle controller

The rule base used in the Fuzzy- PD controller for pendulum angle and cart position controller is shown in table 2 and 3 respectively.

**Table 2.** Rule base for pendulum angle controller

| $e/\dot{e}$ | NM | NS | Z | PS | PM |
|---|---|---|---|---|---|
| NM | - | - | - | - | - |
| NS | - | - | NM | Z | - |
| Z | - | NM | - | PM | - |
| PS | NM | Z | PM | - | - |
| PM | Z | PM | - | - | - |

**Table 3.** Rule base for cart position controller

| $e/\dot{e}$ | N | Z | P |
|---|---|---|---|
| N | N | N | Z |
| Z | N | Z | P |
| P | Z | P | P |

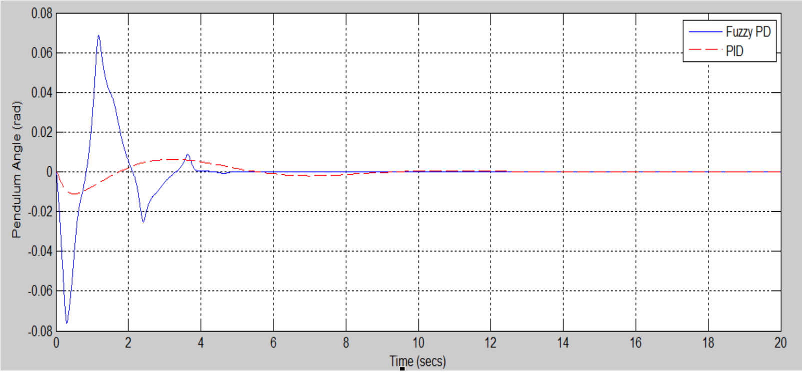


**Fig. 7.** Pendulum angle response of Inverted pendulum

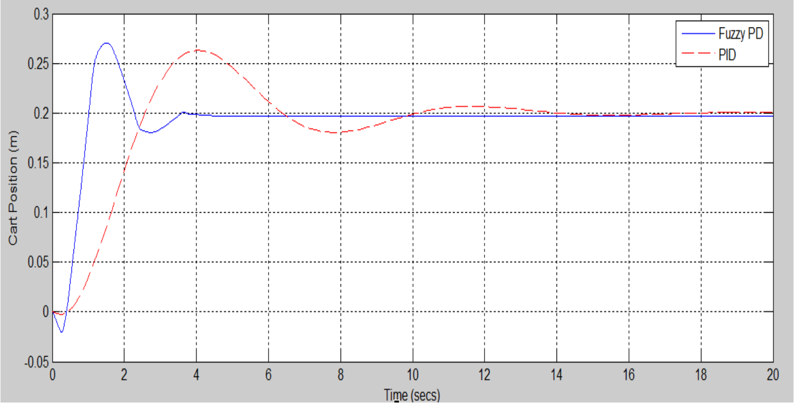


**Fig. 8.** Cart position response of Inverted pendulum

## 5 Conclusion

The objective of this work was to design a controller for stable the inverted pendulum which has been successfully achieved. The fuzzy PD controller is proved to be effective and feasible in both of the angular control of pendulum at upright position and position control of cart to the desired position. As shown in the results, that pendulum angle and cart position both acquire desired position in 3.5 secs. in case of Fuzzy PD controller while 14 secs in PID controller. It is clear from results that fuzzy PD controller stable the Inverted Pendulum more accurately and efficiently than the PID controller.

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