Chapter 2
Equilibrium and the Adjustment Process in a Mixed Oligopoly: A Graphical Explanation

Suzuka Okuyama

Abstract This chapter provides a graphical explanation of a mixed oligopoly model in which private firms maximize their own profits and a public firm maximizes the sum of both the consumer and producer surpluses. In every period, the marginal revenue curve and the after-tax marginal cost curve play important roles in describing the behavior of the private firm. Furthermore, the marginal social benefit curve and the after-tax marginal cost curve contribute to the behavior of the public firm. Both firms react against the supply of other firms in turn in an adjustment process. Finally, the economy reaches equilibrium.

2.1 Introduction

The purpose of this chapter is to present a graphical explanation of a mixed oligopoly, that is, a Cournot–Nash game involving profit-maximizing firms and a social welfare-maximizing firm. Assuming the entry of private firms into a publicly monopolistic market, we depict the equilibrium and the adjustment process to the equilibrium. Although a number of studies have previously addressed mixed oligopolies, few have used graphs, which are helpful for understanding mixed oligopolies.

Over the past few decades, privatization has been brought to the public's attention, and many attempts have been made by scholars to explain this subject. Politically, since the 1980s, many countries have privatized a significant number of public firms to reconstruct public finance. In the United Kingdom, the Thatcher government privatized a wide range of companies, including a public gas company and the waterworks bureau. Similarly, in Japan, the government privatized the Japan Tobacco and Salt Public Company in 1985, the Japanese Railway Company in 1987, and, recently, the Japan Post in 2007. De Fraja (1991) discussed privatization and established the baseline used in this field, following the mixed oligopoly

S. Okuyama (✉)
Osaka University, Osaka, Japan
e-mail: ngp008os@student.econ.osaka-u.ac.jp

M. Yanagihara, M. Kunizaki (eds.), *The Theory of Mixed Oligopoly*, New Frontiers in Regional Science: Asian Perspectives 14, DOI 10.1007/978-4-431-55633-6_2

model developed by De Fraja and Delbono (1989). Since then, many researchers have extended De Fraja's research on privatization. For example, Matsumura (1998) examined a mixed duopoly containing private firms and a privatized firm shared by both the public and private sectors. White (1996) investigated the effect of subsidies in a mixed oligopoly model and clarified that subsidies have a cost distribution effect on production. Lee (2006) considered privatization in the telecommunication industry, where a public firm supplies an essential network service.

Whereas a large number of studies have investigated privatization, few have tried to explain graphically how equilibrium is achieved and how the adjustment process proceeds. Studies on privatization have compared the welfare levels of two situations, mixed and pure oligopolies. This implies that theories of privatization are based on the theories of pure and mixed oligopolies.

There are several studies (e.g., Baldwin 1987 and Nicholson 1972) that explain equilibrium and the adjustment process in a pure oligopoly. However, few studies have attempted to provide a graphical explanation for a mixed oligopoly. Applying the same techniques used by Baldwin (1987) and Nicholson (1972), we graphically depict an equilibrium and the adjustment process in a mixed oligopoly, as introduced in De Fraja (1991). In this setting, it is initially assumed that a public firm monopolistically supplies goods to the market and then private firms enter this market and determine their outputs based on the output of the public firm. The firms continue to play the game until equilibrium is attained.

Obtaining equilibrium in a mixed oligopoly requires two kinds of first-order conditions. One is derived from profit maximization and the other from social welfare maximization. The first-order condition of profit maximization consists of the marginal revenue and the after-tax marginal cost, which is the marginal cost plus the tax rate. As for the latter, that of social welfare maximization consists of the marginal social benefit and after-tax marginal cost. The tax income is used to finance the budget loss of the social welfare maximization firm.

To analyze the adjustment process and equilibrium graphically, we reinterpret first-order conditions as intersections of curves. First, the private firm's quantity is determined to be an intersection of the marginal revenue curve and the after-tax marginal cost curve. These curves have two variables, the private firm's offered price and quantity. Similarly, that of the public firm is determined to be an intersection of the marginal social benefit curve and the after-tax marginal cost curve. These curves also have two variables, the public firm's offered price and quantity.

Illustrating these curves on a plane (with a vertical axis of price and a horizontal axis of quantity) clearly shows the adjustment process and equilibrium in a mixed oligopoly. After entry, each firm repeats its offer of quantity until equilibrium is reached. The reoffering is illustrated as a leftward shift of the marginal revenue curve or the marginal social benefit curve. During the adjustment, incumbent firms decrease their quantities and entry firms increase theirs. When the prices offered by the two firms (public and private) are equal, equilibrium is achieved.

This chapter is organized as follows. Section 2.1 presents the fundamental model of a mixed oligopoly, based on De Fraja (1991). Section 2.2 explains both the adjustment process to reach equilibrium and the equilibrium in a mixed duopoly. Section 2.3 extends the concept to a mixed oligopoly and focuses on the equilibrium, and Sect. 2.4 provides some concluding remarks.

2.2 The Model

We first review the mixed oligopoly model developed by De Fraja (1991). In this model, there are n private firms that maximize their profits and one public firm that maximizes social welfare, defined as the sum of consumer surplus and producer surplus. Let the private firm be firm $i\,(i = 1, \ldots, n)$ and let the public firm be firm 0. These firms play a Cournot–Nash game in which each firm determines its output under the given levels of outputs supplied by the other firms.

These $n + 1$ firms supply their goods to the market, whose inverse demand function is given by

$$p = 1 - Q, \tag{2.1}$$

where p is the market price and Q is the total amount of goods. Assuming symmetry across the private firms and with q_i denoting the supply of firm i and q_0 as that of firm. Q can be expressed as

$$Q = q_0 + \sum_{i=1}^{n} q_i. \tag{2.2}$$

In this model, it is assumed that the cost of a firm is composed of a fixed cost that does not change with the level of output and a variable cost that depends on the output. The fixed cost is the same for both the private firms and the public firm, while the variable cost is not.

We assume that variable costs, which have constant marginal costs, are

$$c_0 > c_1 = c_2 = \cdots = c_n \equiv \widetilde{c}, \tag{2.3}$$

where subscripts represent the firms. The difference in the marginal costs implies that the productivity of the public firm is less efficient than those of the private firms. These marginal costs and the fixed cost provide the cost function of the public firm as follows:

$$c(q_0) = c_0 q_0 + F, \tag{2.4}$$

where F is the fixed cost. Similarly, the cost function of the private firm is

$$c(\widetilde{q}) = \widetilde{c}\,\widetilde{q} + F. \tag{2.5}$$

From (2.1) and (2.5), the profit of the private firm is given by

$$\widetilde{\pi} = p\widetilde{q} - \widetilde{c}\,\widetilde{q} - F - t\widetilde{q}, \tag{2.6}$$

where t represents a specific tax rate that is constant through all periods. As we will see later, because the public firm maximizes social welfare and not profit, there is a deficit that is equal to the fixed cost of the public firm in equilibrium. To finance this budget loss, the government levies a tax on the outputs supplied by all firms. The profit maximization problem of the private firm takes the following form:

$$\frac{\partial \widetilde{\pi}}{\partial \widetilde{q}} = \widetilde{MR} - \widetilde{TMC} = 0 \tag{2.7}$$

where

$$\widetilde{MR} = (1 - q_0 - \sum_{i=1}^{n} q_i) + (-q_i) \tag{2.8}$$

is the marginal revenue of firm i and

$$\widetilde{TMC} = \widetilde{c} + t \tag{2.9}$$

is the after-tax marginal cost of firm i. Regarding tax, an extra unit of production requires the marginal cost plus the tax rate, that is, the after-tax marginal cost.

The right-hand side of (2.8) can be interpreted as follows. First, $(1 - q_0 - \sum_{i=1}^{n} q_i)$ represents the positive effect of an increase in q_i on the revenue of firm i, which is equal to the price. Second, $(-q_i)$ represents the negative effect of a change (decrease) of price on the total revenue. Then, the condition of the profit maximization of (2.7) requires an equalization of (2.8) and (2.9).

The public firm determines its output depending not only on its own profit but also on the profit of the private firm and the consumer surplus. Because the inverse demand function is linear, as in (2.1), the consumer surplus is given by

$$CS = \frac{1}{2}\left(q_0 + \sum_{i=1}^{n} q_i\right)^2. \tag{2.10}$$

As a result, from (2.4), (2.6), and (2.10), social welfare can be represented as follows:

$$S = SB - TTC_n = \left[\frac{1}{2}\left(q_0 + \sum_{i=1}^{n} q_i\right)^2 + \sum_{i=1}^{n} \pi_i + pq_0\right] - (c_0 q_0 + F), \tag{2.11}$$

where S, SB, and TTC_0 represent social welfare, social benefit, and the after-tax cost of the public firm, respectively. SB is defined as social welfare excluding the after-tax marginal cost.

The first-order condition of social welfare maximization then becomes

$$\frac{\partial S}{\partial q_0} = MSB - TMC_0 = 0, \tag{2.12}$$

where

$$MSB = \left(q_0 + \sum_{i=1}^{n} q_i\right) - \sum_{i=1}^{n} q_i + \left[1 - \left(q_0 + \sum_{i=1}^{n} q_i\right) - q_0\right] \tag{2.13}$$

is the marginal social benefit that denotes the additional social benefit brought about by an increase in an incremental unit of output. Furthermore,

$$TMC_0 = c_0 + t \tag{2.14}$$

is the after-tax marginal cost of the public firm. The right-hand side of (2.13) can be interpreted as follows. First, $\left(q_0 + \sum_{i=1}^{n} q_i\right)$ represents the effect of an increase in q_0 on the consumer surplus. Second, $-\sum_{i=1}^{n} q_i$ represents the effect of an increase in q_0 on the revenue of the private firm through the decrease in price. Finally, $\left[1 - \left(q_0 + \sum_{i=1}^{n} q_i\right) - q_0\right]$ represents the effects of an increase in q_0 on the revenue of the public firm, which is equal to the price, and a decrease in q_0 on the revenue of the public firm. It should be noted that the second term and part of the third term cancel the first term. This simply implies that an increase shifts the benefit from the producer surplus to the consumer surplus. As a result, this becomes equivalent to the inverse demand function given by (2.1).

Finally, solving (2.7) and (2.12), the Cournot–Nash equilibrium outputs of these firms can be obtained as follows:

$$q_i = c_0 - \widetilde{c}, \tag{2.15}$$

$$q_0 = 1 - (n+1)c_0 + n\widetilde{c} - t \tag{2.16}$$

2.3 Graphical Explanation

In this section, we graphically represent the Cournot–Nash equilibrium given in (2.15) and (2.16). The merit of the graphical explanation given here is that it can satisfactorily express how equilibrium can be achieved through the adjustment process in a mixed oligopoly. Before turning to the explanation of the mixed oligopoly, it is helpful to consider a mixed duopoly model in which only one private firm (firm 1) and one public firm (firm 0) exist.

In the case of a duopoly, the adjustment process will proceed as follows: originally (or in the 0^{th} period), only the public firm existed in the market and supplied goods monopolistically. In the first period, a private firm enters this market and determines its output by reacting to the output level of the public firm. In the second period, the public firm revises its output level by reacting to the output of the private firm. The behavior of the private firm in the first period and that of the public firm in the second period are defined as the first turn for these two firms, respectively. In the third period, the private firm revises its output, and then in the fourth period, the public firm revises its output. This behavior by the two firms can be defined as the second turn for each firm. The private firm revises its output in every odd-numbered period, and the public firm does so in every even-numbered period. That is, the private firm determines its output in the m^{th} turn in the $2m-1^{\text{th}}$ period, and the public firm determines its output in the m^{th} turn in the $2m^{\text{th}}$ period. The succeeding periods follow this procedure, and the economy finally achieves a Cournot–Nash equilibrium.

2.3.1 From the First to the Third Periods

Let us start with the circumstance in which the public firm dominates the market. The marginal social benefit of the public firm in the 0^{th} period can be written as follows:

$$MSB_0^0 = 1 - q_0^0, \tag{2.13'}$$

where q_0^0 is the level of the output in the 0^{th} period. The superscript denotes the number of turns.[1] From (2.13′) and (2.14), the output of the public firm in the 0^{th} turn can be obtained as

$$q_0^0 = 1 - c_0 - t. \tag{2.17}$$

As shown in Fig. 2.1, with p as the vertical axis and Q as the horizontal axis, the marginal social benefit curve (DD') can be drawn by a straight line with a slope of -1 and an intercept of 1 on the vertical axis. The after-tax marginal cost curve TMC_0 is parallel to the horizontal axis. Therefore, q_0^0 is represented by OA, and the price offered by the public firm is represented by eO; cost curve (TMC_0) is parallel to the horizontal axis.[2] Therefore, q_0^0 is represented by OA, and the price offered by the public firm is represented by eO, which corresponds to TMC_0.

Next, let us consider the entry of the private firm in the first period. Because the after-tax marginal cost of the private firm is lower than that of the public firm, which is equal to the price offered by the public firm as assumed in (2.3), the private

[1] The behavior in the 0^{th} turn is defined as the behavior of the public firm in the 0^{th} period.

[2] The marginal social benefit curve in the 0^{th} period is equivalent to the inverse demand function.

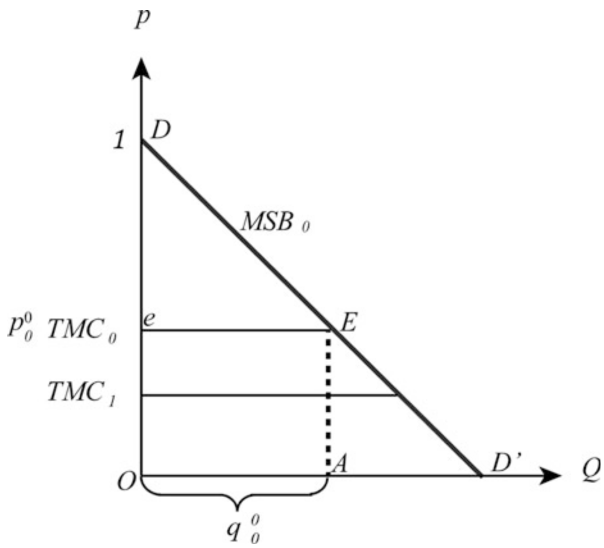

Fig. 2.1 The public firm supplies $q_0^0(OA)$ at $p_0^0(eO)$ in the 0th turn

firm enters the market. The private firm faces its demand curve (ED'), which gives the following marginal revenue:

$$MR_1^1 = 1 - 2q_1^1 - q_0^0, \tag{2.8$'$}$$

as shown by MR_1^1 in Fig. 2.2 and the marginal cost as

$$TMC_1 = \widetilde{c} + t. \tag{2.9$'$}$$

From (2.8′) and (2.9′), the output of the private firm in the first turn q_1^1, can be obtained as

$$q_1^1 = \frac{1}{2}(\widetilde{c} - c_1). \tag{2.18}$$

In Fig. 2.2, the marginal revenue curve $\left(MR_1^1\right)$ can be drawn by a straight line with a slope of -2 and an interception of DD' at E. The after-tax marginal cost curve (TMC_1) can be taken as parallel to the Q axis. The private firm chooses C, which is the intersection of the two curves, and determines its output as AB. Therefore, the price offered by the private firm becomes p_1^1.

The outputs of the public firm given by (2.17) and of the private firm given by (2.18) cannot be consistent with the equilibrium outputs because the prices faced by each firm are different. Thus, the public firm should change its output by responding

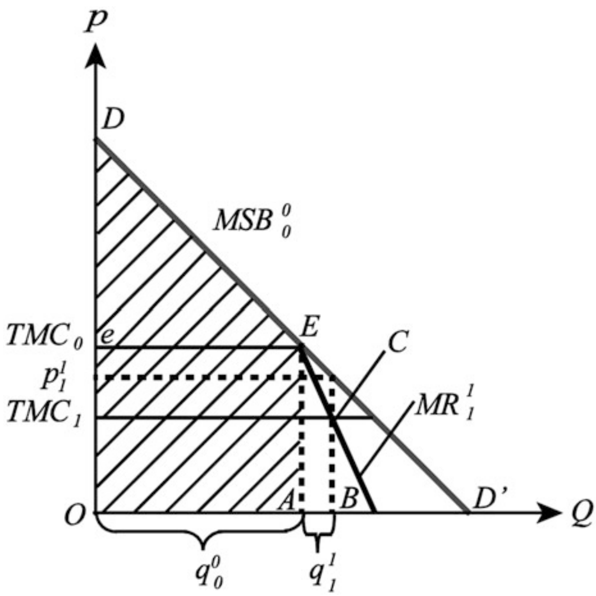

Fig. 2.2 The private firm enters the market in the 1st turn. The private firm supplies $q_1^1(AB)$ at p_1^1

to the outputs of the private firm obtained in (2.18), even though the after-tax marginal cost of the public firm represented by (2.17) is unchanged.

We now consider the behavior of the public firm in the second period. As mentioned above, in this period, the marginal social benefit curve changes in reaction to q_1^1. From this new marginal social benefit curve obtained by substituting (2.18) into (2.13) and the after-tax marginal cost, the output of the public firm in the first turn can be obtained by

$$q_0^1 = (1 - c_0 - t) - \frac{1}{2}(c_0 - \widetilde{c}). \tag{2.19}$$

The adjustment process of how the public firm changes its output is revealed in Fig. 2.3. Now, MR_1^1 in Fig. 2.2 corresponds to MR_1^1 in Fig. 2.3. The quantity OB in Fig. 2.3, which is AB in Fig. 2.2, has shifted left. This determines the length of the inverse demand curve that the public firm faces as GD', denoted by MSB_0^1. As in the 0^{th} turn, the public firm determines its output as BA by choosing E as the intersection of MSB_0^1 and TMC_0, and offers p_0^1, which is unchanged from the 0^{th} period.

It should be noted that in comparison with (2.17), the output of the public firm decreases by $\frac{1}{2}(c_0 - c_1)$ in reaction to the output of the private firm, as given in (2.18). Furthermore, even though the output of the public firm is revised from q_0^0 to q_0^1, the price offered by the public firm has not been altered. This implies that the private firm has an incentive to change the output level in response to the output level of the public firm in the second period given by (2.19).

The output of the private firm in the 3rd period (in the 2nd turn) is obtained by

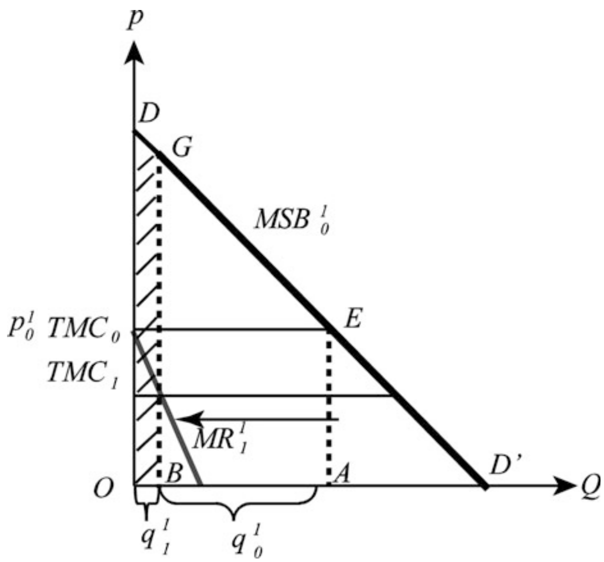

Fig. 2.3 The public firm revises its output in the 1st turn. The public firm revises outputs q_0^1 (AB) at p_0^1 in the 1st turn given q_1^1 determined by the private firm in the previous period. In addition, the public firm supplies at TMC_0

$$q_1^2 = \frac{3}{4}(c_0 - \widetilde{c}). \tag{2.20}$$

The adjustment process that the private firm goes through to change its output in the second turn is shown in Fig. 2.4a. As MSB_0^1 shifts to the left, BA in Fig. 2.2 can be presented as OA in Fig. 2.4a. The inverse demand curve GD' faced by the private firm determines the marginal revenue curve as MR_1^2. The private firm chooses C, which is represented by the intersection of MR_1^2 and TMC_1, and determines its output as AB. The price that the private firm offers is revised asp_1^2.

Regarding the change in the output of the private firm, comparing (2.19) with (2.17), it increases by $\frac{1}{4}(c_0 - \widetilde{c})$. This shows that the private firm gradually increases its share in the market as the public firm loses its market share. In fact, the output of the public firm in the 4th period (in the 2nd turn) becomes

$$q_0^2 = (1 - c_0 - t) - \frac{3}{4}(c_0 - \widetilde{c}) \tag{2.21}$$

which can be represented by BA in Fig. 2.4b. This output is less than (2.19) by just $\frac{1}{4}(c_0 - \widetilde{c})$.

By the same procedure, we obtain the outputs of both firms in the 3rd turn as follows:

$$q_1^3 = \frac{7}{8}(c_0 - \widetilde{c}), \tag{2.22}$$

and

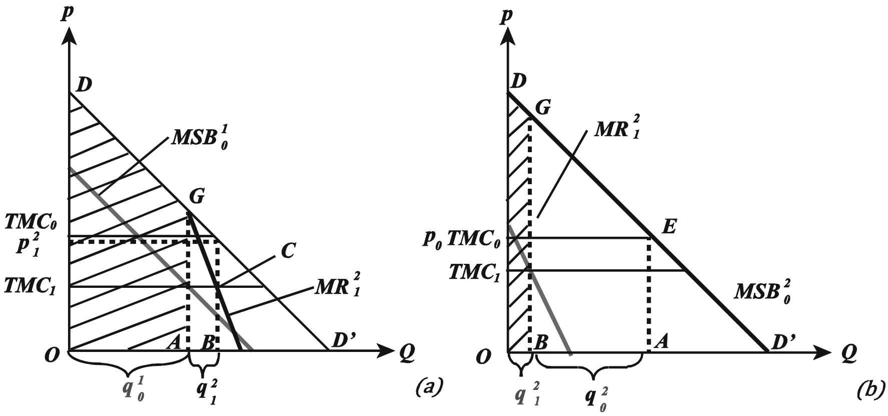

Fig. 2.4 The adjustment process of the 2nd turn. (**a**) The private firm determines its output in the 3rd period. (**b**) The public firm determines its output in the 4th period

$$q_0^3 = (1 - c_0 - t) - \frac{7}{8}(c_0 - c_1). \tag{2.23}$$

Again, it can be acknowledged that an increase in the output of the private firm and, in contrast, the decrease in that of the public firm are $\frac{1}{8}(c_0 - \widetilde{c})$.

2.3.2 From the m^{th} Period to Equilibrium

From the above procedure, we can obtain the outputs of both firms in the m^{th} turn by induction as

$$q_1^m = \frac{2^m - 1}{2^m}(c_0 - \widetilde{c}), \tag{2.24}$$

and

$$q_0^m = (1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \widetilde{c}). \tag{2.25}$$

The adjustment process in which both firms determine their outputs in the m^{th} turn is in Fig. 2.5a, b. Figure 2.5a represents that the private firm determines q_1^m in the $2m - 1^{\text{th}}$ period. The private firm chooses C, which is the intersection of MR_1^m and TMC_1, so that it determines its output as AB and offers p_1^m. Figure 2.5b represents that the public firm determines q_0^m in the $2m^{\text{th}}$ period. The public firm chooses E, which is an intersection of MSB_0^m and TMC_0, the output becomes BA, and the price offered by the public firm is p_0.

We now consider the equilibrium. Although the price offered by the private firm and the one offered by the public firm have not been equal (the public firm's price has been higher than that of the private firm), as m^{th} increases, the price offered by the private firm gradually increases to the (constant) price offered by the public firm. Finally, when the prices offered by both firms are equal, the economy reaches equilibrium. In equilibrium, the outputs of both firms are

$$q_1^* = c_0 - \widetilde{c}, \tag{2.26}$$

and

$$q_0^* = (1 - c_0 - t) - (c_0 - \widetilde{c}) \tag{2.27}$$

as already obtained in (2.15) and (2.16). In Fig. 2.6, (2.26) and (2.27) are represented as to AB and OA, respectively. In this equilibrium, the prices of both firms are equal to p^*.

Because the outputs of both firms in the m^{th} turn can be obtained from (2.24) and (2.25), the total output in equilibrium can be obtained as

$$\begin{aligned} Q^* &= q_1^m + q_0^m = \left[\frac{2^m - 1}{2^m}(c_0 - \widetilde{c})\right] + \left[(1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \widetilde{c})\right] \\ &= (1 - c_0 - t). \end{aligned} \tag{2.28}$$

Because the budget deficit of the public firm is

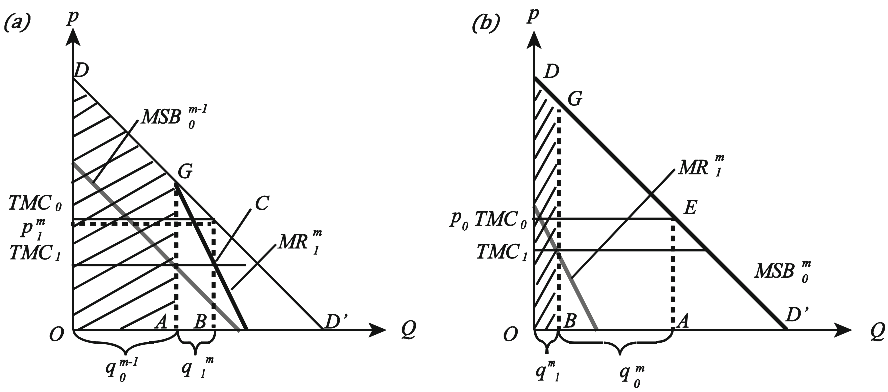

Fig. 2.5 The adjustment process of the m^{th} turn. (**a**) The private firm determines its output in the $2m-1^{\text{th}}$ period. (**b**) The public firm determines its output in the $2m^{\text{th}}$ period

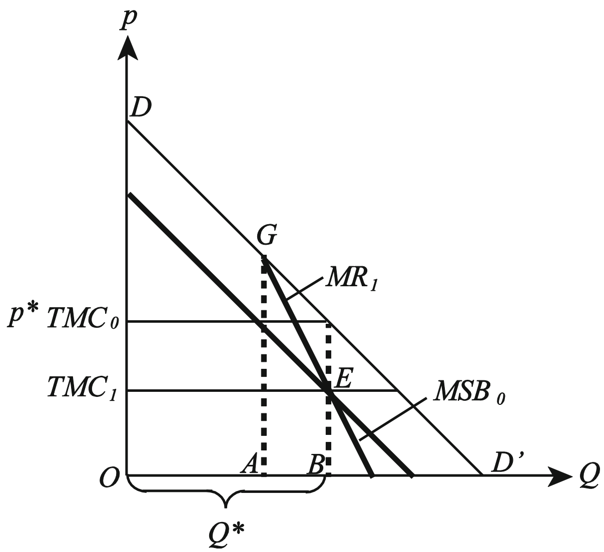

Fig. 2.6 Equilibrium. At equilibrium, the total output is Q^*. The public firm supplies OA and the private firm supplies AB

$$F = tQ \tag{2.29}$$

the tax rate becomes constant in equilibrium and can be obtained as

$$t = \frac{1}{2}\left[1 - c_0 - \sqrt{(1 - c_0)^2 - 4F}\right]. \tag{2.30}$$

If c_0 decreases, (2.28) provides that the total output increases and then the consumer surplus also increases.

Furthermore, (2.26) means that a decrease in c_0 leads to a decrease in the output of the private firm. The equivalency of both firms' marginal costs makes the output of the private firm zero and its profit $-F$. Therefore, the private firm will exit the market. However, the equivalency of marginal costs maximizes the consumer surplus. This means an improvement of the public firm's efficiency is not required to maximize social welfare under the condition that privatization never occurs.

2.3.3 n *Firms Case*

We now turn to the mixed oligopoly in which a public firm and n private firms exist in the economy. We also assume that the public firm and the (group of) private firms determine their outputs in turn. Under these circumstances, the production of one private firm among n firms becomes $\frac{1}{n}$ times that in a mixed duopoly. Therefore, the outputs of one private firm in the m^{th} turn can be obtained as

$$\tilde{q}^m = \frac{1}{n}\left[\frac{2^m - 1}{2^m}(c_0 - \tilde{c})\right], \tag{2.31}$$

and that of the public firm in the m^{th} turn is

$$q_0^m = (1 - c_0 - t) - \sum_{i=1}^{n} \frac{1}{n}\left[\frac{2^m - 1}{2^m}(c_0 - \tilde{c})\right] = (1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \tilde{c}). \tag{2.32}$$

Finally, the output of a private firm and that of a public firm in equilibrium become

$$q^\infty = \frac{1}{n}(c_0 - \tilde{c}), \tag{2.33}$$

and

$$q_0^\infty = (1 - c_0 - t) - (c_0 - \tilde{c}). \tag{2.34}$$

As a result, the total output of the n private firms and the output of the public firm are identical with those in the case of mixed duopoly, seen in (2.26) and (2.27).

2.4 Concluding Remarks

This chapter depicted an equilibrium and the adjustment process in the mixed oligopoly described by De Fraja (1991). The adjustment process to equilibrium can be shown by utilizing the first-order conditions of private and public firms. Specifically, the first-order condition of the private firm is satisfied at the intersection of the marginal revenue curve and the after-tax marginal cost curve. Similarly, the first-order condition of the public firm is satisfied at the intersection of the marginal social benefit curve and the after-tax marginal cost curve. As time proceeds, under a fixed tax rate and marginal costs, the marginal revenue curves and the marginal social benefit curve will change.

By providing a graph that compares a pure oligopoly to a mixed oligopoly, we can understand how private and public firms behave, not only in equilibrium but also in the adjustment process to equilibrium. This graphical explanation contributes to clarifying the effects of the privatization of a public firm.

References

Baldwin L (1987) Market power, competition, and antitrust policy. Irwin Publication in Economics, Homewood
De Fraja G (1991) Efficiency and privatization in imperfectly competitive industries. J Ind Econ 39:311–321
De Fraja G, Delbono F (1989) Alternative strategies of a public enterprise in oligopoly. Oxford Econ Pap 41:302–311
Lee S (2006) Welfare-improving in the telecommunications industry. Contemp Econ Policy 24 (2):237–248
Matsumura T (1998) Partial privatization in mixed duopoly. J Public Econ 70(3):473–483
Nicholson M (1972) Oligopoly and conflict a dynamic approach. Liverpool University Press, Liverpool
White D (1996) Mixed oligopoly, privatization and subsidization. Econ Lett 53:189–195