New Frontiers in Regional Science: Asian Perspectives 14

Mitsuyoshi Yanagihara
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The Theory of Mixed Oligopoly

Privatization, Transboundary Activities, and Their Applications

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Preface

The purpose of this book is to provide a theoretical explanation of mixed oligopolies under different situations. In general, competition among public and private firms in a market can be viewed as a unique example of an oligopoly. However, the roles and the inferences of the public firm can have various implications or effects on the economy. The aim of this book is to therefore serve as a resource and to present recent developments regarding the mixed oligopoly.

When we look at the roles of public firms and the mixed oligopoly, we see the following features in the various fields of economics. First, the history of economic theory recognizes that most of the countries that have experienced the takeoff stage of economic development have adopted incubation policies for new industries. The governments of such countries first established main industries and market activities.

Via these policies, governments attempted to transition from traditional economies to market economies. Similar phenomena are also observed in emerging economies that require the creation of modern industrial sectors. These countries aim to develop their economies and to realize rapid growth. For this purpose, public firms have contributed to industrialization and have captured new technology to enhance economic growth.

With the expansion of the market economy, the role of public firms has changed from market creation to ensuring efficient resource allocation. The public firm is regarded as a policy instrument to restore market failure. When the market faces entry barriers, there are little or no firms to produce goods and occupy the market. The economic analysis of pubic firms shows how the public firm affects and determines the price level of the market.

Stemming from discussions on the implications of public firms, the theory of public firms has been applied to oligopolistic markets where new private firms enter a market monopolized by a public firm. This type of market is recognized as a "mixed oligopoly," where public and private firms compete in the same market. Public economics investigates how the public firm adjusts the distortion resulting from private firms. In this sense, the public firm is justified as a policy instrument to achieve efficient resource allocation.

Furthermore, the expansion of the market economy enhances the entry of new private firms equipped with new production technology. The production cost of such firms is superior to the cost of the public firm. Accordingly, the gap between the production cost of public and private firms becomes significantly larger, and this gap then introduces a further distortion of resource allocation.

The discussion of the privatization of public firms is based on this difference of efficiency between public and private firms. Therefore, the theory of public firms has been expanded to include a new field of industrial organization. In most developed countries, public firms operate under considerable deficit, and thus, privatization is used to alleviate fiscal pressure.

Moreover, the significance of the public firm appears in strategic trade policies, and the mixed oligopoly model is discussed in the field of international economics. Within these studies, the public firm is considered a policy tool to protect the domestic market or social welfare from foreign firms. However, the noncooperative policy games of public firms can result in international conflict, and the coordination of privatization for an open market system becomes a key issue in free trade agreements.

The mixed oligopoly model is appropriate for many economic issues, and in this book, we attempt to consider the mixed oligopoly from several aspects. This book consists of three parts and 13 chapters. We briefly summarize these chapters and present how each chapter investigates the mixed oligopoly and presents analysis results.

Part I introduces the basic framework of a mixed oligopoly. Chapter 1 briefly discusses previous studies and summarizes the properties of a mixed oligopoly. This chapter presents an overview of a mixed oligopoly model and describes the fundamental characteristics of government intervention in an oligopoly.

Chapter 2 provides a graphical explanation of a mixed oligopoly model in which private firms maximize their own profits and the public firm maximizes social welfare. This chapter shows the adjustment process where both firms react against the supply of other firms to reach equilibrium.

Chapter 3 examines the case where the Stackelberg position of a public firm is maintained. Meanwhile, when a public firm acts as a Stackelberg leader before and after privatization, privatization necessarily decreases social welfare irrespective of the number of private firms. Furthermore, when a public firm acts as a Stackelberg follower before and after privatization, privatization decreases social welfare if the number of private firms is relatively small.

Chapter 4 investigates the effect of capital accumulation on partial privatization in a dynamic oligopoly model. It is shown that when the steady state is characterized by a demand-driven equilibrium, partial privatization is adopted and the privatization rate corresponds perfectly to the static model. Furthermore, when a public firm produces Ramsey output, the level of social welfare in a steady state does not depend on the privatization rate and the government adopts a full nationalization policy.

Chapter 5 outlines the basic properties of an international mixed oligopoly and considers the policy implications of privatization in the context of a strategic trade policy. This chapter presents the relationship between the strategic trade policy (i.e., import tariffs) and the degree of privatization and develops an integrated explanation of corporative privatization policy in a two-country mixed oligopoly model.

Part II examines policies on a mixed oligopoly. Chapter 6 considers the optimal privatization policy in an international mixed oligopoly. Allowing for partial privatization and cost asymmetry, this chapter analyzes optimal policies under various tax regimes: arbitrary taxation, origin principle, destination principle, import tariff, and a combination of tax and import tariffs.

Chapter 7 investigates the privatization neutrality theorem when a public firm has an objective other than social welfare maximization. The privatization neutrality theorem claims that when the government provides an optimal subsidy to both public and private firms, social welfare is exactly the same before and after privatization. This chapter considers the validity and limitations of the neutrality theorem in several cases.

Chapter 8 analyzes the effect of domestic lobbying on the optimal degree of privatization and social surplus in a closed, mixed oligopoly model and in an extended two-country model. This chapter shows that lobbying activity leads to overprivatization in a closed economy and may improve social surplus in a two-country economy.

Chapter 9 analyzes how the government's preference affects merger activity between a public firm and a private firm. This chapter shows whether a merger occurs depends on the shareholding ratio of the merged firm and the government's preference for tax revenue. If the government puts a large weight on tax revenue, public and private firms do not merge.

Part III considers further applications of the theory of mixed oligopoly to various economic circumstances. Chapter 10 analyzes a mixed oligopoly model with two asymmetric regions in which the population, the number of private firms, and the individual's shareholding ratios in private firms are different. This chapter shows that social welfare in the case of state ownership is greater than that in local ownership. The effects of distribution of the regional population are also examined.

Chapter 11 investigates a duopolistic long-term care market with uncertainty using a Hotelling-type spatial competition model where care providers decide the quality of the care to attract patients. Three types of competition structures are investigated: duopoly with private nonprofit (NP) providers, a mixed duopoly with an NP provider and a private for-profit (FP) provider, and a duopoly with FP providers. This chapter shows that the equilibrium levels of quality in a mixed duopolistic market are higher than those in an NP duopoly and lower than those in an FP duopoly.

Chapter 12 analyzes how the existence of a corporate social responsibility (CSR) private firm influences the privatization of a public firm in a mixed duopoly model. This chapter concludes that replacing a pure private firm with a CSR private firm increases the consumer surplus, but lowers the privatization ratio of the public firm.

Finally, Chap. 13 considers the market expansion tool of advertising in the mixed oligopoly model. This chapter shows how advertising affects the level of production of both public and private firms. It also illustrates that the public firm plays a critical role in expanding market demand.

These chapters cover a wide range of considerations and discuss various theoretical analyses of the mixed oligopoly. However, some important issues remain unresolved. For example, the presence of public firms prevents entry of private firms. In addition, an empirical investigation of this effect is required when a decision is made to privatize the public firm. Empirical research is also necessary to examine international aspects and quality competition. In addition, various game configurations should be studied to further understand how the first-mover advantage affects the degree of the entry barrier and the number of private firms. Thus, these issues require further analysis and are important to evaluate the existence of the public firm.

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Nagoya, Japan

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Part I
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Chapter 1
Basic Properties of a Mixed Oligopoly Model

Tsuyoshi Shinozaki and Minoru Kunizaki

Abstract In this chapter, we discuss previous studies and summarize the properties of a mixed oligopoly. With our overview of a mixed oligopoly model, we attempt to understand the fundamental characteristics of government intervention within an oligopoly. We consider the partial privatization problem in relation to the Stackelberg leader solution. The second-best outcome can be achieved by partial privatization. We also show that full privatization is not optimal if private firms can enter the oligopolistic market. In a free-entry equilibrium, the government can control excessive entry by imposing an entry tax.

1.1 Introduction

"When we talk about public and private firms, we think of firms which pursue different objectives, the pudding of a private firm is not baked with any social welfare ingredient, whereas social welfare should ultimately be the very raison d'etre of a public firm" (De Fraja and Delbono 1989).

Thus, they explored mixed oligopolies and found that social welfare is higher in a pure private oligopoly than when public firms attempt to maximize social welfare. That result suggests that the privatization of public firms is beneficial if there are more private firms in the market. De Fraja and Delbono (1989, 1990) compared a pure private oligopoly and a mixed oligopoly where the public firm is fully owned by the government. These two situations represent two extremes: whether the public firm is fully privatized or not. Furthermore, the government can control firms through the partial ownership of the public firm or by purchasing a share of the private firm.

Matsumura (1998) went further and explicitly considered the possibility of partial privatization in which the public firm is jointly owned by private and public shareholders to optimize social welfare. Matsumura and Kanda (2005) extended the

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partial privatization model developed by Fershtman (1990) and then illustrated that partial privatization ensures short-run optimal conditions but that full nationalization becomes optimal in a long-run free-entry equilibrium. The differences between these short-run and long-run results occur because of the excessive entry of private firms. Mankiw and Winston (1986) and Suzumura and Kiyono (1987) pointed out that the number of firms exceeds the optimal level in a free-entry equilibrium and that a decrease in private firms improves social welfare, known as the excess entry theorem. Matsumura and Kanda (2005) consequently suggested that a fully nationalized firm can be used as an instrument to restrict entry into the market. Previous studies have largely focused on government interventions in oligopolistic markets and what form public firms should take. As a result, a mixed oligopoly is supported when the number of private firms is small and public firms can restrict the entry of private firms.

In this chapter, we discuss previous research and summarize the properties of a mixed oligopoly. In our overview of a mixed oligopoly model, we attempt to understand the fundamental characteristics of government intervention in an oligopoly. The chapter is structured as follows. In Sect. 1.2, we present the basic model of a mixed oligopoly and its main properties. In Sect. 1.3, we investigate the properties of partial privatization that replicate the Stackelberg leader solution from a short-run perspective. In Sect. 1.4, we derive the optimal form of a public firm in a long-run, free-entry equilibrium. We also show that full nationalization does not ensure a second-best solution. In Sect. 1.5, we consider entry tax as a policy tool, and then the second-best solution is ensured if the government can use privatization and entry tax. Finally, Sect. 1.6 provides a summary and offers a brief outline of the following chapters.

1.2 Basic Mixed Oligopoly Model

To illustrate a mixed oligopoly, we use a simple oligopoly model in which there exist n private symmetric firms and one public firm. Each private firm, $i = 1, \cdots, n$, behaves as a Nash competitor to produce its output, q_i, with the given public firm output, q_0, and the number of firms, n. The profit of the private firms is as follows:

$$\pi_i = pq_i - c(q_i). \tag{1.1}$$

where p represents the price and relates to the total output, Q, via the inverse demand function, $p = p(Q)$, and $c(q_i)$ is the cost function. For simplicity, we assume that public and private firms have the same cost functions, even though De Fraja (1993) addressed the cost difference between public and private firms. By maximizing (1.1), the first-order condition can be obtained as $p + p'q_i - c'(q_i) = 0$. We also impose $p' + q_i p'' < 0$ and $p' - c'' < 0$. These assumptions ensure the stability of the Nash equilibrium among private firms. Because we assume all private firms are identical, the output level of each private firm, $q = q_i$, can be obtained as $q = q(q_0, n)$, with

$$\frac{\partial q}{\partial q_0} = -\frac{1}{n+k} < 0, \ \frac{\partial q}{\partial n} = -\frac{q}{n+k} < 0,$$

where $k = \frac{p' - c''}{p' + p'' q}$. In this model, the market is the same as a pure private oligopoly, with the exception of q_0. Social welfare can be written as follows:

$$W = \int_0^Q p(s)ds - pQ + \pi_0 + n\pi = G(Q) - c(q_0) - nc(q), \tag{1.2}$$

where $G(Q) = \int_0^Q p(s)ds$ and $\pi_0 = pq_0 - c(q_0)$ are the profit of the public firm.

Because the outputs of the private firms are the function of q_0 and n, social welfare changes with the output of the public firm and the number of public firms:

$$\frac{\partial W}{\partial q_0} = [p - c'(q_0)] + n[p - c'(q)]\frac{\partial q}{\partial q_0}, \tag{1.3}$$

$$\frac{\partial W}{\partial n} = [pq - c'(q_0)] + n[p - c'(q)]\frac{\partial q}{\partial n}. \tag{1.4}$$

Next, we obtain the second-best solution with respect to q_0 and n by setting the above equations to zero. In the short run, n is fixed. The second-best condition shows that because an increase in the output of the public firm decreases the outputs of the private firms and $p - c'(q) = -p'q > 0$, the optimal output of the public firm must be smaller than the output of the marginal pricing (MP) level. It also suggests that the public firm is more profitable if the number of private firms is large enough. This point will be discussed in the latter part of this section in our comparison of a mixed oligopoly and a pure private oligopoly. In addition, an increase in the number of private firms may improve social welfare if the private firms are sufficiently profitable. However, the entry of a private firm may harm social welfare if the profit of the private firm were small where the number of firms is large enough.

We now analyze the behavior of the public firm. We assume that the purpose of the public firm is determined by the government and the government cannot directly control the output of the public firm. In this case, the firm is fully nationalized and the output level is set to maximize social welfare as given private outputs. The optimization problem of the public firm is as follows:

$$q_0 = \text{argmax}\ W.$$

The first-order condition is

$$G' - c'(q_0) = p - c'(q_0) = 0.$$

In this case, the public firm behaves as a Nash competitor, and the output level is equal to the MP level. The public firm is a more aggressive producer than the private firms. Using comparative statics, we see that the total output with a fully nationalized firm exceeds the second-best level, and social welfare within a mixed oligopoly is lower than the second-best level because the marginal cost is equal to the market price.

If the public firm is fully privatized, it maximizes its own profit instead of social welfare. This case is the same as a pure private oligopoly. We can compare pure and mixed oligopolies regarding social welfare. From (1.3), a fully mixed oligopoly is superior to a pure private oligopoly where the number of private firms is small. De Fraja and Delbono (1989), however, demonstrated that social welfare in a mixed oligopoly may not be higher than in a pure private oligopoly. We can interpret their result from (1.3), where the MP level harms social welfare. Furthermore, total social welfare may improve if the number of private firms is large enough and if the private firm is still profitable. Thus, the second-best solution is not realized if the public firm behaves as a Nash competitor and is fully nationalized. Although the effect of full privatization on social welfare is ambiguous, it may be superior to a mixed oligopoly if the private firms are highly incubated.

1.3 Partial Privatization

In this section, we assume that the government can partially own a private firm or it can sell some of its equity in a public firm. Such a firm is regarded as a partially privatized firm, and its objective typically combines both public and private interests. To simplify, we assume that the partial public firm considers both social welfare and its own profit and that the weight on both objectives depends on the level of government ownership. This type of firm faces the following optimization problem:

$$q_0 = \text{argmax}\ \theta\pi_0 + (1-\theta)W. \tag{1.5}$$

The first-order condition is

$$\begin{aligned}&(1-\theta)[p - c'(q_0)] + \theta[p + p'q_0 - c'(q_0)]\\&= [p - c'(q_0)] + \theta p' q_0 = 0,\end{aligned} \tag{1.6}$$

where θ is the government's shareholding ratio of the public firm or the privatization ratio. If the firm is fully privatized, θ is equal to unity, and it is zero when the firm is fully nationalized. Given θ, the output reaches an intermediate level (between pure private and fully nationalized levels) if the firm is partially owned by the government.

We know that a mixed oligopoly is beneficial when the number of private firms is small. However, can partial privatization replicate the second-best solution? The answer is yes. Using (1.3) and (1.6),

$$\theta = \frac{nq}{(n+k)q_0} > 0. \tag{1.7}$$

We call this condition the optimal privatization ratio, which achieves the second-best level of output. The properties of this ratio depend on the number of private firms. We have discussed the above short-run results via a comparison of previous research and have identified several properties of a mixed oligopoly.

We now consider government intervention in a mixed oligopoly. In this section, full nationalization and full privatization do not ensure a second-best solution under a fixed number of private firms. If the government indirectly controls the output of the public firm via partial ownership, the output level is delivered to the optimal level. However, partial privatization is only justified with a fixed number of private firms. De Fraja and Delbono (1989) focused on optimal privatization and showed that a mixed oligopoly is not superior to a Stackelberg leader solution where the government directly sets the output and acts as the leader. We too find similar results for optimal partial privatization. In addition, an increase in the number of private firms promotes the total output if the output of the public firm is fixed. In such a case, the output of the public firm must decrease to ensure a second-best solution. The government then enforces the privatization of the firm because entry restrictions must be loosened.

In this section, we showed the existence of the optimal privatization ratio and its implications related to previous studies. We now consider the long-run situation in which new firms enter the market and explain the limitations of the privatization policy.

1.4 Free-Entry Equilibrium and the Privatization Problem

We now turn to the long-run situation of a mixed oligopoly. From a long-run perspective, new firms may enter the market until the profit becomes zero. In this case, the zero profit condition determines the number of private firms regardless of whether the public firm is fully nationalized or privatized. However, because the degree of privatization affects the output of the public firm, the number of private firms also depends on the privatization policy. Regarding the long-run equilibrium, the number of private firms can be written as a function of the output of the public firm using a zero profit condition:

$$\pi = pq - c(q) = 0, \text{ or } p = \frac{c(q)}{q}, \tag{1.8}$$

$$n^* = n(q_0), \ \frac{\partial n^*}{\partial q_0} = -\frac{1}{q}. \tag{1.9}$$

where * denotes the long-run equilibrium variables.

The property of the number of private firms is characterized by the privatization policy. The number becomes larger if privatization is accelerated. Because privatization brings about a decrease in the output of the privatized firm, a new firm can enter the market. In other words, nationalization can be interpreted as a tool to restrict entry.

The free-entry equilibrium in a pure private oligopoly results in inefficient resource allocation. This was shown by Mankiw and Winston (1986) and Suzumura and Kiyono (1987) and is known as the excess entry theorem. This theorem states that the number of firms reaches a certain level depending on the cost function and that each firm produces an output at a higher average cost. Thus, a decrease in the number of firms can lower sunk costs and decrease the average cost. Therefore, government entry restrictions may improve social welfare and restore more efficient resource allocation. If we apply the excess entry theorem to the mixed oligopoly, nationalization or partial privatization can be interpreted as an instrument of entry restriction.

Next, we consider the output and market price in a free-entry equilibrium. Matsumura and Kanda (2005) commented that private output and total output are independent of the output of the public firm. We call such properties the neutrality theorem of privatization. Using comparative statics of the long-run number of private firms, we can check the neutrality theorem as follows:

$$q^* = q^*\left(q_0, \ n^*(q_0)\right), \tag{1.10}$$

$$\frac{dq^*}{dq_0} = 0, \tag{1.11}$$

$$Q^* = q_0 + n^* q^*\left(q_0, \ n^*\right), \tag{1.12}$$

$$dQ^* = \left(1 + q^* \frac{\partial n^*}{\partial q_0}\right) dq_0 = 0. \tag{1.13}$$

We now consider the privatization problem from a long-run perspective and investigate whether the government can maximize social welfare using the output of the public firm. The answer is no because the government only chooses the privatization ratio. We can check this using the following maximization problem:

$$\max_{q_0} W = \int_0^Q p(s)ds - pQ + \pi_0 + n\pi = G(Q) - c(q_0) - nc(q),$$

The first-order condition is

$$-c'(q_0) - c(q)\frac{\partial n^*}{\partial q_0} = \frac{c(q)}{q} - c'(q_0) = p - c'(q_0) = 0. \tag{1.14}$$

In this case, if the government only uses the output of the public firm as a policy instrument, then such a condition can only be satisfied when the public firm is fully nationalized because a fully nationalized firm always provides the MP level of output.

However, with (1.3) and (1.4), full nationalization cannot achieve the second-best solution because the number of firms still exceeds the optimal level. Even though the public firm restricts the number of private firms, the MP level of output of the public firm is still too high. This depends on the number of policy instruments because the government has to control the output of the public firm and the number of private firms to satisfy the second-best situation. The government needs to use another policy instrument to restrict the number of private firms. In the following section, we consider how the government reaches the second-best solution and what type of policy instrument is appropriate.

1.5 Entry Restriction and Partial Privatization

As mentioned above, the second-best solution cannot be realized through a privatization policy because by itself it is not enough. In this section, we introduce an entry tax as an additional policy instrument and analyze the policy effects and social welfare implications.

The profit function is rewritten if the government imposes an entry tax to each firm. We also assume that the entry tax is imposed as a lump sum. The entry tax does not change the output level of the private firm in the short run. However, it affects the number of private firms in the long run.

$$\pi = pq - c(q) - T, \tag{1.15}$$

$$n^* = n^*(q_0, T), \ \frac{\partial n^*}{\partial q_0} = -\frac{1}{q}, \ \frac{\partial n^*}{\partial T} = \frac{n^* + k}{p'q^2(1+k)} < 0, \tag{1.16}$$

$$\frac{dq}{dT} = -\frac{1}{p'q(1+k)} > 0, \ \frac{dQ}{dT} = \frac{k}{p'q(1+k)} < 0. \tag{1.17}$$

The entry tax has some interesting effects not only regarding the number of firms but also on output levels from a long-run perspective. This enables the government to control both levels. The government now faces the following maximization problem:

$$W = G(Q) - c(q_0) - n^* c(q). \tag{1.18}$$

The first-order conditions are

$$\frac{\partial W}{\partial q_0} = -c'(q_0) + \frac{c(q)}{q} = p - \frac{T}{q} - c'(q_0) = 0, \tag{1.19}$$

$$\frac{\partial W}{\partial T} = p\left(n^* \frac{\partial q}{\partial T} + q\frac{\partial n^*}{\partial T}\right) - n^* c'(q)\frac{\partial q}{\partial T} - c(q)\frac{\partial n^*}{\partial T} = T\frac{\partial n^*}{\partial T} - n^* p' q \frac{\partial q}{\partial T} = 0. \tag{1.20}$$

In this case, the second-best solution does not require the MP level output of the public firm. Moreover, the entry tax is a positive value as follows:

$$p - c'(q_0) > 0, \tag{1.21}$$

$$T^* = -\frac{n^* p' q^2}{n^* + k} > 0. \tag{1.22}$$

Because the public firm provides less than the MP level of output if the firm is partially privatized, the second-best condition requires partial privatization, not full nationalization. This is in contrast to when the government only uses a privatization policy. The optimal entry tax is always positive, regardless of whether the public firm is privatized or nationalized. Thus, the government now separately and fully controls the number of firms and the output of the public firm.

De Fraja and Delbono (1989) considered a similar situation, although they did not introduce partial privatization and entry tax. They stated that the government can maximize social welfare if it controls the output of the public firm and the number of private firms like a Stackelberg leader. Because the free-entry equilibrium always exceeds the optimal level, any entry restriction may improve social welfare, and full privatization results in overprovision. However, if the government cannot use the number of private firms and the output of public firms, it is difficult to bring a market solution to second-best level. Our case highlights their replication using an entry tax and partial privatization. We have also shown that a second-best solution cannot be achieved if the government adopts either an entry tax or a partial privatization policy.

Finally, we mentioned several properties of a mixed oligopoly from both short- and long-run perspectives, with reference to previous research including De Fraja and Delbono (1989) and Matsumura and Kanda (2005). The policy implications from our analysis of a mixed oligopoly are broad, taking into account market size and market composition. From a short-run view, a privatization policy is more likely to be effective, but in the long run, the government needs other policy instruments to restrict the entry of firms.

Thus, privatization is regarded as a government intervention in an oligopolistic market. Although privatization affects the market structure and restores social welfare, there are some limitations to improving social welfare via a privatization policy. For instance, the public firm alone is not a strong enough policy instrument to restrict the entry of new firms. Thus, government intervention via a public firm cannot fully restrict the number of private firms, and a further policy tool is necessary to ensure the efficient allocation of resources. We introduced an entry tax to resolve this problem. As a result, a second-best solution is found via an entry tax and a privatization policy.

1.6 Concluding Remarks

In this section, we highlight some remaining issues and discuss the intended extension of the mixed oligopoly model. First, domestic firms sometimes compete with foreign firms. Foreign firms aim to maximize their own profit, and these profits are usually forwarded to the firms' home countries. Because such an erosion of profits harms domestic social welfare, the domestic government has an incentive to use the public firm to prevent such erosions. In this case, Fjell and Pal (1996) considered that the public firm not only acts to restore social welfare but also behaves as a protection tool against foreign firms. This situation is known as an international mixed oligopoly. Traditionally, the import tariff has been recognized as a protection instrument in such cases. If we have such a protection tool, we need to consider whether the tariff rate and privatization are compatible. This problem is worth considering when composing relevant policy in an international economy.

Second, the public firm can be seen as an entry barrier to private firms and prevents private firms' capital accumulation. Furthermore, a disincentive for capital accumulation may harm economic growth. To analyze this situation, we need an intertemporal model of a mixed oligopoly to investigate the causality between a mixed oligopoly and economic growth, as in Futagami et al. (2011). The capital accumulation process relates to the entry of firms in the market, as new firms always bring new capital. The number of firms can then be used as a proxy of capital accumulation. Therefore, both the entry process and privatization policies are related to economic growth.

Third, the present chapter assumed that the government maximizes social welfare or acts as a benevolent agent. In reality, private firms often approach the government to derive more favorite policies via campaign contributions to politicians. In this case, the government has its own payoff function (not maximizing social welfare), which means it is a non-benevolent agent. This political bias via campaign contributions may change the level of privatization, and thus, the market outcomes differ from a benevolent case. A question remains unanswered: which case brings about higher social welfare?

If we consider political bias in an international mixed oligopoly, the intuitive explanation becomes more complex. Domestic firms wish to increase import tariffs to reduce the presence of foreign firms. They also try to privatize public firms via campaign contributions because privatization brings a market share to private firms. This incentive with campaign contributions can also distort domestic social welfare. Thus, further investigations are required to determine the effect of political bias on social welfare.

When oligopolistic markets are considered, Cournot quantity-setting models and Bertrand price-setting models are commonly used as analytical tools. However, the price in both models is determined by the market, even though the market is a pure private or mixed oligopoly. Furthermore, sometimes a public firm competes against a private firm in a market in which the price is restricted. For example, national universities in Japan compete with private universities, and the tuition fees of both are restricted by government regulations. In the healthcare market, both public and private hospitals share patients, but the payments from national health insurance are standardized by the government. Thus, this non-price mixed oligopoly may represent different definitions of privatization and government intervention.

To encompass the various policy implications of privatization, we investigate several extensions of the mixed oligopoly model in the following chapters. Each chapter will show how privatization policy relates to the individual subject. Throughout this book, we seek to identify the significance of privatization and its related meanings.

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Chapter 2
Equilibrium and the Adjustment Process in a Mixed Oligopoly: A Graphical Explanation

Suzuka Okuyama

Abstract This chapter provides a graphical explanation of a mixed oligopoly model in which private firms maximize their own profits and a public firm maximizes the sum of both the consumer and producer surpluses. In every period, the marginal revenue curve and the after-tax marginal cost curve play important roles in describing the behavior of the private firm. Furthermore, the marginal social benefit curve and the after-tax marginal cost curve contribute to the behavior of the public firm. Both firms react against the supply of other firms in turn in an adjustment process. Finally, the economy reaches equilibrium.

2.1 Introduction

The purpose of this chapter is to present a graphical explanation of a mixed oligopoly, that is, a Cournot–Nash game involving profit-maximizing firms and a social welfare-maximizing firm. Assuming the entry of private firms into a publicly monopolistic market, we depict the equilibrium and the adjustment process to the equilibrium. Although a number of studies have previously addressed mixed oligopolies, few have used graphs, which are helpful for understanding mixed oligopolies.

Over the past few decades, privatization has been brought to the public's attention, and many attempts have been made by scholars to explain this subject. Politically, since the 1980s, many countries have privatized a significant number of public firms to reconstruct public finance. In the United Kingdom, the Thatcher government privatized a wide range of companies, including a public gas company and the waterworks bureau. Similarly, in Japan, the government privatized the Japan Tobacco and Salt Public Company in 1985, the Japanese Railway Company in 1987, and, recently, the Japan Post in 2007. De Fraja (1991) discussed privatization and established the baseline used in this field, following the mixed oligopoly

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model developed by De Fraja and Delbono (1989). Since then, many researchers have extended De Fraja's research on privatization. For example, Matsumura (1998) examined a mixed duopoly containing private firms and a privatized firm shared by both the public and private sectors. White (1996) investigated the effect of subsidies in a mixed oligopoly model and clarified that subsidies have a cost distribution effect on production. Lee (2006) considered privatization in the telecommunication industry, where a public firm supplies an essential network service.

Whereas a large number of studies have investigated privatization, few have tried to explain graphically how equilibrium is achieved and how the adjustment process proceeds. Studies on privatization have compared the welfare levels of two situations, mixed and pure oligopolies. This implies that theories of privatization are based on the theories of pure and mixed oligopolies.

There are several studies (e.g., Baldwin 1987 and Nicholson 1972) that explain equilibrium and the adjustment process in a pure oligopoly. However, few studies have attempted to provide a graphical explanation for a mixed oligopoly. Applying the same techniques used by Baldwin (1987) and Nicholson (1972), we graphically depict an equilibrium and the adjustment process in a mixed oligopoly, as introduced in De Fraja (1991). In this setting, it is initially assumed that a public firm monopolistically supplies goods to the market and then private firms enter this market and determine their outputs based on the output of the public firm. The firms continue to play the game until equilibrium is attained.

Obtaining equilibrium in a mixed oligopoly requires two kinds of first-order conditions. One is derived from profit maximization and the other from social welfare maximization. The first-order condition of profit maximization consists of the marginal revenue and the after-tax marginal cost, which is the marginal cost plus the tax rate. As for the latter, that of social welfare maximization consists of the marginal social benefit and after-tax marginal cost. The tax income is used to finance the budget loss of the social welfare maximization firm.

To analyze the adjustment process and equilibrium graphically, we reinterpret first-order conditions as intersections of curves. First, the private firm's quantity is determined to be an intersection of the marginal revenue curve and the after-tax marginal cost curve. These curves have two variables, the private firm's offered price and quantity. Similarly, that of the public firm is determined to be an intersection of the marginal social benefit curve and the after-tax marginal cost curve. These curves also have two variables, the public firm's offered price and quantity.

Illustrating these curves on a plane (with a vertical axis of price and a horizontal axis of quantity) clearly shows the adjustment process and equilibrium in a mixed oligopoly. After entry, each firm repeats its offer of quantity until equilibrium is reached. The reoffering is illustrated as a leftward shift of the marginal revenue curve or the marginal social benefit curve. During the adjustment, incumbent firms decrease their quantities and entry firms increase theirs. When the prices offered by the two firms (public and private) are equal, equilibrium is achieved.

This chapter is organized as follows. Section 2.1 presents the fundamental model of a mixed oligopoly, based on De Fraja (1991). Section 2.2 explains both the adjustment process to reach equilibrium and the equilibrium in a mixed duopoly. Section 2.3 extends the concept to a mixed oligopoly and focuses on the equilibrium, and Sect. 2.4 provides some concluding remarks.

2.2 The Model

We first review the mixed oligopoly model developed by De Fraja (1991). In this model, there are n private firms that maximize their profits and one public firm that maximizes social welfare, defined as the sum of consumer surplus and producer surplus. Let the private firm be firm $i\,(i = 1, \ldots, n)$ and let the public firm be firm 0. These firms play a Cournot–Nash game in which each firm determines its output under the given levels of outputs supplied by the other firms.

These $n + 1$ firms supply their goods to the market, whose inverse demand function is given by

$$p = 1 - Q, \tag{2.1}$$

where p is the market price and Q is the total amount of goods. Assuming symmetry across the private firms and with q_i denoting the supply of firm i and q_0 as that of firm. Q can be expressed as

$$Q = q_0 + \sum_{i=1}^{n} q_i. \tag{2.2}$$

In this model, it is assumed that the cost of a firm is composed of a fixed cost that does not change with the level of output and a variable cost that depends on the output. The fixed cost is the same for both the private firms and the public firm, while the variable cost is not.

We assume that variable costs, which have constant marginal costs, are

$$c_0 > c_1 = c_2 = \cdots = c_n \equiv \widetilde{c}, \tag{2.3}$$

where subscripts represent the firms. The difference in the marginal costs implies that the productivity of the public firm is less efficient than those of the private firms. These marginal costs and the fixed cost provide the cost function of the public firm as follows:

$$c(q_0) = c_0 q_0 + F, \tag{2.4}$$

where F is the fixed cost. Similarly, the cost function of the private firm is

$$c(\widetilde{q}) = \widetilde{c}\,\widetilde{q} + F. \tag{2.5}$$

From (2.1) and (2.5), the profit of the private firm is given by

$$\widetilde{\pi} = p\widetilde{q} - \widetilde{c}\,\widetilde{q} - F - t\widetilde{q}, \tag{2.6}$$

where t represents a specific tax rate that is constant through all periods. As we will see later, because the public firm maximizes social welfare and not profit, there is a deficit that is equal to the fixed cost of the public firm in equilibrium. To finance this budget loss, the government levies a tax on the outputs supplied by all firms. The profit maximization problem of the private firm takes the following form:

$$\frac{\partial \widetilde{\pi}}{\partial \widetilde{q}} = \widetilde{MR} - \widetilde{TMC} = 0 \tag{2.7}$$

where

$$\widetilde{MR} = (1 - q_0 - \sum_{i=1}^{n} q_i) + (-q_i) \tag{2.8}$$

is the marginal revenue of firm i and

$$\widetilde{TMC} = \widetilde{c} + t \tag{2.9}$$

is the after-tax marginal cost of firm i. Regarding tax, an extra unit of production requires the marginal cost plus the tax rate, that is, the after-tax marginal cost.

The right-hand side of (2.8) can be interpreted as follows. First, $(1 - q_0 - \sum_{i=1}^{n} q_i)$ represents the positive effect of an increase in q_i on the revenue of firm i, which is equal to the price. Second, $(-q_i)$ represents the negative effect of a change (decrease) of price on the total revenue. Then, the condition of the profit maximization of (2.7) requires an equalization of (2.8) and (2.9).

The public firm determines its output depending not only on its own profit but also on the profit of the private firm and the consumer surplus. Because the inverse demand function is linear, as in (2.1), the consumer surplus is given by

$$CS = \frac{1}{2}\left(q_0 + \sum_{i=1}^{n} q_i\right)^2. \tag{2.10}$$

As a result, from (2.4), (2.6), and (2.10), social welfare can be represented as follows:

$$S = SB - TTC_n = \left[\frac{1}{2}\left(q_0 + \sum_{i=1}^{n} q_i\right)^2 + \sum_{i=1}^{n} \pi_i + pq_0\right] - (c_0 q_0 + F), \tag{2.11}$$

where S, SB, and TTC_0 represent social welfare, social benefit, and the after-tax cost of the public firm, respectively. SB is defined as social welfare excluding the after-tax marginal cost.

The first-order condition of social welfare maximization then becomes

$$\frac{\partial S}{\partial q_0} = MSB - TMC_0 = 0, \tag{2.12}$$

where

$$MSB = \left(q_0 + \sum_{i=1}^{n} q_i\right) - \sum_{i=1}^{n} q_i + \left[1 - \left(q_0 + \sum_{i=1}^{n} q_i\right) - q_0\right] \tag{2.13}$$

is the marginal social benefit that denotes the additional social benefit brought about by an increase in an incremental unit of output. Furthermore,

$$TMC_0 = c_0 + t \tag{2.14}$$

is the after-tax marginal cost of the public firm. The right-hand side of (2.13) can be interpreted as follows. First, $\left(q_0 + \sum_{i=1}^{n} q_i\right)$ represents the effect of an increase in q_0 on the consumer surplus. Second, $-\sum_{i=1}^{n} q_i$ represents the effect of an increase in q_0 on the revenue of the private firm through the decrease in price. Finally, $\left[1 - \left(q_0 + \sum_{i=1}^{n} q_i\right) - q_0\right]$ represents the effects of an increase in q_0 on the revenue of the public firm, which is equal to the price, and a decrease in q_0 on the revenue of the public firm. It should be noted that the second term and part of the third term cancel the first term. This simply implies that an increase shifts the benefit from the producer surplus to the consumer surplus. As a result, this becomes equivalent to the inverse demand function given by (2.1).

Finally, solving (2.7) and (2.12), the Cournot–Nash equilibrium outputs of these firms can be obtained as follows:

$$q_i = c_0 - \widetilde{c}, \tag{2.15}$$

$$q_0 = 1 - (n+1)c_0 + n\widetilde{c} - t \tag{2.16}$$

2.3 Graphical Explanation

In this section, we graphically represent the Cournot–Nash equilibrium given in (2.15) and (2.16). The merit of the graphical explanation given here is that it can satisfactorily express how equilibrium can be achieved through the adjustment process in a mixed oligopoly. Before turning to the explanation of the mixed oligopoly, it is helpful to consider a mixed duopoly model in which only one private firm (firm 1) and one public firm (firm 0) exist.

In the case of a duopoly, the adjustment process will proceed as follows: originally (or in the 0^{th} period), only the public firm existed in the market and supplied goods monopolistically. In the first period, a private firm enters this market and determines its output by reacting to the output level of the public firm. In the second period, the public firm revises its output level by reacting to the output of the private firm. The behavior of the private firm in the first period and that of the public firm in the second period are defined as the first turn for these two firms, respectively. In the third period, the private firm revises its output, and then in the fourth period, the public firm revises its output. This behavior by the two firms can be defined as the second turn for each firm. The private firm revises its output in every odd-numbered period, and the public firm does so in every even-numbered period. That is, the private firm determines its output in the m^{th} turn in the $2m-1^{\text{th}}$ period, and the public firm determines its output in the m^{th} turn in the $2m^{\text{th}}$ period. The succeeding periods follow this procedure, and the economy finally achieves a Cournot–Nash equilibrium.

2.3.1 From the First to the Third Periods

Let us start with the circumstance in which the public firm dominates the market. The marginal social benefit of the public firm in the 0^{th} period can be written as follows:

$$MSB_0^0 = 1 - q_0^0, \tag{2.13'}$$

where q_0^0 is the level of the output in the 0^{th} period. The superscript denotes the number of turns.[1] From (2.13′) and (2.14), the output of the public firm in the 0^{th} turn can be obtained as

$$q_0^0 = 1 - c_0 - t. \tag{2.17}$$

As shown in Fig. 2.1, with p as the vertical axis and Q as the horizontal axis, the marginal social benefit curve (DD') can be drawn by a straight line with a slope of -1 and an intercept of 1 on the vertical axis. The after-tax marginal cost curve TMC_0 is parallel to the horizontal axis. Therefore, q_0^0 is represented by OA, and the price offered by the public firm is represented by eO; cost curve (TMC_0) is parallel to the horizontal axis.[2] Therefore, q_0^0 is represented by OA, and the price offered by the public firm is represented by eO, which corresponds to TMC_0.

Next, let us consider the entry of the private firm in the first period. Because the after-tax marginal cost of the private firm is lower than that of the public firm, which is equal to the price offered by the public firm as assumed in (2.3), the private

[1] The behavior in the 0^{th} turn is defined as the behavior of the public firm in the 0^{th} period.

[2] The marginal social benefit curve in the 0^{th} period is equivalent to the inverse demand function.

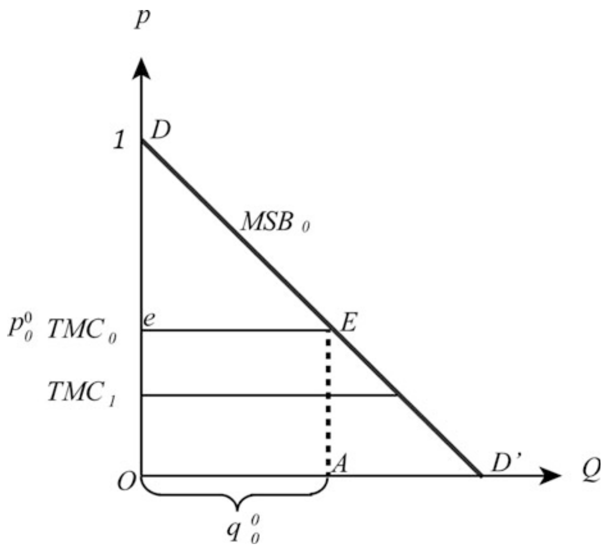

Fig. 2.1 The public firm supplies $q_0^0(OA)$ at $p_0^0(eO)$ in the 0th turn

firm enters the market. The private firm faces its demand curve (ED'), which gives the following marginal revenue:

$$MR_1^1 = 1 - 2q_1^1 - q_0^0, \tag{2.8$'$}$$

as shown by MR_1^1 in Fig. 2.2 and the marginal cost as

$$TMC_1 = \widetilde{c} + t. \tag{2.9$'$}$$

From (2.8′) and (2.9′), the output of the private firm in the first turn q_1^1, can be obtained as

$$q_1^1 = \frac{1}{2}(\widetilde{c} - c_1). \tag{2.18}$$

In Fig. 2.2, the marginal revenue curve $\left(MR_1^1\right)$ can be drawn by a straight line with a slope of -2 and an interception of DD' at E. The after-tax marginal cost curve (TMC_1) can be taken as parallel to the Q axis. The private firm chooses C, which is the intersection of the two curves, and determines its output as AB. Therefore, the price offered by the private firm becomes p_1^1.

The outputs of the public firm given by (2.17) and of the private firm given by (2.18) cannot be consistent with the equilibrium outputs because the prices faced by each firm are different. Thus, the public firm should change its output by responding

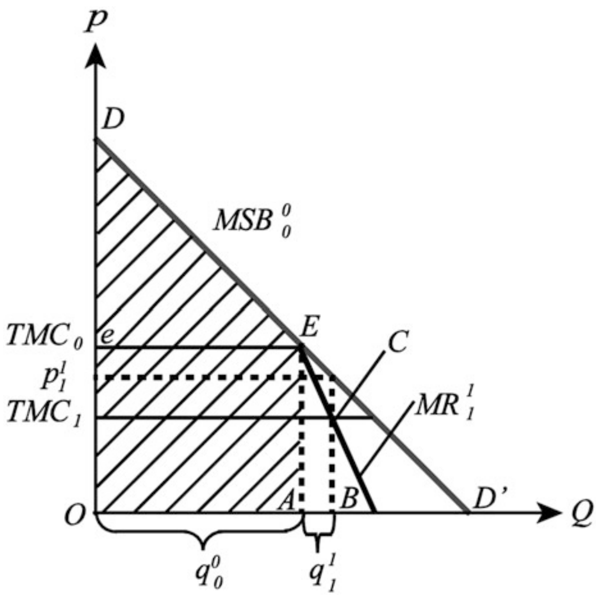

Fig. 2.2 The private firm enters the market in the 1st turn. The private firm supplies $q_1^1(AB)$ at p_1^1

to the outputs of the private firm obtained in (2.18), even though the after-tax marginal cost of the public firm represented by (2.17) is unchanged.

We now consider the behavior of the public firm in the second period. As mentioned above, in this period, the marginal social benefit curve changes in reaction to q_1^1. From this new marginal social benefit curve obtained by substituting (2.18) into (2.13) and the after-tax marginal cost, the output of the public firm in the first turn can be obtained by

$$q_0^1 = (1 - c_0 - t) - \frac{1}{2}(c_0 - \widetilde{c}). \tag{2.19}$$

The adjustment process of how the public firm changes its output is revealed in Fig. 2.3. Now, MR_1^1 in Fig. 2.2 corresponds to MR_1^1 in Fig. 2.3. The quantity OB in Fig. 2.3, which is AB in Fig. 2.2, has shifted left. This determines the length of the inverse demand curve that the public firm faces as GD', denoted by MSB_0^1. As in the 0^{th} turn, the public firm determines its output as BA by choosing E as the intersection of MSB_0^1 and TMC_0, and offers p_0^1, which is unchanged from the 0^{th} period.

It should be noted that in comparison with (2.17), the output of the public firm decreases by $\frac{1}{2}(c_0 - c_1)$ in reaction to the output of the private firm, as given in (2.18). Furthermore, even though the output of the public firm is revised from q_0^0 to q_0^1, the price offered by the public firm has not been altered. This implies that the private firm has an incentive to change the output level in response to the output level of the public firm in the second period given by (2.19).

The output of the private firm in the 3rd period (in the 2nd turn) is obtained by

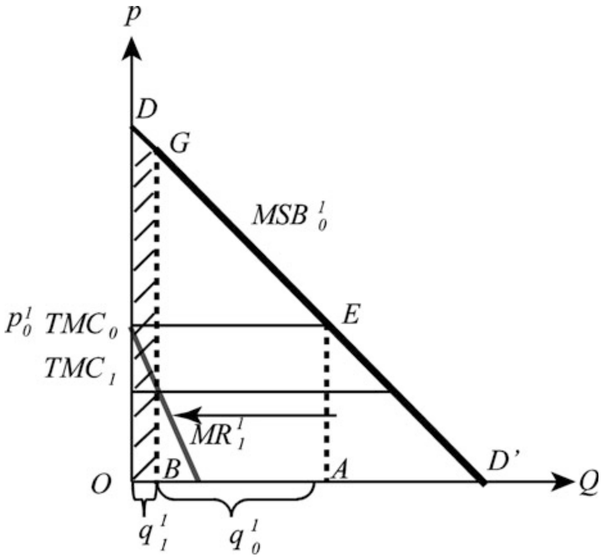

Fig. 2.3 The public firm revises its output in the 1st turn. The public firm revises outputs q_0^1 (AB) at p_0^1 in the 1st turn given q_1^1 determined by the private firm in the previous period. In addition, the public firm supplies at TMC_0

$$q_1^2 = \frac{3}{4}(c_0 - \widetilde{c}). \tag{2.20}$$

The adjustment process that the private firm goes through to change its output in the second turn is shown in Fig. 2.4a. As MSB_0^1 shifts to the left, BA in Fig. 2.2 can be presented as OA in Fig. 2.4a. The inverse demand curve GD' faced by the private firm determines the marginal revenue curve as MR_1^2. The private firm chooses C, which is represented by the intersection of MR_1^2 and TMC_1, and determines its output as AB. The price that the private firm offers is revised asp_1^2.

Regarding the change in the output of the private firm, comparing (2.19) with (2.17), it increases by $\frac{1}{4}(c_0 - \widetilde{c})$. This shows that the private firm gradually increases its share in the market as the public firm loses its market share. In fact, the output of the public firm in the 4th period (in the 2nd turn) becomes

$$q_0^2 = (1 - c_0 - t) - \frac{3}{4}(c_0 - \widetilde{c}) \tag{2.21}$$

which can be represented by BA in Fig. 2.4b. This output is less than (2.19) by just $\frac{1}{4}(c_0 - \widetilde{c})$.

By the same procedure, we obtain the outputs of both firms in the 3rd turn as follows:

$$q_1^3 = \frac{7}{8}(c_0 - \widetilde{c}), \tag{2.22}$$

and

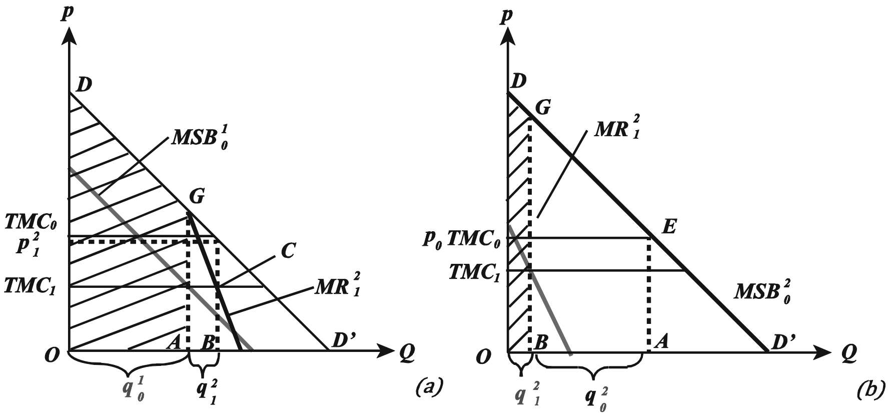

Fig. 2.4 The adjustment process of the 2nd turn. (**a**) The private firm determines its output in the 3rd period. (**b**) The public firm determines its output in the 4th period

$$q_0^3 = (1 - c_0 - t) - \frac{7}{8}(c_0 - c_1). \tag{2.23}$$

Again, it can be acknowledged that an increase in the output of the private firm and, in contrast, the decrease in that of the public firm are $\frac{1}{8}(c_0 - \widetilde{c})$.

2.3.2 From the m^{th} Period to Equilibrium

From the above procedure, we can obtain the outputs of both firms in the m^{th} turn by induction as

$$q_1^m = \frac{2^m - 1}{2^m}(c_0 - \widetilde{c}), \tag{2.24}$$

and

$$q_0^m = (1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \widetilde{c}). \tag{2.25}$$

The adjustment process in which both firms determine their outputs in the m^{th} turn is in Fig. 2.5a, b. Figure 2.5a represents that the private firm determines q_1^m in the $2m - 1^{\text{th}}$ period. The private firm chooses C, which is the intersection of MR_1^m and TMC_1, so that it determines its output as AB and offers p_1^m. Figure 2.5b represents that the public firm determines q_0^m in the $2m^{\text{th}}$ period. The public firm chooses E, which is an intersection of MSB_0^m and TMC_0, the output becomes BA, and the price offered by the public firm is p_0.

We now consider the equilibrium. Although the price offered by the private firm and the one offered by the public firm have not been equal (the public firm's price has been higher than that of the private firm), as m^{th} increases, the price offered by the private firm gradually increases to the (constant) price offered by the public firm. Finally, when the prices offered by both firms are equal, the economy reaches equilibrium. In equilibrium, the outputs of both firms are

$$q_1^* = c_0 - \widetilde{c}, \tag{2.26}$$

and

$$q_0^* = (1 - c_0 - t) - (c_0 - \widetilde{c}) \tag{2.27}$$

as already obtained in (2.15) and (2.16). In Fig. 2.6, (2.26) and (2.27) are represented as to AB and OA, respectively. In this equilibrium, the prices of both firms are equal to p^*.

Because the outputs of both firms in the m^{th} turn can be obtained from (2.24) and (2.25), the total output in equilibrium can be obtained as

$$\begin{aligned} Q^* &= q_1^m + q_0^m = \left[\frac{2^m - 1}{2^m}(c_0 - \widetilde{c})\right] + \left[(1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \widetilde{c})\right] \\ &= (1 - c_0 - t). \end{aligned} \tag{2.28}$$

Because the budget deficit of the public firm is

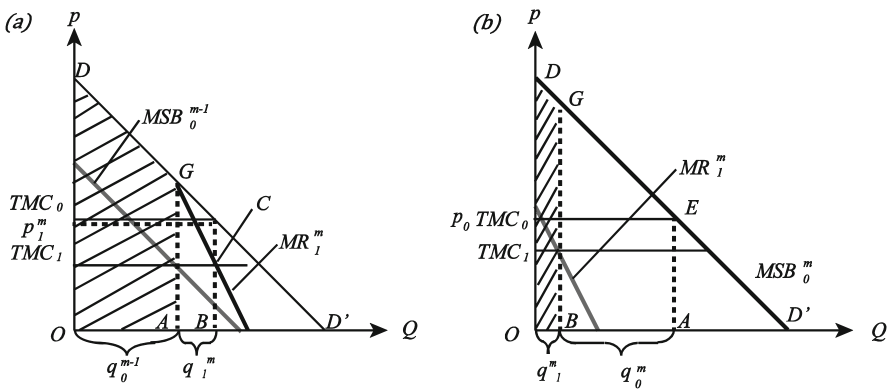

Fig. 2.5 The adjustment process of the m^{th} turn. (**a**) The private firm determines its output in the $2m - 1^{\text{th}}$ period. (**b**) The public firm determines its output in the $2m^{\text{th}}$ period

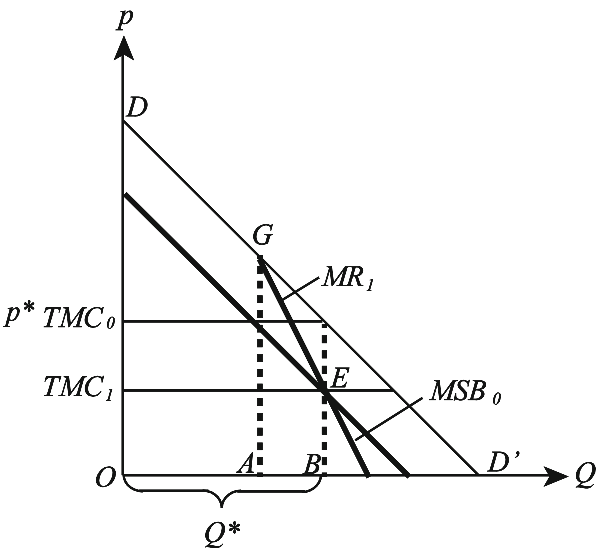

Fig. 2.6 Equilibrium. At equilibrium, the total output is Q^*. The public firm supplies OA and the private firm supplies AB

$$F = tQ \tag{2.29}$$

the tax rate becomes constant in equilibrium and can be obtained as

$$t = \frac{1}{2}\left[1 - c_0 - \sqrt{(1 - c_0)^2 - 4F}\right]. \tag{2.30}$$

If c_0 decreases, (2.28) provides that the total output increases and then the consumer surplus also increases.

Furthermore, (2.26) means that a decrease in c_0 leads to a decrease in the output of the private firm. The equivalency of both firms' marginal costs makes the output of the private firm zero and its profit $-F$. Therefore, the private firm will exit the market. However, the equivalency of marginal costs maximizes the consumer surplus. This means an improvement of the public firm's efficiency is not required to maximize social welfare under the condition that privatization never occurs.

2.3.3 n *Firms Case*

We now turn to the mixed oligopoly in which a public firm and n private firms exist in the economy. We also assume that the public firm and the (group of) private firms determine their outputs in turn. Under these circumstances, the production of one private firm among n firms becomes $\frac{1}{n}$ times that in a mixed duopoly. Therefore, the outputs of one private firm in the m^{th} turn can be obtained as

$$\tilde{q}^m = \frac{1}{n}\left[\frac{2^m - 1}{2^m}(c_0 - \tilde{c})\right], \tag{2.31}$$

and that of the public firm in the m^{th} turn is

$$q_0^m = (1 - c_0 - t) - \sum_{i=1}^{n}\frac{1}{n}\left[\frac{2^m - 1}{2^m}(c_0 - \tilde{c})\right] = (1 - c_0 - t) - \frac{2^m - 1}{2^m}(c_0 - \tilde{c}). \tag{2.32}$$

Finally, the output of a private firm and that of a public firm in equilibrium become

$$q^{\infty} = \frac{1}{n}(c_0 - \tilde{c}), \tag{2.33}$$

and

$$q_0^{\infty} = (1 - c_0 - t) - (c_0 - \tilde{c}). \tag{2.34}$$

As a result, the total output of the n private firms and the output of the public firm are identical with those in the case of mixed duopoly, seen in (2.26) and (2.27).

2.4 Concluding Remarks

This chapter depicted an equilibrium and the adjustment process in the mixed oligopoly described by De Fraja (1991). The adjustment process to equilibrium can be shown by utilizing the first-order conditions of private and public firms. Specifically, the first-order condition of the private firm is satisfied at the intersection of the marginal revenue curve and the after-tax marginal cost curve. Similarly, the first-order condition of the public firm is satisfied at the intersection of the marginal social benefit curve and the after-tax marginal cost curve. As time proceeds, under a fixed tax rate and marginal costs, the marginal revenue curves and the marginal social benefit curve will change.

By providing a graph that compares a pure oligopoly to a mixed oligopoly, we can understand how private and public firms behave, not only in equilibrium but also in the adjustment process to equilibrium. This graphical explanation contributes to clarifying the effects of the privatization of a public firm.

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Chapter 3
Privatization in a Stackelberg Mixed Oligopoly

Kojun Hamada

Abstract This chapter examines whether privatization improves social welfare in a Stackelberg mixed oligopoly. Extending the pioneering study of De Fraja and Delbono (Oxf Econ Pap 41(1):302–311, 1989) to Stackelberg competitions between a public firm and private firms, we investigate whether privatization increases social welfare in a sequential-move game. We consider the different competitive environments in which even after privatization, a public firm's Stackelberg position is maintained. We demonstrate the following results: First, when a public firm acts as a Stackelberg leader before and after privatization, privatization necessarily decreases social welfare irrespective of the number of private firms. Second, even when a public firm acts as a Stackelberg follower before and after privatization, privatization decreases social welfare if the number of private firms is relatively small.

3.1 Introduction

Since the pioneering study of De Fraja and Delbono (1989) identified the possibility that privatization increases social welfare, numerous articles have investigated the effect of privatization on social welfare. Although the existing literature emphasizes that privatizing a public firm can result in a higher level of social welfare, many studies focus on examining the effect of privatization on social welfare within a framework of Cournot competition where both public and private firms choose their output simultaneously.

In this chapter, we examine whether privatization improves social welfare in a Stackelberg mixed oligopoly where a single public firm and a number of private firms engage in quantity competition through the sequential choice of production by both firms. As shown by De Fraja and Delbono (1989), it is a well-established result in a mixed oligopoly that privatizing a public firm can increase social welfare in Cournot competition. Moreover, De Fraja and Delbono (1989) also investigated the effect of privatization on social welfare in a Stackelberg competition and showed

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that privatization decreases social welfare when before privatization, a public firm acts as a Stackelberg leader. However, their social welfare comparison before and after privatization depends on the assumption that both the public and private firms engage in Cournot competition after privatization. In De Fraja and Delbono's (1989) setting, privatizing a public firm brings about two simultaneous but different changes to the public firm. One is the change of the public firm's objective to maximize to its own profit from social welfare. The other is the loss of the leadership position after privatizing. De Fraja and Delbono (1989) showed that both changes after privatization cause a decrease in social welfare. In contrast, we pay attention to Stackelberg competition in which a public firm continues to act as a Stackelberg leader or follower, irrespective of whether privatization occurs, and we compare the social welfare before and after privatization. Because we only wish to measure the effect of the change in the public firm's objective concerning social welfare, we adopt a different competitive environment in which Stackelberg leadership is maintained for a public firm, both before and after privatization. This assumption is in contrast with that in previous studies.[1]

Relatively few articles have examined mixed oligopolies in Stackelberg competition. Although De Fraja and Delbono (1989) considered Stackelberg competition in which a public firm acts as a leader, they only compared social welfare in Stackelberg competition before privatization with that in Cournot competition after privatization. Another exception is Beato and Mas-Colell (1984), who examined a mixed oligopoly in which private and public firms compete in a market including both Cournot and Stackelberg competition. They demonstrated that when the timing of output choice is altered, the relative size of social welfare is also changed. However, they only focused on duopolistic competition between a public and private firm and did not compare the changes in social welfare resulting from privatization. Pal (1998) and Fjell and Heywood (2004) analyzed the endogenous timing of moves in a mixed oligopoly, but did not conduct a social welfare comparison. Furthermore, other studies have paid little attention to the link between the timing of decision-making and social welfare before and after privatization (e.g., Matsumura 2003a, b; Jacques 2004; Lu 2006, 2007).

We compare social welfare before and after privatization when a public firm behaves as a Stackelberg leader or follower irrespective of whether privatization occurs and show the following results. First, when a public firm acts as a Stackelberg leader before and after privatization, privatization necessarily reduces social welfare irrespective of the number of private firms. Second, even when a public firm acts as a Stackelberg follower before and after privatization, privatization decreases social welfare if the number of private firms is relatively small. These results are in stark contrast with those in a Cournot setting, where privatization can result in an increase in social welfare. The results suggest that when we

[1]We omit the details of the survey on mixed oligopoly and privatization. De Fraja and Delbono (1990) reviewed various mixed oligopoly models including different move orders in oligopolistic games. For a recent survey on a mixed oligopoly, see Matsumura and Shimizu (2010).

extensively consider the sequential-move choice of production in oligopolistic competition, the situation in which privatization in a mixed oligopoly enhances social welfare is more limited. Although it has been argued that privatization is welfare enhancing and the reform of state-owned enterprises should occur, the privatization of public firms has not progressed strongly worldwide. However, as this chapter suggests, if privatizing public firms itself brings about a negative effect on social welfare, the lack of progression in moves to privatize public firms and actual resistance to such development might be justified from the viewpoint of national welfare.

The remainder of this chapter is organized as follows. Section 3.2 describes the Stackelberg mixed oligopoly model wherein public and private firms compete in a homogeneous goods market. Section 3.3 compares social welfare before and after privatization when a public firm acts as a Stackelberg leader. Section 3.4 compares social welfare before and after privatization when a public firm acts as a Stackelberg follower. Section 3.5 compares social welfare before privatization in Cournot competition and two Stackelberg competitions. Section 3.6 presents concluding remarks. Proofs can be found in the Appendix.

3.2 The Model

The basic setting used in this study follows that of De Fraja and Delbono (1989). There are $(n+1)$ firms that have an identical production technology in a simple homogeneous product market. One firm is a public firm and the others are private firms. They engage in quantity competition in an oligopolistic market. The objective of the public firm is to maximize social welfare before privatizing and to maximize its own profit after privatizing. The objective of n private firms is to maximize their own firm's profit. The public firm is indexed by firm $i=0$, private firms are indexed by firm $i=\{1,2,\cdots,n\}$, and q_i denotes firm i's output. As we only focus on the symmetric equilibrium among private firms, the output of identical private firms is the same, i.e., $q\equiv q_i$, $i=\{1,2,\cdots,n\}$. The inverse demand function is given by $p=p(Q)=a-Q$, $a>0$, where p denotes the price and $Q\equiv q_0+nq$ the total output. The cost function of firms is denoted by $C=c(q_i)=F+(k/2)q_i^2$, $F\geq 0$, $k>0$. For brevity, we assume that $F=0$. The profit of firm i is denoted by $\pi_i=p(Q)q_i-(k/2)q_i^2$. We denote the profit of identical private firms as $\pi\equiv\pi_i$, $i=\{1,2,\cdots,n\}$. Consumer surplus and producer surplus are denoted by $CS\equiv\int_0^Q p(s)ds-p(Q)Q=(1/2)Q^2$ and $PS\equiv\pi_0+n\pi=p(Q)Q-(k/2)\sum_{i=0}^n q_i^2$, respectively. Social welfare is defined as $W\equiv CS+PS=aQ-(1/2)Q^2-(k/2)\sum_{i=0}^n q_i^2$. For brevity, we ignore the integer problem on the number of firms.

We classify the timing of output choice by the public firm and private firms as follows. Throughout this chapter, we do not deal with the endogenous timing of production to avoid the complexity of analysis. First, in Sect. 3.3, we investigate

Stackelberg competition in which a public firm acts as a leader before and after privatizing. Second, in Sect. 3.4, we proceed to investigate Stackelberg competition in which a public firm acts as a follower before and after privatizing. The equilibrium concept follows the subgame perfect Nash equilibrium (SPNE). We solve the equilibrium by inducing backward.

3.3 When a Public Firm Acts as a Stackelberg Leader

In this section, we derive the Stackelberg equilibrium before and after privatization when a public firm acts as a leader, irrespective of whether to privatize, and compare social welfare before and after privatization. De Fraja and Delbono (1989) previously compared social welfare before and after privatization where the public firm behaves as a Stackelberg leader before privatizing and with the privatized public firm engaging in Cournot competition with private firms. As mentioned in the introduction, when De Fraja and Delbono (1989) assumed that the public firm loses its Stackelberg leader position after privatizing, two key effects occur for the public firm. One is the change in its objective and the other is the loss of leadership position. De Fraja and Delbono (1989) showed that both effects bring about lower social welfare after privatization. However, it is necessary to distinguish between these two effects to focus on the change in the public firm's objective with privatization. Therefore, unlike the assumption in De Fraja and Delbono (1989), we assume that irrespective of whether a firm privatizes, the public firm remains a Stackelberg leader. In the following, we consider the situation in which the public firm's objective changes, from seeking to maximize social welfare before privatization to maximizing its own profit after privatization. The firm's position as a Stackelberg leader does not change. Thus, we focus on the effect of the change in the public firm's objective associated with privatization.

3.3.1 Before Privatization

In the second stage, irrespective of whether to privatize, a private firm maximizes its own profit given the output level of a public firm that has already been determined in the first stage. The first-order condition for a private firm is as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a - Q - q_i - kq_i = 0$$
$$\Leftrightarrow q_i = r_i(q_0, Q_{-i}) \equiv \frac{a - q_0 - Q_{-i}}{k+2}, Q_{-i} \equiv \sum_{j \neq 0, i} q_j. \tag{3.1}$$

As the private firms are identical and $q \equiv q_i$, we arrange the reaction function of the identical private firms (3.1) responding to q_0 as follows:

$$q = r(q_0) \equiv \frac{a - q_0}{n + k + 1}. \tag{3.2}$$

In the first stage, the public firm maximizes social welfare anticipating that the private firms' output in the second-stage subgame satisfies (3.2). The first-order condition for the public firm is as follows:

$$\frac{dW}{dq_0} = (a - Q)[1 + nr'(q_0)] - k[q_0 + nqr'(q_0)] = 0$$
$$\Leftrightarrow q_0 = \frac{\left[(k+1)^2 + nk\right]a}{(k+1)^2 + nk + k(n+k+1)^2}. \tag{3.3}$$

Substituting q_0, which is obtained in (3.3), into (3.2), we obtain the private firms' output as follows:

$$\Leftrightarrow q = \frac{k(n+k+1)a}{(k+1)^2 + nk + k(n+k+1)^2}. \tag{3.4}$$

The SPNE variables when a public leader firm and private follower firms engage in Stackelberg competition before privatization are summarized in Table 3.1.[2]

3.3.2 After Privatization

Even after privatizing, the reaction function of private firms in the second stage (3.2) does not change at all. In the first stage, the public firm maximizes its profit by anticipating that the private firms' output in the second-stage subgame satisfies (3.2). The first-order condition for the public firm is as follows:

[2]Throughout this chapter, we denote the equilibrium variables when a public firm acts as a Stackelberg leader before and after privatization by the superscripts *LB* (leader before) and *LA* (leader after), respectively. Likewise, we use the superscripts *FB* (follower before) and *FA* (follower after) when a public firm acts as a follower before and after privatization, respectively. Superscript *C* denotes the Cournot equilibrium.

Table 3.1 SPNE before privatization when a public firm acts as a leader

Public firm's output	q_0^{LB}	$\frac{Xa}{X+kY^2}$
Private firm's output	q^{LB}	$\frac{kYa}{X+kY^2}$
Total output	Q^{LB}	$\frac{(nkY+X)a}{X+kY^2}$
Price	p^{LB}	$\frac{k(k+1)Ya}{X+kY^2}$
Public firm's profit	π_0^{LB}	$\frac{k\left[(k+1)^2Y^2-n^2\right]a^2}{2\left(X+kY^2\right)^2}$
Private firm's profit	π^{LB}	$\frac{k^2(k+2)Y^2a^2}{2\left(X+kY^2\right)^2}$
Social welfare	W^{LB}	$\frac{[X+nk(Y+1)]a^2}{2\left(X+kY^2\right)}$

$(X \equiv (k+1)^2 + nk = k(Y+1)+1,\ Y \equiv n+k+1)$

Table 3.2 SPNE after privatization when a public firm acts as a leader

Public firm's output	q_0^{LA}	$\frac{(k+1)a}{X+k+1}$
Private firm's output	q^{LA}	$\frac{Xa}{(X+k+1)Y}$
Total output	Q^{LA}	$\frac{[nX+(k+1)Y]a}{(X+k+1)Y}$
Price	p^{LA}	$\frac{(k+1)Xa}{(X+k+1)Y}$
Public firm's profit	π_0^{LA}	$\frac{(k+1)^2a^2}{2(X+k+1)Y}$
Private firm's profit	π^{LA}	$\frac{X^2a^2}{(X+k+1)^2Y^2}$
Social welfare	W^{LA}	$\frac{\left[nX^2(Y+1)+2(k+1)XY^2+(1-k)(k+1)^2Y^2\right]a^2}{2(X+k+1)^2Y^2}$

$(X \equiv (k+1)^2 + nk = k(Y+1)+1,\ Y \equiv n+k+1)$

$$\frac{\partial \pi_0}{\partial q_0} = a - Q - [1 + nr'(q_0)]q_0 - kq_0 = 0$$
$$\Leftrightarrow q_0 = \frac{(k+1)a}{2(k+1)+k(n+k+1)}. \tag{3.5}$$

Substituting q_0 into (3.2), we obtain the private firms' output as follows:

$$q = \frac{\left[(k+1)^2 + nk\right]a}{(n+k+1)[2(k+1)+k(n+k+1)]}. \tag{3.6}$$

The SPNE variables when the privatized public leader firm and private follower firms engage in Stackelberg competition are summarized in Table 3.2.[3]

[3]By tedious calculation, W^{LA} is rearranged as follows: $W^{LA} = \frac{Za^2}{2(X+k+1)^2Y^2}$, where $Z \equiv k^2n^4 + k(k+2)(3k+2)n^3 + (k+1)^2\left(3k^2+11k+4\right)n^2 + (k+1)^3\left(k^2+7k+8\right)n + (k+1)^4(k+3)$.

3.3.3 Social Welfare Comparison When a Public Firm Acts as a Leader

Now we compare social welfare in the SPNE before and after privatization. From Tables 3.1 and 3.2, we provide a social welfare comparison in the following proposition.

Proposition 3.1 *Consider the situation in which a public firm acts as a Stackelberg leader before and after privatization. Social welfare before privatization is not lower than that after privatization, irrespective of the number of private firms. That is,* $W^{LB} \geq W^{LA}\ \forall n \geq 1$.

Proposition 3.1 claims that if the public firm acts as a Stackelberg leader irrespective of whether privatization has occurred, privatization is not desirable from the viewpoint of social welfare improvement. When the public firm retains the leader position to make a decision on output, the change in the firm's maximized objective, from social welfare to profit, never increases the welfare. Thus, this proposition suggests that social welfare may not increase after privatization in a sequential-move game. As far as we consider the simultaneous-move game in Cournot competition, the possibility of social welfare increasing by privatizing only arises when the number of private firms is relatively large.

To explain why privatizing a public leader firm decreases social welfare in a Stackelberg mixed oligopoly, we show the reaction functions of both a public and private firm in Fig. 3.1. The reaction functions of a public firm before and after privatizing are obtained by solving the first-order condition to maximize social welfare and its profit, respectively, as follows:[4]

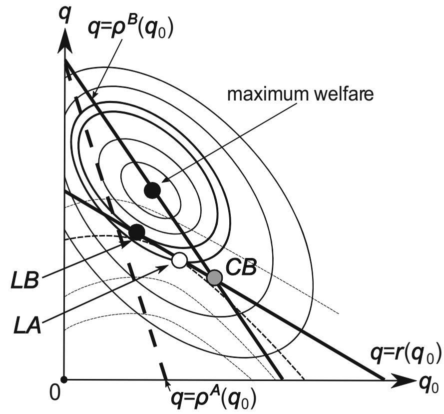
$$\frac{\partial W}{\partial q_0} = a - Q - kq_0 = 0 \Leftrightarrow q_0 = r_0^B(q) \equiv \frac{a - nq}{k+1}. \tag{3.7}$$

$$\frac{\partial \pi_0}{\partial q_0} = a - Q - q_0 - kq_0 = 0 \Leftrightarrow q_0 = r_0^A(q) \equiv \frac{a - nq}{k+2}. \tag{3.8}$$

Arranging $q_0 = r_0^B(q)$ and $q_0 = r_0^A(q)$ with respect to q, the inverse reaction functions are inverted as $q = \rho^B(q_0) \equiv \frac{a-(k+1)q_0}{n}$ and $q = \rho^A(q_0) \equiv \frac{a-(k+2)q_0}{n}$, respectively. The reaction function of the identical private firms is denoted by (3.2). Furthermore, the iso-welfare curve is an oval shape, and the maximum social welfare is attained under the positive firm outputs, which are obtained by maximizing social welfare, $W(q_0, q) \equiv a(q_0 + nq) - (1/2)(q_0 + nq)^2 - (k/2)\left(q_0^2 + nq^2\right)$, with respect to (q_0, q). The maximal solution $\left(q_0^*, q^*\right)$ lies on the public firm's reaction function before privatizing.

[4] We distinguish the reaction functions before and after privatization with the inclusion of the superscripts B (before) and A (after).

Fig. 3.1 SPNE before and after privatization when a public firm is a leader



LB: SPNE before privatization, *LA*: SPNE after privatization, *CB*: Cournot eq. before privatization

Before privatization, a public firm as a Stackelberg leader maximizes social welfare by anticipating the best response of the private firms as followers. Thus, in Fig. 3.1, SPNE before privatization lies on the point that the iso-welfare curve is tangent to the private firm's reaction function. However, after privatizing, the public firm (maximizing its profit) has a different best response from that before privatizing. The iso-profit curve for a public firm is an inverted U shape, and the vertexes are always on the privatized public firm's reaction function. SPNE after privatization lies on the point that the iso-profit curve is tangent to the private firm's reaction function. It should be noted that all the upper contour sets of the iso-welfare curves and those of the iso-profit curves are convex sets. Thus, as shown by the hyperplane separation theorem (which claims that if both disjoint convex sets are open, then there is a hyperplane between them), the private firm's reaction function can always separate the iso-welfare curve in the SPNE before privatization from the iso-profit curve in the SPNE after privatization. Moreover, as social welfare increases, the corresponding iso-welfare curve is located to the upper left, but as the public firm's profit increases, the corresponding iso-profit curve is located to the lower right. Thus, social welfare after privatization can never achieve the levels reached before privatization except in the accidental case where the iso-welfare curve is tangent to the private firm's reaction function and the iso-profit curve is also tangent at the exact same point.

3.4 When a Public Firm Acts as a Stackelberg Follower

In this section, we consider the opposite case to that analyzed in Sect. 3.3.2, where a public firm acts as a Stackelberg follower before and after privatization. It is often assumed that the public firm has an advantage over private firms with regard to the timing of output choice. However, relevant research has yet to present an economically persuasive rationale as to why the public firm must act as a Stackelberg leader. On the contrary, the possibility that the public firm acts as a Stackelberg follower might exist because it has often been pointed out that public firms have slower decision-making capabilities than private firms. Furthermore and surprisingly, it is possible that social welfare before privatization (when the public firm acts as a Stackelberg follower) is greater than that when it competes in a Cournot fashion or even when it acts as a Stackelberg leader. We will show this result in Sect. 3.5.

Therefore, in this section, we examine social welfare when a public follower firm and private leader firms engage in Stackelberg competition before and after privatization. In the next section, we look at Stackelberg competition in which the public firm is a follower. We show that social welfare is higher when the public firm is a Stackelberg follower than when the public firm is a leader or under Cournot competition.

3.4.1 Before Privatization

In the second stage before privatizing, the public firm maximizes social welfare given the output levels of private firms that have already been determined in the first stage. The first-order condition for a public firm is as follows:

$$\frac{\partial W_0}{\partial q_0} = a - Q - kq_0 = 0 \Leftrightarrow q_0 = r_0^B\left(\{q_i\}_{i=1}^n\right) \equiv \frac{a - \sum_{i=1}^n q_i}{k+1}. \tag{3.9}$$

Note that $\frac{\partial r_0^B}{\partial q_i} = -\frac{1}{k+1}$.

In the first stage, each private firm maximizes its profit by anticipating that the public firm's output in the second-stage subgame satisfies (3.9). The first-order condition for a private firm is as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a - Q - \left(1 + \frac{\partial r_0^B}{\partial q_i}\right)q_i - kq_i = 0$$
$$\Leftrightarrow q_i = \frac{a - Q_{-(0,i)}}{k+3}, \; Q_{-(0,i)} \equiv \sum\nolimits_{j \neq 0,i} q_j. \tag{3.10}$$

As private firms are identical, we obtain the SPNE output of private firms as follows:

$$q = \frac{a}{n+k+2}. \tag{3.11}$$

Table 3.3 SPNE before privatization when a public firm acts as a follower

Public firm's output	q_0^{FB}	$\frac{(k+2)a}{(k+1)(Y+1)}$
Private firm's output	q^{FB}	$\frac{a}{Y+1}$
Total output	Q^{FB}	$\frac{(Y+nk+1)a}{(k+1)(Y+1)}$
Price	p^{FB}	$\frac{k(k+2)a}{(k+1)(Y+1)}$
Public firm's profit	π_0^{FB}	$\frac{k(k+2)^2a^2}{2(k+1)^2(Y+1)^2}$
Private firm's profit	π^{FB}	$\frac{k(k+3)a^2}{2(k+1)(Y+1)^2}$
Social welfare	W^{FB}	$\frac{\left[(k+1)n^2+(k+1)(k+4)n+(k+2)^2\right]a^2}{2(k+1)(Y+1)^2}$

$(Y \equiv n + k + 1)$

Substituting q, which is obtained in (3.11), into (3.9), we obtain the public firm's output as follows:

$$q_0 = \frac{(k+2)a}{(k+1)(n+k+2)}. \tag{3.12}$$

The SPNE variables when private leader firms and a public follower firm engage in Stackelberg competition before privatization are summarized in Table 3.3.

3.4.2 After Privatization

In the second stage after privatizing, the reaction function of the public firm maximizing its profit changes as follows:

$$\frac{\partial \pi_0}{\partial q_0} = a - Q - q_0 - kq_0 = 0 \Leftrightarrow q_0 = r_0^A\left(\{q_i\}_{i=1}^n\right) \equiv \frac{a - \sum_{i=1}^n q_i}{k+2}. \tag{3.13}$$

Note that $\frac{\partial r_0^A}{\partial q_i} = -\frac{1}{k+2}$.

In the first stage, each private firm maximizes its profit by anticipating that the public firm's output in the second-stage subgame satisfies (3.13). The first-order condition for a private firm is as follows:

$$\frac{\partial \pi_i}{\partial q_i} = a - Q - \left(1 + \frac{\partial r_0^A}{\partial q_i}\right)q_i - kq_i = 0 \Leftrightarrow q_i = \frac{(k+1)\left(a - Q_{-(0,i)}\right)}{k^2+4k+2}. \tag{3.14}$$

Substituting $q_i \equiv q$ into (3.14), we obtain identical SPNE outputs for the private firms:

Table 3.4 SPNE after privatization when a public firm acts as a follower

Public firm's output	q_0^{FA}	$\frac{(k^2+3k+1)a}{(k+2)(X+n+k)}$
Private firm's output	q^{FA}	$\frac{(k+1)a}{X+n+k}$
Total output	Q^{FA}	$\frac{[n(k+1)(k+2)+(k+1)^2+k]a}{(k+2)(X+n+k)}$
Price	p^{FA}	$\frac{(k+1)(k^2+3k+1)a}{(k+2)(X+n+k)}$
Public firm's profit	π_0^{FA}	$\frac{(k^2+3k+1)^2a^2}{2(k+2)(X+n+k)^2}$
Private firm's profit	π^{FA}	$\frac{(k+1)^2(k^2+4k+2)a^2}{2(k+2)(X+n+k)^2}$
Social welfare	W^{FA}	$\frac{Va^2}{2(k+2)^2(X+n+k)^2}$

$(X \equiv (k+1)^2 + nk,$
$V \equiv (k+1)^2(k+2)^2n^2 + (k+1)(k+1)^2(k^2+5k+2)n$
$+(k+3)(k^2+3k+1)^2)$

$$q = \frac{(k+1)a}{n(k+1) + k^2 + 3k + 1}. \tag{3.15}$$

Substituting q into (3.13), we obtain the public firm's output as follows:

$$q_0 = \frac{(k^2+3k+1)a}{(k+2)[n(k+1) + k^2 + 3k + 1]}. \tag{3.16}$$

The SPNE variables when a privatized public follower firm and private leader firms engage in Stackelberg competition are summarized in Table 3.4.

3.4.3 *Social Welfare Comparison When a Public Firm Acts as a Follower*

Now we compare social welfare in SPNE before and after privatization. From Tables 3.3 and 3.4, we provide a social welfare comparison in the following proposition.

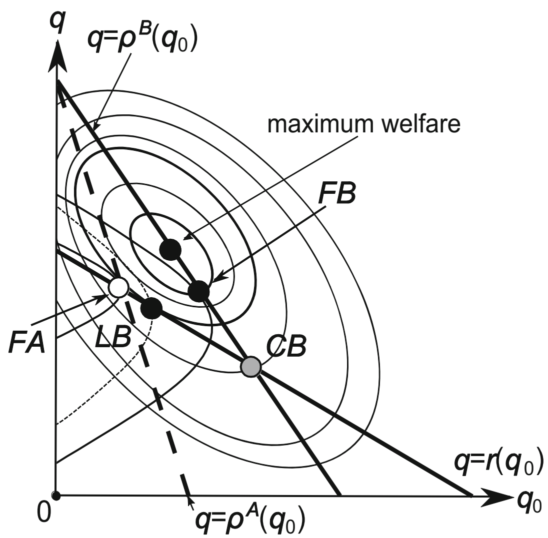
Proposition 3.2 *Consider the situation in which a public firm acts as a Stackelberg follower before and after privatization. If the number of private firms is less than 12, social welfare before privatization is higher than that after privatization irrespective of the value of the cost coefficient k. That is, if* $n = \{1, 2, \cdots, 11\}$, $W^{FB} \geq W^{FA}\ \forall k > 0$.

Proposition 3.2 suggests the possibility that privatization decreases social welfare even though the public firm acts as a Stackelberg follower, irrespective of whether privatization occurs. In particular, when the number of private firms is relatively small ($n = \{1, 2, \cdots, 11\}$), privatization certainly leads to a decrease in social welfare. If a public follower firm competes with a small number of private firms, as is usually presumed in a mixed oligopoly, it is unlikely that privatization is desirable from the perspective of social welfare improvement. However, unlike the case in which a public firm behaves as a Stackelberg leader, whether privatization increases social welfare depends on the number of private firms. If the number is sufficiently large, that is, larger than 12, privatization might improve social welfare although the exact result depends on the size of the cost coefficient, k.

This proposition suggests that the result that social welfare might increase after privatization is quite limited, even in a sequential-move game in which the public firm behaves as a Stackelberg follower. Furthermore, this result is similar to that when we consider the simultaneous-move game under Cournot competition—the possibility of social welfare improving by privatizing only occurs when the number of private firms is relatively large. Thus, when Stackelberg competition occurs in a mixed oligopoly, privatization decreases social welfare in many cases (especially when there are just a few private firms in the market), irrespective of whether the public firm acts as a leader or a follower.

To explain the reason why privatizing a public follower firm decreases social welfare in a Stackelberg mixed oligopoly when the number of private firms is small, we show the reaction functions of both a public and private firm in Fig. 3.2. The reaction functions of the public firm before and after privatizing are $q_0 = r_0^B(q)$ and $q_0 = r_0^A(q)$, respectively, which satisfy (3.7) and (3.8). The inverse reaction functions are inverted as $q = \rho^B(q_0) \equiv \frac{a-(k+1)q_0}{n}$ and $q = \rho^A(q_0) \equiv \frac{a-(k+2)q_0}{n}$. The reaction function of the identical private firms, $q = r(q_0)$, is denoted by (3.2).

Fig. 3.2 SPNE before and after privatization when a public firm is a follower



FB: SPNE before privatization, FA: SPNE after privatization
LB: SPNE before privatization, CB: Cournot eq. before privatization

The iso-welfare curve is an oval shape and the maximum social welfare is attained under positive firm outputs. The maximal solution (q_0^*, q^*) lies on the public firm's reaction function before privatizing. Before privatization, identical private firms as Stackelberg leaders maximize their profits by anticipating the best response of the public firm as a follower. The iso-profit curve for a private firm is an inverted U shape along the vertical axis, and as the iso-profit curve approaches the vertical axis, the profit becomes larger. In Fig. 3.2, the SPNE before and after privatization lies on the point that an iso-profit curve is tangent to the public firm's reaction function before privatization. In contrast, after privatizing, the public firm maximizing its profit has a different best response from that before privatizing. As the slope of the reaction function after privatization is steeper than that before privatization, the private firm can attain a larger profit after privatizing than before.

It should be noted that as the number of private firms n becomes larger, the slopes of the reaction functions, $q = \rho^B(q_0)$, $q = \rho^A(q_0)$, and $q = r(q_0)$, become gentler, and the oval of the iso-welfare curve becomes flatter. Under the flat oval, it is possible that the social welfare after privatization is higher than that before privatization. However, when the iso-welfare oval is vertical, social welfare before privatization exceeds that after privatization.

3.5 Social Welfare Comparison Before Privatization

We compare social welfare before privatizing in the following three cases. The timing of output choice by the public firm and the private firms differs in each scenario. First, we consider the case where the public firm and private firms engage in Cournot competition. The reaction functions of the public firm and private firms, $q_0 = r_0^B(q_0)$ and $q = r(q_0)$, were presented earlier in (3.7) and (3.2) in Sect. 3.3, and as such we obtain the Cournot equilibrium outputs by solving the simultaneous equations (3.7) and (3.2) with respect to (q_0, q). We summarize the Cournot equilibrium variables in Table 3.5.

Table 3.5 Cournot equilibrium before privatization

Public firm's output	q_0^{CB}	$\frac{(k+1)a}{X}$
Private firm's output	q^{CB}	$\frac{ka}{X}$
Total output	Q^{CB}	$\frac{(nk+k+1)a}{X}$
Price	p^{CB}	$\frac{k(k+1)a}{X}$
Public firm's profit	π_0^{CB}	$\frac{k(k+1)^2a^2}{2X^2}$
Private firm's profit	π^{CB}	$\frac{k^2(k+2)a^2}{2X^2}$
Social welfare	W^{CB}	$\frac{\left[(k+1)^3+nk\left(nk+k^2+4k+2\right)\right]a^2}{2X^2}$

$(X \equiv (k+1)^2 + nk)$

The SPNE before privatization with a public firm as a leader (follower) is shown above in Sect. 3.3.1 (Sect. 3.4.1). We compare the three kinds of social welfare in Cournot competition and two Stackelberg competition scenarios when the public firm is a leader or follower to obtain the following proposition.

Proposition 3.3 *Compare social welfare before privatization when the public firm engages in Cournot competition and when it acts as a Stackelberg leader or follower. Social welfare before privatization when the public firm is a follower is higher than when it is a leader, and it is higher than when it faces Cournot competition. These results are satisfied irrespective of the number of private firms. That is,* $W^{FB} > W^{LB} > W^{CB}\ \forall n \geq 1$.

In Proposition 3.3, it is perhaps surprising that when a public firm behaves as a follower, social welfare is higher than when it behaves as a leader. The reason why a public follower is preferred to a public leader from the viewpoint of social welfare is as follows: provided that all firms have quadratic cost functions, the difference in production between a public firm and private firms brings about production inefficiency. As the public firm usually produces more than private firms to enhance the consumer surplus, its production costs, which are higher than that of private firms, increase the total production costs and decrease the welfare. When the public firm is a follower, private firms enjoy a first-mover advantage, and the increase in production by private firms reduces the public firm's output under strategic substitutes. The excess supply by the public firm is inhibited and leads to improved social welfare. See also Fig. 3.2, where $W^{FB} > W^{LB} > W^{CB}$ is confirmed in the iso-welfare curves.

As is usually expected, the public firm as a Stackelberg leader acquires a first-mover advantage by moving from the simultaneous choice of output to the first-mover choice. In this sense, it is appropriate that the existing literature assumes the public firm is the first mover. However, within mixed oligopolistic competition, unlike pure competition between private firms, the public firm as the Stackelberg follower can acquire higher social welfare than when it is the leader. The result that the public firm can enjoy the second-mover advantage is quite counterintuitive. The reason why this result arises is that the second-mover welfare maximizer is better at adjusting total quantity by observing the outputs of the first-mover private firms. Therefore, the result of Proposition 3.3 justifies our setting in which the public firm becomes a Stackelberg follower.

Thus, the difference in the results of Propositions 3.1 and 3.2 can be explained by the difference in advantageous moves before and after privatization. Before privatization, as shown in Proposition 3.3, there exists a second-mover advantage for the public firm. However, after privatization, pure private competition starts, and the public firm as a second mover loses its advantageous position because a first-mover advantage exists in quantity competition between profit-maximizing firms. When there are relatively few private firms in the market, the welfare loss associated with the loss of second-mover advantage becomes large.

3.6 Concluding Remarks

In this chapter, we extended the results of De Fraja and Delbono (1989) to the sequential-move game between a public firm and private firms before and after privatization, and we obtained some additional and novel results. We demonstrated that in many cases, privatization does not improve social welfare. We first showed that when a public firm acts as a Stackelberg leader before and also after privatization, privatization never improves social welfare (Proposition 3.1). Second, we demonstrated that even when a public firm remains a Stackelberg follower before and after privatization, welfare deterioration occurs with privatization if the number of private firms is relatively small (Proposition 3.2). Third, comparing social welfare before privatization when a public firm engages in Cournot competition and when it acts as a Stackelberg leader or a follower, we clarify that social welfare before privatization when the public firm is a follower is higher than when it is a leader, and it is higher than when it faces Cournot competition (Proposition 3.3).

Our results suggest that the possibility of improving social welfare by privatization might be more restricted than previously indicated in earlier studies, which emphasized the positive effect of privatization on social welfare in mixed oligopolistic Cournot competition. Thus, our results are partially compatible with the fact that there are still many public firms that have not been privatized in various countries around the world, including in developed capitalist countries. The existence of public firms can be justified if the transition to privatization brings about lower social welfare in mixed oligopolistic markets.

Finally, we conclude our chapter by discussing possible extensions. We did not consider the endogenization of the timing of output choice by firms in our study. There is a strong relation between a firm's timing of the decision-making and the firm's objective. If the public firm has a second-mover advantage and the private firms have a first-mover advantage, Stackelberg competition in which the public firm is a follower and private firms are the leader would be chosen endogenously. Therefore, future research should investigate the endogenized timing of output choice within a mixed oligopoly framework.

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Appendix

A.3.1 Proof of Proposition 3.1

By simple calculation, the following equation is satisfied:

$$W^{LB} \geq W^{LA} \Leftrightarrow \frac{X+nk(Y+1)}{X+kY^2} \geq \frac{nX^2(Y+1)+2(k+1)XY^2+(1-k)(1+k)^2Y^2}{(X+k+1)^2Y^2}$$
$$\Leftrightarrow (X+k+1)^2Y^2[X+nk(Y+1)]\left(X+kY^2\right)$$
$$\times\left[nX^2(Y+1)+2XY^2(k+1)(1-k)(k+1)^2Y^2\right] \geq 0$$

$\Leftrightarrow \left[(k+1)^3 - n^2k\right]^2 \geq 0$. The equality holds only when $n = \left(\frac{(k+1)^3}{k}\right)^{\frac{1}{2}}$.

A.3.2 Proof of Proposition 3.2

By direct calculation, the following relationship is satisfied: $W^{FB} \gtrless W^{FA} \Leftrightarrow$

$$\frac{(k+1)n^2+(k+1)(k+4)n+(k+2)^2}{(k+1)(Y+1)^2}$$
$$\gtrless \frac{(k+1)^2(k+2)^2n^2+(k+1)(k+2)^2\left(k^2+5k+2\right)n+(k+3)\left(k^2+3k+1\right)^2}{(k+2)^2(X+n+k)^2}$$

$\Leftrightarrow D(n,k) \equiv (k+1)\left(k^3+2k^2-k-1\right)n^2 - (k+1)(k+2)\left(3k^2+8k+2\right)n - (k+2)^2 \left(k^2+3k+1\right)^2 \lesseqgtr 0$. k^+ denotes the positive real-valued solution of the cubic equation, $f(k) \equiv k^3+2k^2-k-1=0$, which is approximately 0.8019. When $k=k^+$, because $f(k^+)=0$ holds, $D(n,k^+)$ is a linear function of n and $D(n,k^+)=-52.23n-128.71<0 \forall n$. Thus, when $k=k^+$, it is necessarily satisfied that $W^{FB}>W^{FA}$. When $k \neq k^+$, i.e., $f(k) \neq 0$, $D(n,k)$ is a quadratic equation of n. Denote two solutions of $D(n,k)=0$ with respect to n as $n^+(k)$ and $n^-(k)$, where $n^+(k)>n^-(k)$.[5] Note that $n^-(k)<0$ for all $k>0$. As $f(k)<0$ holds when $k \in (0,k^+)$, $D(n,k)$ is a quadratic function of n in which the coefficient of the square of n is negative and $n^+(k)<0$ also holds. Thus, when $k \in (0,k^+)$, $D(n,k)$ is always negative for all $n>0$ and $W^{FB}>W^{FA}$ holds. However, as $f(k)>0$ holds when $k>k^+$, $D(n,k)$ is a quadratic function of n in which the coefficient of the square of n is positive and $n^+(k)>0$ holds. In this case, if $n \in (0,n^+(k))$, $D(n,k)$ is negative and as a result, $W^{FB}>W^{FA}$ is satisfied; if $n>n^+(k)$, $D(n,k)>0$ and $W^{FB}<W^{FA}$ is possible to occur. Note that $n^+(k)$ has the minimum value when $k>k^+$. When $k=k^{\min} \approx 3.489$, the

[5] $n^+(k) \equiv \frac{(k+1)(k+2)\left(3k^2+8k+2\right)+\sqrt{Z(k)}}{2(k+1)\left(k^3+2k^2-k-1\right)}$ and $n^-(k) \equiv \frac{(k+1)(k+2)\left(3k^2+8k+2\right)-\sqrt{Z(k)}}{2(k+1)\left(k^3+2k^2-k-1\right)}$, where $Z(k) \equiv (k+1)^2 (k+2)^2\left(3k^2+8k+2\right)^2+4(k+1)\left(k^3+2k^2-k-1\right)\ (k+2)^2\left(k^2+3k+1\right)^2$.

minimum value is $n^{+\min} \equiv n^{+}(k^{\min}) \approx 11.216$. Therefore, for all values of k, if $n = \{1, 2, \cdots, 11\}$, it is necessarily satisfied that $D(n,k)$ is negative, that is, $W^{FB} > W^{FA}$. When $n \geq 12$, $W^{FB} > W^{FA}$ if $n < n^{+}(k)$ and otherwise vice versa.

A.3.3 Proof of Proposition 3.3

By direct calculation, the following equations are satisfied: $W^{FB} > W^{LB}$ $\Leftrightarrow \frac{(k+1)n^2+(k+1)(k+4)n+(k+2)^2}{(k+1)(Y+1)^2} > \frac{X+nk(Y+1)}{X+kY^2} \Leftrightarrow nk(n+2k+3) > 0$ and $W^{LB} > W^{CB}$ $\Leftrightarrow \frac{X+nk(Y+1)}{X+kY^2} > \frac{(k+1)^3+nk\left(nk+k^2+4k+2\right)}{X^2} \Leftrightarrow n^2k^2 > 0$.

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Chapter 4
Physical Capital Accumulation and Partial Privatization

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Abstract This chapter investigates the effect of capital accumulation on partial privatization. We extend the dynamic oligopoly model proposed by Cellini and Lambertini in 1998 to a dynamic mixed oligopoly model. We show that (i) when a steady state is characterized by a demand-driven (i.e., static) equilibrium, partial privatization is adopted and the privatization ratio perfectly corresponds to the static model; (ii) when a public firm produces Ramsey output, the level of social welfare in a steady state does not depend on the privatization ratio; and (iii) when a private firm produces Ramsey output, the government adopts a full nationalization policy. The results of (ii) and (iii) are in contrast with the static result that partial privatization is optimal.

4.1 Introduction

In this chapter, we investigate the effect of capital accumulation on privatization. Historically, governments in developed and developing countries protected infant industries in the process of industrialization. During the eighteenth century in the United Kingdom and the United States, governments protected industries by imposing high tariffs. The Japanese government offered protection via the selling of national factories to the private sector. For example, the Tomioka silk mill, which is now registered as a world heritage site, was founded in the process of modernization and auctioned to the private sector.

The relationship between privatization and economic growth has been extensively investigated from an empirical viewpoint rather than a theoretical perspective. Plane (1997) showed that privatization increases the economic growth rate using a sample of 35 developing countries from 1988 to 1992. Mackenzie (1998)

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also found that the reason for the increase in economic growth rate with privatization stems from the change of the composition of government expenditure.

From a theoretical perspective, there exist numerous studies about the relationship between capital accumulation and imperfect competition using a partial equilibrium model. Cellini and Lambertini (1998) proposed a dynamic capital accumulation game in a Cournot oligopoly with capital accumulation based on a Ramsey model. They demonstrated that two types of equilibrium exist. One is a demand-driven equilibrium, which corresponds to a static equilibrium, and the other is a Ramsey golden rule equilibrium, which corresponds to a dynamic equilibrium. They also compared the steady-state level of social welfare under a social planning solution with one under a decentralized solution.

This dynamic duopoly model has been extended to various economic situations. Baldini and Lambertini (2011) analyzed the effect of a profit tax as a domestic policy, and Calzolari and Lambertini (2006) investigated whether the tariff–quota equivalence theorem holds in international economics. Each proved that static results may not survive in a dynamic situation because of the capital accumulation process.

To our knowledge, no previous study has analyzed the relationship between capital accumulation and privatization. We extend the oligopoly model proposed by Cellini and Lambertini (1998) to a mixed oligopoly model. We show the following results: (i) when a steady state is characterized by a demand-driven (i.e., static) equilibrium, partial privatization is adopted and the privatization ratio perfectly corresponds to the static model; (ii) when a public firm produces a Ramsey golden rule equilibrium, the level of social welfare in the steady state does not depend on the privatization ratio; and (iii) when a private firm produces Ramsey output, the government adopts a fully nationalized policy. The results of (ii) and (iii) are in contrast to the static result that partial privatization is optimal.

The structure of this chapter is as follows. In the next section, we construct the basic model. In Sect. 4.3, we examine the properties of dynamic optimization and a steady-state solution. Section 4.4 illustrates the dynamic optimal privatization policy, and Sect. 4.5 discusses the possibility of the expanding the model in future research.

4.2 The Model

Consider a mixed duopoly market for a single homogeneous good where private and public firms compete in a Cournot market. Time starts in period t and lasts forever, $t \in [0, \infty)$. The structure of the game is as follows. In the first stage, the government chooses the privatization ratio in the leader's position, so as to maximize social welfare, under the follower's output decision. In the second stage, each firm strategically sets its output level.

As in Calzolari and Lambertini (2006, 2007), we assume a linear inverse demand function in period t as

$$p_t = a - q_{0,t} - q_{1,t}, \tag{4.1}$$

where p_t, $q_{0,t}$, and $q_{1,t}$ denote the price, the quantity sold by the public firm (or, equivalently, the quantity of demand for the public firm's output), and the one sold by the private firm, respectively.

Following Cellini and Lambertini (1998), we assume that each firm $i = 0, 1$ accumulates capital stock, $k_{i,t}$,

$$\frac{dk_{i,t}}{dt} = f(k_{i,t}) - q_{i,t} - \delta k_{i,t}, \tag{4.2}$$

where $\delta \in (0, 1)$ denotes the capital depreciation rate. The output, $y_{i,t}$, is produced by the production function $y_{i,t} = f(k_{i,t})$ with the properties $f' > 0$ and $f'' < 0$. Thus, excess output is reinvested into this production process.

Each firm aims to maximize the discount value of instantaneous profit. The instantaneous profit function of a private firm in period t is $\pi_{1,t} = p_t q_{1,t} - c_1 q_{1,t}$, where $c_i > 0$ denotes the marginal cost of the production of firm i. Thus, the maximization problem of the private firm is

$$\begin{aligned} \max_{q_{1,t}} \Pi_{1,t} &= \int_0^\infty e^{-\rho t} \pi_{1,t} dt \\ \text{subject to } \frac{dk_{1,t}}{dt} &= f(k_{1,t}) - q_{1,t} - \delta k_{1,t}. \end{aligned} \tag{4.3}$$

The manager of the public firm maximizes the discounted value of the weighted average of the profit of both firms in period t, $\pi_{0,t} = p_t q_{0,t} - c_0 q_{0,t}$, and social welfare in period t, $W_t = CS_t + \pi_{0,t} + \pi_{1,t}$;

$$\begin{aligned} V_t = \int_0^\infty e^{-\rho t}[(1-\theta)(CS_t + \pi_{0,t} + \pi_{1,t}) + \theta \pi_{0,t}]dt, \\ \text{subject to } \frac{dk_{0,t}}{dt} = f(k_{0,t}) - q_{0,t} - \delta k_{0,t}, \end{aligned} \tag{4.4}$$

where $CS_t = \frac{(q_{0,t}+q_{1,t})^2}{2}$ represents the consumer surplus in period t and θ denotes the privatization ratio. For simplicity, we include the following assumption reflecting the real situation.

Assumption 4.1: Efficiency Condition Between Private and Public Firms The productivity of the private firm is more efficient than that of the public firm: $c_0 > c_1 = 0$.

4.3 Dynamic Optimization

This section considers the intertemporal maximization problem of both firms as an open-loop strategy and derives the stability condition of the dynamic system and both outputs in a steady state.

4.3.1 The Maximization Problem of the Private Firm

The Hamiltonian of the private firm is given by

$$H_{1,t} = e^{-\rho t}\left\{\pi_{1,t} + \lambda_{1,t}\left[f(k_{1,t}) - q_{1,t} - \delta k_{1,t}\right]\right\},$$

where $\lambda_{1,t} = \mu_{1,t}e^{\rho t}$, and $\mu_{1,t}$ denotes the costate variable of capital stock in the present value. The first-order conditions of this problem are given by

$$\frac{\partial H_{1,t}}{\partial q_{1,t}} = e^{-\rho t}\left(a - q_{0,t} - 2q_{1,t} - \lambda_{1,t}\right) = 0, \tag{4.5}$$

$$\frac{\partial \lambda_{1,t}}{\partial t} = [(\delta + \rho) - f'(k_{1,t})]\lambda_{1,t}, \tag{4.6}$$

$$\frac{dk_{1,t}}{dt} = f(k_{1,t}) - q_{1,t} - \delta k_{1,t},$$

$$\lim_{t\to\infty} \mu_{1,t}k_{1,t} = 0,$$

where (4.5) implies that if there is net marginal revenue $a - q_{0,t} - 2q_{1,t} > 0$, the value of capital stock, $\lambda_{1,t}$, should increase. From (4.5) and (4.6), the Euler equation in terms of private firms' output becomes[1]

$$\frac{\partial q_{1,t}}{\partial t} = -\frac{1}{2}\left\{\frac{dq_{0,t}}{dt} + [\delta + \rho - f'(k_{1,t})]\left(a - q_{0,t} - 2q_{1,t}\right)\right\}. \tag{4.7}$$

The dynamic reaction function of the private firm has two effects: (i) the *net marginal revenue effect*, $[\delta + \rho - f'(k_{1,t})](a - q_{0,t} - 2q_{1,t})$, and (ii) the *dynamic reaction effect*, $\frac{dq_{0,t}}{dt}$. The net marginal revenue effect means that when the capital level satisfies dynamic efficiency, $\delta + \rho < f'(k_{1,t})$, the private firm tends to increase its output. The dynamic reaction effect means that when the public firm increases its output in period t, $\frac{dq_{0,t}}{dt} > 0$, the output of the private firm tends to decrease because the market share decreases.

[1]See Appendix A.4.1 for the derivation.

4.3.2 The Maximization Problem of the Public Firm

The Hamiltonian of the public firm is given by

$$H_{0,t} = e^{-\rho t}\{(1-\theta)(CS_t + \pi_{0,t} + \pi_{1,t}) + \theta\pi_{0,t} + \lambda_{0,t}[f(k_{0,t}) - q_{0,t} - \delta k_{0,t}]\}$$

where $\lambda_{0,t} = \mu_{0,t}e^{\rho t}$, and $\mu_{0,t}$ denotes the costate variable of the capital stock of the public firm in the present value. The first-order conditions of this problem are

$$\frac{\partial H_t}{\partial q_{0,t}} = e^{-\rho t}[a - c_0 - (1+\theta)q_{0,t} - q_{1,t} - \lambda_{0,t}] = 0, \tag{4.8}$$

$$\frac{\partial \lambda_{0,t}}{\partial t} = [(\delta+\rho) - f'(k_{0,t})]\lambda_{0,t}, \tag{4.9}$$

$$\frac{dk_{0,t}}{dt} = f(k_{1,t}) - q_{0,t} - \delta k_{0,t},$$

$$\lim_{t\to\infty} \mu_{0,t}k_{0,t} = 0,$$

where (4.8) implies that if there is a net marginal revenue, $a - c_0 - (1+\theta)q_{0,t} - q_{1,t} > 0$, the value of capital stock, $\lambda_{0,t}$, should increase. From (4.8) and (4.9), the Euler equation in terms of public firms' output becomes[2]

$$\frac{\partial q_{0,t}}{\partial t} = -\frac{1}{1+\theta}\left\{\frac{dq_{1,t}}{dt} + [\delta + \rho - f'(k_{0,t})][a - c_0 - (1+\theta)q_{0,t} - q_{1,t}]\right\}. \tag{4.10}$$

This dynamic reaction function produces the same effects as the private dynamic reaction function: (i) the *net marginal revenue effect*, when the capital level satisfies dynamic efficiency, $\delta + \rho < f'(k_{0,t})$, and there exists positive (negative) net marginal revenue, $a - [q_{1,t} + (1+\theta)q_{0,t} + c_0] > (<)0$, then the public firm tends to increase (decrease) its output and (ii) the *dynamic reaction effect*, when the public firm increases its output in period t, $\frac{dq_{1,t}}{dt} > 0$, the output of the private firm tends to decrease because the market share decreases.

Note that the structure of the dynamic reaction of the public firm differs from that of the private firm. The reason for this is that the public firm cares about its own profit *and* social welfare. Thus, when the private firm increases its output, the output of the public firm decreases. Moreover, we find that the change in the privatization ratio has two opposite effects: if the government adopts a low privatization ratio corresponding to the nationalization policy, then (i) the net marginal revenue effect, $\frac{dq_{0,t}}{dt}$, weakens and (ii) the dynamic reaction effect strengthens.

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[2]See Appendix A.4.2 for the derivation.

4.3.3 Local Stability

We now investigate the stability of this dynamic system. The dynamic system comprises four equations: the capital accumulation equation of both firms (4.2) and the Euler equations of both firms (4.7) and (4.10). Noting that this system is characterized by asymmetric dynamic equations regarding the outputs and capital stock of both firms, we linearize these dynamic equations for $q_{0,t}$, $q_{1,t}$, $k_{0,t}$, and $k_{1,t}$ around the neighborhood of the steady state as follows:

$$\begin{bmatrix} \frac{dq_{0,t}}{dt} \\ \frac{dq_{1,t}}{dt} \\ \frac{dk_{0,t}}{dt} \\ \frac{dk_{1,t}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{2(1+\theta)\{f'(k_0^*)-(\delta+\rho)\}}{1+2\theta} & -\frac{2\{f'(k_0^*)-(\delta+\rho)\}}{1+3\theta+2\theta^2} & 0 & 0 \\ \frac{(1+\theta)\{f'(k_1^*)-(\delta+\rho)\}}{2+4\theta} & \frac{2(1+\theta)\{f'(k_1^*)-(\delta+\rho)\}}{1+2\theta} & 0 & 0 \\ -1 & 0 & f'(k_0)-\delta & 0 \\ 0 & -1 & 0 & f'(k_1^*)-\delta \end{bmatrix} \begin{bmatrix} q_{0,t}-q_0^* \\ q_{1,t}-q_1^* \\ k_{0,t}-k_0^* \\ k_{1,t}-k_0^* \end{bmatrix}.$$

The eigenvalues of this Jacobian are given by $\zeta_1 = \frac{1}{1+2\theta}[2(\delta+\rho) - f'(k_0) - f'(k_1) - \sqrt{D}$, $\zeta_2 = \frac{1}{1+2\theta}[2(\delta+\rho) - f'(k_0) - f'(k_1) + \sqrt{D}$, $\zeta_3 = f'(k_0) - \delta > 0$ and $\zeta_4 = f'(k_1) - \delta > 0$, where $D \equiv (\delta+\rho)^2 + (1+\theta)^2\left[f'(k_0)^2 + f'(k_1)^2\right] - (\delta+\rho)[f'(k_0) + f'(k_1)] + f'(k_0)f'(k_1)[1 + 2\theta(2+\theta)] > 0$. Thus, when the former two eigenvalues, ζ_1 and ζ_2, are both negative, the steady-state equilibrium of this system is characterized by saddle-point stability. We define the steady-state variables as q_0^*, q_1^*, k_0^*, and k_1^*, satisfying $\frac{dq_{0,t}}{dt} = \frac{dq_{1,t}}{dt} = \frac{dk_{0,t}}{dt} = \frac{dk_{1,t}}{dt} = 0$. Thus, starting from a given initial level of capital stock, this dynamic system converges to the steady state along with a saddle path.

Next, we see the properties of the outputs in the steady-state equilibrium. Substituting the condition of the steady state, $\frac{dq_{0,t}}{dt} = \frac{dq_{1,t}}{dt} = \frac{dk_{0,t}}{dt} = \frac{dk_{1,t}}{dt} = 0$, into (4.7) and (4.10), we find that there are two types of equilibrium, a demand-driven equilibrium and a Ramsey equilibrium, as demonstrated by Cellini and Lambertini (1998) and Calzolari and Lambertini (2006, 2007):

$$q_0^{d,*} = \frac{a - 2c_0}{1+\theta} \text{ or } f'(k_0^{R,*}) = \delta + \rho, \tag{4.11}$$

$$q_1^{d,*} = \frac{a\theta + c_0}{1+\theta} \text{ or } f'(k_1^{R,*}) = \delta + \rho, \tag{4.12}$$

where $q_i^{d,*}$ denotes the steady-state output of the demand-driven equilibrium and $f'\left(k_i^{R,*}\right) = \delta + \rho, \ (i = 0, 1)$ determines the steady-state level of capital at the Ramsey equilibrium. The former steady-state output of the demand-driven equilibrium has the same properties of the static Cournot equilibrium solution in a mixed oligopoly model. Furthermore, an increase in the marginal cost of the public firm, c_0, decreases the output of the public firm and increases the output of the private firm. Regarding θ, as the privatization process progresses, $q_0^{d,*}$ decreases.

Following Calzolari and Lambertini (2006), we can confine the discussion to the two types of output where the equilibrium solution satisfies $q_i^{R,*} = f\left(k_i^{R,*}\right)$ or the demand-driven equilibrium, $q_i^{d,*}$. As a result, three kinds of steady-state equilibrium emerge.

The first is the demand-driven outputs of the public and private firms, $q_0^{d,*}$ and $q_1^{d,*}$, respectively. In this case, both firms do not produce at full capacity. The second case is the Ramsey output of the public firm and the demand-driven output of the private firm, $q_0^{R,*}$ and $q_1^{d,*}$, respectively. From (4.11) and (4.12), when the marginal cost of the public firm, c_0, is small, the equilibrium output of the public firm may reach Ramsey equilibrium, while the equilibrium output of the private firm is in the demand-driven equilibrium because the output of the private firm decreases when c_0 becomes smaller. The third case is the Ramsey output of the private firm and the demand-driven output of the public firm, $q_1^{R,*}$ and $q_0^{d,*}$, respectively, because of a larger marginal cost, c_0. Thus, in contrast with symmetric models such as Cellini and Lambertini (1998), there exist three types of equilibrium as in Lemma 4.1.

Lemma 4.1 *In the steady state satisfying the demand-driven equilibrium or Ramsey equilibrium, the set of possible solutions is* $\{ q_0^{d,*}, q_1^{d,*} \}$, $\{ q_0^{R,*}, q_1^{d,*} \}$, *and* $\{ q_1^{R,*}, q_0^{d,*} \}$, *given the privatization ratio.*

4.4 Dynamic Optimal Privatization Ratio

We now investigate the optimal privatization ratio in the steady state corresponding to the three kinds of steady-state equilibria in Lemma 4.1. The objective function of the government is assumed to be a social welfare function that is evaluated in the steady state as follows:

$$W = CS + \pi_0 + \pi_1 = \frac{(q_0 + q_1)^2}{2} + (p - c_0)q_0 + pq_1. \tag{4.13}$$

Thus, the government maximizes (4.13) to choose the privatization ratio, θ, for each equilibrium in the steady state.

4.4.1 Demand-Driven Equilibrium

First, we investigate the optimal privatization ratio in the demand-driven equilibrium. The steady-state output of the public and private firms are $q_0^{d,*} = \frac{a-2c_0}{1+\theta}$ and $q_1^{d,*} = \frac{a\theta+c_0}{1+\theta}$, respectively. Thus, the maximization problem becomes

$$\max_{\theta} W^d = \frac{a^2(1+\theta)(1+3\theta) + {c_0}^2(2+8\theta) - 2a(c_0 + 3c_0\theta)}{2(1+2\theta)^2}.$$

Therefore, we can derive the optimal privatization ratio as

$$\theta^{d,*} = \frac{c_0}{a - 4c_0}. \tag{4.14}$$

This optimal privatization ratio corresponds to the outcome obtained in the static models. That is, when the marginal cost of the public firm, c_0, increases, the government promotes privatization. Substituting (4.14) into the objective function, W^d, we get the steady-state level of social welfare under the optimal privatization ratio:

$$W^{d,\theta} = \frac{a^2 - 2ac_0 + 4{c_0}^2}{2}.$$

The optimal outputs of the public and private firms under the optimal privatization ratio become $q_0^{d,\theta} = a - 4c_0$ and $q_1^{d,\theta} = 2c_0$, respectively. To guarantee $q_0^{d,\theta} \geq 0$, we confirm the range of the parameters, $a \geq 4c_0$. The reason why the optimal privatization level corresponds to the static result is that both firms are unable to engage in full-capacity production in the steady state. This means that capital accumulation does not affect the privatization policy. Thus, the government has to set the privatization ratio to correspond to the static policy.

4.4.2 The Ramsey Output of the Public Firm

Second, we consider the case of the Ramsey output of the public firm and the demand-driven output of the private firm. In this case, the optimal output of the public and private firms become $q_0^{R0} = f\left(k_0^{R,*}\right)$ and $q_1^{R0} = \frac{a-q_0^{R0}}{2}$, where q_0^{R0} and q_1^{R0} denote the output of the public and private firms, respectively, corresponding to the Ramsey output of public firm. Using the definitions of the consumer surplus and the profits of both firms, we obtain the steady-state level of social welfare, W^{R0}, corresponding to the Ramsey output of the public firm as

$$W^{R0} = \frac{3a^2 + 2aq_0^{R0} - q_0^{R0}\left(8c_0 + q_0^{R0}\right)}{8}. \tag{4.15}$$

We find that steady-state level of welfare does not depend on the privatization ratio. The reason for this is that the pubic firm is not affected by any privatization policy because the public firm produces its output at a modified golden rule, $f'\left(k_0^{R,*}\right) = \delta + \rho$. The output level is Ramsey output, that is, this output level is on the edge of the capacity of production.[3]

4.4.3 Ramsey Output of the Private Firm

Finally, we consider the case of the demand-driven output of the public firm and the Ramsey output of the private firm. In this case, the optimal outputs of the public and the private firms become $q_0^{R1} = \frac{a-c_0-q_1^{R0}}{1+\theta}$ and $q_1^{R0} = f\left(k_1^{R,*}\right)$, respectively, corresponding to the Ramsey output of the public firm. The steady-state level of social welfare becomes

$$W^{R0} = \frac{1}{2(1+\theta)^2}\left\{a^2 - 2ac_0 + {c_0}^2 + 2c_0q_1^{R0} + 2\theta\left[(a-c_0)^2 + 2c_0q_1^{R0}\right] + \theta^2(a-c_0)q_1^{R0}\right\}. \tag{4.16}$$

Maximizing (4.16), the government sets the privatization ratio, $\theta^{R1} = 0$. The private firm produces an efficient level of output at q_1^{R1} in the steady state. Noting that the government is a Stackelberg leader, the government can control the public firm's behavior. Therefore, if the government adopts a full nationalization policy, $\theta^{R1} = 0$, the public firm maximizes social welfare in the steady state.

The relationship between the level of social welfare in the steady state and the privatization ratio is depicted in Fig. 4.1, where we adopt a Cobb–Douglas production function, $q_1 = Ak_1^{\alpha}$. When the government adopts a full nationalization policy, consumer surplus is maximized and the profits of the private and public firms are minimized as in Fig. 4.2.[4] The level of consumer surplus and the profits of the private and public firms under a full nationalization policy are, respectively,

[3]Of course, the public and private firms' output on the transitional path is affected by the privatization policy in (4.7) and (4.10).

[4]The parameter $c_0 = \frac{1}{30}, \delta = \frac{1}{7}, \rho = \frac{1}{8}, a = 12$ is used in Lambertini and Palestini (2009) and $\mathrm{A} = 1, \alpha = \frac{1}{3}$ is used in Weil (2009). A is set to 1 for analytical convenience.

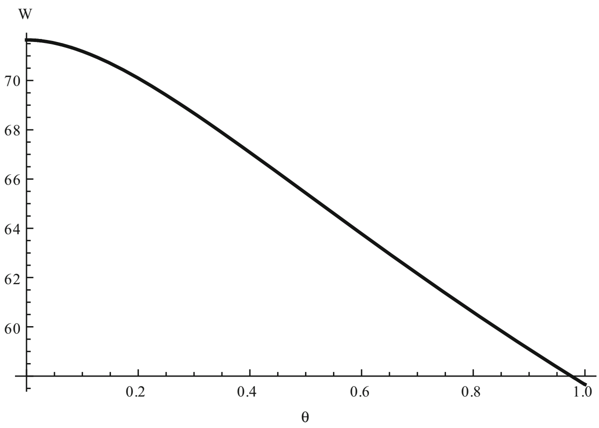

Fig. 4.1 Steady-state level of welfare and privatization ratios

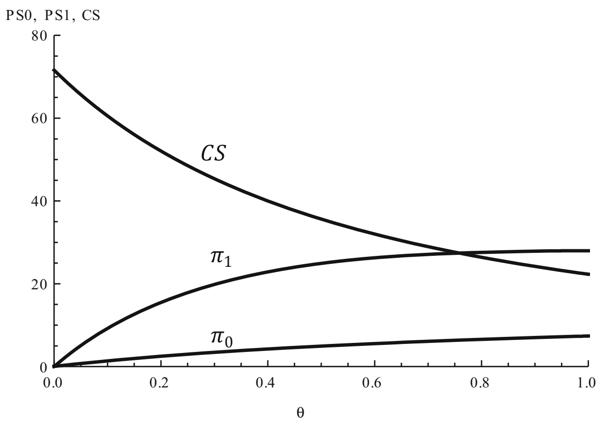

Fig. 4.2 Steady-state level of consumer surplus and privatization ratio

$$CS^{R0} = \frac{(a - c_0)^2}{8},\ \pi^{R0} = 0, \text{ and } \pi^{R0} = c_0 q_1^{R1},$$

The above discussion can be summarized as follows:

Proposition 4.1
In a dynamic mixed oligopoly model:

(i) *When the public and private firms produce demand-driven (i.e., static) outputs, the government adopts a partial privatization policy.*
(ii) *When the public firm produces Ramsey output, the steady-state level of social welfare does not depend on the privatization ratio.*
(iii) *When the private firm produces Ramsey output, the government adopts a full nationalization policy,* $\theta^{R1} = 0$.

4.5 Discussion and Remaining Issues

In this chapter, we investigated the effect of capital accumulation on partial privatization. We found three main results. First, when a steady state is characterized by a demand-driven equilibrium, partial privatization is adopted and the privatization ratio corresponds to the static model. Second, when a public firm produces Ramsey output, the level of social welfare in a steady state does not depend on the privatization ratio. Third, when a private firm produces Ramsey output, the government adopts a full nationalization policy.

We now discuss the possibility of extending this model to various types of dynamic effects. First, we consider another kind of capital accumulation. Calzolari and Lambertini (2006, 2007) used the following dynamic equation of capital:

$$\frac{dk_{i,t}}{dt} = I_{i,t} - \delta k_{i,t},$$

where $I_{i,t}$ is the investment carried out by the firms in period t. This capital accumulation equation is referred to as a Nerlove and Arrow capital accumulation. In this case, even though the output converges to a demand-driven one, it may depend on the capital accumulation level. Therefore, the optimal privatization ratio obtained in this dynamic model also differs from the one derived by a static model.

Second, we should investigate the properties of the dynamic path under the optimal privatization ratio. In this chapter, we only focused on the properties of the privatization policy in a steady state. However, the dynamic path of the optimal privatization ratio may have important implications for economic policy in developing countries.

Third, we could also consider the dynamic general equilibrium model as shown in Futagami et al. (2011). If we analyze the optimal privatization problem within a general equilibrium framework, the implications of the optimal privatization ratio change because the economic agent should always satisfy the modified golden rule. Thus, there is scope for further research. These problems concerning the effect of the dynamics in a mixed oligopoly can be developed further.

Appendix

A.4.1 Derivation of the Reaction Function of Private Firms

From (4.5), the optimal reaction function, $q^r_{1,t}$, becomes

$$q^r_{1,t} = \frac{a - q^r_{0,t} - c_0 - \lambda_{1,t}}{2}. \tag{A4.1}$$

Differentiating (A4.1) with respect to time, we obtain $\frac{dq^r_{1,t}}{dt} = -\frac{1}{2}\left(\frac{dq^r_{0,t}}{dt} + \frac{d\lambda_{1,t}}{dt}\right)$.

Substituting (4.6) into this, we obtain $\frac{dq_{1,t}^r}{dt} = -\frac{1}{2}\left\{\frac{dq_{0,t}^r}{dt} + \lambda_{1,t}[\delta + \rho - f'(k_{1,t})]\right\}$. Using $\lambda_{1,t} = a - q_{0,t}^r - 2q_{1,t}^r - c_0$, we obtain (4.7).

A.4.2 Derivation of the Reaction Function of the Public Firm

From (4.8), an optimal reaction function becomes

$$q_{0,t}^r = \frac{a - q_{1,t}^r - c_0 - \lambda_{1,t}}{1 + \theta}. \tag{A4.2}$$

Differentiating (A4.2) with respect to time, we obtain $\frac{dq_{0,t}^r}{dt} = -\frac{1}{1+\theta}\left(\frac{dq_{1,t}^r}{dt} + \frac{d\lambda_{0,t}}{dt}\right)$. Substituting (4.9) into this, we obtain $\frac{dq_{0,t}^r}{dt} = -\frac{1}{1+\theta}\left\{\frac{dq_{1,t}^r}{dt} + \lambda_{0,t}[\delta + \rho - f'(k_{0,t})]\right\}$. Using $\lambda_{1,t} = a - (1+\theta)q_{0,t}^r - q_{1,t}^r - c_0$, we obtain (4.10).

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Chapter 5
International Mixed Oligopoly

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Abstract This chapter outlines the basic properties of an international mixed oligopoly and considers the policy implications of privatization in the context of a strategic trade policy. We show here that the government sets a low level of privatization to reduce the profit of foreign firms in a non-corporative equilibrium. In a free-entry equilibrium, the number of private firms affects the degree of privatization in an international mixed oligopoly model. We present the complimentary relationship between import tariffs and the degree of privatization. Furthermore, we consider a corporative privatization policy. Regarding a non-corporative equilibrium, a higher degree of privatization in both countries improves global welfare.

5.1 Introduction

In this chapter, we illustrate the basic properties of an international mixed oligopoly and consider the policy implications of privatization in the context of a strategic trade policy. There are numerous studies on privatization policies in an international oligopoly. For example, Fjell and Pal (1996) and Pal and White (1998) considered the welfare effects of privatization, comparing a fully nationalized firm with a fully privatized firm. They showed that the privatization of public firms improves welfare when the government optimally sets the production subsidy. In a domestic context, Matsumura (1998) and Matsumura and Kanda (2005) extended the mixed oligopoly, which can accommodate full privatization and nationalization, to a partial mixed oligopoly problem. In that case, the government seeks to maximize social welfare by sharing the ownership of the relevant firm with the private sector.

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In an international context, the partial privatization model can also include a strategic trade policy problem. For example, Chao and Yu (2006) considered the relationship between a strategic trade policy and partial privatization. In such models, the public firm is a strategic policy instrument for trade protection because it slows the outflow of foreign profit. However, it is recognized that a fully nationalized firm is not satisfied with simply maximizing social welfare, representing the possibility of establishing a partially nationalized or partially privatized firm.

The partial privatization model is also applied to policy harmonization in a two-country mixed oligopoly. Han and Ogawa (2008) considered overnationalization stemming from a strategic policy game between two countries. They suggested that the corporative setting of privatization in both countries leads to higher welfare than a non-corporative equilibrium.

In this chapter, we summarize the basic results of previous studies on international mixed oligopolies and consider various policy implications of privatization. We present the relationship between a strategic trade policy (i.e., an import tariff) and the degree of privatization. We also develop an integrated explanation of a corporative privatization policy in the two-country mixed oligopoly model.

This chapter is organized as follows. Section 5.2 presents a simple international mixed oligopoly model comprising a public firm, n domestic private firms and m foreign private firms. We show that the fully nationalized firm is more aggressive than in a domestic-mixed oligopoly case. This results in friction and validates partial privatization. We also consider a free-entry equilibrium in comparison with a regulated market equilibrium where the number of firms is fixed. Section 5.3 introduces import tariffs as a strategic policy instrument in a mixed oligopolistic market. We consider how the level of the import tariff affects the degree of privatization if the government can control both policy instruments. We also show that the degree of privatization is determined by the import tariff rate and that the public firm is overnationalized (undernationalized) if the tariff rate is set lower (higher) than the optimal tariff rate. Section 5.4 examines in detail a corporative privatization policy in a two-country mixed oligopoly. We try to develop a synthetic explanation of this problem and consider both integrated and segmented international markets. We show that the public firm is always overnationalized in a non-corporative equilibrium, whether the market is integrated or segmented. Section 5.5 summarizes the main results of the chapter and discusses future research directions.

5.2 The Model

Consider a domestic industry consisting of n domestic private firms, m foreign private firms, and one domestic public firm producing a homogeneous good. The output of the domestic private firm is denoted by $q_{d,i}, i = 1 \cdots n$, the output of the foreign private firm is denoted by $q_{f,j}, j = 1 \cdots m$, and that of the domestic public firm is denoted by q_0. Total output is $Q = q_0 + \sum_{i=1}^{n} q_{d,i} + \sum_{j=1}^{m} q_{f,j}$. The consumer's utility can be written as $U = G(Q) + X$, where X is the consumption of perfect competitive goods served as a numeraire and $G(Q)$ is the subutility, which is assumed to be strictly concave. The consumer maximizes the utility subject to a budget constraint $M = pQ + X$, where M is income taken as given and p denotes the consumer price of Q. Thus, the inverse demand function is described as $p(Q) = G'(Q)$.

The profit of the domestic private firms, $\pi_{d,i}$, and the foreign firms, $\pi_{f,i}$, can be written as follows:

$$\pi_{d,i} = p(Q)q_{d,i} - c(q_{d,i}), \quad i = 1\cdots n, \tag{5.1}$$

$$\pi_{f,j} = p(Q)q_{f,j} - c(q_{f,j}), \quad j = 1\cdots m, \tag{5.2}$$

where $c(\cdot)$ is the common cost function of the private firms. We assume here that all private firms have the same cost function. The public firm also produces goods with the same cost function, $c(q_0)$. The profit of the public firm, π_0, is denoted by

$$\pi_0 = p(Q)q_0 - c(q_0). \tag{5.3}$$

The social welfare of the domestic country, W, is written as follows:

$$W = G(Q) - \sum_{i=0}^{n} c(q_{d,i}) - p(Q)\sum_{j=1}^{m} q_{f,j}. \tag{5.4}$$

We consider a private Cournot–Nash game among private firms with the given output of the public firm. Each private firm chooses its output to maximize its profit. The first-order conditions are represented by

$$p(Q) + p'(Q)q_{d,i} - c'(q_{d,i}) = 0, \quad i = 1\cdots n, \tag{5.5}$$

$$p(Q) + p'(Q)q_{f,j} - c'(q_{f,j}) = 0, \quad j = 1\cdots m. \tag{5.6}$$

Here, (5.5) and (5.6) mean $MR = MC$. Because all private firms are identical, the output level of each private firm, $q = q_{d,i} = q_{f,j}$, can be obtained as

$$q = q(\tilde{n},\ q_0), \quad \tilde{n} = n + m, \tag{5.7}$$

$$\frac{\partial q}{\partial q_0} = -\frac{1}{\tilde{n} + g} < 0, \quad \frac{\partial q}{\partial \tilde{n}} = -\frac{q}{\tilde{n} + g} < 0, \tag{5.8}$$

where $g = \frac{p' - c''}{p' + p''q}$. We assume g is positive. This condition ensures the stability of the private oligopoly market as a given output of the public firm.[1] In (5.8), $\frac{\partial q}{\partial q_0} < 0$

[1] A similar condition is imposed in Seade (1980).

and $\frac{\partial q}{\partial \widetilde{n}} < 0$ imply that the reaction function of the private firm has a negative slope with respect to the output of the public firm and the number of private firms.

5.2.1 Partial Privatization in a Regulated Market

We now consider the output level of the public firm. If the firm is fully privatized, the public firm maximizes its own profit (5.3). The first-order condition of the firm must be equal to that of the private firm:

$$p + p'q_0 - c'(q_0) = 0. \tag{5.9}$$

If the public firm is fully nationalized, the public firm maximizes domestic social welfare (5.4). In this case, the first-order condition is changed as follows:

$$G' - c'(q_0) - mp'q = p - c'(q_0) - mp'q = 0, \tag{5.10}$$

where $p < c'(q_0)$. As we saw, the output of the public firm under full privatization is the same as that in a pure private oligopoly. In contrast, full nationalization brings about an aggressive output that exceeds the marginal cost pricing level. This implies that the public firm protects the outflow of foreign firms' profits.

To determine whether full privatization or full nationalization maximizes social welfare defined as (5.4), we derive the first-order condition of the public firm's socially optimal level of output as follows:

$$[p - c'(q_0) - mp'q] - p'q(n + \widetilde{n}m)\frac{\partial q}{\partial q_0} = 0, \tag{5.11}$$

where $\frac{\partial q}{\partial q_0} < 0$. The terms in the squared bracket of (5.11), $p - c'(q_0) - mp'q$, must be positive because the latter terms, $-p'q(n + \widetilde{n}m)\frac{\partial q}{\partial q_0}$, are negative. This means that the output level of the public firm is larger than that of the private firms, as the marginal cost is lower than the marginal revenue. However, the optimal output level is not larger than that under full nationalization. This tells us that full privatization or nationalization does not achieve social welfare maximization. Therefore, the government can change the privatization level to bring the public firm's output to the optimum level.

The optimal condition (5.11) must satisfy the following relation:

$$p - c'(q_0) \lesseqgtr 0 \Leftarrow gm - n \gtreqless 0. \qquad (5.12)$$

If the number of foreign firms, m, is large, the public firm produces a higher level of output to reduce the share of the foreign firms. In contrast, if the number of domestic firms, n, is sufficiently larger than the number of foreign firms, the output level of the public firm is less aggressive when its output does not exceed the marginal cost pricing level, $p = c'(q_0)p = c'(q_0)$. The optimal output level is lower than the output in the case of full nationalization.

If the government intervenes in the public firm via a share of ownership, the optimization problem of the public firm becomes

$$q_0^* = \text{argmax}\theta\pi_0 + (1 - \theta)W,$$

where θ represents the share of government ownership in the public firm. If θ is equal to zero (unity), the government fully nationalizes (privatizes) the public firm. When the government can control the output level by setting θ, the first-order condition of public firm is as follows:

$$[p - c'(q_0) - mp'q] + \theta[p'q_0 + mp'q] = 0. \qquad (5.13)$$

The former terms in the squared bracket, $p - c'(q_0) - mp'q$, must be positive, because the latter terms, $\theta[p'q_0 + mp'q]$, are always negative when $0 < \theta \leq 1$. This means that the partially privatized firm produces an intermediate level of output between full privatization and full nationalization. Therefore, the government can achieve the optimal welfare level by setting the privatization level as

$$\theta^* = \frac{(n + \tilde{n}m)}{\left(\frac{q_0}{q} + m\right)(\tilde{n} + g)}. \qquad (5.14)$$

It is obvious that the privatization ratio reaches the same level as in a domestic-mixed oligopoly case if there are no foreign firms. Thus, partial privatization can be interpreted as the government achieving a Stackelberg leader solution to change the ownership share of the public firm.[2]

From the above discussion, we summarize that, first, full nationalization is too aggressive compared with partial privatization and, second, partial privatization is preferable if the government can control the output of the public firm via an ownership share in the public firm. In Chap. 1 on domestic-mixed oligopolies, it was shown that the free-entry equilibrium presents an alternative scenario in the privatization problem in which full nationalization is supported. We now examine a similar situation where firms have free entry into an international mixed oligopoly.

[2] Unfortunately, in the present study, we were unable to derive the relative difference of privatization levels between domestic and international cases.

5.2.2 Partial Privatization in a Free-Entry Equilibrium

To consider the number of firms, including domestic and foreign firms, we impose an additional assumption: domestic firms and foreign firms both have free entry into the market. Under this assumption, there are two cases; that is, $d\widetilde{n} = dn$ or $d\widetilde{n} = dm$ at $\pi = 0$. In both cases, the optimal solution of the public firm's output is as follows:

$$p - c'(q_0) = 0. \tag{5.15}$$

This is a familiar marginal cost pricing setting. That is, a fully privatized firm and a nationalized firm cannot set the same level from (5.9) and (5.10). In this case, the government is only concerned with the level of the consumer surplus because the profit of the foreign firms is always zero at the free-entry equilibrium.

5.2.3 Excess Entry and Privatization

At the free-entry equilibrium, the public firm just concentrates on the output to achieve the marginal cost pricing level. However, does the number of firms exceed the optimal level? Matsumura and Kanda (2005) showed that the number of firms always exceeds the social optimal level at the free-entry equilibrium in a closed economy. This is similar to the excess entry theorem of Mankiw and Winston (1986) and Suzumura and Kiyono (1987).

In our model, if only the new entry of the domestic firm is allowed, there is the possibility of excess entry of private firms. There are two procedures for examining the excess entry problem. One directly evaluates social welfare and the number of firms, as used by Matsumura and Kanda (2005). The other uses an entry tax on private firms and evaluates the sign of the optimal entry tax. If the entry tax does not affect social welfare, the number of private firms does not exceed the optimal level. However, an excessive entry of firms occurs when the optimal entry tax becomes positive. We use the second procedure to evaluate the excessive entry problem.

We assume that the government imposes an entry tax, such as a lump sum tax, T, on existing firms in the market. In this case, the first-order condition of profit maximization does not change by the introduction of the entry tax. However, the profit level becomes lower, which leads to a decrease in the number of private firms. We describe these properties as follows:

$$\pi = pq - c(q) - T = 0. \tag{5.16}$$

From (5.16), we can solve the number of firms $\widetilde{n}^* = \widetilde{n}^*(q_0, T)$. Comparative statistics gives us

$$\frac{\partial \widetilde{n}^*}{\partial q_0} = -\frac{1}{q},$$

$$\frac{\partial \widetilde{n}^*}{\partial T} = \frac{\widetilde{n}^* + g}{p' q^2 (1+g)} < 0,$$

$$\frac{dq}{dT} = -\frac{1}{p' q (1+g)} > 0\ ,$$

and

$$\frac{dQ}{dT} = \frac{k}{p' q (1+g)} < 0.$$

5.2.4 Optimal Entry Tax

We now turn to the optimal output of the public firm and the optimal entry tax. The social welfare function is equal to gross benefit minus total production cost:

$$\begin{aligned} W &= G(Q) - c(q_0) - nc(q_d) - mpq_f + mT \\ &= G(Q) - c(q_0) - \widetilde{n}^* c(q). \end{aligned} \tag{5.17}$$

The optimal conditions of output and entry tax are written as follows:

$$\frac{\partial W}{\partial q_0} = p - \frac{T}{q} - c'(q_0) = 0, \tag{5.18}$$

$$\frac{\partial W}{\partial T} = T\frac{\partial \widetilde{n}^*}{\partial T} - \widetilde{n}^* p' q \frac{\partial q}{\partial T} = 0, \tag{5.19}$$

$$\theta^* = \frac{\widetilde{n}^* m + \widetilde{n}^* + gm}{(\widetilde{n}^* + g)\left(m + \frac{q_0}{q}\right)} \in (0,1), \text{ and } T^* = -\frac{\widetilde{n}^* p' q^2}{\widetilde{n}^* + g} > 0. \tag{5.20}$$

The first-order condition of the public firm, (5.18), is modified from the marginal cost pricing level. As a result, the optimal entry tax becomes positive and reduces the number of private firms. Then the free-entry equilibrium without entry tax has an inefficient number of private firms. Under a positive entry tax, the public firm is privatized to a higher level to reduce its output rather than the marginal cost pricing level, as each private firm produces a higher output because of the entry tax. A fully nationalized firm must satisfy the output level of the marginal cost pricing level. However, (5.18) requires less output than the marginal cost pricing level. Thus, the government should privatize rather than fully nationalize. When the government has two policy instruments, θ and T, the first-best solution can be achieved by controlling the number of firms and the output of the public firm.

5.3 Strategic Trade Policy and Privatization

In the previous section, foreign firms do not receive differential treatment from the domestic government. Thus, the foreign firm is only affected by the privatization policy. In reality, most foreign firms face differential treatment in the market. For example, the foreign firm exports their goods to the home country, and the domestic government imposes an import tariff on the goods of the foreign firms. Within the strategic trade policy problem, governments typically use a tariff to protect their own domestic market. A number of studies examine strategic trade policies in an international mixed oligopoly.

Chao and Yu (2006) showed the relationship between tariffs and the degree of privatization by changing the number of foreign firms. They demonstrated that the optimal tariff rate is lower at a low level of privatization and a large number of foreign firms. Lee et al. (2013) showed the optimal combination of tariff rate and degree of privatization. They illustrated that the optimal degree of privatization has a positive relation with the tariff rate and that a tariff rate reduction results in a lower level of privatization. Wang et al. (2014) considered a strategic trade policy in a free-entry equilibrium. They found that the privatization level and tariff rate at a free-entry equilibrium are lower than a short-run equilibrium.

The above studies highlighted the positive relation between the optimal tariff and degree of privatization. Thus, if the government uses a tariff rate as a protection against import levels, the tariff causes a weak incentive to nationalize public firms to intervene in the output of foreign firms. In this section, we describe such a relation between privatization and a strategic trade policy by referring to previous research.

We can then determine the effect of the import tariff on private firms. For simplicity, we assume a linear demand function, $p = a - bQ$, and a quadratic cost function, $c(q) = f + \frac{kq^2}{2}$ where a, b,f, and $k > 0$. Because the tariff, $t > 0$, is only imposed on the output of foreign firms, the profit function and the first-order condition of the foreign firm change as follows:[3]

[3] The output of the private firms is obtained as follows: $\begin{bmatrix} b(n+1)+k & bm \\ bn & b(m+1)+k \end{bmatrix} \begin{bmatrix} q_d \\ q_f \end{bmatrix} = \begin{bmatrix} a - bq_0 \\ a - bq_0 - t \end{bmatrix}$, where $q_d = \Delta^{-1}[(a - bq_0)(b+k) + tbm]$, $q_f = \Delta^{-1}\left[(a - bq_0)(b+k) - t[bn + (b+k)]\right]$, $\Delta = (b+k)[b(n+m) + (b+k)]$ and $q_d - q_f = t[b(n+m) + (b+k)] = \frac{t\Delta}{(b+k)}$

$$\pi_f = pq_f - f - \frac{kq_f^2}{2} - tq_f,$$
$$p + p'q_f - kq_f - t = 0.$$

When the public firm is fully privatized, the profit function and the first-order condition are the same as for the domestic private firms. When the public firm is fully or partially nationalized, its objective function must change. Social welfare now includes tariff revenue, such as

$$\begin{aligned} W(n,\ m,\ t,\ q_0) &= G(Q) - pQ + \pi_0 + n\pi_d + tmq_f \\ &= G(Q) - c(q_0) - nc(q_d) - mpq_f + tmq_f. \end{aligned}$$

Differentiating social welfare of the domestic country by q_0,

$$\frac{\partial W}{\partial q_0} = \left[p - kq_0 + mbq_f\right] + \left[n(p - kq_d) + mbq_f\right]\frac{\partial q_d}{\partial q_0} + mt\frac{\partial q_f}{\partial q_0} = 0. \qquad (5.21)$$

If the tariff rate is zero, as we have already mentioned in the previous section, the optimal degree of nationalization is less than one. Therefore, imposing a tariff rate reduces (increases) the degree of nationalization (privatization). In this first-order condition (5.21), as the third term, $mt\frac{\partial q_f}{\partial q_0}$, is negative with a positive tariff rate, the output level of the public firm is lower than if the tariff is not imposed. Therefore, the import tariff stimulates privatization. A detailed discussion of optimal tariff and domestic taxes is given in Chap. 6.

5.4 Harmonization Problem

We now turn to the harmonization policy between the two countries. From the above discussion, each country sets underprivatization within a strategic privatization policy. Han and Ogawa (2008) considered this problem with a symmetric integrated market. They found that coordinated privatization brings Pareto improvement to both countries. As mentioned, domestic and foreign governments always try to prevent the erosion of profits from their own countries. Therefore, both governments choose underprivatization. We will next investigate in which direction a harmonization policy improves the welfare of both countries. Thus, we compare two cases: a segmented market and an integrated market. In this section, we show that in the symmetric case, the welfare of both countries always improves if both countries coordinate to achieve a higher level of privatization. Furthermore, in the asymmetric case, the welfare of both countries improves if each country seeks contrasting privatization policies; that is, one country adopts a higher level of privatization and the other seeks a higher level of nationalization.

5.4.1 Segmented Market

We first consider the coordination problem with a segmented market in which each country has a separate market and both countries' firms compete within the market. For simplicity, in each market there is a domestic firm, a foreign firm, and a public firm. Both countries' social welfare functions are as follows:

$$W = W\left(q_0,\, q_0^*\right) = G(Q) - pQ + \pi_0 + \pi_d + \pi_d^*,$$
$$W^* = W^*\left(q_0,\, q_0^*\right) = G\left(Q^*\right) - p^*Q^* + \pi_0^* + \pi_f + \pi_f^*,$$

where * denotes the foreign country's variables. The total derivatives of both countries' welfare with the public firm outputs are as follows:

$$dW = A_1 dq_0 + A_2 dq_0^*,$$
$$dW^* = A_1^* dq_0 + A_2^* dq_0^*,$$

where

$$A_1 = \left[p - c'(q_d) - p'q_f\right]\frac{\partial q_h}{\partial q_0} - p'q_f\frac{\partial q_f}{\partial q_0} + p - c'(q_0) - p'q_f,$$
$$A_2 = \left[p^* - c'\left(q_d^*\right)\right]\frac{\partial q_d^*}{\partial q_0^*} + p^{*\prime}\left[1 + \frac{\partial q_d^*}{\partial q_0^*} + \frac{\partial q_f^*}{\partial q_0^*}\right] < 0,$$
$$A_1^* = \left[p - c'\left(q_f\right)\right]\frac{\partial q_f}{\partial q_0} + p'\left[1 + \frac{\partial q_d}{\partial q_0} + \frac{\partial q_f}{\partial q_0}\right] < 0,$$

and

$$A_2^* = \left[p^* - c'\left(q_f^*\right) - p^{*\prime}q_d^*\right]\frac{\partial q_f^*}{\partial q_0} - p^{*\prime}q_d^*\frac{\partial q_d^*}{\partial q_0^*} + p^* - c'\left(q_0^*\right) - p^{*\prime}q_d^*.$$

The maximization problem of global welfare is

$$\max_{q_0,\, q_0^*} W + W^*.$$

Thus, the first-order conditions are given by

$$A_1 + A_1^* = 0, \quad A_2 + A_2^* = 0.$$

Instead of global welfare maximization, if both governments act as Nash players, each government sets the output of the public firm as $A_1 = A_2^* = 0$. This Nash equilibrium shows that the output level of the public firm is lower than the optimal global level. This means that both countries adopt underprivatization rather than global optimization.

Regarding the Nash policy game, it is clear that cooperative privatization in both countries improves global welfare. At the policy equilibrium, $A_1 = A_2^* = 0$, $A_2 = \frac{dW}{dq_0^*} < 0$, and $A_1^* = \frac{dW^*}{dq_0} < 0$. This means that the non-corporative decision of privatization results in a higher level of nationalization because of attempts to prevent the erosion of foreign firms' profits. If both countries have nationalized firms, a different cooperative device or negotiation may result in a coordinated privatization policy.

In an asymmetric case, the resulting coordination of privatization is the same as in the symmetric case because $A_2 < 0$ and $A_1^* < 0$ must be satisfied in the asymmetric case. Regarding the Nash equilibrium, enhancing privatization in both countries always improves the welfare of both countries.

5.4.2 *Integrated Market*

We have focused on segmented markets, but a similar policy problem also exists in an integrated market where both countries' firms compete in the same market. To consider the privatization problem in an integrated market, we use the following model (e.g., Keen and Lahiri 1993). Domestic and foreign market demand and supply are expressed as follows:

$$D = a - bp,\ D^* = a^* - b^* p^*,\ Q = q_{0d} + q_d,\ Q^* = q_{0f} + q_f.$$

The market-clearing conditions of the integrated market are

$$D + D^* = Q + Q^*,\ \bar{p} = p = p^*.$$

Both countries face the world market price, $\bar{p}$, and the inverse demand function becomes

$$\bar{p} = \alpha - \beta\left(Q + Q^*\right),\quad \alpha = \frac{a + a^*}{b + b^*},\quad \beta = \frac{1}{b + b^*}.$$

For simplicity, we assume a quadratic cost function, $c(q) = f + \frac{kq_i^2}{2}$, as before. The profit function of each firm becomes

$$\pi_i = \bar{p} q_i - f - \frac{kq_i^2}{2}.$$

The social welfare of both countries is defined as

$$W = G(\overline{p}) - \overline{p}D + \pi_0 + \pi,$$
$$W^* = G^*(\overline{p}) - \overline{p}D^* + \pi_0^* + \pi^*.$$

The changes of social welfare in each country are as follows:

$$dW = B_1 dq_0 + B_2 dq_0^*,$$
$$dW^* = B_1^* dq_0 + B_2^* dq_0^*,$$

where

$$B_1 = \frac{b}{1+\gamma}\left[(D-Q) - \frac{bq}{b+k}\right] + (\overline{p} - kq_0),$$
$$B_2 = \frac{b}{1+\gamma}\left[(D-Q) - \frac{bq}{b+k}\right],$$
$$B_1^* = \frac{b}{1+\gamma}\left[\left(D^* - Q^*\right) - \frac{bq^*}{b+k}\right]$$

and

$$B_2^* = \tfrac{b}{1+\gamma}\left[\left(D^* - Q^*\right) - \tfrac{bq^*}{b+k}\right] + \left(\overline{p} - kq_0^*\right).$$

The maximization of global welfare is

$$\max_{q_0, q_0^*} W + W^* .$$

Thus, the first-order conditions are given by

$$B_1 + B_1^* = 0,\ B_2 + B_2^* = 0.$$

Instead of a segmented market, the privatization level is determined by the total effect of the welfare change of both countries in the integrated market. There is no reason to set the same privatization level in both countries. The reason for this is that if one country has a trade surplus and the other has a trade deficit, then the privatization level depends on the trade position. For example, if one country has a large trade surplus, it has to seek a high level of nationalization. In contrast, the other country faces a large trade deficit, which means that the public firm is subject to a higher level of privatization. However, in a symmetric case, $D - Q = D^* - Q^* = 0$, the result is the same as that in a segmented market.

We now consider the case where both countries act as Nash players. Each country sets the output of the public firm as $B_1 = B_2^* = 0$. In this case, global optimization is not possible. However, this Nash equilibrium shows that the output level of the public firm is lower or higher than the global optimization level, which depends on the trade position of each country.

Regarding the Nash policy game, to improve both countries' welfare, the country with the higher trade surplus must increase the privatization level, and the country with the trade deficit must increase the nationalization level. This is in contrast to a segmented market where privatization always improves the social welfare of both countries. Underprivatization in both countries only occurs in a symmetric case where there is no trade surplus and deficit. Therefore, coordinated privatization holds only in the symmetric case.

In the integrated market, various coordinated privatization policies could apply, and these depend on the trade position in both countries. Thus, coordinated privatization does not support global welfare. In this case, the country with the trade surplus (deficit) may become better (worse) off. Furthermore, it is difficult to adopt a simultaneous privatization policy because each country has a different payoff via the privatization policy. If both countries implement a privatization policy, we need to introduce a transfer from the country with the surplus to the country with the deficit to weaken the trade effect. However, we are unable to analyze such a complicated situation in our model.

5.5 Conclusion

In this chapter, we considered several aspects of a mixed oligopoly in an international setting. First, we illustrated the basic properties of an international mixed oligopoly. We also considered the differences between a short-run and long-run equilibrium in an international mixed oligopoly. In the long-run sense, even though the output level of the pubic firm satisfies the optimal level, the number of private firms still exceeds the optimal number. Thus, an entry tax is a powerful instrument to control the number of private firms. We also presented the relation between the strategic trade policy (i.e., import tariff) and the degree of privatization. Chapter 6 will analyze this issue in greater detail.

We also explained a corporative privatization policy in a two-country mixed oligopoly model. We found that corporative privatization policy only holds in a limited number of situations, including a symmetric case.

Of course, it is not enough to simply provide a comprehensive analysis of an international mixed oligopoly, even though we did consider several aspects of international mixed oligopolies. For example, we were unable to investigate the effects of a domestic tax/subsidy policy on privatization. It is recognized some tax instruments do not affect the output of fully nationalized firm; therefore, the government cannot control the output of the public firm. However, if the tax system adjusts the level of total output, the level of privatization may not matter. We need to investigate which tax instrument is more effective and how the privatization level is changed.

The international oligopoly model could also be extended, especially the two-country model, to consider several policy instruments. Our analysis only focused on the degree of privatization and identified those factors that result in overprivatization or underprivatization. If we introduce other policy instrument, it is unclear whether a privatization policy would be effective. These issues will be addressed in the following chapters, which provide a further analysis of privatization policies and other policy instruments.

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Part II
Policies

Chapter 6
Optimal Partial Privatization in an International Mixed Oligopoly Under Various Tax Principles

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Abstract This chapter considers optimal privatization policy in an international mixed oligopoly. Allowing for partial privatization and cost asymmetry, we analyze the optimal policies under various tax regimes: arbitrary taxation, origin principle, destination principle, import tariffs, and a combination of tax and import tariffs. Our main results are as follows. First, when the government can arbitrarily levy taxes on a public firm's output, maximum welfare is independent of the degree of privatization as long as the public firm is at least partially privatized. Second, under tax schemes that restrict freedom of taxation, an optimal privatization policy depends on tax regimes and cost asymmetry. Third, the elimination of import tariffs and the privatization of public firms improve both domestic welfare and a foreign competitor's profit if a production subsidy is introduced in exchange for tariff elimination. Our results suggest that fiscal incentives such as tax and subsidies are superior in maximizing welfare when compared with managerial incentives such as public ownership.

6.1 Introduction

Mixed markets are widely observed in both developed and developing countries.[1] For example, postal services, financial institutions, and public utilities are often operated by government, while private firms produce similar goods and services in

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[1] Kowalski et al. (2013) calculated the weighted share of public firms for sales, profits, assets, and market values among a country's top ten companies in 38 developed or developing countries. They pointed out that although the public enterprise sector in the OECD area has become significantly

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the market. In developing countries especially, public firms often compete with foreign rivals as well as domestic private firms.

In a market with international trade, the government has an incentive to use public firms as an instrument of strategic trade policy as well as other policy devices such as tariffs and subsidies. In contrast, the primary role of public firms in a closed economy is to mitigate allocative inefficiency caused by imperfect competition. Fjell and Pal (1996) argued that in an international mixed market, the outputs of public firms are at levels where their marginal costs exceed price. That is, the behavior of public firms has a similar effect to the rent shifting discussed in Brander and Spencer (1984).

Recently, arguments concerning free trade agreements and Economic Partnership Agreements (EPA) have focused on the preferential treatment given to public firms.[2] The OECD (2012) stresses fair treatment based on the concept of competitive neutrality, which implies that both public and private firms compete under the same external environment. Tax neutrality, which requires the equal treatment of public and private firms in taxation, is essential for competitive neutrality. Therefore, it is worth considering the linkage between fiscal incentives (e.g., taxation) and managerial incentives (e.g., state ownership) in an international setting.

In this chapter, we focus on the optimal policy in the private ownership of the public firm under various feasibility sets of taxation. Previous analyses mainly considered production subsidies and import tariffs as policy instruments (e.g., Pal and White 1998; Chao and Yu 2006; Yu and Lee 2011; Han 2012; Lee et al. 2013). For example, Pal and White (1998) argued that the privatization of a public firm improves national welfare when the government can set production subsidies at an optimal level. As pointed out by the OECD (2012), public firms often receive preferential treatment in taxation relative to private firms. To consider such a situation, we investigate the optimal policy where the government can arbitrarily levy a commodity tax. We also consider the optimal policy under the destination principle. Production subsidies can be regarded as an (negative) origin-based tax. Previous studies on tax coordination have clearly shown that welfare effects of policy reforms such as tax harmonization depend on the tax principle (e.g., Keen and Lahiri 1998; Keen et al. 2002).[3]

Considering optimal ownership for public firms, we allow for the partial privatization of a public firm and for cost asymmetry between a public firm and private firms. Matsumura (1998) argued that partial privatization is optimal in a mixed duopoly consisting of domestic firms. In an international setting, Yu and Lee (2011)

smaller, among emerging countries including Brazil, Russia, India, Indonesia, and China, state presence in the economy remains significant.

[2] According to Solano and Sennekamp (2006), provisions for state enterprises and state monopolies that contain specific regulations for such firms were found in 55 of the 86 regional trade agreements they investigated.

[3] Mujumdar and Pal (1998) considered the effect of indirect taxation on welfare and government revenue in a mixed oligopoly. They argued that the results substantially differ from that of a pure oligopoly.

showed that full privatization is optimal when a production subsidy is employed as a strategic variable and that partial privatization is optimal under an import tariff regime. In their analysis, production technologies were identical among existing firms. Chang (2005) and Wang et al. (2009) argued that the optimal policy depends on the degree of relative inefficiency of the public firm. However, their analysis focused on an industry consisting of a domestic public firm and a foreign private firm. We consider a mixed market consisting of a domestic private firm, a foreign private firm, and a domestic public firm, following Yu and Lee (2011) and Han (2012).

In the later sections, the following results will be shown. First, an optimal privatization policy heavily depends on both tax principles and cost asymmetries. In general, greater flexibility in taxation reduces the significance of partially privatizing a public firm in industrial policy. Second, national welfare is independent from the degree of privatization as long as the public firm is not wholly nationalized and can be taxed freely. Third, full nationalization is welfare inferior to partial or full privatization (regardless of its degree of privatization) when the government can freely tax the public firm. The last two findings are valid in the model consisting of general demand and cost functions that satisfies appropriate assumptions. Finally, the transition from an import tariff regime to the origin principle not only increases domestic welfare but also the profit of the foreign private firm.

The chapter is organized as follows. In Sect. 6.2 we show the analytical framework and provide preliminary results. In Sect. 6.3 we analyze the effects of privatization on welfare under commodity taxation. In Sect. 6.4 we compare the level of welfare among tax regimes discussed in Sect. 6.3. The main findings of this chapter are summarized in Sect. 6.5.

6.2 The Model and Preliminary Results

The model used here is provided by Yu and Lee (2011), which is a simplified version of Fjell and Pal (1996) and Pal and White (1998). While they have assumed linear demand and quadratic cost functions, we begin the analysis by supposing the general demand and cost functions.

Consider an industry consisting of one domestic private firm denoted by d, one foreign private firm denoted by f, and one domestic public firm denoted by 0, producing homogeneous goods. The output of firm i is denoted by q_i, for $i = d, f, 0$. Total output is $Q = q_d + q_f + q_0$. Domestic demand arises from a utility function of the consumer, represented by $U = u(Q) + X$, where X is the consumption of perfect competitive goods served as a numeraire. We assume that $u(Q)$ is strictly concave. The consumer maximizes utility subject to a budget constraint $M = pQ + X$, where M is income and p denotes a consumer price of Q. Thus, the inverse demand function is described as $p(Q) = u'(Q)$.

The profits of firms can be written as follows:

$$\pi^i = p(Q)\,q_i - C_i - t_i q_i, \quad \text{for} \quad i = d, f, 0, \tag{6.1}$$

where $C_i = c_i(q)$ is the cost function of firm i. For analytical simplicity, we assume that $c_i(q)$ is strictly convex. A commodity tax t_i is imposed on the output produced by firm i.

Assuming that tax revenue is returned to the consumer via a lump sum, we can write domestic income as $M = r + \pi^d + \pi^0 + t_d q_d + t_f q_f + t_0 q_0$, where r is fixed factor income. Domestic welfare is given by

$$W = u(Q) - c_d(q_d) - c_0(q_0) - \left[p(Q) - t_f\right] q_f, \tag{6.2}$$

where we omit fixed factor income.

In addition to tax rates, the government can decide the degree of privatization for the public firm. Let $\theta \in [0, 1]$ be the shareholding ratio of the public firm or the privatization ratio. That is, $\theta = 1(0)$ implies that the public firm is fully privatized (nationalized). If $\theta \in (0, 1)$, the private sector partly owns the public firm. That is, this represents the degree of privatization.

The objective function of the public firm can be written as the weighted sum of welfare and profit:

$$V = \theta\pi^0 + (1 - \theta)W. \tag{6.3}$$

In what follows, we consider a two-stage game. In the first stage, the government chooses the tax rates and the privatization ratio to maximize domestic welfare. In the second stage, taking the tax rates and privatization ratio as given, each private firm chooses its output to maximize profit, and the public firm chooses its output to maximize its objective function.

Consider the second stage of the game. Each firm maximizes its objective function under a Cournot conjecture. The first-order conditions are given by

$$p + p' q_i - c_i' - t_i = 0, \quad \text{for} \quad i = d, f, \tag{6.4}$$

$$p + p' q_0 z - c_0' - \theta t_0 = 0, \tag{6.5}$$

where $z \equiv 1 - (1 - \theta)\left(q_0 + q_f\right)/q_0$.[4]

From (6.4) and (6.5), it should be noted that if $\theta = 0$, the tax levied on the public firm has no effect on the decision of all firms. This means that the government cannot use t_0 as a strategic device in the presence of a wholly nationalized firm.

[4] In what follows, we do not exclude the possibility that the profit of the public firm becomes negative at the equilibrium. Indeed, in the model consisting of linear demand and quadratic cost functions discussed in Sect. 3, a public firm that emphasizes welfare may fall into deficit.

Lemma 6.1 *If the public firm is fully nationalized, the commodity tax imposed on the public firm's output has no effect on welfare.*

From (6.4) and (6.5), we can define the optimal response functions under Nash conjectures. The optimal response function of the domestic private firm can be written as $q_d = \phi^d(q_f, q_0; t_d)$, and its slopes are represented as $\partial\phi^d/\partial q_f \equiv \phi_f^d = -\pi_{df}^d/\pi_{dd}^d = -\pi_{d0}^d/\pi_{dd}^d \equiv \phi_0^d$, where $\pi_{dd}^d = 2p' + p''q_d - c_d'' < 0$ follows from the second-order condition and $\pi_{df}^d = \pi_{d0}^d = p' + p''q_d$. For the foreign private firm, we can similarly define its optimal response function such that $q_f = \phi^f(q_d, q_0; t_f)$ with slopes of $\phi_d^f = \phi_0^f$. We assume diminishing marginal revenue for an increase in total outputs: $p' + p''q_i < 0$ for $i = h, f$, known as a Hahn condition (e.g., Hahn 1962; Dixit 1984). Under this assumption, the optimal response functions of private firms have negative slopes. For the public firm, the optimal response function, $q_0 = \phi^0(q_d, q_f; t_0, \theta)$, has a different slope for a change in each firm's output. That is, $\phi_d^0 = -V_{0d}/V_{00}$ and $\phi_f^0 = -V_{0f}/V_{00}$ where $V_{00} = (1+\theta)p' + zp''q_0 - c_0'' < 0$ from the second-order condition, $V_{0d} = p' + zp''q_0$, and $V_{0f} = \theta p' + zp''q_0$. As shown in Delbono and Scarpa (1995), ϕ^0 may be upward sloping.

By totally differentiating (6.4) and (6.5), we obtain the following system:

$$\mathbf{D}\tilde{\mathbf{q}} = \mathbf{I}_\theta \tilde{\mathbf{t}} + \mathbf{b}d\theta, \tag{6.6}$$

where

$$\mathbf{D} \equiv \begin{bmatrix} \pi_{dd}^d & \pi_{df}^d & \pi_{d0}^d \\ \pi_{fd}^f & \pi_{ff}^f & \pi_{f0}^f \\ V_{0d} & V_{0f} & V_{00} \end{bmatrix}, \tilde{\mathbf{q}} \equiv \begin{bmatrix} dq_d \\ dq_f \\ dq_0 \end{bmatrix}, \mathbf{I}_\theta \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \theta \end{bmatrix},$$

$$\tilde{\mathbf{t}} \equiv \begin{bmatrix} dt_d \\ dt_f \\ dt_0 \end{bmatrix}, \text{ and } \mathbf{b} \equiv \begin{bmatrix} 0 \\ 0 \\ -p'(q_f + q_0) + t_0 \end{bmatrix}.$$

In what follows, we assume that the slopes of the optimal response functions are not too steep so that the stability conditions are satisfied[5]:

[5] See Appendix A.6.1. Together with the convexity of the cost function and diminishing marginal revenue of the private firms, $\left|\phi_f^d \phi_d^f\right| < 1$ is met. However, we cannot exclude the possibility of $|\Delta_2| > 1$ without any restriction of ϕ_i^0.

$$|\Delta_1| \equiv \left|\phi_f^d \phi_d^f\right| < 1, \text{ and } |\Delta_2| \equiv \left| \frac{\phi_d^0\left(\phi_0^d + \phi_f^d \phi_0^f\right) + \phi_f^0\left(\phi_0^f + \phi_d^f \phi_0^d\right)}{1 - \phi_f^d \phi_d^f} \right| < 1.$$

Under this assumption, $\mathbf{D}$ is invertible, and its determinant is negative: $\det\mathbf{D} = \pi_{dd}^d \pi_{ff}^f V_{00}(1 - \Delta_1)(1 - \Delta_2) < 0$. Thus, the output of each firm can be written as a function of the tax vector, $\mathbf{t} = \left(t_d, t_f, t_0\right)$, and the privatization ratio, θ: $q_i = q_i(\mathbf{t}, \theta)$ for $i = d, f, 0$. In addition, welfare is also given as a function of $\mathbf{t}$ and θ as $W = W(\mathbf{t}, \theta)$.

Comparative statics yield the following system:

$$\widetilde{\mathbf{q}} = \mathbf{D}^{-1}\left(\mathbf{I}_\theta \widetilde{\mathbf{t}} + \mathbf{b} d\theta\right), \tag{6.7}$$

where

$$\mathbf{D}^{-1} = \frac{1}{(1 - \Delta_1)(1 - \Delta_2)} \begin{bmatrix} \frac{1 - \phi_0^f \phi_f^0}{\pi_{dd}^d} & \frac{\left(1 + \phi_0^f\right)\phi_f^d}{\pi_{ff}^f} & \frac{\left(1 + \phi_0^f\right)\phi_0^d}{V_{00}} \\ \frac{\left(1 + \phi_0^d\right)\phi_d^f}{\pi_{dd}^d} & \frac{1 - \phi_0^d \phi_d^0}{\pi_{ff}^f} & \frac{\left(1 + \phi_0^d\right)\phi_0^f}{V_{00}} \\ \frac{\phi_d^0 + \phi_d^f \phi_f^0}{\pi_{dd}^d} & \frac{\phi_f^0 + \phi_f^d \phi_d^0}{\pi_{ff}^f} & \frac{1 - \phi_f^d \phi_d^f}{V_{00}} \end{bmatrix}.$$

We turn to the first stage of the game. First, we consider the optimal privatization ratio without tax. That is, the feasibility set for the policy instruments can be written as $\Gamma_{-t} = \{(\mathbf{t}, \theta) : \mathbf{t} = \mathbf{0}, \ \theta \in [0, 1]\}$. In such a situation, it is plausible that partial privatization is optimal to maximize welfare. Indeed, we have the following result, as shown in Matsumura (1998), for a closed market:

Lemma 6.2 *Suppose that all outputs are untaxed and that the output of each private firm is positive at the equilibrium. Partial privatization is optimal if either of the following conditions is met*:

(i) *Production technologies are identical among all firms.*
(ii) *Production technologies are identical among private firms and the demand function takes a linear form.*

Proof See Appendix A.6.2.

We now turn to optimal taxes under a given privatization ratio. Suppose that the government can arbitrarily set all taxes to maximize welfare, which is referred to as *Regime A*. The feasibility set can be written as $\Gamma_A = \{(\mathbf{t}, \theta) : \theta \in [0, 1]\}$. We also define the feasibility set in which full nationalization is excluded as $\Gamma_A^P = \{(\mathbf{t}, \theta)$

$: \theta \in (0, 1]\}$; this will be discussed further below. Optimal taxes are obtained as the solution of the following first-order condition:

$$\mathbf{w}_t = \mathbf{w}_q \mathbf{D}^{-1} \mathbf{I}_\theta + \mathbf{w}_f = \mathbf{0}, \tag{6.8}$$

where $\mathbf{w}_t \equiv [\partial W/\partial t_d,\ \partial W/\partial t_f, \partial W/\partial t_0]$, $\mathbf{w}_q \equiv [p - c_d' - p' q_f,\ \ -p' q_f + t_f,$ $p - c_0' - p' q_f]$, and $\mathbf{w}_f \equiv [0,\ q_f, 0]$. Decomposing $\mathbf{D}^{-1}$ into the column vectors, for example, $\mathbf{D}^{-1} = [\mathbf{d}_1,\ \mathbf{d}_2,\ \mathbf{d}_3]$, we have the first-order conditions as $\mathbf{w}_q \mathbf{d}_1 = 0$, $\mathbf{w}_q \mathbf{d}_2 + q_f = 0$, and $\theta \mathbf{w}_q \mathbf{d}_3 = 0$. Solving the first-order conditions, we obtain an implicit form of the optimal tax vector $\mathbf{t}^A = \left(t_d^A, t_f^A, t_0^A\right)$ as follows:

$$t_d^A = p' q_d - p'' q_f^2, \tag{6.9}$$

$$t_f^A = -q_f \left(p' - p'' q_f - c_f''\right), \tag{6.10}$$

$$t_0^A = \frac{1}{\theta}\left(z p' q_0 - p'' q_f^2\right), \tag{6.11}$$

for $\theta \neq 0$.[6] From (6.9) and (6.10), if $|p''|$ is sufficiently small, then the commodity tax on the domestic goods is negative and that on import goods is positive. In what follows, we assume that the optimal tax vector is uniquely determined: $t_i = t_i(\theta)$, for $\theta \in [0, 1]$ and $i = d, f$, and $t_0 = t_0(\theta)$ for $\theta \in (0, 1]$.

We now consider the optimal privatization ratio. Using the envelope theorem, the effect of the change in the privatization ratio on welfare can be represented as follows:

$$\frac{dW}{d\theta} = \mathbf{w}_q \mathbf{D}^{-1} \mathbf{b} = \left[-p'\left(q_f + q_0\right) + t_0\right] \times \mathbf{w}_q \mathbf{d}_3. \tag{6.12}$$

From (6.12) we have the following result:

Proposition 6.1 *Suppose that the government can arbitrarily control t_d, t_f, and t_0. The maximum welfare is independent of the privatization ratio as long as the public firm is at least partially privatized in the sense of $\theta \in (0, 1]$.*

Proof From (6.8), if the tax on the public firm's output is optimally set, $\theta \mathbf{w}_q \mathbf{d}_3 = 0$. Thus, $dW/d\theta = 0$holds as long as $\theta \neq 0$.

Proposition 6.1 states that the privatization ratio is irrelevant to welfare as long as the government can strategically use the tax levied on the public firm. This result

[6] If $\theta = 0$, optimal taxes are implicitly given as $t_d^A = p'\left(q_d + q_f\right) + c_0'' q_f \left(\pi_{df}^f / V_{00}\right)$ and $t_f^A = -\left(p' - c_f''\right) q_f + p'' q_f^2 \left(\pi_{df}^f - V_{00}\right) / V_{00}$.

is valid if t_d and/or t_f are restricted by certain rates including zero. Let $\Gamma_j^P = \{(\mathbf{t}, \theta) : t_j = \bar{t}_j,\ \theta \in (0, 1]\}$ for $j = d, f$ and $\Gamma_0^P = \{(\mathbf{t}, \theta) : t_d = \bar{t}_d, t_f = \bar{t}_f,\ \theta \in (0, 1]\}$ be the feasible sets of the government under restricted taxation. Let $\widetilde{W}_j^P \equiv \max_{(\mathbf{t},\theta)\in\Gamma_j^P} W(\mathbf{t}, \theta)$ for $j = d, f, 0$ be the maximum welfare over the feasibility set Γ_j^P. We obtain the following results:

Corollary 6.1 Let $\hat{\Gamma}_j^P$ be the feasibility set that limits θ in Γ_j^P to $\widehat{\theta}$. For any $\hat{\theta} \in (0, 1]$, $\widetilde{W}_j^P = \hat{W}_j^P$ holds, where $\hat{W}_j^P \equiv \max_{(\mathbf{t},\hat{\theta})\in\Gamma_j^P} W(\mathbf{t}, \hat{\theta})$.

Proof Even if taxes other than t_0 are restricted, $\mathbf{w}_q\mathbf{d}_3 = 0$ is the first-order condition for t_0. Thus, the same argument as Proposition 6.1 can be applied.

The intuitions of Proposition 6.1 and its corollary are simple. Under the feasibility sets of Γ_A and Γ_j^P, the government has two independent policy instruments, namely, tax and ownership, to control the output of the public firm. Thus, for a given privatization ratio, the government can find a tax rate that makes the public firm produce a desirable quantity.[7] If the public firm is wholly nationalized, the tax on the public firm's output has no effect on welfare: the government loses two policy devices to directly control the public firm in one stroke.

Let $\mathbf{t}^{\mathrm{N}} = \left(t_d^N, t_f^N\right)$ be the optimal commodity tax under $\theta = 0$. Let q_i^N be the output of firm $i (= d, f, 0)$ and Q^N be the total output under $\mathbf{t}^N$ and $\theta = 0$. The welfare consequence of partial privatization can be summarized by the following result:

Proposition 6.2 Suppose that $q_f > 0$ holds for all possible equilibria. Full nationalization of the public firm is welfare inferior to partial and full privatization.

Proof Suppose that the public firm is fully privatized and that the government imposes commodity taxes on the domestic and foreign private firms at the rate of $\mathbf{t}^N$. In addition, the government levies a tax on the public firm's output at the rate of $\hat{t}_0 = p'(Q^N)\left[q_0^N + q_f^N\right]$. In this situation, the first-order condition of the privatized firm for profit maximization becomes the same as that of the public firm before privatization. The output of each firm does not change before and after privatization; thus, $W(\mathbf{t}^N, \hat{t}_0, 1) = W(\mathbf{t}^N, 0)$ holds. Because $(\mathbf{t}^N, \hat{t}_0, 1) \in \Gamma_A^P$, we have $\max_{(\mathbf{t},\theta)\in\Gamma_A^P} W(\mathbf{t}, \theta) \geq W(\mathbf{t}^N, \hat{t}_0, 1) = W(\mathbf{t}^N, 0)$. Furthermore, it is confirmed that for $\theta = 1$, $\mathbf{w}_t \neq 0$ holds at $\mathbf{t} = (\mathbf{t}^N, \hat{t}_0)$. This implies that the tax vector $(\mathbf{t}^N, \hat{t}_0)$ is not an optimal solution to maximize welfare. Thus, we can conclude that $\max_{(\mathbf{t},\theta)\in\Gamma_A} W(\mathbf{t}, \theta) > W(\mathbf{t}^N, 0)$ holds.

[7] One might think that in the absence of tax on the public firm, the government can achieve the same welfare as *Regime A* by setting an appropriate privatization ratio. However, because the privatization ratio is restricted as $\theta \in [0, 1]$, we cannot exclude the possibility of a corner solution.

The intuition of Proposition 6.2 is straightforward. From Lemma 6.1, the government cannot directly control the behavior of the fully nationalized public firm: the government has only two effective instruments, namely, t_d and t_f. In addition, because of the Nash conjecture, the decision of the public firm becomes suboptimal.

As with Proposition 6.1, the superiority of the pure oligopolistic market over the market with a fully nationalized firm holds even if the commodity taxes imposed on the firms other than the public firm are restricted. Let $\Gamma_j^N = \{(\mathbf{t}, \theta) : t_j = \bar{t}_j,\ \theta = 0\}$ for $j = d,f$ and $\Gamma_0^N = \{(\mathbf{t}, \theta) : t_d = \bar{t}_d, t_f = \bar{t}_f,\ \theta = 0\}$ be the feasibility sets for the government in the presence of a fully nationalized public firm. Let $\widetilde{W}_j^N = \max_{(\mathbf{t},\theta)\in\Gamma_j^N} W(\mathbf{t}, \theta)$ for $j = d,f, 0$ be the maximum welfare over the feasibility set Γ_j^N. We obtain the following results:

Corollary 6.2 $\widetilde{W}_j^P > \widetilde{W}_j^N$ holds for $j = d,f, 0$.

Proof Applying an argument similar to that in Proposition 6.2, we can confirm that the equilibrium outputs in Γ_j^N are replicated in Γ_j^P.

Pal and White (1998) have shown that privatization improves domestic welfare when the government introduces optimally set production subsidies. They derived their results using linear demand and quadratic cost functions. Our results suggest that the result in Pal and White (1998) can be extended to general demand and cost functions. If the cost functions are identical between public and private firms, as assumed by Pal and White (1998), then the government imposes uniform commodity taxes on domestic private firms and privatized firms under a pure oligopoly. This is the special case described in Corollary 6.2.

6.3 Optimal Partial Privatization Under Various Tax Principles

Our discussion so far suggests that the privatization of a public firm is desirable in the presence of international trade. However, the result obtained here relies on the assumption that the government can arbitrarily set tax—in the real world, the government may not be able to freely do so for practical reasons. In addition to *Regime A*, we consider an optimal privatization policy under four tax regimes: origin principle, destination principle, import tariff, and the combination of a commodity tax and import tariff.

In this section, we focus on the case where the inverse demand function is linear: $p(Q) = a - Q$, where $a > 0$. We also assume that the cost functions of the firms are quadratic: $C_i = F + (1/2)k_i q^2$ for $i = d,f, 0$. To simplify the analysis, we assume that $F = 0$ and $k_d = k_f = 1$. Additionally, $k_0 = \rho(> 0)$, where ρ represents the technological differences between the public and private firms.

The structure of the game is the same as that in the previous section. For the given commodity taxes and privatization ratio, each firm produces its output as follows:

$$q_d(\mathbf{t},\theta) = \frac{2a(\theta+\rho) - (3+2\theta+3\rho)t_d + (1+\rho)t_f + 2\theta t_0}{2(3\theta+4\rho+3)}, \tag{6.13}$$

$$q_f(\mathbf{t},\theta) = \frac{(\theta+\rho)(2a+t_d) - (3\theta+3\rho+2)t_f + 2\theta t_0}{2(3\theta+4\rho+3)}, \tag{6.14}$$

$$q_0(\mathbf{t},\theta) = \frac{(3-\theta)(2a+t_d) + (3\theta-1)t_f - 8\theta t_0}{2(3\theta+4\rho+3)}. \tag{6.15}$$

From (6.13), (6.14), and (6.15), we can confirm that $\partial q_i/\partial t_i < 0$, $\partial q_i/\partial t_j > 0$, and $\partial Q/\partial t_i < 0$ for $i, j = d, f, 0$, and $i \neq j$. Furthermore, $\partial q_d/\partial t_0 = \partial q_f/\partial t_0$ holds so that the tax on the public firm equally affects the outputs of the private firms. If the cost functions are completely symmetric, then $\partial q_d/\partial t_f|_{\theta,\rho=1} = \partial q_0/\partial t_f|_{\theta,\rho=1}$ and $\partial q_f/\partial t_d|_{\theta,\rho=1} = \partial q_0/\partial t_d|_{\theta,\rho=1}$ hold. In addition, for the change in the privatization ratio, $\partial q_d/\partial\theta = \partial q_f/\partial\theta$ holds.

National welfare can be written as follows:

$$W(\mathbf{t},\theta) = aQ - \frac{1}{2}Q^2 - \frac{1}{2}(q_d)^2 - \frac{1}{2}\rho(q_0)^2 - \left(a - Q - t_f\right)q_f. \tag{6.16}$$

The government sets the tax rate and the privatization ratio according to the feasibility set to maximize welfare.

We now consider the optimal policy under the various regimes. As a benchmark, we begin with the case in which the government can arbitrarily decide all taxes: *Regime A*. The feasibility set of the policy instruments is represented as $\Gamma_A = \{(\mathbf{t},\theta) : \theta \in [0, 1]\}$. As shown in Proposition 6.2, because the optimal policy is $\theta \neq 0$, we exclude the case of full nationalization.

Taking account of the feasibility set and inserting (6.13), (6.14), and (6.15) into (6.16), we obtain the optimal taxes t_i^A for $i = d, f, 0$ as summarized in Table 6.1. The optimal tax for the domestic private firm is always negative because the optimal policy is subsidy. In contrast, the optimal tax for the foreign private firm is positive. These results are not surprising under a strategic trade policy.

In contrast, the optimal tax for the public firm depends on the privatization ratio. If $\theta < \rho/(\rho+4)$ holds, then t_0^A is positive. Furthermore, if the level of state ownership in the public firm is relatively high in the sense of $\theta < \rho/(3\rho+4)$, then the optimal tax rate for the public firm is higher than that of the foreign private

Table 6.1 Optimal policies under arbitrary taxation

Optimal tax	Optimal privatization ratio
$t_d^A = -a\frac{4\rho}{9\rho+4}$, $t_f^A = a\frac{2\rho}{9\rho+4}$, $t_0^A = -a\frac{4\theta-(1-\theta)\rho}{\theta(9\rho+4)}$	$\theta \in (0,\ 1]$

Table 6.2 Optimal policies under the origin principle

Optimal tax	Optimal privatization ratio	
$T_O^* = -\frac{10a\rho}{17\rho+8}$	$\theta_O^* = \frac{\rho}{8-7\rho}$	$for\ \rho \in (0,\ 1)$
$T_O^* = -2a\frac{14\rho+5\rho^2+21}{102\rho+17\rho^2+81}$	$\theta_O^* = 1$	$for\ \rho \in [1,\ 1.2956]$
$T_O^* = -2a\rho\frac{5\rho+6}{27\rho+17\rho^2+9}$	$\theta_O^* = 0$	$for\ \rho \in [1.2957,\ \infty)$

firm. This fact implies that when the government can use tax on a public firm as a policy instrument, the ownership structure plays a role that seemingly ensures a competitive condition between public and foreign private firms.

Inserting optimal taxes into (6.13), (6.14), and (6.15), we obtain the output of each firm as $q_d^A = 4a\rho/(9\rho+4), q_f^A = a\rho/(9\rho+4)$, and $q_0^A = 4a/(9\rho+4)$. This implies that the marginal costs are equalized between the domestic private firm and the public firm. That is, inefficiency arising from cost asymmetry between domestic firms is corrected by taxes.

We now turn to the origin principle, which is referred to as *Regime O*. Under this regime, commodity taxes are imposed on the goods produced by domestic firms. Let T_O be the commodity tax rate according to the origin principle. The feasibility set can be written as $\Gamma_O = \{(\mathbf{t}, \theta) : t_d = t_0 = T_O,\ \ t_f = 0,\ \ \theta \in [0, 1]\}$.

Table 6.2 represents the optimal national tax T_O^* and the optimal privatization ratio θ_O^*. The optimal tax is negative regardless of the inefficiency of the public firm. This is a standard result in imperfect competition. An optimal privatization policy depends on the degree of efficiency of the public firm as summarized in the following proposition:

Proposition 6.3 *Suppose that the government imposes a commodity tax according to the origin principle.*

(i) *Partial privatization is optimal if the public firm is efficient in the sense of* $\rho \in (0, 1)$.
(ii) *Full privatization is optimal if the public firm is not too inefficient in the sense of* $\rho \in [1,\ 1.2956]$.
(iii) *Full nationalization is optimal if the public firm is inefficient in the sense of* $\rho \geq 1.2957$.

Proof See Appendix A.6.3.

Yu and Lee (2011) and Han (2012) proved the optimality of full privatization for the case of $\rho = 1$. Proposition 6.3 (ii) and (iii) show a sharp contrast with the untaxed case in which partial privatization is optimal (Lemma 6.2). In the present regime, either full privatization or full nationalization is optimal under the plausible condition that the public firm is less efficient. Under $\rho > 1$, the optimal privatization ratio is a corner solution so that the optimal policy is discontinuously changed from $\theta = 1$ from $\theta = 0$ as increasing in ρ.

The intuition behind Proposition 6.2 can be explained as follows. First, it should be noted that the origin-based tax does not discriminate between domestic private

Table 6.3 Optimal policies under the destination principle

Optimal tax	Optimal privatization ratio	
$T_D^* = \frac{a\rho}{9\rho+5}$	$\theta_D^* = \frac{2\rho}{3\rho+5}$	for $\rho \in (0, \infty)$

firms and public firms as long as $\theta \neq 0$. Suppose that the initial situation is a pure oligopoly. When the public firm is efficient, the output of the domestic private firm is less than that of the public firm. This implies that the marginal cost of the domestic private firm is greater than that of the public firm because $p'(q_d - q_0) = c_d' - c_0'$ holds from the first-order conditions of the firms. Thus, an increase in the public firm's output at the expense of a reduction of the domestic private firm's output improves welfare because of the convexity of the cost functions. Hence, it will be desirable that the government gives the public firm a further incentive to increase its output, which means an increase in the state ownership of the public firm.

In contrast, if the public firm is inefficient in the sense of $\rho > 1$, the output of the public firm should be restrained. For this purpose, the government has two choices. First, the output of the public firm can be decreased by an increase in the private ownership of the public firm. Thus, full privatization is desirable. In this case, origin-based tax can be used to shift the rent of the foreign private firm. Second, because the tax on the private firm's output has no direct effect on the behavior of a fully nationalized public firm, in this case, the origin-based tax gives the domestic private firm an incentive to increase its output while the public firm does not change its behavior. Thus, by fully nationalizing the public firm, the government can only use the (negative) origin-based tax for the domestic private firm.

We now turn to the destination principle, which is referred to as *Regime D*. Under the destination principle, commodity taxes are imposed on goods consumed by domestic residents at a uniform rate. Let T_D be the tax rate under the destination principle. The feasibility set is $\Gamma_D = \{(\mathbf{t}, \theta) : t_d = t_f = t_0 = T_D,\ \ \theta \in [0, 1]\}$.

Table 6.3 shows the optimal tax T_D^* and privatization ratio θ_D^* under the destination principle. In contrast to the origin principle, the optimal tax is strictly positive. We have the following proposition for the optimal privatization ratio:

Proposition 6.4 *Suppose that the government imposes a commodity tax according to the destination principle. Partial privatization is optimal.*

Proof Inserting (6.13), (6.14), and (6.15) into (6.16) and taking account of $(\mathbf{t}, \theta) \in \Gamma_D$, we obtain the first-order conditions as $\partial W / \partial T_D = 0$ and $\partial W / \partial \theta = 0$. Solving the first-order conditions, we have the optimal policies represented in Table 6.3. In addition, it is confirmed that $W = W(T_D, \theta)$ is concave around (T_D^*, θ_D^*).

The intuition behind the result is explained as follows. Both destination-based tax and state ownership do not discriminate between foreign firms and domestic private firms. Although an increase in state ownership negatively affects the

Table 6.4 Optimal policies under the import tariff

Optimal tax	Optimal privatization ratio	
$\tau_I^* = 14a\frac{\rho}{41\rho+20}$	$\theta_I^* = \frac{9\rho}{5\rho+20}$	for $\rho \in (0,\ 5)$
$\tau_I^* = 2a\frac{18\rho+7\rho^2+15}{134\rho+41\rho^2+105}$	$\theta_I^* = 1$	for $\rho \in [5,\ \infty)$

domestic private firm's output, total domestic outputs increase.[8] Thus, the optimal policy is a combination of a positive commodity tax, which reduces imports, and an appropriate privatization ratio, which increases domestic outputs. In this regime, the public firm produces outputs where its marginal cost is equal to the market price. In contrast, the private firms' marginal costs are below the market price. Overall, the strategic use of tax and privatization is limited under the destination principle.

We now consider the situation where the government imposes an import tariff, which is referred to as *Regime I*. When the government's only strategy is an import tariff, denoted by τ_I, the feasibility set can be written as $\Gamma_I = \{(\mathbf{t}, \theta) : t_f = \tau_I,\ \ t_d = t_0 = 0,\ \ \theta \in [0, 1]\}$.

The optimal policy is summarized in Table 6.4. The optimal tariff is strictly positive. The optimal ownership is basically partial privatization, as shown by Yu and Lee (2011) and Han (2012) in the case of $\rho = 1$. However, cost asymmetry affects optimal ownership.

Proposition 6.5 *Suppose that the government optimally levies the import tariff.*

(i) *Partial privatization is optimal if the public firm is not too inefficient in the sense of* $\rho < 5$.
(ii) *Full privatization is optimal if* $\rho \geq 5$.

Proof Inserting $t_d = t_f = 0$ into (6.16), we can write welfare as $W = W(\tau_I, \theta)$. Solving $\partial W/\partial \tau_I = \partial W/\partial \theta = 0$, we have the optimal tariff and the privatization ratio for $\theta \geq 0$. Furthermore, $W(\tau_I, \theta)$ is concave around the optimal tariff and privatization ratio so that the second-order condition is satisfied. Let $\widehat{\tau}(\theta) = \arg\max_{\tau_I} W(\tau_I, \theta)$ be the optimal tariff for the given θ. For $\rho \geq 5$, because $dW[\hat{\tau}(\theta), \theta]/d\theta > 0$ holds for $\theta \in [0, 1]$, full privatization is optimal.

The intuition of Proposition 6.4 is as follows. As indicated in numerous studies on strategic trade policy, the strategic use of import tariffs has rent-shifting effects. However, in the absence of domestic tax, the government has no corrective device to reduce the inefficiency arising from imperfect competition. An increase in the tariff reduces the total output.[9] Thus, public ownership is required to mitigate

[8] From (6.13), (6.14), and (6.15), we have $\partial q_d/\partial \theta|_{T_D = T_D^*} = [a(3+\rho) + 3T_D^*(1+\rho)]/A^2 > 0$ and $\partial(q_d + q_0)/\partial \theta|_{T_D = T_D^*} = -[3a(3+\rho) + 3T_D^*(1+\rho)]/A^2 < 0$, where $A \equiv 3\theta + 4\rho + 3$.

[9] From (6.13), (6.14), and (6.15), we have $\partial Q/\partial t_f|_{\theta=1} = -(\rho+1)/[2(2\rho+3)]$.

Table 6.5 Optimal policies under the combination of commodity tax and import tariff

Optimal tax	Optimal privatization ratio	
$T_C^* = -\frac{4a\rho}{9\rho+4},\ \tau_C^* = \frac{6a\rho}{9\rho+4}$	$\theta_C^* = \frac{\rho}{4-3\rho}$	if $\rho \in (0, 1)$
$T_C^* = -\frac{4a(2\rho+\rho^2+5)}{54\rho+9\rho^2+41},\ \tau_C^* = \frac{2a(10\rho+3\rho^2+11)}{54\rho+9\rho^2+41}$	$\theta_C^* = 1$	*if* $\rho \in [1, 1.7183]$
$T_C^* = -\frac{2a\rho(2\rho+3)}{15\rho+9\rho^2+5},\ \tau_C^* = \frac{2a\rho(3\rho+4)}{15\rho+9\rho^2+5}$	$\theta_C^* = 0$	*if* $\rho \in [1.7184, \infty)$

inefficiency due to the imperfect competition. However, an increase in public ownership has an equally negative effect on the outputs of both domestic and foreign private firms, although total outputs are increased.[10] As a result, the efficiency of domestic production is impaired. Thus, except for the case in which the public firm is too inefficient, partial privatization is optimal in the import tariff regime.

Finally, we consider the optimal policy where the government uses a combination of the destination-based tax and the import tariff, which is referred to as *Regime C*. Let T_C and τ_C be the commodity tax and import tariff under this regime, respectively. The feasibility set is $\Gamma_C = \{(\mathbf{t}, \theta) : t_d = t_0 = T_C,\ t_f = \tau_C + T_C,\ \theta \in [0, 1]\}$.

The optimal taxes and privatization ratio are represented in Table 6.5. As expected, the optimal combination of fiscal incentives consists of negative commodity tax and positive import tariff. The optimal privatization ratio depends on the efficiency of the public firm, which is summarized by the following proposition:

Proposition 6.6 *Suppose that the government levies an import tariff and a domestic commodity tax.*

(i) *Partial privatization is optimal if the public firm is efficient in the sense of* $\rho < 1$.
(ii) *Full privatization is optimal if the public firm is not too inefficient in the sense of* $\rho \in [1, 1.7183]$.
(iii) *Full nationalization is optimal if the public firm is significantly less efficient in the sense of* $\rho \in [1.7184, \infty)$.

Proof See Appendix A.6.4.

The intuition behind Proposition 6.5 is basically the same as that of Proposition 6.2. As with the origin principle, this regime also produces different welfare implications for privatization, as opposed to those of the destination principle. Because the government can use import tariffs in addition to a commodity tax, the range of cost differentials to be privatized is relatively wide.

[10] For a given θ, the optimal tariff is $\tau_I(\theta) = \frac{6\rho+7\rho^2+\theta(9\rho+3)+\theta^2(3\rho+12)}{59\rho+41\rho^2+\theta(66\rho+48)+\theta^2(9\rho+36)+21}$. Inserting this expression into (6.13), (6.14), and (6.15), we have $\partial Q/\partial\theta < 0$ and $\partial q_d/\partial\theta > 0$.

6.4 Welfare Comparisons

We compare the welfare levels under the regimes considered so far. Welfare ordering in the various regimes partly follows the size of the feasibility sets. We can easily confirm that $\Gamma_J \subset \Gamma_C \subset \Gamma_A$ for $J = O, D, I$. Thus, *Regime A* dominates the other regimes regarding welfare. In addition, *Regime C* provides greater welfare than the origin, destination, and import tariff regimes.

In Table 6.6, $\widetilde{W}_J \equiv \max_{(\mathbf{t},\theta)\in\Gamma_J} W$ and $\widetilde{\pi}_J^f$, respectively, denote the maximum welfare and profit of the foreign private firm, when the government optimally sets the policy instruments. A direct comparison shows that $\widetilde{W}_A \geq \widetilde{W}_C > \widetilde{W}_O > \widetilde{W}_I > \widetilde{W}_D$ holds for $\rho \in (0, \infty)$. Yu and Lee (2011) and Han (2012) have shown that $\widetilde{W}_C > \widetilde{W}_O > \widetilde{W}_I$ when $\rho = 1$. Our result shows that welfare orderings are robust for cost asymmetries. The fact that $\widetilde{W}_A = \widetilde{W}_C$ holds for $\rho \in (0, 1)$ results from Proposition 6.1. However, $\widetilde{W}_A > \widetilde{W}_C$ for $\rho > 1$ arises from the constraint that $\theta \leq 1$. Thus, as instruments for strategic trade policy, tax incentives are superior to public firm ownership.

In contrast, a direct comparison of $\widetilde{\pi}_J^f$ produces the orderings of the foreign private firm's profit, which depend on cost asymmetries.

Table 6.6 Welfare comparisons

	Domestic welfare	Foreign profit	
Regime A	$\widetilde{W}_A = \frac{a^2(5\rho+4)}{2(9\rho+4)}$	$\widetilde{\pi}_A^f = \frac{3}{2}\left(\frac{a\rho}{9\rho+4}\right)^2$	for $\rho \in (0, \infty)$
Regime O	$\widetilde{W}_O = \frac{a^2(9\rho+8)}{2(17\rho+8)}$	$\widetilde{\pi}_O^f = \frac{3}{2}\left(\frac{3a\rho}{17\rho+8}\right)^2$	for $\rho \in (0, 1)$
	$\widetilde{W}_O = \frac{a^2(54\rho+9\rho^2+73)}{2(102\rho+17\rho^2+81)}$	$\widetilde{\pi}_O^f = \frac{3}{2}\left[\frac{3a(6\rho+\rho^2+1)}{102\rho+17\rho^2+81}\right]^2$	for $\rho \in [1, 1.2956]$
	$\widetilde{W}_O = \frac{9a^2(\rho+1)^2}{2(27\rho+17\rho^2+9)}$	$\widetilde{\pi}_O^f = \frac{3}{2}\left[\frac{3a\rho(\rho+1)}{27\rho+17\rho^2+9}\right]^2$	for $\rho \in [1.2957, \infty)$
Regime D	$\widetilde{W}_D = \frac{a^2(4\rho+5)}{2(9\rho+5)}$	$\widetilde{\pi}_D^f = \frac{3}{2}\left(\frac{2a\rho}{9\rho+5}\right)^2$	for $\rho \in (0, \infty)$
Regime I	$\widetilde{W}_I = \frac{a^2(21\rho+20)}{2(41\rho+20)}$	$\widetilde{\pi}_I^f = \frac{3}{2}\left(\frac{5a\rho}{41\rho+20}\right)^2$	for $\rho \in (0, 5)$
	$\widetilde{W}_I = \frac{a^2(78\rho+21\rho^2+85)}{2(134\rho+41\rho^2+105)}$	$\widetilde{\pi}_I^f = \frac{3}{2}\left[\frac{a(14\rho+5\rho^2+5)}{134\rho+41\rho^2+105}\right]^2$	for $\rho \in [5, \infty)$
Regime C	$\widetilde{W}_C = \frac{a^2(5\rho+4)}{2(9\rho+4)}$	$\widetilde{\pi}_C^f = \frac{3}{2}\left(\frac{a\rho}{9\rho+4}\right)^2$	for $\rho \in (0, 1)$
	$\widetilde{W}_C = \frac{a^2(30\rho+5\rho^2+37)}{2(54\rho+9\rho^2+41)}$	$\widetilde{\pi}_C^f = \frac{3}{2}\left[\frac{a(6\rho+\rho^2+1)}{54\rho+9\rho^2+41}\right]^2$	for $\rho \in [1, 1.7183]$
	$\widetilde{W}_C = \frac{5a^2(\rho+1)^2}{2(15\rho+9\rho^2+5)}$	$\widetilde{\pi}_C^f = \frac{3}{2}\left[\frac{a\rho(\rho+1)}{15\rho+9\rho^2+5}\right]^2$	for $\rho \in [1.7184, \infty)$

$$\begin{aligned} &\tilde{\pi}_D^f > \tilde{\pi}_O^f > \tilde{\pi}_I^f > \tilde{\pi}_C^f = \tilde{\pi}_A^f, \quad \text{if } \rho \in (0,\ 1], \\ &\tilde{\pi}_D^f > \tilde{\pi}_O^f > \tilde{\pi}_I^f > \tilde{\pi}_C^f > \tilde{\pi}_A^f, \quad \text{if } \rho \in \left(1,\ 1.7183\right], \\ &\tilde{\pi}_D^f > \tilde{\pi}_O^f > \tilde{\pi}_I^f > \tilde{\pi}_A^f > \tilde{\pi}_C^f, \quad \text{if } \rho \in [1.7184,\ \infty). \end{aligned} \tag{6.17}$$

These orderings are not the complete reverse of the welfare orderings, which suggests that regime switching may increase welfare in line with the Pareto principle. Noting that $\tilde{\pi}_O^f > \tilde{\pi}_I^f$ for $\forall \rho \in (0,\ \infty)$ and $\tilde{\pi}_A^f > \tilde{\pi}_C^f$ for $\forall \rho \in (1.7184,\ \infty)$ hold, we obtain the following result.

Proposition 6.7 *Both domestic welfare and a foreign firm's profit are increased by*:

(i) *A transition from Regime I to Regime O.*
(ii) *A transition from Regime C to Regime A if the public firm is inefficient in the sense of* $\rho \geq 1.7184$.

Proof The claims follow from direct comparisons of welfare and profit.

Proposition 6.7(i) shows a possibility that trade liberalization and privatization are compatible. Suppose that the initial situation as in *Regime I* and the public firm is inefficient in the sense of $\rho \in \left(1,\ 1.2956\right]$. In this case, the optimal policy is a positive tariff and partial privatization as shown in Proposition 6.5. Starting from here, eliminating the tariff and fully privatizing the public firm improves both domestic welfare and the foreign firm's profit if the domestic government is allowed to impose an optimal origin-based tax in exchange for eliminating the tariff.

If the public firm is too inefficient in the sense of $\rho \geq 1.2957$, the optimal policy under *Regime O* is the full nationalization of the public firm. However, if the change in regime improves efficiency, then trade liberalization and privatization are still beneficial for both countries. From Table 6.1 we can show that $\widetilde{W}_O\big|_{\rho=1} > \widetilde{W}_I\big|_{\rho=\rho_0}$ and $\tilde{\pi}_O^f\Big|_{\rho=1} > \tilde{\pi}_I^f\Big|_{\rho=\rho_0}$ hold for $\rho_0 > 1$.

Proposition 6.7(ii) states that two different tax rates for the public firm and the domestic private firm may improve both domestic welfare and the foreign firm's profit if the public firm is too inefficient. Under *Regime C*, the disparities in marginal costs between the public and private firms induce a significant welfare loss so that the government has to restrain total outputs. In contrast, production efficiency is achieved under *Regime A* by discriminated taxation for the public firm and the domestic private firm.

6.5 Concluding Remarks

In this chapter we analyzed an optimal privatization policy for a public firm under various tax principles in the presence of international trade. In addition to production subsidies and import tariffs, which were considered in previous literature, we considered a destination-based commodity tax and a tax system that applies different tax rates for each firm. Furthermore, we allowed for asymmetric costs between the public and private firms.

When a government can apply preferential treatment to a public firm, tax incentives (e.g., production subsidy) and managerial incentives (represented by ownership by the public firm) are relatively equivalent, as long as the public firm is at least partially privatized. This result suggests that the government has an incentive to partly privatize public firms. For example, suppose that in the negotiation of an EPA, preferential treatment to a public firm must be reduced somewhat. In such a case, the government can maintain the same welfare level as before the agreement by adjusting the private ownership of the public firm. In this sense, to ensure the effectiveness of free trade agreements and EPAs, the transparency of the policies applied to the public firm will become an important issue.

We also found that the optimal degree of privatization depends on both the tax regime to be applied and the cost asymmetry. In an international oligopolistic market, the government has to solve multiple problems to maximize welfare: mitigate consumers' losses because of imperfect competition, ensure efficient production among firms that have different technology, and shift the profit of foreign competitors. The existing tax systems analyzed herein restrict the possibility that the government will completely solve the above problems. Partial privatization can contribute to mitigate inefficiencies; however, this role is limited because the government only offers managerial incentives within nationalization and full privatization. Thus, the assurance of freedom in taxation is desirable from the perspective of national welfare.

The present analysis implies that in certain situations, trade liberalization and privatization are compatible to improve both domestic and foreign welfare. In particular, eliminating import tariffs and fully privatizing public firms increase both domestic welfare and the profits of foreign competitors, if the government can introduce a production subsidy in exchange for tariff elimination. In the process of trade liberalization, it is often argued that all preferential treatments for domestic industries should be eliminated. However, as long as the market is oligopolistic, Pigouvian intervention is still desirable.

Because the present analysis is based on a very simple framework, there exists the opportunity for further research. This chapter only considers Cournot competition, and as such different types of competition may alter the results. It is plausible that the number of private firms affects the optimal policy. In this chapter, we focused on a public firm operating in a domestic market. As pointed out in Kowalski et al. (2013), the activities of overseas public firms have increased in recent times. Further investigation of this phenomenon can offer richer insights into international mixed oligopolies.

Appendix

A.6.1 Stability Condition

Consider a simple adjustment process according to the Nash conjecture: both domestic and foreign private firms respond to a change in the output of the public firm. Once the private firms have responded, the public firm adjusts its output subject to the optimal response function. First, regarding the change in the public firm's output, the outputs of the private firms are adjusted as follows:

$$\begin{bmatrix} dq_d/dq_0 \\ dq_f/dq_0 \end{bmatrix} = \frac{1}{1-\phi_f^d\phi_d^f}\begin{bmatrix} \phi_0^d + \phi_f^d\phi_0^f \\ \phi_0^f + \phi_d^f\phi_0^d \end{bmatrix}. \tag{A6.1}$$

Thus, stability requires $\left|\phi_f^d\phi_d^f\right| < 1$. In such a case, the optimal response functions of the private firms are given by $q_d = \omega^d(q_0)$ and $q_f = \omega^f(q_0)$. Next, the public firm adjusts its outputs according to the following adjustment process:

$$q_{0(t+1)} = \phi^0\left[\omega^d\left(q_{0(t)}\right), \omega^f\left(q_{0(t)}\right)\right], \tag{A6.2}$$

where a subscript(s) denotes the time period. The approximation of the change in output around the equilibrium (denoted by q_0^*) is represented by

$$\Delta q_{0(t+1)} = \left(\phi_d^0\omega^{d'} + \phi_f^0\omega^{f'}\right)\Delta q_{0(t)}, \tag{A6.3}$$

where $\Delta q_{0(s)} \equiv q_{0(s)} - q_0^*$. Thus, $|\Delta_2| = \left|\phi_d^0\omega^{d'} + \phi_f^0\omega^{f'}\right| < 1$ is required for the stability.

A.6.2 Proof of Lemma 6.2

The effect of the change in the privatization ratio on welfare can be written as

$$\begin{aligned}\frac{\partial W(0,\theta)}{\partial\theta} &= \mathbf{w}_q\mathbf{D}^{-1}\mathbf{b}\\ &= \frac{(p')^2(q_f+q_0)}{(1-\Delta_1)(1-\Delta_2)V_{00}}\Big[(q_f+q_d)\left(1+\phi_0^f\right)\phi_0^d+q_f(1+\phi_0^d)\phi_0^f\\ &\quad +(q_f+zq_0)\left(1-\phi_f^d\phi_d^f\right)\Big],\end{aligned} \tag{A6.4}$$

where $\mathbf{w}_q$ denotes the national marginal benefits of an increase in each firm's output, which is represented as $\mathbf{w}_q\big|_{\mathbf{t}=0} = -p'\left[q_d+q_f,\ q_f,\ q_f+q_0z\right]$. Identical technologies among private firms imply that in the equilibrium, two private firms have the same level of output, denoted by $\widetilde{q}(=q_d=q_f)$. Thus, the optimal response functions also have same slopes denoted as $\widetilde{\phi}_q\left(=\phi_0^d=\phi_f^d=\phi_0^f=\phi_d^f\right)$. Solving $\partial W(0,\theta)/\partial\theta=0$, we have an implicit form of the optimal privatization ratio θ_{-t}^* as follows:

$$\theta_{-t}^* = \left(\frac{-\widetilde{\phi}_q}{1-\widetilde{\phi}_q}\right)\left(\frac{3\widetilde{q}}{q_0+\widetilde{q}}\right). \tag{A6.5}$$

Under condition (i), because $\partial q_0/\partial\theta<0$ and $\partial q_i/\partial\theta>0$ for $i=d,f$ hold from (6.7), $q_0\geq\widetilde{q}$ holds for $\theta\in[0,1]$. Thus, if condition (i) is met, $\widetilde{\phi}_q/\left(1-\widetilde{\phi}_q\right)\in(0,\ 1/2)$ and $3\widetilde{q}/(q_0+\widetilde{q})\in\left(0,\ 3/2\right]$. Therefore, $\theta_{-t}^*\in(0,\ 3/4)$ under condition (i). We turn to condition (ii). Linear demand function implies $\widetilde{\phi}_q<-1/2$, where strict inequality follows from the convexity of the cost function. In this case, $\widetilde{\phi}_q/\left(1-\widetilde{\phi}_q\right)\in(0,\ 1/3)$ and $3\widetilde{q}/(q_0+\widetilde{q})\in\left(0,\ 3\right]$. Thus, $\theta_{-t}^*\in(0,\ 1)$ holds under condition (ii). From (A6.4), it is confirmed that $\partial W(0,0)/\partial\theta>0$ and $\partial W(0,1)/\partial\theta<0$ hold if either condition (i) or (ii) is met.

A.6.3 Proof of Proposition 6.3

Taking account of the feasibility set, we obtain the optimal tax for the given θ:

$$T_O(\theta) = -2a(\Lambda_O)^{-1}\left[3(3\rho+8)\theta^2-(\rho+3)\theta+(5\rho+6)\rho\right], \tag{A6.6}$$

where $\Lambda_O\equiv 9\ (9\rho+8)\theta^2-6\rho\theta+(17\rho+7)\rho+9>0$ for $\theta\in[0,1]$. Inserting (A6.6) into (6.16), we can write welfare as $W_O=W_O(\theta)$ in which origin-based tax is optimally chosen. The first-order condition for θ is as follows:

$$\frac{dW_O(\theta)}{d\theta} = a^2(\Lambda_O)^{-2}(7\theta\rho - 8\theta + \rho)\left(6\theta\rho - 7\rho^2 + 9\right) = 0. \tag{A6.7}$$

Thus, we have two roots: $\theta_O^1 = \rho/(8 - 7\rho)$ for $\rho \neq 8/7$ and $\theta_O^2 = (7\rho^2 - 9)/(6\rho)$. However, the second-order condition implies that θ_O^2 gives the minimum solution. Hence, $\theta_O^* = \theta_O^1$ for $\rho \in \left(0, 1\right]$. In (A6.7), it is confirmed that $dW_O/d\theta > 0$ holds for $\theta \in [0, 1]$ and $\rho \in \left(1, 3/\sqrt{7}\right)$. That is, $\theta_O^* = 1$ is optimal for $\rho \in \left(1,\ 3/\sqrt{7}\right)$. For $\rho \geq 3/\sqrt{7}$, θ_O^2 becomes nonnegative and $\theta_O^1 \notin [0, 1]$. Thus, θ_O^* is either zero or unity. A direct calculation yields $W_O(1) - W_O(0) > 0$ for $\rho \in (1,\ 1.2956)$ and $W_O(1) - W_O(0) < 0$ for $\rho \in (1.2957, \infty)$. Together with these facts described above, the claims are proved.

A.6.4 Proof of Proposition 6.6

For the given privatization ratio, the optimal taxes can be written as follows:

$$T_C(\theta) = -2a(\Lambda_C)^{-1}[3(\rho + 4)\theta^2 - 2(\rho + 1)\theta + (2\rho + 3)\rho], \tag{A6.8}$$

$$\tau_C(\theta) = 2a(\Lambda_C)^{-1}[(9\rho + 12)\theta^2 - (3\rho + 1)\theta + (3\rho + 4)\rho], \tag{A6.9}$$

where $\Lambda_C \equiv (45\rho + 36)\theta^2 - 6\rho\theta + (9\rho + 15)\rho + 5 > 0$ for $\theta \in [0, 1]$. The second-order condition is satisfied because $W(T_C, \tau_C, \theta)$ is concave in T_C and τ_C for $\theta \in [0, 1]$. Inserting (A6.8) and (A6.9) into (6.10), we can write the welfare as $W_C = W_C(\theta)$ in which the taxes are optimally chosen. The first-order condition for θ is as follows:

$$\frac{dW_C(\theta)}{d\theta} = a^2(\Lambda_C)^{-2}[(3\theta\rho - 4\theta + \rho)(6\theta\rho - 3\rho^2 + 5)] = 0. \tag{A6.10}$$

Although $\theta_C^1 = \rho/(4 - 3\rho)$ for $\rho \neq 4/3$ and $\theta_C^2 = (3\rho^2 - 5)/(6\rho)$ are two roots of (A6.10), θ_C^2 gives a minimum value. Hence, $\theta_C^* = \theta_C^1$ for $\rho \in \left(0, 1\right]$. In contrast, $dW_C/d\theta > 0$ holds for $\theta \in [0, 1]$ and $\rho \in \left(1, \sqrt{5/3}\right)$. That is, $\theta_O^* = 1$ is optimal for $\rho \in \left(1, \sqrt{5/3}\right)$. For $\rho \geq \sqrt{5/3}$, θ_C^2 is nonnegative and $\theta_C^1 \notin [0, 1]$: θ_C^* is either zero or unity. We obtain $W_C(1) - W_C(0) > 0$ for $\rho \in \left(1,\ 1.7183\right]$, and $W_C(1) - W_C(0) < 0$ for $\rho \in \left[1.7184, \infty\right)$.

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Chapter 7
Privatization Neutrality Theorem When a Public Firm Maximizes Objectives Other than Social Welfare

Kojun Hamada

Abstract This chapter investigates the privatization neutrality theorem when a public firm has a different objective from social welfare maximization. The privatization neutrality theorem claims that when the government gives the optimal subsidy to both public and private firms, social welfare is exactly the same before and after privatization. We demonstrate that if the discriminatory subsidy scheme is adopted to public and private firms, the privatization neutrality theorem can be recovered in a variety of situations. Especially, we obtain a seemingly paradoxical result as follows: When a public firm incorrectly recognizes that a subsidy to firms by a government directly affects the welfare size, the privatization neutrality necessarily holds. In contrast, when a public firm correctly recognizes that a subsidy affects only income distribution but not social welfare itself, the situation in which the neutrality holds is limited.

7.1 Introduction

The privatization neutrality theorem is a theoretical consequence in mixed oligopolies and has been examined by many scholars interested in determining whether privatization leads to an increase in social welfare. The privatization neutrality theorem insists that when a government provides the optimal subsidy to both public and private firms in a mixed oligopoly, social welfare does not change; that is, it is exactly the same before and after privatization. Most previous studies on this theorem have focused on a uniform subsidy policy to public and private firms because a uniform subsidy makes for a simple analysis. Furthermore, in situations in which all firms enjoy identical technology, we can show privatization neutrality even if only paying attention to uniform subsidy. However, public and private firms have different objectives, and a government or regulatory authority generally implements regulatory policies while fully recognizing the difference in objectives.

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If we focus on such differences between firms, it is important for researchers to investigate the situation in which regulatory authorities apply a discriminatory subsidy policy to public and private firms to improve social welfare. Moreover, some recent studies insist that when a discriminatory subsidy is adopted, the privatization neutrality theorem can be recovered unless public and private firms are identical, which is in stark contrast with the results shown in the earlier literature. In this chapter, we investigate the privatization neutrality theorem when a public firm has an objective that differs from social welfare maximization. By incorporating the discriminatory subsidy into the model, we clarify whether the privatization neutrality theorem continues to hold when there is a divergence of objectives between the government and a public firm.

White's (1996) seminal study first showed that if uniform subsidies are used before and after privatization, social welfare is unchanged by privatization. Since then, a considerable number of studies have investigated the so-called privatization neutrality theorem in various extended situations. Poyago-Theotoky (2001) showed that even when a public firm behaves as a Stackelberg leader, the privatization neutrality theorem holds. Myles (2002) generalized the neutrality result by extending the linear demand and the quadratic cost specified in Poyago-Theotoky (2001) to include general functions of demand and cost. Tomaru and Saito (2010) investigated endogenous timing and privatization in the resulting Stackelberg duopolies, where the public firm is the leader and private firms are followers. Social welfare was not affected. In addition to the above studies, extensive research has demonstrated that the privatization neutrality theorem continues to hold in different situations. For example, Tomaru (2006) adopted the partial privatization approach. Kato and Tomaru (2007) introduced nonprofit-maximizing private firms. Hashimzade et al. (2007) extended the analysis to product differentiation. All of these studies verified the robustness of the privatization neutrality theorem.[1]

However, there are several economic situations in which the privatization neutrality theorem is not satisfied. By tedious calculation, we can confirm that if a public firm has a different cost structure from private firms, this theorem does not hold. Likewise, when a public firm behaves as a Stackelberg leader before and after privatization, a different order of action between public and private firms brings about the difference in social welfare before and after privatization. Fjell and Heywood (2004) considered such a sequential-move situation where a public firm continues to act as a leader irrespective of whether privatization occurs, and they showed that the privatization neutrality theorem collapses.[2] Several previous studies mentioned that firm asymmetry between public and private firms, which is caused by the difference in cost and/or the different orders of the steps to produce,

[1]As a comprehensive survey of privatization neutrality theorem, see Tomaru (2014, Ch. 4).

[2]In contrast to Fjell and Heywood (2004) who stated that privatization is not welfare neutral, Matsumura and Okumura (2013) found that privatization neutrality holds if an output floor is introduced instead. Although their result suggests that neutrality can be recovered using an output floor regulation and the economic situation in which the privatization neutrality theorem is robust is enlarged, such a regulatory scheme, however, might seem too artificial.

hinders privatization neutrality. Cato and Matsumura (2013) considered a free entry market in which private firms freely enter a market and privatization neutrality is not satisfied. Zikos (2007) and Gil-Moltó et al. (2011) introduced R&D competition and showed that the existence of other choice variables in addition to production level undermines privatization neutrality. The privatization neutrality theorem no longer holds with taxation distortion, which is necessary to fund a subsidy (Matsumura and Tomaru 2013), for the existence of foreign private firms (Matsumura and Tomaru 2012) and the divergence of objectives between the government and a public firm (Kato 2008).

Since White (1996) first examined the optimal subsidy given to both public and private firms in a uniform specific manner, the majority of extant studies have assumed a uniform subsidy to public and private firms. If a public firm has a different cost structure and/or different timing of production and the government that implements the subsidy policy correctly recognizes the difference between firms, it would be quite natural for such an authority to adopt a discriminatory subsidy scheme for each firm to achieve social welfare maximization. By explicitly introducing a discriminatory subsidy policy into the model, Hamada (2016) first demonstrated that if different subsidy rates are applied between public and private firms, even when there is firm asymmetry, the privatization neutrality theorem continues to hold. This result implies that the privatization neutrality theorem can be recovered even in a situation with firm asymmetry, and maximized social welfare can be always achieved by adopting a discriminatory subsidy policy without relying on any artificial regulation such as an output floor. The effectiveness of the discriminatory subsidy to maximize social welfare and achieve the privatization neutrality theorem needs to be verifiable, even in other contexts within a mixed oligopoly.

In the context of firm regulation, conflict of interest exists between regulating authorities and regulated firms. As represented in Laffont and Tirole (1993), existing regulatory economics have investigated how regulatory authorities should implement the optimal policy to regulated firms mainly in a principal–agent framework with asymmetric information. A considerable number of studies assume conflicting interests between the regulator and regulated firms and allow different targeting policies depending on the characteristics of the firms, for example, through a menu of contracts. Although asymmetric information is not dealt with in the present study, we focus on the situation in which the public firm has a different objective from the government pursuing social welfare maximization. We examine whether a discriminatory subsidy between public and private firms can completely overcome the conflict of interest between the government and public firm. As one of the few exceptions, Kato (2008) examined how the divergence of objectives between the government and a public firm collapses the privatization neutrality theorem. However, he only focused on a uniform subsidy and investigated a scenario different from ours in which the government itself pursues a distortive objective other than social welfare with a preference bias toward tax revenue expansion. In that study, it is the public firm that aims to maximize social welfare. By considering a discriminatory subsidy policy in a case where the government maximizes social welfare, we investigate whether and how a more finely tuned subsidy instrument can bring about welfare improvement.

In this chapter, in contrast to the results of the existing literature that assumed a uniform subsidy to both public and private firms and claimed that this theorem does not hold, we demonstrate that if a discriminatory subsidy scheme is adopted for public and private firms, the privatization neutrality theorem can be recovered in many situations in which a public firm pursues various objectives other than social welfare. More specifically, we present the following seemingly paradoxical results. When a public firm incorrectly recognizes that subsidization causes the distortion of income distribution, the privatization neutrality theorem necessarily holds. However, when it correctly recognizes that subsidization does not affect welfare size at all, the situation in which the neutrality holds is limited. Our results suggest that even when differences in recognition exist between the government and a public firm with respect to the effect of a subsidy on social welfare, the government can achieve social welfare maximization in a mixed oligopoly by appropriately implementing a discriminatory subsidy policy. By utilizing the difference in the recognition of the effectiveness of the subsidy, the government can achieve first-best social welfare by setting an optimal discriminatory subsidy for both a public firm and a private firm.

The remainder of this chapter is organized as follows. Section 7.2 describes a mixed duopoly model in which a public firm and a private firm engage in quantity competition in a homogeneous good market. Section 7.3 presents the benchmark result when before privatization, both a government and a public firm aim to maximize social welfare and, after privatizing, the public firm purses its own profit maximization. Section 7.4 demonstrates the main result that when a public firm has a different objective from social welfare maximization before privatizing, the privatization neutrality theorem continues to hold by adopting a discriminatory specific subsidy scheme. Section 7.5 provides some concluding remarks.

7.2 The Model

Consider a duopolistic market of a homogeneous good in which a public firm and a private firm engage in Cournot quantity competition. We denote the index of firms by firm $i = 0, 1$. The public firm is indexed by firm 0, and before privatizing it aims to maximize social welfare or other objectives than social welfare as explained below. After privatizing, the public firm seeks to maximize its own profit. The private firm is indexed by firm 1 and aims to maximize its own profit. Both the public firm and private firm have identical technology. Furthermore, q_i denotes the output of firm i. The inverse demand function is assumed to be linear, that is, $p = p(Q) = a - Q;\ a > 0$, where $Q \equiv q_0 + q_1$ is the total output and p is the price.

The cost function of the firm is denoted by $C = c(q_i) = F + (k/2)q_i^2;$ $F \geq 0, k > 0$. For brevity and without loss of generality, we assume no fixed cost, that is, $F = 0$. The government maximizes social welfare and uses a subsidy as a policy instrument, with s_i denoting the specific subsidy per unit given to firm i. Thus, s_0 and s_1 denote the subsidy to the public firm and the private firm, respectively. Although previous studies only focused on uniform specific subsidies, that is, $s_0 = s_1 \equiv s$, we deal with a discriminatory specific subsidy that differs between a public firm and a private firm to clarify the effect of discrimination. The profit of firm i is $\pi_i = p(Q)q_i - (k/2)q_i^2 + s_i q_i = (a - Q + s_i)q_i - (k/2)q_i^2$. Consumer surplus and producer surplus are $CS \equiv \int_0^Q p(x)dx - p(Q)Q = (1/2)Q^2$ and $PS \equiv \pi_0 + \pi_1 = p(Q)Q - (k/2)\left(q_0^2 + q_1^2\right) + s_0 q_0 + s_1 q_1$, respectively. Social welfare is defined by the sum of the consumer surplus and producer surplus net total subsidy, that is, $W \equiv CS + PS - s_0 q_0 - s_1 q_1 = aQ - (1/2)Q^2 - (k/2)\left(q_0^2 + q_1^2\right)$.

The timing of the mixed oligopoly consists of a two-stage game as follows. In the first stage, the welfare-maximizing government determines the optimal discriminatory specific subsidy levels to both public and private firms (s_0, s_1). After the first stage, both firms observe the optimal discriminatory subsidy levels (s_0, s_1) and engage in Cournot quantity competition in the second stage. The equilibrium concept follows the subgame perfect Nash equilibrium (SPNE). We solve the equilibrium by backward induction.

In the subsequent two sections, we investigate whether privatization neutrality holds in each situation. In Sect. 7.3, we examine the benchmark case in which the government and public firm aim to maximize social welfare when a discriminatory subsidy is adopted and confirm that the privatization neutrality theorem holds, as has been shown in previous studies. In Sect. 7.4, we consider the case in which the public firm has objectives other than social welfare maximization before privatizing and demonstrate the main result on privatization neutrality.

7.3 Previous Results for the Privatization Neutrality Theorem

Since White (1996) first clarified the privatization neutrality theorem, most of his successive studies have assumed that the government gives a uniform specific subsidy to both public and private firms. In this benchmark section, allowing the government to implement a discriminatory subsidy for both firms, we confirm that when a public firm maximizes social welfare before privatization, the neutrality theorem holds, and the resulting discriminatory subsidy levels are the same between the public firm and the private firm. To solve the SPNE by backward induction, we first consider Cournot competition between public and private firms in the second stage after the government provides the discriminatory specific subsidies, s_0 and s_1, to the public firm and private firm, respectively. We then consider the first stage in which the government correctly inducing the outcome of the second-stage subgame thereby solves the optimal discriminatory subsidy levels to maximize social welfare. We present the equilibrium outcome before and after privatization in Sects. 7.3.1 and 7.3.2, respectively.

7.3.1 Before Privatization

In the second stage before privatization, the first-order condition for a public firm to maximize social welfare is as follows:

$$\frac{\partial W}{\partial q_0} = a - Q - kq_0 = 0 \Leftrightarrow q_0 = r_0(q_1) \equiv \frac{a - q_1}{k + 1}. \tag{7.1}$$

Note that the reaction function $q_0 = r_0(q_1)$ does not depend on its own subsidy level s_0 because s_0 obviously only affects the distribution of social welfare between the economic entities but not the amount of social welfare itself. The first-order condition for a private firm to maximize its own profit is as follows:

$$\frac{\partial \pi_1}{\partial q_1} = (a - Q + s_1) - q_1 - kq_1 = 0 \Leftrightarrow q_1 = r_1(q_0) \equiv \frac{a + s_1 - q_0}{k + 2}. \tag{7.2}$$

In contrast to the public firm, the reaction function of the private firm $q_1 = r_1(q_0)$ depends on its own subsidy level s_1.

By solving the simultaneous equations (7.1) and (7.2), we obtain the outputs for the public and private firms and the total output in the second stage before privatization as follows:

$$(q_0(s_1),\ q_1(s_1)) = \left(\frac{(k+1)a - s_1}{k^2 + 3k + 1}, \frac{ka + (k+1)s_1}{k^2 + 3k + 1}\right), \tag{7.3}$$

$$Q(s_1) = q_0(s_1) + q_1(s_1) = \frac{(2k+1)a + ks_1}{k^2 + 3k + 1}. \tag{7.4}$$

The Cournot–Nash equilibrium variables in the second stage before privatization are summarized in Table 7.1.[3]

Table 7.1 shows that the equilibrium variables (except for the public firm's profit) do not depend on the public firm's subsidy s_0. The following equations hold: $p^B = kq_0^B,\ p^B + s_1 = (k+1)q_1^B,\ \pi_0^B = s_0 q_0^B + (k/2)\left(q_0^B\right)^2$, and $\pi_1^B = \frac{k+2}{2}\left(q_1^B\right)^2$. Whether q_0^B or q_1^B is larger depends on the relative size of s_1 because $q_0^B \gtrless q_1^B$ if and only if $a \gtrless (k+2)s_1$. Likewise, whether π_0^B or π_1^B is larger also depends on the relative size of s_1. As already explained, social welfare does not depend on s_0. This

[3]Throughout this chapter, we denote the equilibrium variables before and after privatization with the superscripts B (before) and A (after).

Table 7.1 Cournot–Nash equilibrium in the second stage before privatization

Public firm's output	q_0^B	$\frac{(k+1)a-s_1}{k^2+3k+1}$
Private firm's output	q_1^B	$\frac{ka+(k+1)s_1}{k^2+3k+1}$
Total output	Q^B	$\frac{(2k+1)a+ks_1}{k^2+3k+1}$
Price	p^B	$\frac{k[(k+1)a-s_1]}{k^2+3k+1}$
Public firm's profit	π_0^B	$\frac{\left[k(k+1)a+2\left(k^2+3k+1\right)s_0-ks_1\right][(k+1)a-s_1]}{2\left(k^2+3k+1\right)^2}$
Private firm's profit	π_1^B	$\frac{(k+2)[ka+(k+1)s_1]^2}{2\left(k^2+3k+1\right)^2}$
Social welfare	W^B	$(1/2)\left(Q^B\right)^2+\pi_0^B+\pi_1^B-s_0q_0^B-s_1q_1^B$

implies that a government's subsidy policy for a public firm does not function at all to maximize social welfare.

The derivatives of outputs with respect to s_1 are as follows:

$$\begin{aligned}&\left(q_0^B(s_1)\right)'=-\frac{1}{k^2+3k+1}<0,\quad \left(q_1^B(s_1)\right)'=\frac{k+1}{k^2+3k+1}>0,\\&(Q_B(s_1))'=\frac{k}{k^2+3k+1}>0.\end{aligned} \tag{7.5}$$

Thus, the increase in the private firm's subsidy s_1 before privatization brings about an increase in the private firm's output, total output, and profit and an increase in consumer surplus. In contrast, it decreases the public firm's output and price. Whether the increase in s_1 increases the public firm's profit and social welfare depends on the relative size of s_1 itself.

In the first stage, the government sets the subsidy level of the private firm to maximize social welfare. As social welfare does not depend on the subsidy level of the public firm, s_0 is not endogenously determined. By substituting (7.3) and (7.4), social welfare is described as a function of s_1 as follows:

$$W_B(s_1)=\frac{1}{2}(Q(s_1))^2+\pi_0^B(s_1)+\pi_1^B(s_1)-s_0q_0^B-s_1q_1^B. \tag{7.6}$$

Noting that $\left(\pi_0^B(s_1)\right)'=\left(s_0+kq_0^B\right)\left(q_0^B\right)'<0$ and $\left(\pi_1^B(s_1)\right)'=(k+2)q_1^B\left(q_1^B\right)'>0$, we derive the first-order condition for the government as follows:[4]

[4] In this chapter, we present the calculation process because the derivation of the optimal subsidy is quite complicated.

$$
\begin{aligned}
\left(W^B\right)' &= Q^B\left(Q^B\right)' + \left(\pi_0^B\right)' + \left(\pi_1^B\right)' - s_0\left(q_0^B\right)' - q_1^B - s_1\left(q_1^B\right)' = 0 \\
\Leftrightarrow s_1 &= \frac{Q^B\left(Q^B\right)' + \left(s_0 + kq_0^B\right)\left(q_0^B\right)' + (k+2)q_1^B\left(q_1^B\right)' - s_0\left(q_0^B\right)' - q_1^B}{\left(q_1^B\right)'} \\
&= \frac{kQ^B - kq_0^B + (k+1)(k+2)q_1^B - \left(k^2+3k+1\right)q_1^B}{k+1} = q_1^B \\
\Leftrightarrow s_1^{B*} &= \frac{a}{k+2}.
\end{aligned} \tag{7.7}
$$

The optimal subsidy level of the private firm is uniquely determined. In contrast, the subsidy level of the public firm only affects the distribution of social welfare among the economic agents and does not affect the size of the welfare at all. Thus, s_0 is not endogenously determined. By substituting the optimal private firm's subsidy before privatization, that is, s_1^{B*}, into the equilibrium variables in Table 7.1, we obtain the SPNE variables as shown in Table 7.2.

The following relationships regarding the SPNE variables hold: $s_1^{B*} = q_0^B = q_1^B$, $p^B = kq_0^B, p^B + s_1^{B*} = (k+1)q_1^B, \pi_0^B = s_0q_0^B + (k/2)\left(q_0^B\right)^2$, and $\pi_1^B = \frac{k+2}{2}\left(q_1^B\right)^2$. In this setting, both firms produce the same amount, and the optimal subsidy level of the private firm is equal to the firm's output. Although the public firm's profit π_0^B depends on the subsidy level of the public firm, s_0 is not endogenously determined. Above all, it should be noted that marginal cost pricing is realized, that is, $p^B = c'\left(q_i^B\right)$. Thus, the maximized social welfare achieves the first-best level.

Before privatization, a public firm chooses its own output that satisfies the marginal cost pricing to maximize social welfare. Furthermore, private firms are induced with an appropriate subsidy by the government to produce an output level that satisfies the marginal cost pricing.

7.3.2 *After Privatization*

After privatizing, the public firm maximizes its own profit. Thus, in the second stage after privatization, the first-order condition for the public firm is the same as that for the private firm. The first-order condition of the public firm is as follows:

$$
\frac{\partial \pi_0}{\partial q_0} = (a - Q + s_0) - q_0 - kq_0 = 0 \Leftrightarrow q_0 = \frac{a + s_0 - q_1}{k+2}. \tag{7.8}
$$

In contrast to the case before privatization, note that the reaction function of the public firm after privatization depends on its own subsidy level s_0. The reaction function of the private firm does not change before and after privatization and it satisfies (7.2). By solving (7.8) and (7.2) with regard to q_0 and q_1, we obtain the outputs for both the public and private firms and the total output in the second stage after privatization, as follows:

Table 7.2 SPNE before privatization

Optimal public firm's subsidy	s_0^{B*}	Indeterminate
Optimal private firm's subsidy	s_1^{B*}	$\frac{a}{k+2}$
Public firm's output	q_0^B	$\frac{a}{k+2}$
Private firm's output	q_1^B	$\frac{a}{k+2}$
Total output	Q^B	$\frac{2a}{k+2}$
Price	p^B	$\frac{ka}{k+2}$
Public firm's profit	π_0^B	$\frac{[ka+2(k+2)s_0]a}{2(k+2)^2}$
Private firm's profit	π_1^B	$\frac{a^2}{2(k+2)}$
Social welfare	W^B	$\frac{a^2}{k+2}$

$$\begin{aligned}&\left(q_0^A(s_0,s_1),\ q_1^A(s_0,s_1)\right)\\&\quad=\left(\frac{(k+1)a+(k+2)s_0-s_1}{(k+1)(k+3)},\ \frac{(k+1)a-s_0+(k+2)s_1}{(k+1)(k+3)}\right),\end{aligned} \tag{7.9}$$

$$Q^A(s_0,s_1)=\frac{2a+s_0+s_1}{k+3}. \tag{7.10}$$

The Cournot–Nash equilibrium variables in the second stage after privatization are summarized in Table 7.3.

In Table 7.3, the equilibrium variables depend on both firms' subsidy levels (s_0, s_1). The following equations hold: $p^A+s_0=(k+1)q_0^A$, $p^A+s_1=(k+1)q_1^A$, $\pi_0^A=\frac{k+2}{2}\left(q_0^A\right)^2$, and $\pi_1^A=\frac{k+2}{2}\left(q_1^A\right)^2$. Whether q_0^A or q_1^A is larger is completely determined by the relative size of s_0 and s_1. That is, $q_0^A \gtreqless q_1^A$ if and only if $s_0 \gtreqless s_1$. Likewise, the magnitude relation between π_0^A and π_1^A is also determined by the relative size of s_0 and s_1. The derivatives of outputs with respect to s_0 and s_1 are as follows:

$$\frac{\partial q_0^A}{\partial s_0}=\frac{k+2}{(k+1)(k+3)}>0,\quad \frac{\partial q_1^A}{\partial s_0}=-\frac{1}{(k+1)(k+3)}<0,\quad \frac{\partial Q^A}{\partial s_0}=\frac{1}{k+3}>0, \tag{7.11}$$

$$\frac{\partial q_0^A}{\partial s_1}=-\frac{1}{(k+1)(k+3)}<0,\quad \frac{\partial q_1^A}{\partial s_1}=\frac{k+2}{(k+1)(k+3)}>0,\quad \frac{\partial Q^A}{\partial s_1}=\frac{1}{k+3}>0. \tag{7.12}$$

Thus, the increase in the public firm's subsidy s_0 after privatization brings about an increase in the public firm's output, total output, and profit, as well as an increase in consumer surplus. It also results in a decrease in the private firm's output, price, and profit. Whether the increase in s_0 increases social welfare depends on the relative size of s_0. In contrast, the increase in the private firm's subsidy s_1 after

Table 7.3 Cournot–Nash equilibrium in the second stage after privatization

Public firm's output	q_0^A	$\frac{(k+1)a+(k+2)s_0-s_1}{(k+1)(k+3)}$
Private firm's output	q_1^A	$\frac{(k+1)a-s_0+(k+2)s_1}{(k+1)(k+3)}$
Total output	Q^A	$\frac{2a+s_0+s_1}{k+3}$
Price	p^A	$\frac{(k+1)a-s_0-s_1}{k+3}$
Public firm's profit	π_0^A	$\frac{(k+2)[(k+1)a+(k+2)s_0-s_1]^2}{2(k+1)^2(k+3)^2}$
Private firm's profit	π_1^A	$\frac{(k+2)[(k+1)a-s_0+(k+2)s_1]^2}{2(k+1)^2(k+3)^2}$
Social welfare	W^A	$(1/2)\left(Q^A\right)^2+\pi_0^A+\pi_1^A-s_0q_0^A-s_1q_1^A$

privatization brings about an increase in its output, total output, and profit, as well as an increase in consumer surplus. Furthermore, it decreases the public firm's output, price, and profit. Whether the increase in s_1 increases social welfare depends on the relative size of s_1.

In the first stage, the government sets the subsidy levels of both public and private firms to maximize social welfare. By substituting (7.9) and (7.10) into social welfare, social welfare is described as a function of s_0 and s_1 as follows:

$$\begin{aligned} W^A(s_0,s_1) = \frac{1}{2}\Big(Q^A(s_0,s_1)\Big)^2 + \pi_0^A(s_0,s_1) + \pi_1^A(s_0,s_1) \\ -s_0q_0^A(s_0,s_1) - s_1q_1^A(s_0,s_1)\ . \end{aligned} \tag{7.13}$$

Noting that $\frac{\partial \pi_0^A}{\partial s_0} = (k+2)q_0^A\frac{\partial q_0^A}{\partial s_0} > 0, \frac{\partial \pi_1^A}{\partial s_0} = (k+2)q_1^A\frac{\partial q_1^A}{\partial s_0} < 0, \frac{\partial \pi_0^A}{\partial s_1} = (k+2)q_0^A \frac{\partial q_0^A}{\partial s_1} < 0$, and $\frac{\partial \pi_1^A}{\partial s_1} = (k+2)q_1^A\frac{\partial q_1^A}{\partial s_1} > 0$, we derive the first-order conditions of the government with regard to s_0 and s_1 as follows:

$$\begin{aligned} \frac{\partial W^A}{\partial s_0} &= Q^A\frac{\partial Q^A}{\partial s_0} + \frac{\partial \pi_0^A}{\partial s_0} + \frac{\partial \pi_1^A}{\partial s_0} - q_0^A - s_0\frac{\partial q_0^A}{\partial s_0} - s_1\frac{\partial q_1^A}{\partial s_0} = 0 \\ \Leftrightarrow\ s_0 &= \frac{Q^A\frac{\partial Q^A}{\partial s_0} + (k+2)q_0^A\frac{\partial q_0^A}{\partial s_0} + (k+2)q_1^A\frac{\partial q_1^A}{\partial s_0} - q_0^A - s_1\frac{\partial q_1^A}{\partial s_0}}{\frac{\partial q_0^A}{\partial s_0}} \\ &= \frac{(k+1)Q^A + (k+2)^2q_0^A - (k+2)q_1^A - (k+1)(k+3)q_0^A + s_1}{k+2} \\ &= \frac{(k+2)q_0^A - q_1^A + s_1}{k+2} \Leftrightarrow (k+2)s_0 - s_1 = (k+2)q_0^A - q_1^A. \end{aligned} \tag{7.14}$$

$$\begin{aligned}\frac{\partial W^A}{\partial s_1} &= Q^A\frac{\partial Q^A}{\partial s_1}+\frac{\partial \pi_0^A}{\partial s_1}+\frac{\partial \pi_1^A}{\partial s_0}-s_0\frac{\partial q_0^A}{\partial s_1}-q_1^A-s_1\frac{\partial q_1^A}{\partial s_1}=0\\ \Leftrightarrow s_1 &= \frac{Q^A\frac{\partial Q^A}{\partial s_1}+(k+2)q_0^A\frac{\partial q_0^A}{\partial s_1}+(k+2)q_1^A\frac{\partial q_1^A}{\partial s_1}-s_1\frac{\partial q_1^A}{\partial s_1}-q_1^A}{\frac{\partial q_1^A}{\partial s_1}}\\ &= \frac{(k+1)Q^A-(k+2)q_0^A+(k+2)^2q_1^A+s_0-(k+1)(k+3)q_1^A}{k+2}\\ &= \frac{-q_0^A+(k+2)q_1^A+s_0}{k+2} \quad \Leftrightarrow s_0-(k+2)s_1=q_0^A-(k+2)q_1^A.\end{aligned} \tag{7.15}$$

By arranging the final equations of (7.14) and (7.15) with respect to s_0 and s_1, we obtain the two following equations:

$$s_0=q_0^A=\frac{(k+1)a+(k+2)s_0-s_1}{(k+1)(k+3)}\Leftrightarrow \left(k^2+3k+1\right)s_0+s_1=(k+1)a. \tag{7.16}$$

$$s_1=q_1^A=\frac{(k+1)a-s_0+(k+2)s_1}{(k+1)(k+3)}\Leftrightarrow s_0+\left(k^2+3k+1\right)s_1=(k+1)a. \tag{7.17}$$

By solving the simultaneous equations (7.16) and (7.17) with respect to s_0 and s_1, we obtain the welfare-maximizing subsidy levels of both firms (s_0^{A*}, s_1^{A*}) as follows:

$$\left(s_0^{A*}, s_1^{A*}\right)=\left(\frac{a}{k+2},\frac{a}{k+2}\right). \tag{7.18}$$

Both firms' subsidy levels are identical, that is, $s_0^{A*}=s_1^{A*}$. By substituting the optimal subsidy levels after privatization (s_0^{A*}, s_1^{A*}) in Table 7.3, we obtain the SPNE variables as shown in Table 7.4.

The optimal subsidy levels of both firms after privatization are uniquely determined in contrast to the public firm's subsidy level before privatization. The following relationships regarding the SPNE variables hold: $s_0^{A*}=s_1^{A*}=q_0^A=q_1^A$, $p^A+s_0^{A*}=(k+1)q_0^A$, $p^A+s_1^{A*}=(k+1)q_1^A$, and $\pi_0^A=\pi_1^A=\frac{k+2}{2}\left(q_i^A\right)^2$. It is clear that both firms produce the same amount, and the optimal subsidy levels are also equal between firms because both the public and private firms are identical after privatization. Although before privatization s_0^B is not determined endogenously and as a result π_0^B is also indeterminate, after privatization, the optimal subsidy level of the public firms s_0^{A*} and also its profit π_0^A are determined.

It should be noted that even after privatization marginal cost pricing is realized, that is, $p^A=c'\left(q_i^A\right)$, and the first-best maximized social welfare is achieved in the

Table 7.4 SPNE after privatization

Optimal public firm's subsidy	s_0^{A*}	$\frac{a}{k+2}$
Optimal private firm's subsidy	s_1^{A*}	$\frac{a}{k+2}$
Public firm's output	q_0^A	$\frac{a}{k+2}$
Private firm's output	q_1^A	$\frac{a}{k+2}$
Total output	Q^A	$\frac{2a}{k+2}$
Price	p^A	$\frac{ka}{k+2}$
Public firm's profit	π_0^A	$\frac{a^2}{2(k+2)}$
Private firm's profit	π_1^A	$\frac{a^2}{2(k+2)}$
Social welfare	W^A	$\frac{a^2}{k+2}$

equilibrium. After privatizing, both the public firm and private firm are induced with a subsidy to produce an output level that satisfies the marginal cost pricing.

From the above result, we confirm that even when adopting a discriminatory subsidy, the government sets a uniform subsidy for each firm and the introduction of the uniform subsidy is justifiable to maximize social welfare.

7.3.3 The Privatization Neutrality Theorem

Comparing the SPNE before and after privatization, we can confirm that the privatization neutrality theorem is satisfied. In the benchmark case where the public firm maximizes social welfare before privatizing, the privatization neutrality theorem is summarized as follows.

Proposition 7.1 *Suppose that the government sets the optimal discriminatory specific subsidy for a public firm and a private firm before and after privatization. The optimal subsidy level of the private firm, the public firm's and private firm's outputs, and social welfare are equal before and after privatization. Moreover, the outputs of both the public firm and private firm are identical. That is,* $s_1^{B*} = s_1^{A*}$, $\left(q_0^B, q_1^B\right) = \left(q_0^A, q_1^A\right)$, *and* $W^B = W^A$. *The optimal subsidy level of the public firm* s_0^{A*} *is uniquely determined only after privatizing.*

Proof These results are obviously derived by comparing the SPNE variables in Table 7.2 with those in Table 7.4.

Proposition 7.1 is a benchmark result that has been already shown in previous studies. It claims that when the government appropriately sets optimal subsidy levels to both public and private firms, first-best social welfare can be achieved, and the levels of social welfare before and after privatization are equal. Moreover, coincidentally, the optimal subsidy level before and after privatization is the same. Proposition 7.1 holds for the following reasons. The optimal level of social welfare is achieved when marginal cost pricing is implemented. The government can adjust each firm's output and implement marginal cost pricing by subsidizing both firms at an appropriate level. Because the public firm maximizes social welfare before privatizing, the output level that satisfies marginal cost pricing is fulfilled on its own without any policy induction.

In the next section, we consider the case in which the public firm maximizes objectives other than social welfare and investigate whether the privatization neutrality theorem continues to hold under a discriminatory subsidy policy.

7.4 When a Public Firm Maximizes Objectives Other than Social Welfare

In this section, we consider the case in which a public firm has an objective other than social welfare maximization. Previous studies have investigated whether privatization is neutral from the viewpoint of social welfare in a mixed oligopoly in which private firms have objectives other than profit maximization. For example, Kato and Tomaru (2007) clarified that even when private firms pursue general purposes other than profit maximization (e.g., total revenue maximization or total cost minimization), the privatization neutrality theorem continues to hold. They insist that for the neutrality theorem to hold, it is essential that the objective of the public firm is to maximize social welfare irrespective of the objectives of private firms. Therefore, in the present study, we do not investigate what happens when the objective of a private firm is generalized. Instead, we only focus on the situation in which a public firm pursues a general objective other than social welfare, while the private firm seeks to maximize its own profit as usual.[5]

First, we describe the generalized objective of the public firm before and after privatization. As social welfare is denoted by $W = (1/2)Q^2 + \pi_0 + \pi_1 - (s_0 q_0 + s_1 q_1)$, we represent the generalized objective of the public firm as follows:

$$V \equiv \alpha CS + \beta_0 \pi_0 + \beta_1 \pi_1 - \gamma (s_0 q_0 + s_1 q_1). \tag{7.19}$$

The objective of the public firm depends on four weighted parameters $(\alpha, \beta_0, \beta_1, \gamma)$. Furthermore, α, β_0, β_1, and γ denote the weights of the objective function on consumer surplus, public firm's profit, private firm's profit, and payment to the subsidy (or tax burden if the subsidy is covered by tax), respectively. The above formulation can cover a variety of purposes of the public firm. For example, when $(\alpha, \beta_0, \beta_1, \gamma) = (1, 0, 0, 0)$ and (0, 1, 1, 0), the public firm maximizes

[5]A recent study by Kato (2008) focused on the divergence of government and public firm objectives and investigated the optimal subsidy level and the maximized social welfare level when neutrality does not hold. However, that model assumed that the government itself pursues an objective other than social welfare, while the public firm maximizes social welfare under a uniform subsidy scheme. This differs from our setting in which the government maximizes social welfare and the public firm pursues other objectives.

the consumer surplus and producer surplus, respectively. When $(\alpha, \beta_0, \beta_1, \gamma) = (1, 1, 1, 1)$, the objective is the same as social welfare maximization before privatization as usual. When $(\alpha, \beta_0, \beta_1, \gamma) = (0, 1, 0, 0)$, it is the same as the public firm's profit maximization after privatization as usual, as shown in Sect. 7.3. Moreover, $(\alpha, \beta_0, \beta_1, \gamma) = (\alpha, 1, \alpha, \alpha)$ implies that the public firm is partially privatized because its objective is the weighted average between the social welfare and the public firm's profit by the ratio of $(\alpha, 1 - \alpha)$.[6] Furthermore, when $(\alpha, \beta_0, \beta_1, \gamma) = (1, 1, 1, 0)$, it implies that the public firm maximizes social welfare without taking a welfare loss (caused by a subsidy payment) into consideration. When $(\alpha, \beta_0, \beta_1, \gamma) = (0, 0, 0, 1)$, the public firm aims to minimize the subsidy payment (or equivalently, maximize the tax revenue).

In the benchmark analysis in Sect. 7.3, we compared only two cases, $(\alpha, \beta_0, \beta_1, \gamma) = (1, 1, 1, 1)$ and $(\alpha, \beta_0, \beta_1, \gamma) = (0, 1, 0, 0)$, and showed that social welfare does not change before and after privatization. Tomaru's (2006) seminal study demonstrated that even if we consider partial privatization as the generalized case, that is, $(\alpha, \beta_0, \beta_1, \gamma) = (\alpha, 1, \alpha, \alpha)$, social welfare does not change before and after privatization. As other examples, if the public firm cares more (less) about tax revenue, parameter γ becomes larger (smaller) than β_0.

In the above setting regarding the public firm's objective function, we investigate whether the privatization neutrality theorem holds. To solve the SPNE by backward induction, in Sect. 7.4.1, we first consider Cournot competition between a public firm and a private firm in the second stage. In Sect. 7.4.2, we consider the first stage in which the welfare-maximizing government sets optimal discriminatory subsidy levels.

7.4.1 The Second Stage

In the second stage, before privatization, the public firm maximizes the objective denoted by (7.19) with respect to q_0. In this chapter, we limit the discussion to the situation where the equilibrium output is the interior solution to simplify the analysis. Noting that as $CS = (1/2)Q^2$, $\pi_0 = (a - Q + s_0)q_0 - (k/2)q_0^2$, and $\pi_1 = (a - Q + s_1)q_1 - (k/2)q_1^2$, these derivatives with respect to q_0 are $\frac{\partial CS}{\partial q_0} = Q$, $\frac{\partial \pi_0}{\partial q_0} = a - Q + s_0 - q_0 - kq_0$, and $\frac{\partial \pi_1}{\partial q_0} = -q_1 < 0$; the first-order condition for the public firm is derived as follows:

[6] Note that as the objective is social welfare (public firm's profit) maximization when $(\alpha, \beta_0, \beta_1, \gamma) = (1, 1, 1, 1)$ ($(0, 1, 0, 0)$), we obtain $(\alpha, 1, \alpha, \alpha)$ by taking a weighted average among them at a rate of $(\alpha, 1 - \alpha)$.

$$\begin{aligned}&\frac{\partial V}{\partial q_0} = \alpha \frac{\partial CS}{\partial q_0} + \beta_0 \frac{\partial \pi_0}{\partial q_0} + \beta_1 \frac{\partial \pi_1}{\partial q_0} - \gamma s_0 = 0\\&\Leftrightarrow \alpha Q + \beta_0[a - Q + s_0 - (k+1)q_0] - \beta_1 q_1 - \gamma s_0 = 0\\&\Leftrightarrow q_0 = r_0(q_1) \equiv \frac{\beta_0 a - (\gamma - \beta_0)s_0 + (\alpha - \beta_0 - \beta_1)q_1}{\beta_0(k+2) - \alpha}.\end{aligned} \tag{7.20}$$

If $\alpha \neq \beta_0 + \beta_1$, the reaction function depends on q_1. Moreover, if $\beta_0 \neq \gamma$, the reaction function of the public firm, (7.20), depends on its own subsidy level s_0. This implies that if $\beta_0 = \gamma$, the subsidy level s_0 only affects the distribution between the public firm and other economic agents without altering the social welfare level.

In contrast, suppose that the fully privatized public firm maximizes its own profit. As the public firm's profit is $\pi_0 = (a - Q + s_0)q_0 - (k/2)q_0^2$, the first-order condition for the fully privatized public firm is given as follows:

$$\frac{\partial \pi_0}{\partial q_0} = (a - Q + s_0) - q_0 - kq_0 = 0 \Leftrightarrow q_0 = r_0(q_1) \equiv \frac{a + s_0 - q_1}{k+2}. \tag{7.21}$$

The reaction function depends on s_0. However, this case is a specific case among more generalized ones because $(\alpha, \beta_0, \beta_1, \gamma) = (0, 1, 0, 0)$. Thus, in the following analysis, we do not distinguish between before and after privatization and derive the equilibrium under generalized parameters, $(\alpha, \beta_0, \beta_1, \gamma)$.

The private firm maximizes its own profit. The first-order condition for a private firm is as follows:

$$\frac{\partial \pi_1}{\partial q_1} = (a - Q + s_1) - q_1 - kq_1 = 0 \Leftrightarrow q_1 = r_1(q_0) \equiv \frac{a + s_1 - q_0}{k+2}. \tag{7.22}$$

The reaction function of the private firm, (7.22), depends on its own subsidy level s_1.

By solving the reaction functions (7.20) and (7.22) with respect to q_0 and q_1, we derive the equilibrium variables in Table 7.5. The following equations hold: $p + s_1 = (k+1)q_1$, $\pi_0 = [p + s_0 - (k/2)q_0]q_0$, $\pi_1 = \frac{k+2}{2}q_1^2$, and $q_0 = B_0/C_0$. Whether q_0 or q_1 is larger depends on the relative sizes of s_0 and s_1. Likewise, the magnitude relationship between π_0 and π_1 also depends on (s_0, s_1).

It should be noted that when $\beta_0 = \gamma$, the equilibrium variables in the second stage (except for the public firm's profit) are independent of the public firm's subsidy level s_0 as shown in Table 7.5. The reason is that $\beta_0 = \gamma$ implies that the government and public firm correctly evaluate the impacts that the subsidy and tax have on social welfare and assigns an appropriate weight to tax and subsidy on social welfare. When $\beta_0 = \gamma$, the public firm properly recognizes that the same amount of tax levied from an economic entity subsidizes the public firm, and the total amount of social welfare does not change at all. Thus, the subsidy level to the public firm s_0 only affects income distribution and not welfare size.

Table 7.5 Cournot–Nash equilibrium in the second stage in the general case

Public firm's output	q_0	$\frac{[\alpha+\beta_0(k+1)-\beta_1]a-(\gamma-\beta_0)(k+2)s_0+(\alpha-\beta_0-\beta_1)s_1}{C_0}$
Private firm's output	q_1	$\frac{[\beta_0(k+1)-\alpha]a+(\gamma-\beta_0)s_0+[\beta_0(k+2)-\alpha]s_1}{C_0}$
Total output	Q	$\frac{[2\beta_0(k+1)-\beta_1]a-(\gamma-\beta_0)(k+1)s_0+[\beta_0(k+1)-\beta_1]s_1}{C_0}$
Price	p	$\frac{(k+1)[\beta_0(k+1)-\alpha]a+(\gamma-\beta_0)(k+1)s_0-[\beta_0(k+1)-\beta_1]s_1}{C_0}$
Public firm's profit	π_0	$\frac{A_0 B_0}{2C_0^2}$
Private firm's profit	π_1	$\frac{(k+2)\{[\beta_0(k+1)-\alpha]a+(\gamma-\beta_0)s_0+[\beta_0(k+2)-\alpha]s_1\}^2}{2C_0^2}$
Social welfare	W	$(1/2)Q^2+\pi_0+\pi_1-s_0q_0-s_1q_1$

$$A_0 \equiv [\beta_0(k+1)(k+2)+\beta_1 k-\alpha(3k+2)]a + \left[\beta_0(k+2)^2-2\beta_1+\gamma\left(k^2+4k+2\right)-2\alpha(k+1)\right]s_0-[\alpha k+(\beta_0-\beta_1)(k+2)]s_1$$
$$B_0 \equiv [\alpha+\beta_0(k+1)-\beta_1]a-(\gamma-\beta_0)(k+2)s_0+(\alpha-\beta_0-\beta_1)s_1$$
$$C_0 \equiv \beta_0(k+1)(k+3)-\beta_1-\alpha(k+1)$$

In contrast, when $\beta_0 \neq \gamma$, the public firm does not adequately recognize the weight of the tax and subsidy on social welfare, and the government and the public firm achieve different levels of recognition with respect to the effect of the subsidy on social welfare. Thus, the difference in their objectives distorts decision-making. When $\beta_0 < \gamma$, the public firm underestimates the effect of the subsidy policy because, for example, due to the distortion on taxation, the public firm recognizes that the amount of the tax levied from others is less than the subsidy paid to the public firm and that subsidization causes the inefficiency on income distribution. In contrast, when $\beta_0 > \gamma$, the public firm places greater weight on the public firm's profit than the government and overestimates the effectiveness of the subsidy. When $\beta_0 \neq \gamma$, a conflict of interest on taxation or subsidization between the government and the public firm arises, and the public firm that overestimates or underestimates the effect of the subsidy on social welfare adjusts its output depending on the given subsidy level. In sum, when $\beta_0 = \gamma$, the equilibrium variables (except for the public firm's profit) do not depend on s_0, and when $\beta_0 \neq \gamma$, all the equilibrium variables including social welfare depend on both firms' subsidy levels (s_0, s_1).

7.4.2 *The First Stage*

In the first stage, the government provides a discriminatory specific subsidy to a public firm and a private firm (s_0, s_1) to maximize social welfare. Because whether social welfare is a function of s_0 depends on whether $\beta_0 = \gamma$ holds, we divide the following analysis into two cases, $\beta_0 = \gamma$ and $\beta_0 \neq \gamma$, and examine them in order.

7.4.2.1 When a Public Firm Correctly Recognizes the Effect of the Subsidy ($\beta_0 = \gamma$)

When a public firm correctly recognizes that subsidization does not change the level of social welfare at all, social welfare only depends on s_1 as follows:

$$W(s_1) = \frac{1}{2}(Q(s_1))^2 + \pi_0(s_1) + \pi_1(s_1) - s_0 q_0 - s_1 q_1. \tag{7.23}$$

Therefore, q_0, q_1, Q, and π_1 do not depend on s_0. It should be noted that because $\pi_0 = [p + s_0 - (k/2)q_0]q_0$, social welfare also does not depend on s_0. In this case, the optimal subsidy level to a public firm is indeterminate from the viewpoint of social welfare maximization, which is similar to the result before privatizing in Sect. 7.3.1. Here we solve the optimal subsidy to the private firm s_1^*. By tedious calculation, we obtain the derivatives of the equilibrium variables with respect to s_1 as follows:

$$\begin{aligned} q_0'(s_1) &= \frac{\alpha - \beta_0 - \beta_1}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ q_1'(s_1) &= \frac{\beta_0(k+2) - \alpha}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ Q'(s_1) &= \frac{\beta_0(k+1) - \beta_1}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ p'(s_1) &= -Q', \\ \pi_0'(s_1) &= (p + s_0 - kq_0)q_0' - q_0 Q', \pi_1'(s_1) = (k+2)q_1 q_1'. \end{aligned} \tag{7.24}$$

The first-order condition for the government with respect to s_1 is as follows:[7]

$$\begin{aligned} W'(s_1) &= QQ' + \pi_0' + \pi_1' - s_0 q_0' - q_1 - s_1 q_1' = 0 \\ \Leftrightarrow\ s_1 &= \frac{QQ' + (p + s_0 - kq_0)q_0' - q_0 Q' + (k+2)q_1 q_1' - s_0 q_0' - q_1}{q_1'} \\ &= \frac{[a - (k+1)q_0]q_0' + (k+3)q_1 q_1' - q_1}{q_1'} \\ &= \frac{(\alpha - \beta_0 - \beta_1)a - (\alpha - \beta_0 - \beta_1)(k+1)q_0 + [\beta_0(k+3) + \beta_1 - 2\alpha]q_1}{\beta_0(k+2) - \alpha} \\ \Leftrightarrow\ s_1^* &= \frac{[\beta_0(k+1) - \alpha][\beta_0(k+1) - \beta_1] + (\alpha - \beta_0 - \beta_1)(\beta_1 - 2\alpha)k}{[\beta_0(k+2) - \alpha][\beta_0 k(k+3) - 2\beta_1 - \alpha(k-1)] + (\alpha - \beta_0 - \beta_1)^2(k+1)} a. \end{aligned} \tag{7.25}$$

[7]Throughout the analysis, we assume that the second-order condition is satisfied. Depending on the value of the parameters, $(\alpha, \beta_0, \beta_1, \gamma)$, it arises that the second-order condition is not satisfied, and the optimal subsidy level is a corner solution. We exclude such an uninteresting case from the analysis.

By substituting the optimal private firm's subsidy level obtained in (7.25) into the equilibrium variables in Table 7.5, we obtain the SPNE variables. However, because the equilibrium output is too complicated to describe, we omit the SPNE outcome in this study. In general, welfare size and SPNE outcomes differ depending on the parameters $(\alpha, \beta_0, \beta_1, \gamma)$, and the privatization neutrality theorem does not hold.

Therefore, we consider under which conditions the privatization neutrality theorem holds when $\beta_0 = \gamma$. To show the welfare neutrality when the public firm aims for social welfare maximization, suppose that the optimal private firm's subsidy level that we solved in (7.25) is equal to the optimal level we presented in Sect. 7.3, that is, $s_1^* = \frac{a}{k+2}$. From this equality, we obtain the following condition:

$$\begin{aligned} s_1^* &= \frac{[\beta_0(k+1)-\alpha][\beta_0(k+1)-\beta_1] + (\alpha-\beta_0-\beta_1)(\beta_1-2\alpha)k}{[\beta_0(k+2)-\alpha][\beta_0 k(k+3) - 2\beta_1 - \alpha(k-1)] + (\alpha-\beta_0-\beta_1)^2(k+1)} a \\ &= \frac{a}{k+2} \Leftrightarrow (\beta_0+\beta_1-2\alpha)[k(k+3)\alpha - \beta_0(k^2+2k-1) - \beta_1(k^2+3k+1)] \\ &= 0 \Rightarrow \beta_0+\beta_1 = 2\alpha. \end{aligned} \tag{7.26}$$

Thus, if $\beta_0 + \beta_1 = 2\alpha$, the optimal subsidy level is equal to that when the privatization neutrality theorem holds. When substituting this optimal subsidy level into the SPNE variables in Table 7.5, we can confirm that we derive the same equilibrium outcome as the SPNE outcome before privatization in Table 7.2. We summarize the SPNE outcome in Table 7.6.

Only when $\beta_0 + \beta_1 = 2\alpha$, the maximized social welfare is attained, and the privatization neutrality is satisfied. If $\beta_0 = \beta_1 \equiv \beta$, that is, the public firm puts equal weight to both firms' profits on social welfare, this condition is rewritten as $\alpha = \beta$. In other words, if the public firm regards consumer surplus and producer surplus to be equally important, the privatization neutrality theorem recovers, and the government can achieve maximized social welfare. In particular, two representative examples are $(\alpha, \beta_0, \beta_1, \gamma) = (1, 0, 0, 0)$ and $(1, 1, 1, 1)$. The former (the latter) implies that the public firm pursues consumer surplus (social welfare) maximization. In the former case, the public firm does not put any weight on producer surplus so that $2\alpha > \beta_0 + \beta_1 = 0$ holds. When the public firm pursues consumer surplus maximization, neutrality does not hold.

In contrast, when $(\alpha, \beta_0, \beta_1, \gamma) = (1, 1, 1, 1)$, as already shown in Sect. 7.3, the government and public firm have a common objective—social welfare maximization. As consumer surplus and producer surplus are assigned an equal weight, neutrality holds. However, it should be noted that when $\beta_0 = \gamma$, the region of parameters in which privatization neutrality holds is quite limited. If $\beta_0 = \beta_1 \equiv \beta = \gamma$, the condition in which neutrality holds degenerates into $\alpha = \beta$, which indicates that the public firm is a social welfare maximizer and has a common interest with the government. It suggests that when $\beta_0 = \gamma$, neutrality

Table 7.6 SPNE when $\beta_0 + \beta_1 = 2\alpha$ and $\beta_0 = \gamma$

Optimal public firm's subsidy	s_0^*	Indeterminate
Optimal private firm's subsidy	s_1^*	$\frac{a}{k+2}$
Public firm's output	q_0	$\frac{a}{k+2}$
Private firm's output	q_1	$\frac{a}{k+2}$
Total output	Q	$\frac{2a}{k+2}$
Price	p	$\frac{ka}{k+2}$
Public firm's profit	π_0	$\frac{[ka+2(k+2)s_0]a}{2(k+2)^2}$
Private firm's profit	π_1	$\frac{a^2}{2(k+2)}$
Social welfare	W	$\frac{a^2}{k+2}$

cannot be satisfied in many situations in which the public firm has various objectives.

7.4.2.2 When a Public Firm Incorrectly Recognizes the Effect of Subsidy ($\beta_0 \neq \gamma$)

When a public firm incorrectly recognizes that subsidization changes welfare size, social welfare depends on (s_0, s_1) as follows:

$$\begin{aligned} W(s_0, s_1) = &\frac{1}{2}\Big(Q(s_0, s_1)\Big)^2 + \pi_0(s_0, s_1) + \pi_1(s_0, s_1) \\ &-s_0 q_0(s_0, s_1) - s_1 q_1(s_0, s_1). \end{aligned} \tag{7.27}$$

Here we solve the optimal subsidy levels for both public and private firms (s_0^*, s_1^*). By tedious calculation, we obtain the partial derivatives of the equilibrium variables with respect to s_0 and s_1 as follows:

$$\begin{aligned} \frac{\partial q_0}{\partial s_0} &= -\frac{(\gamma - \beta_0)(k+2)}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ \frac{\partial q_1}{\partial s_0} &= \frac{\gamma - \beta_0}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ \frac{\partial Q}{\partial s_0} &= -\frac{(\gamma - \beta_0)(k+1)}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\ \frac{\partial p}{\partial s_0} &= -\frac{\partial Q}{\partial s_0}, \end{aligned} \tag{7.28}$$

$$\begin{aligned}
\frac{\partial \pi_0}{\partial s_0} &= (p + s_0 - kq_0)\frac{\partial q_0}{\partial s_0} + \left(1 - \frac{\partial Q}{\partial s_0}\right) q_0, \; \frac{\partial \pi_1}{\partial s_0} = (k+2)q_1 \frac{\partial q_1}{\partial s_0}, \\
\frac{\partial q_0}{\partial s_1} &= \frac{\alpha - \beta_0 - \beta_1}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\
\frac{\partial q_1}{\partial s_1} &= \frac{\beta_0(k+2) - \alpha}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\
\frac{\partial Q}{\partial s_1} &= \frac{\beta_0(k+1) - \beta_1}{\beta_0(k+1)(k+3) - \beta_1 - \alpha(k+1)}, \\
\frac{\partial p}{\partial s_1} &= -\frac{\partial Q}{\partial s_1}, \\
\frac{\partial \pi_0}{\partial s_1} &= (p + s_0 - kq_0)\frac{\partial q_0}{\partial s_1} - q_0 \frac{\partial Q}{\partial s_1}, \\
\frac{\partial \pi_1}{\partial s_1} &= (k+2)q_1 \frac{\partial q_1}{\partial s_1}.
\end{aligned} \tag{7.29}$$

The partial derivatives with respect to s_1 are the same as those in (7.24).

The first-order conditions for the government with respect to s_0 and s_1 are as follows:

$$\begin{aligned}
\frac{\partial W}{\partial s_0} &= Q\frac{\partial Q}{\partial s_0} + \frac{\partial \pi_0}{\partial s_0} + \frac{\partial \pi_1}{\partial s_0} - q_0 - s_0 \frac{\partial q_0}{\partial s_0} - s_1 \frac{\partial q_1}{\partial s_0} = 0 \\
&\Leftrightarrow \; [a - (k+1)q_0]\frac{\partial q_0}{\partial s_0} = [s_1 - (k+3)q_1]\frac{\partial q_1}{\partial s_0} \\
&\Leftrightarrow \; s_1 - q_1 = -(k+2)[a - (k+1)q_0 - q_1].
\end{aligned} \tag{7.30}$$

$$\begin{aligned}
\frac{\partial W}{\partial s_1} &= Q\frac{\partial Q}{\partial s_1} + \frac{\partial \pi_0}{\partial s_1} + \frac{\partial \pi_1}{\partial s_0} - s_0 \frac{\partial q_0}{\partial s_1} - q_1 - s_1 \frac{\partial q_1}{\partial s_1} = 0 \\
\Leftrightarrow \; s_1 &= \frac{Q\frac{\partial Q}{\partial s_1} + (p + s_0 - kq_0)\frac{\partial q_0}{\partial s_1} - q_0\frac{\partial Q}{\partial s_1} + (k+2)q_1\frac{\partial q_1}{\partial s_1} - s_0\frac{\partial q_0}{\partial s_1} - q_1}{\frac{\partial q_1}{\partial s_1}} \\
&= \frac{[a - (k+1)q_0]\frac{\partial q_0}{\partial s_1} + (k+3)q_1\frac{\partial q_1}{\partial s_1} - q_1}{\frac{\partial q_1}{\partial s_1}} \\
&= \frac{(\alpha - \beta_0 - \beta_1)[a - (k+1)q_0] - [2\alpha - \beta_0(k+3) - \beta_1]q_1}{\beta_0(k+2) - \alpha} \\
\Leftrightarrow \; & [\beta_0(k+2) - \alpha](s_1 - q_1) = (\alpha - \beta_0 - \beta_1)[a - (k+1)q_0 - q_1].
\end{aligned} \tag{7.31}$$

By solving the simultaneous equations (7.30) and (7.31) with respect to (s_0, s_1), we obtain the following optimal subsidy levels:[8]

[8]See Appendix A.7.1 for the derivative process.

Table 7.7 SPNE when $\beta_0 \neq \gamma$

Optimal public firm's subsidy	s_0^*	$\frac{(2\alpha-\beta_0-\beta_1)a}{(\gamma-\beta_0)(k+2)}$
Optimal private firm's subsidy	s_1^*	$\frac{a}{k+2}$
Public firm's output	q_0	$\frac{a}{k+2}$
Private firm's output	q_1	$\frac{a}{k+2}$
Total output	Q	$\frac{2a}{k+2}$
Price	p	$\frac{ka}{k+2}$
Public firm's profit	π_0	$\frac{[2(2\alpha-\beta_0-\beta_1)+k(\gamma-\beta_0)]a^2}{2(\gamma-\beta_0)(k+2)^2}$
Private firm's profit	π_1	$\frac{a^2}{2(k+2)}$
Social welfare	W	$\frac{a^2}{k+2}$

$$s_0^* = \frac{(2\alpha - \beta_0 - \beta_1)a}{(\gamma - \beta_0)(k+2)},\ s_1^* = \frac{a}{k+2} = q_0 = q_1. \tag{7.32}$$

By (7.32), we confirm that both firms' outputs are identical, that is, $q_0 = q_1$, and the private firm's subsidy level is equivalent to the firm's output. This subsidy level is equal to that when the neutrality theorem holds. Moreover, when $\beta_0 + \beta_1 = 2\alpha$, the public firm's subsidy level is zero. In contrast, when $\beta_1 + \gamma = 2\alpha$, s_0^* is equal to s_1^*, and the optimal subsidy level is virtually uniform. For example, when the public firm is partially privatized, that is, when $(\alpha, \beta_0, \beta_1, \gamma) = (\alpha, 1, \alpha, \alpha)$, $\beta_1 + \gamma = 2\alpha$ holds, and the uniform subsidy scheme can achieve welfare neutrality, as shown in Tomaru (2006). By substituting the optimal subsidy levels obtained in (7.32) into the equilibrium variables in Table 7.5, we obtain the SPNE variables as shown in Table 7.7.

By choosing the optimal subsidy levels for both the public and private firms in an appropriate manner, the government can achieve maximized social welfare at any time when $\beta_0 \neq \gamma$. Moreover, the private firm's optimal subsidy level is always the same, and it does not depend on the weighting parameters on the public firm's objective. By making the government adjust the public firm's subsidy at an optimal level that is parameter dependent, the privatization neutrality theorem necessarily holds.

7.4.3 *The Generalized Privatization Neutrality Theorem*

Based on the results of the previous subsection, as summarized in Tables 7.6 and 7.7, we can summarize the generalized privatization neutrality theorem as the situation when a public firm pursues an objective different to that of the social welfare-maximizing government.

Proposition 7.2 *Suppose that a public firm has an objective other than social welfare maximization and the government sets the optimal discriminatory subsidy for both public and private firms*:

(i) *When the public firm appropriately evaluates income distribution by subsidization, the privatization neutrality theorem does not hold in general. More precisely, when $\beta_0 = \gamma$, neutrality only holds if $\beta_0 + \beta_1 = 2\alpha$.*
(ii) *When the public firm incorrectly recognizes that subsidization affects welfare size, that is, $\beta_0 \neq \gamma$, the privatization neutrality theorem necessarily holds.*

Proposition 7.2(i) implies that even if the government optimally sets discriminatory subsidies to public and private firms, the privatization neutrality theorem might not be satisfied. Even when the public firm properly recognizes that the subsidy only affects income redistribution and not the size of social welfare, there exists the case in which the government cannot achieve maximized social welfare. In contrast, Proposition 7.2(ii) implies that when the public firm overestimates or underestimates the income redistribution effect caused by subsidization, the government can necessarily achieve maximized social welfare by implementing a discriminatory subsidy scheme. At first glance, this result seems paradoxical because this proposition claims that if a conflict of interest exists between the regulated public firm and government, the optimal first-best welfare level can be always implemented with a discriminatory subsidy policy but also vice versa. The reason why this proposition comes out is that when the public firm and the government have a common objective, the public firm's subsidy does not work at all and the policy instruments available to the government are limited. In this case, only when the public firm is also a social welfare maximizer can the first-best social welfare be achieved without any subsidy to the public firm, and privatization neutrality holds. In contrast, when a discrepancy exists between the goals of the government and regulated public firm, the government can utilize the difference in their objectives as an incentive scheme for the public firm to choose the appropriate quantity level. When the subsidy to the public firm functions effectively, first-best social welfare is necessarily guaranteed with the appropriate setting of the discriminatory subsidy to both public and private firms.

7.5 Concluding Remarks

This chapter investigated whether privatization neutrality holds when a public firm has an objective other than social welfare maximization and the government sets the optimal discriminatory subsidy for both public and private firms in a mixed oligopoly. We demonstrated that when the government adopts a discriminatory subsidy scheme for both a public firm and a private firm, welfare neutrality can be recovered in many situations. Notably, when a public firm incorrectly recognizes that subsidization causes income redistribution, the government can necessarily achieve maximized social welfare via the difference in recognition. In such a case, privatization neutrality necessarily holds. In contrast, when a public firm correctly recognizes that subsidization only affects income distribution and not social welfare, the economic situation in which neutrality holds is considerably limited. Our results suggest that if a government can effectively implement a discriminatory subsidy scheme for a public firm and a private firm as a regulatory instrument, the government can achieve the optimal result irrespective of the objective of the pubic firm. Together with the results of previous research in which the objective of the private firms does not matter for welfare neutrality to hold, our result suggests that irrespective of the objectives of both public and private firms in a mixed oligopoly, privatization neutrality can be supported in considerably broader economic circumstances.

Finally, we finish with a brief discussion of extension. First, this study only focused on the situation in which the public firm has a different objective to that of the government seeking social welfare maximization. As a further extension, it is natural that we extend the analysis to the situation in which the government also pursues objectives other than social welfare maximization, as already examined in several previous studies. For example, Kato (2008) investigated what happens when the government is also concerned about tax revenue as well as social welfare, while the public firm aims to maximize social welfare. However, such an extension can further complicate the analysis. In addition, according to our present investigation, if the government also has a different objective from social welfare maximization, the privatization neutrality theorem does not hold in most situations. Second, there will be room to incorporate asymmetric information into the model. Whether the government sets a uniform or discriminatory subsidy scheme might depend on the ability of the government or regulatory authorities to gather information about regulated firms. Since the seminal work of Laffont and Tirole (1993), numerous studies have investigated optimal regulation schemes under asymmetric information. If the appropriate choice among policy instruments, for example, a uniform or discriminatory subsidy scheme, depends on the information structure of the government, this will be a further important issue to explicitly incorporate into the model (e.g., the difference in information structures under asymmetric information). Third, our results show that when the government can effectively use policy instruments in a mixed oligopoly, a first-best result is achievable. From a mathematical viewpoint, the privatization neutrality theorem could be categorized as an envelope theorem. Thus, future research should investigate whether the privatization neutrality theorem under a discriminatory subsidy could prove to be an envelope theorem.

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Appendix

Derivation of the Optimal Subsidy Levels (s_0^*, s_1^*) when $\boldsymbol{\beta_0 \neq \gamma}$

From the two equations, (7.30) and (7.31), $\{(\alpha-\beta_0-\beta_1)+(k+2)[\beta_0(k+2)-\alpha]\}[a-(k+1)q_0-q_1]=0$ holds. Because generally $(\alpha-\beta_0-\beta_1)+(k+2)[\beta_0(k+2)-\alpha]$ is not equal to zero for any $k>0$, $a-(k+1)q_0-q_1=0$ holds. Thus, it is confirmed that $s_1=q_1$ and $q_0=\frac{a-q_1}{k+1}=\frac{a-s_1}{k+1}$. When arranging the latter equation, we obtain the following equations:

$$
\begin{aligned}
q_0 &= \frac{[\alpha+\beta_0(k+1)-\beta_1]a-(\gamma-\beta_0)(k+2)s_0+(\alpha-\beta_0-\beta_1)s_1}{\beta_0(k+1)(k+3)-\beta_1-\alpha(k+1)}=\frac{a-s_1}{k+1}\\
&\Leftrightarrow\ (\gamma-\beta_0)(k+1)(k+2)s_0+[\beta_1-\beta_0(k+1)](k+2)s_1\\
&=[2(k+1)\alpha-2\beta_0(k+1)-\beta_1 k]a,
\end{aligned}
\tag{A.7.1}
$$

$$
\begin{aligned}
q_1 &= \frac{[\beta_0(k+1)-\alpha]a+(\gamma-\beta_0)s_0+[\beta_0(k+2)-\alpha]s_1}{\beta_0(k+1)(k+3)-\beta_1-\alpha(k+1)}=s_1\\
&\Leftrightarrow (\gamma-\beta_0)s_0+[\alpha k-\beta_0(k^2+3k+1)+\beta_1]s_1\\
&=[\alpha-\beta_0(k+1)]a.
\end{aligned}
\tag{A.7.2}
$$

By solving the simultaneous equations, (A.7.1) and (A.7.2), we obtain the optimal subsidy levels as follows:

$$
s_0^*=\frac{(2\alpha-\beta_0-\beta_1)a}{(\gamma-\beta_0)(k+2)},\ s_1^*=\frac{a}{k+2}=q_0=q_1. \tag{A.7.3}
$$

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Chapter 8
Political Economic Analysis of Privatization

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Abstract This chapter analyzes the effect of domestic lobbying on the optimal degree of privatization and social surplus in a closed mixed oligopoly model and an extended two-country model. We find that lobbying activity leads to overprivatization in a closed economy and may improve social welfare in a two-country economy. When each country's benevolent government determines the optimal privatization level, the privatization level always leads to underprivatization. This means an open trade policy leads to underprivatization. However, our results show that overprivatization may also exist in an open economy.

8.1 Introduction

This chapter analyzes the effect of domestic lobbying on the optimal degree of privatization and social surplus in a mixed oligopoly model. Matsumura (1998) found that "neither full privatization (the government does not hold any shares) nor full nationalization (the government holds all of the shares) is optimal under moderate conditions." Thus, the government can achieve the optimal allocation by determining the privatization ratio as in Chap. 1. However, previous studies have not explained why the optimal privatization policy does not proceed if the above circumstance prevails. We argue that lobbying activity is one of the reasons for this problem. To our knowledge, no studies to date analyze a mixed oligopoly with the effect of lobbying activity from a theoretical perspective. From an empirical point of view, as in Ang and Boyer (2007), special interest groups in public firms lobby politicians and political parties and induce desirable policies via campaign contributions.

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Except for the studies of mixed oligopoly, numerous studies have investigated the effect of lobbying activity. Grossman and Helpman (1994) analyzed the effect of lobbying on trade protection using a small-country model and found that trade protection, or the optimal tariff rate, should be higher in industries characterized by lobbying. Goldberg and Maggi (1999) verified the same by empirical investigation. Moreover, Kagitani (2008) extended the lobbying model to a strategic trade policy.[1]

Based on these considerations, we analyze the effect of lobbying activity on privatization policy and social welfare using a closed model and an extended two-country model. We find that lobbying activity leads to overprivatization in a closed economy and may improve social welfare in a two-country economy.

As in Han and Ogawa (2008) and Chap. 5, when each country's government can determine the optimal privatization level, underprivatization always emerges as a strategic policy. This implies that an open trade policy is a factor in underprivatization. Thus, they point out the need to increase the level of privatization as a coordination policy of the two countries. However, our results show that overprivatization is still possible based on the political behavior of the policymakers.

The structure of this chapter is as follows. In the next section, we construct a closed-economy model with or without lobbying activity by private firms. In Sect. 8.3, we extend the closed-economy model to that of a two-country model. Section 8.4 discusses the policy implications and explores future research possibilities.

8.2 Closed Economy

We describe a mixed oligopoly model with lobbying activity by a private firm. This section investigates the effect of lobbying activity on the optimal privatization policy in a closed economy following Han and Ogawa (2008), and Sect. 8.3 extends this model to a two-country model.

8.2.1 Basic Setting in a Closed Economy

Consider a market in a closed economy served by a partially privatized firm (firm 0) jointly owned by the government and the private sector and a pure private firm (firm 1). Both firms produce a homogenous good, with q_0 and q_1 representing the quantity of output of the public firm and the private firm, respectively. The firms face the inverse demand function $p = 1 - 2Q$, where p and $Q \equiv q_0 + q_1$ denote the price of goods and the aggregate output in this market. Both firms have identical cost

[1] Bernheim and Whinston (1986) and Dixit et al. (1997) developed the lobbying model to extend the theory of common agency.

functions, $c_j = \frac{1}{2}q_j,\ j = 0, 1$; thus, the revenue function of each firm becomes $\pi_j = \{1 - 2(q_0 + q_1)\}q_j - \frac{1}{2}q_j,\ j = 0, 1$. Assuming the demand function is linear, consumer surplus, CS, becomes $CS = Q^2$. Therefore, social welfare is $W = CS + \pi_0 + \pi_1$.

Following Matsumura (1998), the government owns a share of the partially privatized firm, $1 - \theta \in [0, 1]$. Here θ can be seen as the degree of privatization: $\theta = 1$ means a fully privatized firm, and $\theta = 0$ means a fully nationalized firm. Thus, the partially privatized firm's objective function becomes the sum of social welfare and the producer surplus:

$$V = \theta \pi_0 + (1 - \theta)W.$$

Maximizing the above objective function of the public firm and the revenue function of the private firm, we obtain the equilibrium output of the partially privatized firm, q_0^*, the private firm, q_1^*, and the equilibrium price, p^*, as

$$q_0^* = \frac{3}{11 + 10\theta}, \quad q_1^* = \frac{1 + 2\theta}{11 + 10\theta}, \tag{8.1}$$

and

$$p^* = \frac{3(1 + 2\theta)}{11 + \theta}$$

where the effect of privatization on the equilibrium outputs of both firms and the equilibrium price are $\frac{dq_0^*}{d\theta} = -\frac{30}{(11+10\theta)^2} < 0$, $\frac{dq_1^*}{d\theta} = \frac{12}{(11+10\theta)^2} > 0$, and $\frac{dp^*}{d\theta} = \frac{36}{(11+10\theta)^2} > 0$, respectively.

Using (8.1), the producer surplus (or profits) of both firms and the consumer surplus become

$$\pi_0^* = \frac{9(1 + 4\theta)}{2(11 + 10\theta)^2}, \quad \pi_1^* = \frac{5(1 + 2\theta)^2}{2(11 + 10\theta)^2}, \tag{8.2}$$

and

$$CS^* = \frac{4(2 + \theta)^2}{(11 + 10\theta)^2}.$$

The effect of privatization on the producer surplus of both firms and the consumer surplus can be also calculated as

$$\begin{aligned}\frac{d\pi_0^*}{d\theta} &= \frac{36(3-5\theta)}{(11+10\theta)^2},\\ \frac{d\pi_1^*}{d\theta} &= \frac{60(1+2\theta)}{(11+10\theta)^2} > 0,\end{aligned} \tag{8.3}$$

and

$$\frac{dCS^*}{d\theta} = -\frac{72(2+\theta)}{(11+10\theta)^2} < 0.$$

From (8.2), equilibrium social welfare can be obtained as $W^* = \frac{23+2\theta(22+7\theta)}{(11+10\theta)^2}$, and then we can obtain the optimal privatization ratio, which satisfies the maximization condition $\frac{dW^*}{d\theta} = 0$ as $\theta^* = \frac{2}{11}$ under a closed economy.

Lemma 8.1 *Optimal privatization ratio in a closed economy* (*Han and Ogawa* 2008).

In a closed economy, partial privatization is the optimal policy, and the privatization ratio is $\theta^* = \frac{2}{11}$.

8.2.2 Political Economic Model in a Closed Economy

In this subsection, we analyze the effect of lobbying by a private firm on the optimal privatization ratio. From the above analysis, the private firm has an incentive to influence the degree of privatization because an increase in θ decreases the output of the public firm and increases the revenue of the private firm. We assume that the private firm can provide contributions, Z, to the policymakers in return for influencing the privatization ratio, θ. Thus, the private firm offers a differentiable contribution schedule for the privatization ratio, $Z(\theta)$, to the policymakers ($Z'(\theta) > 0$). As a result, the payoff for the private firm is

$$\Pi_1 = \pi_1 - Z(\theta). \tag{8.4}$$

As in Cai and Li (2014), policymakers care about the level of the campaign contribution and the social surplus because the number of votes depends not only on the size of the campaign contribution but also on the public endorsement. Thus, the objective function of the policymaker is the sum of the consumer surplus, the producer surplus of the public firm, the producer surplus of the private firm, and the political contributions:

$$G(\theta) = CS + \pi_0 + \Pi_1 + \gamma Z(\theta) = CS + \pi_0 + \pi_1 + (\gamma - 1)Z(\theta), \tag{8.5}$$

where $\gamma(>1)$ denotes the weight that policymakers put on the political contributions of private firms.

Next, we consider the amount of political contributions from the private firm. Following Grossman and Helpman (1994), we focus on a truthful contribution schedule: $Z(\theta) = \max\{0, \pi_1(\theta) - b\}$. The maximization condition of the private firm has to satisfy

$$\frac{\partial \pi_1}{\partial \theta} = \frac{\partial Z}{\partial \theta}, \tag{8.6}$$

and when this condition is satisfied, $Z > 0$.

We consider a three-stage game. In the first stage, the private firm offers a campaign contribution schedule to the policymaker. In the second stage, the policymaker determines the privatization level. In the third stage, the private firm and the public firm compete in a market. The game is solved by backward induction.

8.2.3 Optimal Privatization Policy and Lobbying Activity in a Closed Economy

In the third stage, both firms choose their outputs to maximize their revenue given in (8.1). In the second stage, the policymaker chooses the optimal privatization policy. In the first stage, the two firms determine the contribution schedule.

The policymaker determines the optimal privatization policy to maximize (8.5) subject to (8.6). Then we obtain

$$(\gamma - 1)\frac{d\pi_1(\theta)}{d\theta} + \frac{dW(\theta)}{d\theta} = \frac{12(10\gamma\theta + 5\gamma - 21\theta - 3)}{(11 + 10\theta)^3}.$$

From this equation, the equilibrium privatization ratio, θ^{pc}, becomes

$$\theta^{pc} = \frac{5\gamma - 3}{21 - 10\gamma} > \theta^*. \tag{8.7}$$

Thus, we find that the lobbying activity leads to overprivatization in a closed economy. Differentiating (8.7) by γ, the effect of the policymaker's preference on the privatization ratio is $\frac{\partial \theta^{pc}}{\partial \gamma} = \frac{75}{(21-10\gamma)^2} > 0$. This implies that the private firm can induce the policymaker to choose a preferable policy by providing campaign contributions.

Proposition 8.1 *The optimal privatization policy in a political economic equilibrium in a closed economy.*

In a political equilibrium with lobbying activity by a private firm, the privatization level is higher than the optimal level.

8.3 Two-Country Model

In this section, we extend the closed-economy model to a two-country model as in Chap. 5 and Han and Ogawa (2008). Thus, the basic model is the same as Sect. 8.2 except that there are two symmetric countries, d and f.

8.3.1 Basic Setting of a Two-Country Model

In each country, there is a single public firm and a single private firm. Firms in each country produce homogenous goods and compete in a Cournot fashion in a single integrated market. The inverse demand function of the integrated market is given by $p = 1 - 2\left(q_0^d + q_1^d + q_0^f + q_1^f\right)$, where p and q_j^i represent the market price and the amount of goods sold by firm j in country i.

As with the closed economy, we assume that both firms in each country have identical cost functions, $c_j^i = \frac{1}{2}q_j^i$, $i = d, f$, and $j = 0, 1$. Thus, the revenue function of both firms in each country becomes $\pi_j^i = \left[1 - 2\left(q_0^d + q_1^d + q_0^f + q_1^f\right)\right]q_j^i - \frac{1}{2}q_j^i$ and $j = 0, 1$. Note that the consumer surplus, CS^i, becomes $CS^i = \left(Q^i\right)^2 = 0.25Q^2$ because we assume that the two countries are identical and have $Q^d = Q^f$. Thus, social welfare in country i is given by $W^i = CS^i + \pi_0^i + \pi_1^i$.

The objective function of the manager of the public firm in each country becomes the sum of social welfare and the producer surplus of the firms of its country, $V^i = \theta^i\pi_0^i + \left(1 - \theta^i\right)W^i$, where θ^i represents the privatization ratio in country i. From the revenue maximization of the public and private firms, the outcomes of each firm in Nash equilibrium are

$$q_0^d = \frac{4 + \theta^f - \theta^d}{4\left(4 + \theta^d + \theta^f\right)}, \quad q_0^f = \frac{4 + \theta^d - \theta^f}{4\left(4 + \theta^d + \theta^f\right)}, \tag{8.8}$$

and

$$q_1^d = q_1^f = \frac{2 + \theta^d + \theta^f}{4(4 + \theta^d + \theta^f)}.$$

Substituting (8.8) to the inverse demand function, the world price becomes

$$p^w = \frac{1}{2} - \frac{1}{4 + \theta^d + \theta^f}. \tag{8.9}$$

Using (8.8) and (8.9), we obtain the equilibrium producer surplus (or revenue) of the firms and the consumer surplus in each country by

$$\begin{aligned} \pi_0^d &= \frac{(4 + 3\theta^d + 5\theta^f)(4 + \theta^f - \theta^d)}{32(4 + \theta^d + \theta^f)^2}, \\ \pi_0^f &= \frac{(4 + 3\theta^f + 5\theta^d)(4 + \theta^d - \theta^f)}{32(4 + \theta^d + \theta^f)^2} \\ \pi_1^d = \pi_1^f &= \frac{3(2 + \theta^d + \theta^f)^2}{32(4 + \theta^d + \theta^f)^2}, \end{aligned} \tag{8.10}$$

and

$$CS^d = CS^f = \frac{(6 + \theta^d + \theta^f)^2}{16(4 + \theta^d + \theta^f)^2}.$$

Differentiating (8.8), (8.9), and (8.10) with respect to the privatization ratio, we find that the effects of the privatization level on the output of all firms, the world price, the consumer surplus, and the producer surplus of all firms in both countries are as follows:

$$\begin{aligned}
&\frac{\partial q_0^i}{\partial \theta^i} = -\frac{4+\theta^{-i}}{(4+\theta^i+\theta^{-i})^2} < 0,\\
&\frac{\partial q_0^{-i}}{\partial \theta^i} = \frac{\theta^{-i}}{2(4+\theta^i+\theta^{-i})^2} > 0,\\
&\frac{\partial q_1^i}{\partial \theta^i} = \frac{\partial q_1^{-i}}{\partial \theta^i} = \frac{1}{2(4+\theta^i+\theta^{-i})^2} > 0,\\
&\frac{\partial p^w}{\partial \theta^i} = \frac{1}{(4+\theta^i+\theta^{-i})^2} > 0,\\
&\frac{\partial CS^i}{\partial \theta^i} = \frac{\partial CS^{-i}}{\partial \theta^i} = -\frac{6+\theta^i+\theta^{-i}}{4(4+\theta^i+\theta^{-i})^3} < 0,\\
&\frac{\partial \pi_0^i}{\partial \theta^i} = \frac{8-(\theta^{-i})^2-14\theta^i-\theta^{-i}(2+3\theta^i)}{8(4+\theta^i+\theta^{-i})^3},\\
&\frac{\partial \pi_0^{-i}}{\partial \theta^i} = \frac{3(\theta^{-i})^2-(2-\theta^i)\theta^{-i}+2(4+\theta^i)}{8(4+\theta^i+\theta^{-i})^3} > 0,
\end{aligned} \tag{8.11}$$

and

$$\frac{\partial \pi_1^i}{\partial \theta^i} = \frac{\partial \pi_1^{-i}}{\partial \theta^i} = \frac{3\left(2+\theta^i+\theta^{-i}\right)}{8\left(4+\theta^i+\theta^{-i}\right)^3} > 0,$$

where superscript $-i$ means the country's variables except for country i. From (8.11), the increase of the degree of domestic privatization decreases the consumer surplus in each country and increases the foreign firms' producer surplus. Therefore, the domestic policymaker has an incentive to decrease the privatization level.[2]

Using (8.10), social welfare in each country becomes

$$W^i = \frac{25+13\theta^i+\theta^{-i}(13+3\theta^i+2\theta^{-i})}{8(4+\theta^i+\theta^{-i})^2}. \tag{8.12}$$

Each benevolent policymaker maximizes (8.12) by choosing θ^i given by θ^{-i}. Thus, we obtain the first-order condition as follows:

$$\frac{dW^i}{d\theta^i} = \frac{13\theta^i+3\theta^i\theta^{-i}+\left(\theta^{-i}\right)^2+\theta^{-i}-2}{4+\theta^i+\theta^{-i}} = 0.$$

Solving this equation for θ^i, the optimal reaction function for the privatization policy of country i becomes $\theta^i = \frac{\left(1-\theta^{-i}\right)\left(2+\theta^{-i}\right)}{13+3\theta^{-i}}$. Using the symmetric assumption of the two countries, we obtain the optimal privatization level in Nash equilibrium:

[2] (8.11) means that an increase in θ^i decreases q_0^i and increases q_0^{-i} and p^w.

$$\theta^d = \theta^f = \frac{1}{4}\left(\sqrt{57} - 7\right) \tag{8.13}$$

Lemma 8.2 (*Proposition 1 in Han and Ogawa* (2008)).
The extent of privatization in an international mixed oligopoly with two countries is smaller than that in a mixed oligopoly with a single domestic market.

As mentioned above, privatization increases the producer surplus of the domestic public firm and the foreign private firm and decreases the consumer surplus.[3] Thus, the domestic policymaker prevents the flow of the domestic surplus to foreign producers. The coordinated problem provides the following first-order condition:

$$\frac{dW^w}{d\theta^i} = \frac{dW^i}{d\theta^i} + \frac{dW^{-i}}{d\theta^i} = \frac{\left(\theta^{-i}\right)^2 - 5\theta^i - \left(1+\theta^i\right)\theta^{-i} + 2}{4 + \theta^i + \theta^{-i}} = 0.$$

By solving this, the optimal privatization levels in the two countries' economies become

$$\theta^d = \theta^f = \theta^{w*} = \frac{1}{3}. \tag{8.14}$$

By comparing (8.13) and (8.14), we obtain Lemma 8.3.

Lemma 8.3 (*Proposition 2 in Han and Ogawa* (2008)).
When lobbying activity is prohibited, underprivatization occurs in an international mixed oligopoly with two countries.

8.3.2 Optimal Privatization Policy in a Two-Country Political Economic Model

In this section, we illustrate the effect of domestic lobbying activity on the privatization policy in a two-country setting.[4] To compare results, the basic setting is the same as in Sect. 8.3.1.

As in Sect. 8.2, the payoff of the private firm in each country is

$$\Pi_1^i = \pi_1^i - Z^i\left(\theta^i\right), \tag{8.15}$$

[3] See (8.11).

[4] In this chapter, we do not examine international lobbying or cross-border lobbying.

where $Z(\theta^i)$ represents the contribution schedule that the private firms optimally choose. The policymaker chooses the privatization level to maximize the weighted sum of social welfare, $CS^i + \pi_0^i + \Pi_1^i$, and $Z^i(\theta^i)$ as follows:

$$\begin{aligned} G^i\left(\theta^i\right) &= CS^i\left(\theta^i\right) + \pi_0^i\left(\theta^i\right) + \Pi_1^i\left(\theta^i\right) + \gamma^i Z^i\left(\theta^i\right), \\ &= CS^i\left(\theta^i\right) + \pi_0^i\left(\theta^i\right) + \pi_1^i\left(\theta^i\right) + (\gamma^i - 1)Z^i\left(\theta^i\right), \end{aligned} \tag{8.16}$$

where $\gamma^i(> 1)$ denotes the weight that the policymakers in each country put on the political contributions of domestic private firms. As in Sect. 8.2.3, private firms should select the following contribution schedule: $Z^i\left(\theta^i\right) = \max\left\{0, \pi_1^i\left(\theta^i\right) - b^i\right\}$. Thus, we obtain the maximization condition of the domestic lobbying agent in country i as

$$\frac{\partial \pi_1^i}{\partial \theta^i} = \frac{\partial Z^i}{\partial \theta^i}, \tag{8.17}$$

when this condition is satisfied, $Z^i > 0$.

8.3.3 Inference of Lobbying Activity into the Privatization Policy in a Two-Country Model

Policymakers in each country maximize the objective function (8.16) by choosing the privatization level subject to (8.17):

$$(\gamma^i - 1)\frac{d\pi_1^i(\theta^i)}{d\theta^i} + \frac{dW^i}{d\theta^i} = \frac{3\gamma^i(2 + \theta^i + \theta^{-i}) - (3\theta^{-i} + 16)\theta^i - (\theta^{-i} + 2)^2}{8(4 + \theta^i + \theta^{-i})^3} = 0.$$

By solving this first-order condition for θ^i, the reaction function of country i becomes $\theta^{pc,i} = \frac{\left(\theta^{-i}\right)^2 + 4\theta^{-i} - 3\theta^{-i}\gamma^i - 6\gamma^i + 4}{3\gamma^i - 3\theta^{-i} - 16}$. Under the symmetric assumption, the privatization level in a two-country economy becomes

$$\theta^{pc,i} = \frac{1}{4}\left(\sqrt{3}\sqrt{28 - 12\gamma^i + 3(\gamma^i)^2} + 3\gamma^i - 10\right) \gtreqless \theta^{w*}. \tag{8.18}$$

Thus, we find that this Nash equilibrium privatization level is affected by the weight of the campaign contribution, $\gamma^i > 0$. When the policymaker has a strong interest in campaign contributions compared with the flow of domestic social surplus, the policymaker increases the privatization level to acquire campaign contributions.

Proposition 8.2 *Optimal privatization policy in a political economic equilibrium in a two-country model.*

When a private firm lobbies policymakers and policymakers' interests are stronger (*weaker*) *than 25/18, underprivatization* (*overprivatization*) *occurs in an international mixed oligopoly with two countries.*

Proof By solving (8.18) to $\gamma^i > 0$, we obtain $\gamma^i = 25/18$.

This proposition implies that the strategic effect shown in Lemma 8.3 is canceled out by the domestic lobbying effect, which is shown in Proposition 8.1. When the politicians' interests are 25/18, the privatization level also corresponds to the optimal privatization level, θ^{w*}.

Corollary 8.1 *The Nash equilibrium privatization level in a two-country model corresponds to the socially optimal level when politicians' interests are 25/18.*

8.4 Discussion and Remaining Issues

This chapter analyzed the effect of lobbying activity by private firms on the optimal privatization level. As a result, we obtained two main results. First, in a closed economy, lobbying activity leads to overprivatization. Second, in a two-country economy, if and only if policymakers have a strong (weak) interest in campaign contributions, lobbying activity leads to overprivatization (underprivatization).

8.4.1 Implications of Lobbying Activity in a Closed Economy

In a closed economy, policymakers can control the output of the public firm by choosing the privatization level; here they choose lower partial privatization. However, private firms do not seek such lower partial privatization because it results in a greater output by public firms. As a result, this conflict between private and public leads to political pressure. Thus, private firms can increase their own revenue by providing campaign contributions. However, the size of the revenue depends on the interests of the policymaker. A higher level of privatization and higher revenue can be achieved when the policymaker has a high interest in campaign contributions.

The privatization level induced by lobbying activity leads to overprivatization rather than a socially optimal level. Thus, a high interest in campaign contributions by the policymaker reduces social welfare rather than increases the revenue of private firms.

8.4.2 Implications of Lobbying Activity in an Open Economy

In an open economy, two opposite effects are evident: the strategic effect (in a pure market), which was explained in Sect. 8.2, and the political effect, explained in Sect. 8.3. The former is caused by the strategic policy in each country. As in Han and Ogawa (2008) and Chap. 5, an open trade policy creates an outflow of domestic surplus. A benevolent policymaker chooses a low level of privatization to prevent such an outflow. The latter effect is caused by the lobbying activity of private firms. When a policymaker receives a campaign contribution, the private firm can increase its revenue by increasing the level of privatization.

Thus, we find that the optimal privatization policy is determined by two contrasting effects: the strategic effect reduces the privatization level, and the political effect increases the privatization level. Based on these considerations, when these two effects are canceled out, the privatization level corresponds to the socially optimal privatization level. This means that although only benevolent policymakers provide a low level of privatization, if a policymaker has an interest in obtaining campaign contributions, social surplus may increase in a two-country economy.

8.4.3 Other Types of Lobbying

Finally, we consider the possibility of extending this model to incorporate various types of lobbying effects.

First, we consider the case of lobbying competition between public and private firm. This differs from monopsonistic lobbying as in this chapter because the equilibrium output of the mixed oligopoly is characterized by asymmetric outputs. This situation induces different levels of equilibrium campaign contributions.

Second, we consider the effect of a change in the number of private firms. When private firms can increase their profits by providing campaign contributions, they always lobby policymakers. However, in general, lobbying itself is not conducted by one firm but by a number of firms. From this perspective, as in Mitra (1999) and Kagitani (2008), we can consider how a lobby is endogenously formed using a model with the organizational costs of lobby because the properties of lobbying are characterized by public goods that cause the free-rider problem.

Third, we consider the possibility of lobbying by foreign firms to domestic policymakers as in Huang et al. (2015). In this case, lobbying competition occurs between domestic and foreign firms, and the policymaker's objective function becomes $W^h = W^{h,\text{only}} + \varphi Z^f$, where the domestic social surplus, campaign contribution, and the preference parameter of the foreign campaign contribution are $W^{h,only}$, Z^f, and φ, respectively. Such foreign contributions will induce a higher level of privatization to increase the profits of foreign firms.

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Chapter 9
Government Preference and Merger

Hideya Kato

Abstract This chapter analyzes how the government's preference affects merger activity between a public firm and a private firm. We show that whether the merger occurs depends on the share ownership ratio of the merged firm and the government's preference for tax revenue. If the government puts a large weight on tax revenue, public and private firms will not merge.

9.1 Introduction

Seminal studies on mergers in mixed oligopolies include those of Bárcena-Ruiz and Garzón (2003) and Méndez-Naya (2008). These two studies deal with a merger between a public and private firm, using the partial privatization model constructed by Matsumura (1998). Bárcena-Ruiz and Garzón (2003) assumed that when a private and public firm merge, they establish a jointly owned multiproduct firm. The decision to merge depends on the degree to which goods are substitutes and on the percentage of the shares that the government owns in the multiproduct firm. In contrast, Méndez-Naya (2008) compared two oligopoly cases: one concerned a mixed oligopoly with a public firm and n identical private firms and the other case concerned $n - 1$ private firms and the merger of a private firm and a public firm. Méndez-Naya (2008) examined the effects of the number of private firms and the percentage of the shares of the merged firm on mergers.[1]

White (1996) and Fjell and Heywood (2004) incorporated government policy into the mixed oligopoly model. They assumed that the government and public firms are interested in social welfare and showed that the government should subsidize production in a mixed oligopoly. In contrast, Kato (2008) focused on the fact that the objective of the government may differ from that of the public firm and assumed that the government cares not only about social welfare but also about

[1] In addition to these papers, there are also studies on merger activities in a mixed oligopoly model, including Nakamura and Inoue (2007), Kamijo and Nakamura (2009), and Andree (2013).

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tax revenue. Under this assumption, it is shown that the government's optimal policy is to tax production.

In this chapter, we incorporate government preference into a mixed duopoly model where merger activity is considered. We follow the assumption of Kato (2008) and show that a merger depends not only on the percentage of the shares of the merged firm but also on the government preference for tax revenue. In particular, we show that if the government is concerned with tax revenue rather than social welfare, public and private firms will not merge.

This chapter is organized as follows. Section 9.2 describes the model. In Sect. 9.3, we solve equilibrium outcomes in a mixed duopoly and a merged monopoly. Section 9.4 compares these equilibrium outcomes and Sect. 9.5 concludes.

9.2 The Model

Consider an industry consisting of a public firm (firm 0) and a private firm (firm 1). Let q_0 be the output of the public firm, q_1 be the output of the private firm, and $Q(= q_0 + q_1)$ be the total output. We assume that the public and private firms produce a homogeneous good. Its price is given by the inverse demand function $p = 1 - Q$. Under this demand function, consumer surplus is given by $CS = Q^2/2$. A specific tax is imposed on both firms.

As is usually assumed (e.g., De Fraja and Delbono 1989), both firms have identical technologies, which are represented by the following quadratic cost function:

$$c(q_i) = F + kq_i^2/2, \quad i = 0, 1, \tag{9.1}$$

where F is a fixed cost. For simplicity, we assume that $F = 0$ and $k = 1$. Then, the profit function of firm i is given by

$$\pi_i = (1 - Q)q_i - q_i^2/2 - tq_i, \quad i = 0, 1, \tag{9.2}$$

where t is the specific tax rate. The private firm chooses output level q_1 to maximize its own profit π_1 given by (9.2). In contrast, the public firm chooses output level q_0 to maximize the sum of the consumer and producer surplus, which is eliminated by tax revenue as follows:

$$U = \frac{Q^2}{2} + (1 - Q)Q - \frac{q_0^2 + q_1^2}{2} - T, \tag{9.3}$$

where $T = tQ$. In reality, a government and a public firm play different roles in the governmental organization. In general, the main purpose of the public firm is not expected to be tax collection. Thus, we assume the public firm is not concerned about the tax revenue but rather the sum of the consumer and producer surplus.

In this chapter, following Kato (2008), we assume that the government has a preference. That is, it has its own objective function, which is different from that of the public firm, and is given by

$$W = U + (1 + \alpha)T, \tag{9.4}$$

where α is the parameter representing the weight of the government preference for tax revenue. Because we are interested in the case where the government puts a larger weight on T than on U, we set $\alpha \geq 0$. If $\alpha = 0$, the government puts the same weight on U and T. In this case, the government is a benevolent one that maximizes social welfare. In contrast, the greater α becomes, the more the government cares about T. In particular, if α approaches infinity, the government can be seen as a Leviathan government that cares only about T.

We assume that when the public and the private firms merge, the government owns $\theta(\in [0, 1])$ percent of the shares of the merged firm (hereinafter, "share ownership ratio of the merged firm").[2] Following Bárcena-Ruiz and Garzón (2003), we consider that the merged firm maximizes the weighted average of the government's payoff and profit. Therefore, the merged firm chooses output levels of q_0 and q_1 to maximize the following objective function:

$$V = \theta U + (1 - \theta)(\pi_0 + \pi_1). \tag{9.5}$$

It should be noted that (9.5) can be rewritten as $V = \theta CS + (\pi_0 + \pi_1)$. That is, parameter θ denotes the degree that the government is concerned about consumer surplus. If the share ownership ratio is $\theta = 1$, the objective function of the merged firm is the sum of the consumer and producer surplus: the merged firm has the same objective function as the public firm. If $\theta = 0$, the merged firm is a pure private firm that maximizes the sum of the profits of both firms. If $\theta \in (0, 1)$, the merged firm is jointly owned by the public and private firms.

The equilibrium outcomes are derived from a framework of a simple three-stage game. In the first stage, the government and the private firm decide whether to merge. In the second stage, the government imposes a tax on the firms. In the third stage, the firms determine their output. If both firms merge, the merged firm makes a production decision in a monopoly market. However, if they do not merge, both firms simultaneously set their output in a mixed duopoly market. To obtain the subgame perfect Cournot–Nash equilibrium, we solve the model by backward induction.

[2] The share ownership ratio of the merged firm in this chapter corresponds to the shareholding ratio referred to in other chapters.

9.3 Equilibrium Outcomes

We consider the following two cases in turn: a mixed duopoly in which the public and private firm do not merge and a merged monopoly in which both firms merge and then the government and the private firm owns θ percent and $1-\theta$ percent of the shares of the merged firm, respectively. In what follows, we denote the mixed duopoly case and the merged monopoly case by the superscripts MD and ME, respectively.

9.3.1 Mixed Duopoly

In this subsection, we consider the case of a mixed duopoly where the public and the private firm do not merge. First, we solve the output levels of both firms. The private firm chooses the level q_1, and maximizes its own profit, given by (9.2). The public firm chooses output level q_0, maximizing the sum of the consumer and producer surplus, given by (9.3). Then, solving these maximization problems under Nash competition, we obtain the following output levels of the public and private firms:

$$q_0 = \frac{2(1-t)}{5}, \tag{9.6}$$

$$q_1 = \frac{(1-t)}{5}. \tag{9.7}$$

It is clear that the output of the public firm is greater than that of the private one. Thus, in equilibrium, the public firm has higher marginal and total costs than the private one because the cost functions of both firms are quadratic and identical. The outputs of both firms decrease with the specific tax rate, t. From (9.6) and (9.7), the consumer and producer surpluses can be obtained as $CS = 9(1-t)^2/50$ and $PS = 7(1-t)^2/50$, respectively. Both of these surpluses also decrease with t. Because of this feature, the sum of both surpluses, U, strictly decreases with t.

We now turn to the second stage of the game in the case of a mixed duopoly. The government sets t to maximize (9.4), taking the outputs of both firms, (9.6) and (9.7), into consideration. From (9.3), (9.4), (9.6), and (9.7), the government's payoff can be rewritten as follows:

$$W = \frac{(1-t)[8 + (7 + 15\alpha)t]}{25}. \tag{9.8}$$

When the government chooses t to maximize (9.8), the optimal specific tax rate in the mixed duopoly can be obtained as follows:

$$t^{MD} = \frac{-1 + 15\alpha}{2(7 + 15\alpha)}. \tag{9.9}$$

It is clear that the optimal specific tax rate increases with α. This is because the greater α is, the more the government values tax revenue. Furthermore, if $\alpha > 1/15$, the optimal specific tax rate becomes positive; if $0 \leq \alpha < 1/15$, it becomes negative and the government subsidizes both firms; and if $\alpha = 1/15$, the optimal specific tax rate becomes zero.

Finally, we obtain the equilibrium values. Substituting (9.9) into (9.6) and (9.7), the outputs of the public and private firms and the total output in the equilibrium can be derived as follows:

$$q_0^{MD} = \frac{3(1 + \alpha)}{7 + 15\alpha}, \tag{9.10}$$

$$q_1^{MD} = \frac{3(1 + \alpha)}{2(7 + 15\alpha)}, \tag{9.11}$$

$$Q^{MD} = \frac{9(1 + \alpha)}{2(7 + 15\alpha)}. \tag{9.12}$$

We can see that the outputs of both firms and the total output all decrease with α. This is because as we have seen in (9.9), an increase in α increases t.

Under the equilibrium, the consumer surplus, the profits of both firms, and the government's payoff can be given by

$$CS^{MD} = \frac{81(1 + \alpha)^2}{8(7 + 15\alpha)^2}, \tag{9.13}$$

$$\pi_1^{MD} = \frac{27(1 + \alpha)^2}{8(7 + 15\alpha)^2}, \tag{9.14}$$

$$\pi_0^{MD} = \frac{9(1 + \alpha)^2}{2(7 + 15\alpha)^2}, \tag{9.15}$$

$$W^{MD} = \frac{9(1 + \alpha)^2}{4(7 + 15\alpha)}. \tag{9.16}$$

It should be noted that, from (9.13), (9.14), and (9.15), the consumer surplus and the profits of both firms decrease with α. In contrast, from (9.16), if the weight parameter of the government preference for tax revenue is large (small) enough, $\alpha > 1/15$ ($\alpha < 1/15$), the government's payoff increases (decreases) with α.

These features can be explained by focusing on the output levels of both firms. As mentioned above, an increase in α decreases total output via an increase in t. This effect causes an increase in price that consumers pay and a decrease in price that firms receive and therefore decreases both the consumer and producer surplus.

Furthermore, it increases the tax revenue because of the increase in t. As a result, the sum of the consumer and producer surplus (which means a payoff for the public firm) decreases with α, while the payoff for the government increases with α because of the positive effect on tax revenue.

9.3.2 Merger

In this subsection, we consider the case in which both firms merge: the merged firm becomes a monopoly firm. The merged firm chooses the levels q_0 and q_1 to maximize (9.5). Solving this problem, we obtain the output as follows:

$$q_0^{ME} = q_1^{ME} = \frac{1-t}{5-2\theta}. \tag{9.17}$$

It should be noted that the outputs of the merged firm decrease with the specific tax rate, t, and increase with the share ownership ratio, θ. It is straightforward to consider the former effect. That is, the merged firm decreases output with the specific tax rate increase. In contrast, the latter effect can be explained as follows. As the share ownership ratio increases, the merged firm is more concerned about the consumer surplus relative to the producer surplus. Therefore, the merged firm increases its total output because the consumer surplus increases with a decrease in market price.

Then, the government sets the specific tax rate to maximize (9.4), taking the output of the merged firm into consideration. From (9.3), (9.4), and (9.17), the government's payoff can be written as follows:

$$W = \frac{(1-t)[7+(3+10\alpha)t-4\theta(1+t\alpha)]}{(5-2\theta)^2}. \tag{9.18}$$

When the government chooses t to maximize the government's payoff, (9.18), the optimal specific tax rate is given by

$$t^{ME} = \frac{-2+2\theta(1-\alpha)+5\alpha}{3+2(5-2\theta)\alpha}. \tag{9.19}$$

If the weight of the government preference for tax revenue, α, is $\alpha < 2(1-\theta)/(5-2\theta)$, the government subsidizes both firms; if $\alpha > 2(1-\theta)/(5-2\theta)$, the government imposes a tax on both. In addition, the optimal specific tax rate increases with α and θ. When α increases, the government increases the specific tax rate because it takes a greater interest in the tax revenue. Conversely, when θ increases, the merged firm must increase its total output to increase the consumer surplus. Therefore, when θ is large, the government can impose a higher rate of tax on the merged firm.

Using (9.19), we obtain the following equilibrium outcome under the merged scenario:

$$q_0^{ME} = q_1^{ME} = \frac{(1+\alpha)}{3+(10-4\theta)\alpha}, \tag{9.20}$$

$$Q^{ME} = \frac{2(1+\alpha)}{3+(10-4\theta)\alpha}, \tag{9.21}$$

$$CS^{ME} = \frac{2(1+\alpha)^2}{[3+(10-4\theta)\alpha]^2}, \tag{9.22}$$

$$\pi^{ME} = \frac{(5-4\theta)(1+\alpha)^2}{[3+(10-4\theta)\alpha]^2}, \tag{9.23}$$

$$W^{ME} = \frac{(1+\alpha)^2}{3+(10-4\theta)\alpha}. \tag{9.24}$$

Thus, all the equilibrium outcomes depend on parameters α and θ.

Let us look closely at the features of the equilibrium outcomes. We first examine the effects of α on the equilibrium outcome. Total output, consumer surplus, and the profit of the merged firm all decrease with α. These results are explained as follows. Because an increase in α increases the specific tax rate, the merged firm decreases output and profit. This decrease in output causes a decrease in the consumer surplus via an increase in price. In contrast, if α is large (small) enough, $\alpha > 2(1-\theta)/(5-2\theta)$ ($\alpha < 2(1-\theta)/(5-2\theta)$), the government's payoff increases (decreases) with α. This result is the same feature as in the case of a mixed duopoly.

Next, we examine the effects of θ on equilibrium outcomes. An increase in θ causes two effects: a direct positive effect on total output by increasing the weight to consumer surplus and an indirect negative effect on total output through an increase in the specific tax rate, as in (9.19). Because the former direct effect denominates the latter indirect effect, an increase in θ increases total output. This results in an increase in the consumer surplus and a decrease in the profit of the merged firm.[3]

9.4 Comparison

Finally, we compare the equilibrium outcomes in a mixed duopoly and a merged monopoly.

[3]Strictly speaking, an increase in θ brings about a negative effect on the profit of the merged firm through an increase in the tax rate.

The private firm has an incentive to merge if its own profit obtained in a merged monopoly, $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right)$, is greater than that in a mixed duopoly, π_1^{MD}. Let θ_P denotes the value of parameter θ such that $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right)$ and π_1^{MD} are equal (see Appendix). Then, the value of θ_P is given by

$$\theta_P = \frac{441+1809\alpha+1755\alpha^2-(7+15\alpha)\sqrt{535+1992\alpha+1845\alpha^2}}{392+1680\alpha+1692\alpha^2}. \tag{9.25}$$

From (9.25), we have the following proposition:

Proposition 9.1 *If θ is small enough ($0 \le \theta < \theta_P$), the private firm has an incentive to merge: $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) > \pi_1^{MD}$. If θ is large enough ($\theta_P < \theta \le 1$), the private firm does not have an incentive to merge: $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) < \pi_1^{MD}$.*

Proof It is clear that if $\theta = 1$, then $\pi_1^{MD} - (1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) = 27(1+\alpha)^2/\left[8(7+15\alpha)^2\right] > 0$ holds $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) < \pi_1^{MD}$, and if $\theta = 0$, then $\pi_1^{MD} - (1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) = -(1+\alpha)^2(1717+6780\alpha+6300\alpha^2)/\left[8(3+10\alpha)^2(7+15\alpha)^2\right] < 0$ holds $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) > \pi_1^{MD}$. Because $\partial[(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right)]/\partial\theta < 0$ and $\partial\pi_1^{MD}/\partial\theta = 0$, there exists θ_P such that $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) = \pi_1^{MD}$ and when $0 \le \theta < \theta_P$, $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) > \pi_1^{MD}$ and when $\theta_P < \theta \le 1$, $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) < \pi_1^{MD}$.

We summarize these features in Fig. 9.1. The curve in Fig. 9.1 represents $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) = \pi_1^{MD}$. When $\alpha = 0$, $\theta_P = \left(441-7\sqrt{535}\right)/392 \approx 0.712$, and for $\alpha \ge 0$, this curve is moderately downward sloping. If α approaches infinity, θ_P approaches $5\left(39-\sqrt{205}\right)/188 \approx 0.656$. In the region below the curve, the private firm wishes to merge because $(1-\theta)\left(\pi_0^{ME}+\pi_1^{ME}\right) > \pi_1^{MD}$ holds. In contrast, the private firm does not wish to merge in the region above the curve.

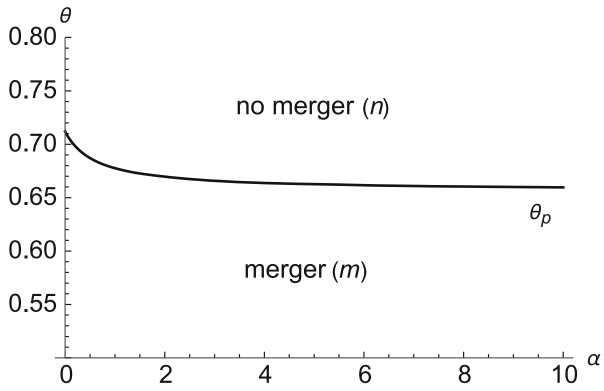


Fig. 9.1 Profit of the private firm

The government wishes to merge if the government's payoff in the merged monopoly, W^{ME}, is greater than that in the mixed duopoly, W^{MD}. Let θ_G denote the value of parameter θ such that $W^{ME} = W^{MD}$. From (9.16) and (9.24), the value of θ_G is given by

$$\theta_G = \frac{5}{6} - \frac{1}{36\alpha}. \tag{9.26}$$

It can be verified that θ_G is a concave function of the parameter α: $\partial\theta_G/\partial\alpha = 1/(36\alpha^2) > 0$ and $\partial^2\theta_G/\partial\alpha^2 = -1/(18\alpha^3) < 0$. If α approaches zero, θ_G approaches $-\infty$. If α approaches infinity, θ_G approaches $5/6 \approx 0.833$. If $\alpha = 1/30 \approx 0.033$, $\theta_G = 0$. These features are illustrated in Fig. 9.2. As discussed above, in the region above the curve, the government seeks to merge because $W^{ME} > W^{MD}$ holds. In contrast, the government does not seek to merge in the region below the curve.

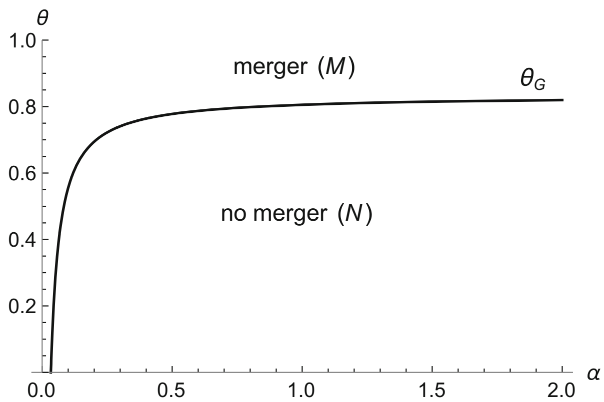
We summarize the above findings as the following proposition.

Proposition 9.2 *If θ is small enough ($0 \le \theta < \theta_G$), the government does not have an incentive to merge: $W^{ME} < W^{MD}$. If θ is large enough ($\theta_G < \theta \le 1$), the government has an incentive to merge: $W^{ME} > W^{MD}$.*

By superimposing the curves in Figs. 9.1 and 9.2, we obtain Fig. 9.3. The two curves intersect at $(\alpha, \theta) = \left(\left(7+\sqrt{105}\right)/84, 5/6 - 7/\left(21 + 3\sqrt{105}\right)\right) \approx (0.205, 0.698)$. From Fig. 9.3, we obtain the following proposition.

Proposition 9.3 *The government wants to merge, and the private firm does not if $\theta > \max\{\theta_P, \theta_G\}$ (Region I). For $\alpha > \left(7+\sqrt{105}\right)/84$, neither the private firm nor the government wants to merge when $\theta_P < \theta < \theta_G$ (Region II). Both the private firm and the government want to merge when $\alpha < \left(7+\sqrt{105}\right)/84$ and $\theta_G < \theta < \theta_P$ (Region III). The government does not want to merge, and the private firm does if $\theta < \min\{\theta_P, \theta_G\}$ (Region IV).*

Fig. 9.2 Government preference



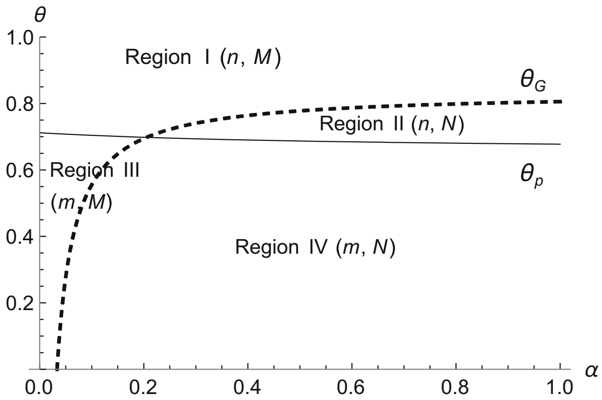


Fig. 9.3 Comparison of profit and government preference

Thus, if the government is a Leviathan government ($\alpha > \left(7 + \sqrt{105}\right)/84$), both firms do not merge. In contrast, if the government is benevolent, $\alpha = 0$, the condition for merger requires that the share rate in the merged firm is not large enough, $\theta < \theta_P$. However, if the government attaches some value to tax revenue, $1/30 < \alpha < \left(7 + \sqrt{105}\right)/84$, the condition for merger requires that the government have an intermediate-level share rate in the merged firm, $\theta_G < \theta < \theta_P$.

9.5 Conclusion

In this chapter, we considered the government's preference in a merger model in a mixed duopoly. In this setting, we demonstrated that a merger between a public firm and a private firm depends on the government's preference and the ownership ratio. Furthermore, the merger depends not only on the percentage of the share ownership ratio of the merged firm but also on the government's preference for tax revenue. We also verified that a merger between a public and a private firm will not occur if the government is concerned about tax revenue rather than social welfare. In addition, if the government owns a high number of shares in the merged firm, the merger will not occur.

Appendix

The difference between π_1^{MD} and $(1-\theta)\left(\pi_0^{ME} + \pi_1^{ME}\right)$ is given by

$$\pi_1^{MD} - (1-\theta)\left(\pi_0^{ME} + \pi_1^{ME}\right) = -\frac{(1+\alpha)^2\left(A - 72B\theta + 16C\theta^2\right)}{8(7+15\alpha)^2[3+(10-4\theta)\alpha]^2},$$

where $A \equiv 1717 + 6780\alpha + 6300\alpha^2$, $B \equiv (49 + 201\alpha + 195\alpha^2)$, and $C \equiv (98 + 420\alpha + 423\alpha^2)$. When π_1^{MD} and $(1-\theta)\left(\pi_0^{ME} + \pi_1^{ME}\right)$ are equal, $A - 72B\theta + 16C\theta^2 = 0$, which is a quadratic equation with respect to θ, must be satisfied. Denoting solutions of this equation with θ_P, the two solutions are given by

$$\theta_P^- = \frac{441 + 1809\alpha + 1755\alpha^2 - (7 + 15\alpha)\sqrt{535 + 1992\alpha + 1845\alpha^2}}{392 + 1680\alpha + 1692\alpha^2},$$
$$\theta_P^+ = \frac{441 + 1809\alpha + 1755\alpha^2 + (7 + 15\alpha)\sqrt{535 + 1992\alpha + 1845\alpha^2}}{392 + 1680\alpha + 1692\alpha^2}.$$

For $\alpha \geq 0$, it holds that although the former solution takes on a range of values between 0 and 1, the latter solution does not. Therefore, the former solution is an adequate solution.

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Part III
Further Applications

Chapter 10
Regional Differences and Privatization

Hideya Kato and Mitsuyoshi Yanagihara

Abstract This chapter analyzes a mixed oligopoly model with two asymmetric regions with different populations, number of private firms, and shareholding ratios of private firms. We consider three cases: local ownership where the public firms are owned by the local government, state ownership where they are owned by the national government, and private ownership where they are owned by the private sector. We show that social welfare in the case of state ownership is greater than that in local ownership. Even in the case of local ownership, the government can replicate the equilibrium outcomes in the state ownership case by dividing the regions into optimal population sizes. We also show that if the number of private firms is large enough, although the public firms should be privatized from a national perspective, social welfare in its region may decrease because of privatization.

10.1 Introduction

Although most studies on mixed oligopoly have mainly focused on domestic economy (De Fraja and Delbono 1989; Matsumura 1998), some have investigated international mixed oligopolies. Fjell and Pal (1996) and Pal and White (1998) considered foreign private firms in a single domestic market. Those studies considered scenarios involving a public firm and private firms and the strategic interaction between the two.

Bárcena-Ruiz and Garzón (2005a, b) and Han and Ogawa (2008) extended the domestic market to an international market in which there is free trade and economic integration. Those studies examined the strategic interaction between the governments (public firms) of two symmetric countries.[1]

[1] While Bárcena-Ruiz and Garzón (2005a, b) only considered full privatization, Han and Ogawa (2008) investigated partial privatization.

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In this chapter, we consider a mixed oligopoly model with two asymmetric regions with different population sizes, number of private firms, and shareholding ratios of private firms. Under such circumstances, this chapter considers the following three cases: local ownership, state ownership, and private ownership. First, the local ownership case is that of public firms that noncooperatively choose output to maximize social welfare in their own regions. Second, the state ownership case is that where public firms cooperatively choose outputs to maximize the sum of social welfare in both regions. This case represents a total ownership case in which public firms cooperatively choose outputs to maximize the sum of both surpluses. Third, the private ownership case is one where all firms are private firms that choose outputs to maximize their own profit. This case represents a pure private oligopoly, implying the situation after the privatization of public firms.

The main results of this chapter are as follows. First, social welfare in the state ownership case is greater than that in the local ownership case. Second, if the number of private firms is large enough, public firms should be privatized from a national perspective. However, social welfare in a region might decrease because of privatization. Last, in the local ownership case, the government can replicate the equilibrium outcomes in the state ownership case by dividing the regions into optimal population sizes.

This chapter is organized as follows. Section 10.2 describes the model. In Sect. 10.3, we obtain the equilibrium outcomes for the three cases. Section 10.4 compares these equilibrium outcomes, and Sect. 10.5 considers an optimal market size in the regions. Section 10.6 concludes.

10.2 The Model

Consider two regions m $(m = A,\ B)$ in a country. There is one public firm (firm 0) in each region, n_A private firms in region A and n_B private firms in region B. Thus, there are two public firms and $N(\equiv n_A + n_B)$ private firms in the country.[2] All firms produce a homogeneous good that is freely traded across the two regions. We assume that there is no trade cost and no price discrimination between individuals within the different regions.

There are L identical populations in the country, and a and $L - a$ populations are distributed in regions A and B, respectively. The individual demand function is given by $x = 1 - p$, where x and p denote demand and price, respectively. Without loss of generality, we assume that the population of the country, L, is normalized to one. Therefore, the inverse demand functions in regions A and B can be given by $p_A = 1 - x_A/a$ and $p_B = 1 - x_B/(1 - a)$, where p_A and p_B are the price of the good in regions A and B and x_A and x_B are the demand for the good in regions A and B, respectively. This inverse demand function requires $a \in (0,\ 1)$. Because we consider

[2] Here, regions are represented by a subscript.

an integrated economy, the price for the good in both regions becomes equal to that in the country: $p_A = p_B = p$. The inverse demand function in the country (for both regions) is given by $p = 1 - x_A - x_B$. In equilibrium, $x_A + x_B = Q_A + Q_B$ should hold, where Q_A and Q_B denote total output in regions A and B, respectively.

All firms produce a homogeneous good using identical technologies, represented by the following cost function:

$$c(q_{Ai}) = F + kq_{Ai}^2/2, \quad i = 0, \cdots, n_A,$$
$$c(q_{Bj}) = F + kq_{Bj}^2/2, \quad j = 0, \cdots, n_B,$$

where F denotes the fixed cost. In what follows, we assume $F = 0$ and $k = 1$ with no loss of generality. Therefore, the profit of each firm is given by

$$\pi_{Ai} = (1 - Q)q_{Ai} - q_{Ai}^2/2, \quad i = 0, \cdots, n_A, \tag{10.1}$$
$$\pi_{Bj} = (1 - Q)q_{Bj} - q_{Bj}^2/2, \quad j = 0, \cdots, n_B, \tag{10.2}$$

where $Q \equiv Q_A + Q_B$. We assume that the profits of all private firms are partially distributed to the individuals in the country: the shareholding ratios of the private firms, which individuals own in regions A and B, are ε and $1 - \varepsilon$, respectively.[3]

Next, we obtain the levels of the consumer surplus, producer surplus, and social welfare. It should be noted that the larger the population size is, the greater the surpluses and welfare become. Therefore, we must evaluate them in per capita terms to avoid overestimations. In the following analyses, we evaluate the levels or the amounts in per capita terms. The consumer surplus of regions A and B and total consumer surplus in the country (both as per capita values) are as follows:

$$CS_A = CS_B = CS = Q^2/2. \tag{10.3}$$

The consumer surpluses per capita of region A, region B, and the country become identical under free trade.

Social welfare is defined as the sum of the consumer surplus and the producer surplus. We assume that the private firms are partially owned by the residents in both regions, while the public firms located in region A (B) are perfectly owned by residents in region A (B). From (10.1), (10.2), and (10.3), social welfare per capita in regions A and B and in the country are given by

[3] Parameter ε can be regarded as an average shareholding ratio in all private firms owned by individuals in region A because all the private firms are identical and obtain the same profit.

$$W_A = \frac{Q^2}{2} + \frac{\pi_{A0}}{a} + \frac{\varepsilon}{a}\left(\sum_{i=1}^{n_A}\pi_{Ai} + \sum_{j=1}^{n_B}\pi_{Bj}\right), \tag{10.4}$$

$$W_B = \frac{Q^2}{2} + \frac{\pi_{B0}}{(1-a)} + \frac{(1-\varepsilon)}{(1-a)}\left(\sum_{i=1}^{n_A}\pi_{Ai} + \sum_{j=1}^{n_B}\pi_{Bj}\right), \tag{10.5}$$

$$W = aW_A + (1-a)W_B = \frac{Q^2}{2} + \sum_{i=0}^{n_A}\pi_{Ai} + \sum_{j=0}^{n_B}\pi_{Bj}, \tag{10.6}$$

respectively. In this model, it should be noted that the following three parameters—population, number of firms, and shareholding ratio—are asymmetric in each region.

10.3 Equilibrium Outcomes

In this section, we consider the three cases described in the introduction: local ownership (Case L), state ownership (Case S), and private ownership (Case P). These cases are represented by the superscripts L, S, and P, respectively.

10.3.1 Local Ownership

The public firm 0 located in region m chooses an output level to maximize social welfare in region m. The private firm in region m chooses an output level to maximize its own profit. Cournot competition between these firms leads to the following outputs levels:

$$q_{Ai}^L = q_{Bj}^L = \frac{1}{6+N}, \quad i = 1, \cdots, n_A, \quad j = 1, \cdots, n_B, \tag{10.7}$$

$$q_{A0}^L = \frac{2+\phi}{2(6+N)}, \tag{10.8}$$

$$q_{B0}^L = \frac{6-\phi}{2(6+N)}, \tag{10.9}$$

where $\phi \equiv a(4+N) - \varepsilon N$. For interior solutions, we assume $-2 < \phi < 6$. All private firms choose the same level of output because there is no trade cost and all firms have the same technologies. The outputs of the private firms and the public firms decrease with the number of private firms, $N(= n_A + n_B)$, regardless of the firms' locations.

While an increase in a increases q_{A0}^L and decreases q_{B0}^L, an increase in ε decreases q_{A0}^L and increases q_{B0}^L. These results are interpreted as follows. The public firm in region A (region B) increases (decreases) output so that it responds to increased (decreased) demand that is caused by an increase (decrease) in the population. In contrast, an increase in ε results in a decrease (an increase) in the output of public firm A (public firm B) to increase (decrease) the profits of the private firms received by individuals in region A (region B).

The total output of the firms in regions A and B and the total output in the country can be given by:

$$Q_A^L = \frac{2(1+n_A)+\phi}{2(6+N)}, \tag{10.10}$$

$$Q_B^L = \frac{2(3+n_B)-\phi}{2(6+N)}, \tag{10.11}$$

$$Q^L = \frac{4+N}{6+N}. \tag{10.12}$$

The total output of each region increases with the number of firms in its own region: $\partial Q_A^L/\partial n_A > 0$ and $\partial Q_B^L/\partial n_B > 0$. In contrast, the total output of each region decreases with the number of firms in the other region: $\partial Q_A^L/\partial n_B < 0$ and $\partial Q_B^L/\partial n_A < 0$. As a result, the total output of the country increases with the number of firms: $\partial Q^L/\partial n_A = \partial Q^L/\partial n_B = 2/(6+N)^2 > 0$. This occurs because the effect of the number of firms in its own region on the total output in that region dominates the effect on the total output in the other region.

Substituting (10.10), (10.11), and (10.12) into the inverse demand, the equilibrium price in the country (regions A and B) becomes

$$p^L = \frac{2}{6+N}. \tag{10.13}$$

From (10.13), we can obtain the following demand in both regions A and B:

$$x_A^L = \frac{a(4+N)}{6+N}, \tag{10.14}$$

$$x_B^L = \frac{(1-a)(4+N)}{6+N}. \tag{10.15}$$

Using (10.10), (10.11), (10.14), and (10.15), import (or export) levels in regions A and B are

$$x_A^L - Q_A^L = \frac{-2(1+n_A)+\chi}{2(6+N)}, \tag{10.16}$$

$$x_B^L - Q_B^L = \frac{2(1+n_A)-\chi}{2(6+N)}, \tag{10.17}$$

where $\chi \equiv a(4+N)+\varepsilon N$. If a and ε are large (small) enough, $a(4+N)+\varepsilon N > (<) 2(1+n_A)$, region A (region B) has an excess demand and imports goods from region B (region A). The import levels of each region decrease with the number of firms in its own region and increase with the number of firms in the other region: $\partial\left(x_A^L - Q_A^L\right)/\partial n_A < 0, \partial\left(x_B^L - Q_B^L\right)/\partial n_B < 0, \partial\left(x_B^L - Q_B^L\right)/\partial n_A > 0$, and $\partial\left(x_A^L - Q_A^L\right)/\partial n_B > 0$. This occurs because the total output in each region increases with the number of private firms in its own region, as seen above, while the total demand in the region remains unchanged.

Finally, we obtain the following equilibrium outcomes:

$$CS_A^L = CS_B^L = CS^L = \frac{(4+N)^2}{2(6+N)^2}, \tag{10.18}$$

$$PS_A^L = \frac{-(2-\phi)^2 + 16 + 12\varepsilon N}{8a(6+N)^2}, \tag{10.19}$$

$$PS_B^L = \frac{-(2-\phi)^2 + 16 + 12(1-\varepsilon)N}{8(1-a)(6+N)^2}, \tag{10.20}$$

$$PS^L = \frac{-(2-\phi)^2 + 16 + 6N}{4(6+N)^2}, \tag{10.21}$$

$$W_A^L = \frac{(4+N)^2}{2(6+N)^2} + \frac{-(2-\phi)^2 + 16 + 12\varepsilon N}{8a(6+N)^2}, \tag{10.22}$$

$$W_B^L = \frac{(4+N)^2}{2(6+N)^2} + \frac{-(2-\phi)^2 + 16 + 12(1-\varepsilon)N}{8(1-a)(6+N)^2}, \tag{10.23}$$

$$W^L = \frac{(4+N)^2}{2(6+N)^2} + \frac{-(2-\phi)^2 + 16 + 6N}{4(6+N)^2}. \tag{10.24}$$

10.3.2 State Ownership

In this section, we consider a state ownership case in which the public firms of regions A and B implement a coordinated activity. While the private firms choose a level of output to maximize their own profits, both public firms coordinate to choose

q_{A0} and q_{B0} to maximize the sum of social welfares in regions A and B, represented by W. Thus, we obtain the following outputs:

$$q_{Ai}^S = q_{Bj}^S = \frac{1}{6+N}, \quad i = 1, \cdots, n_A, \quad j = 1, \cdots, n_B, \tag{10.25}$$

$$q_{A0}^S = q_{B0}^S = \frac{2}{6+N}. \tag{10.26}$$

Furthermore, (10.25) implies that all private firms choose the same level of output because they are identical. Both public firms choose the same level of output unlike Case L as shown in Sect. 10.3.1. Because the cost function is quadratic, marginal costs increase in the output. Therefore, both public firms can produce efficiently by sharing outputs: each firm should produce half of the total output in a coordinated manner. In addition, it should be noted that the outputs of the private and public firms do not depend on a and ε and decrease with the total number of private firms, N, regardless of the location of the firms.

The total outputs in regions A and B and the total output in the country are given by

$$Q_A^S = \frac{2+n_A}{6+N}, \tag{10.27}$$

$$Q_B^S = \frac{2+n_B}{6+N}, \tag{10.28}$$

$$Q^S = \frac{4+N}{6+N}. \tag{10.29}$$

The total output of each region increases with the number of firms in its own region: $\partial Q_A^S/\partial n_A > 0$ and $\partial Q_B^S/\partial n_B > 0$. In contrast, the total outputs of each region decrease with the number of firms in the other region: $\partial Q_A^S/\partial n_B < 0$ and $\partial Q_B^S/\partial n_A < 0$. In sum, the total output of the country increases with the number of firms: $\partial Q^S/\partial n_A = \partial Q^S/\partial n_B = 2/(6+N)^2 > 0$. These features are the same in the case of a non-coordinated equilibrium, Case L.

As in Case L, demand in regions A and B is given by (10.14) and (10.15) because the demand side remains unchanged in Case S. From(10.14), (10.15), (10.27), and (10.28), the import levels in regions A and B are as follows:

$$x_A^S - Q_A^S = \frac{a(4+N) - 2 - n_A}{6+N}, \tag{10.30}$$

$$x_B^S - Q_B^S = \frac{2 + n_A - a(4+N)}{6+N}. \tag{10.31}$$

We then obtain the outcomes under the coordinated equilibrium:

$$CS_A^S = CS_B^S = CS^S = \frac{(4+N)^2}{2(6+N)^2}, \tag{10.32}$$

$$PS_A^S = \frac{4+3\varepsilon N}{2a(6+N)^2}, \tag{10.33}$$

$$PS_B^S = \frac{4+3(1-\varepsilon)N}{2(1-a)(6+N)^2}, \tag{10.34}$$

$$PS^S = \frac{8+3N}{2(6+N)^2}, \tag{10.35}$$

$$W_A^S = \frac{(4+N)^2}{2(6+N)^2} + \frac{4+3\varepsilon N}{2a(6+N)^2}, \tag{10.36}$$

$$W_B^S = \frac{(4+N)^2}{2(6+N)^2} + \frac{4+3(1-\varepsilon)N}{2(1-a)(6+N)^2}, \tag{10.37}$$

$$W^S = \frac{(4+N)^2}{2(6+N)^2} + \frac{8+3N}{2(6+N)^2}. \tag{10.38}$$

10.3.3 Private Ownership

In this subsection, we consider the private ownership case in which all firms are private, representing the situation after privatization. Here, without loss of generality, we assume that only one privatized firm in each region is owned by individuals living in the region where it is located.

In Case P, all firms choose a level of output to maximize its own profit. By solving this problem, we obtain the following levels of output:

$$q_{A0}^P = q_{B0}^P = q_{Ai}^P = q_{Bj}^P = \frac{1}{4+N}, \quad i = 1, \cdots, n_A, \quad j = 1, \cdots, n_B. \tag{10.39}$$

All firms produce the same level of output under this privatized oligopoly. It is clear that the output decreases with the number of firms, N. The total outputs in regions A and B and the total output in the country can be given by

$$Q_A^P = \frac{1+n_A}{4+N}, \tag{10.40}$$

$$Q_B^P = \frac{1+n_B}{4+N}, \tag{10.41}$$

$$Q^P = \frac{2+N}{4+N}. \tag{10.42}$$

Because the demand of each region in Case *P* is the same as that in Case *L* and Case *S*, import levels in regions *A* and *B* are

$$x_A^P - Q_A^P = \frac{-1 - n_A + a(2+N)}{4+N}, \tag{10.43}$$

$$x_B^P - Q_B^P = \frac{1 + n_A - a(2+N)}{4+N}. \tag{10.44}$$

Finally, we obtain the outcomes in Case *P*:

$$CS_A^P = CS_B^P = CS^P = \frac{(2+N)^2}{2(4+N)^2}, \tag{10.45}$$

$$PS_A^P = \frac{3(1+\varepsilon N)}{2a(4+N)^2}, \tag{10.46}$$

$$PS_B^P = \frac{3[1+(1-\varepsilon)N]}{2(1-a)(4+N)^2}, \tag{10.47}$$

$$PS^P = \frac{3(2+N)}{2(4+N)^2}, \tag{10.48}$$

$$W_A^P = \frac{a(2+N)^2 + 3(1+\varepsilon N)}{2a(4+N)^2}, \tag{10.49}$$

$$W_B^P = \frac{(1-a)(2+N)^2 + 3[1+(1-\varepsilon)N]}{2(1-a)(4+N)^2}, \tag{10.50}$$

$$W^P = \frac{(2+N)(5+N)}{2(4+N)^2}. \tag{10.51}$$

10.4 Comparisons

In this section, we compare the outcomes of the three cases obtained in Sect. 10.3.

10.4.1 Comparison of Equilibrium Outcomes in Case L and Case S

First, let us compare outputs under the mixed oligopoly in Case L and Case S. The differences in total outputs can be calculated as follows:

$$Q_A^L - Q_A^S = \frac{\phi - 2}{2a(6+N)}, \tag{10.52}$$

$$Q_B^L - Q_B^S = \frac{2 - \phi}{2(1-a)(6+N)}, \tag{10.53}$$

$$Q^L = Q^S = \frac{4+N}{6+N}. \tag{10.54}$$

If a is large enough ($\phi > 2$), $Q_A^L > Q_A^S$ and $Q_B^L < Q_B^S$. This result can be interpreted as follows. From (10.7) and (10.25), the outputs of the private firms under Case L and Case S are the same: $q_{Ai}^S = q_{Bj}^S = q_{Ai}^L = q_{Bj}^L$. Although the outputs of each public firm are different under both cases, the total outputs of the public firms are the same: $q_{A0}^L + q_{B0}^L = q_{A0}^S + q_{B0}^S = 4/(6+N)$. As a result, the total outputs in the regions are different under both systems, and the total outputs in the country are identical.

Given the above comparison of output under Case L (the uncoordinated case) and Case S (the coordinated case), we can compare the consumer surplus and the producer surplus. There is no difference in consumer surplus:

$$CS_A^L = CS_A^S = CS_B^L = CS_B^S = CS^L = CS^S = \frac{(4+N)^2}{2(6+N)^2}. \tag{10.55}$$

This is because the total outputs in both cases are identical and the market is integrated.

The difference in producer surplus is given by

$$PS_A^L - PS_A^S = \frac{-(2-\phi)^2}{8a(6+N)^2} < 0, \tag{10.56}$$

$$PS_B^L - PS_B^S = \frac{-(2-\phi)^2}{8(1-a)(6+N)^2} < 0, \tag{10.57}$$

$$PS^L - PS^S = \frac{-(2-\phi)^2}{4(6+N)^2} < 0. \tag{10.58}$$

These results are interpreted as follows. As mentioned above, the private firms choose the same level of output in both cases. In addition, in both cases, the total outputs are the same, which, in turn, are priced the same. As a result, the sum of the profits of the private firms is the same in both cases. That is, the difference in the producer surpluses in both cases depends not on the profits of the private firms but on those of the public firms: $\sum_{i=1}^{n_A} \pi_{Ai}^L = \sum_{i=1}^{n_A} \pi_{Ai}^S$, $\sum_{j=1}^{n_B} \pi_{Bj}^L = \sum_{j=1}^{n_B} \pi_{Bj}^S$, and $\pi_{A0}^L - \pi_{A0}^S = \pi_{B0}^L - \pi_{B0}^S = -(2-\phi)^2/\left[8(6+N)^2\right] < 0$. In this model, production efficiency requires that total output be divided as equally as possible among firms under identical quadratic production functions. Thus, both public firms can produce efficiently by producing half the outputs each in a coordinated manner: $\left(q_{A0}^N + q_{B0}^N\right)/2 = q_{A0}^S = q_{B0}^S = 2/(6+N)$.[4]

From the difference in the consumer and producer surpluses under both cases, we see that the difference in social welfare depends on the producer surpluses. That is, the differences in social welfare in regions A and B and in the country are given by

$$W_A^L - W_A^S = \frac{-(2-\phi)^2}{8a(6+N)^2} < 0, \tag{10.59}$$

$$W_B^L - W_B^S = \frac{-(2-\phi)^2}{8(1-a)(6+N)^2} < 0, \tag{10.60}$$

$$W^L - W^S = \frac{-(2-\phi)^2}{4(6+N)^2} < 0. \tag{10.61}$$

Social welfare in regions A and B and in the country is greater in Case S. Therefore, Case S results in greater social welfare than Case L.

The above results can be summarized as the following proposition:

Proposition 10.1 *Social welfare in the state ownership case is greater than that in the local ownership case. This result can be applied at a regional level as well as a country level.*

10.4.2 Comparison of Equilibrium Outcomes in Case S *and Case* P

In the previous section, we showed that social welfare in Case S is larger than that in Case L: the equilibrium outcome achieved in Case S Pareto dominates that in Case L. Therefore, in this section, we compare the equilibrium outcomes in Case S and Case P.

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[4] From (10.8), (10.9), and (10.26), we obtain $q_{A0}^L - q_{A0}^S = (\phi - 2)/[2(6+N)]$ and $q_{B0}^L - q_{B0}^S = (2-\phi)/[2(6+N)]$. By substituting these into the cost function, we obtain (10.56), (10.57), and (10.58). This proves that the above explanation is correct.

By comparing the output levels in Case P and in Case S, we see that $q_{Ai}^S = q_{Bj}^S < q_{A0}^P = q_{B0}^P = q_{Ai}^P = q_{Bj}^P < q_{A0}^S = q_{B0}^S$. In Case S, the public firm produces more than the private firms because the public firm cares about the producer surplus *and* the consumer surplus. The public firm produces more and the private firms produce less in Case S.

Based on the above results, we calculate the difference in total outputs in region A and region B in Case S and Case P given by

$$Q_A^S - Q_A^P = \frac{2 - n_A + n_B}{(4+N)(6+N)}, \tag{10.62}$$

$$Q_B^S - Q_B^P = \frac{2 + n_A - n_B}{(4+N)(6+N)}, \tag{10.63}$$

$$Q^S - Q^P = \frac{4}{(4+N)(6+N)} > 0. \tag{10.64}$$

These differences depend on the number of private firms in each region, n_A and n_B. If $n_A > 2 + n_B$ ($n_A < 2 + n_B$), the total output in region A is greater (smaller) in Case P. If $n_B > 2 + n_A$ ($n_B < 2 + n_A$), the total output in region B is greater (smaller) in Case P. Thus, if the number of private firms in one region is relatively large compared with that in the other region, the total output in the region is greater in Case P. As a result, the total output in the country is greater in Case S.

The difference in consumer surplus is given by

$$CS_A^S - CS_A^P = CS_B^S - CS_B^P = CS^S - CS^P = \frac{4\left[(4+N)^2 - 2\right]}{(4+N)^2(6+N)^2} > 0. \tag{10.65}$$

Because the total output in the country is greater than in Case S, the consumer surplus in regions A and B and in the country is also greater in Case S.

The profit of the private firms in Case S is smaller than that in Case P: $\pi_{Ai}^S = \pi_{Bj}^S < \pi_{Ai}^P = \pi_{Bj}^P$. The difference is given by $\pi_{Ai}^S - \pi_{Ai}^P = \pi_{Bj}^S - \pi_{Bj}^P = -6(5+N)/\left[(4+N)^2(6+N)^2\right] < 0$. In contrast, the difference in the profits of the public firm in both cases is given by

$$\pi_{A0}^S - \pi_{A0}^P = \pi_{B0}^S - \pi_{B0}^P = \frac{(N-2)^2 - 48}{2(4+N)^2(6+N)^2}. \tag{10.66}$$

These differences depend on the number of private firms in a country, N. If the number of private firms in the country is $N > 2\left(1 + 2\sqrt{3}\right) \approx 8.928$, the profit of the firm is greater in Case S, $\pi_{A0}^S > \pi_{A0}^P$, and $\pi_{B0}^S > \pi_{B0}^P$, and vice versa. Therefore, the differences in the producer surplus per capita in regions A and B and in the country are given by

$$PS_A^S - PS_A^P = \frac{(N-2)^2 - 48 - 12\varepsilon N(5+N)}{2a(4+N)^2(6+N)^2}, \tag{10.67}$$

$$PS_B^S - PS_B^P = \frac{(N-2)^2 - 48 - 12(1-\varepsilon)N(5+N)}{2(1-a)(4+N)^2(6+N)^2}, \tag{10.68}$$

$$PS^S - PS^P = -\frac{5N^2 + 34N + 44}{(4+N)^2(6+N)^2} < 0. \tag{10.69}$$

The producer surplus per capita in the country is larger in Case P. However, we cannot see whether $PS_A^S - PS_A^P$ and $PS_B^S - PS_B^P$ decrease or increase in N because they depend on ε.

Last, we compare social welfare levels and obtain the following:

$$W_A^S - W_A^P = \frac{8a[(4+N)^2 - 2] - 12\varepsilon N(5+N) + N^2 - 4N - 44}{2a(4+N)^2(6+N)^2}, \tag{10.70}$$

$$W_B^S - W_B^P = \frac{8(1-a)[(4+N)^2 - 2] - 12(1-\varepsilon)N(5+N) + N^2 - 4N - 44}{2(1-a)(4+N)^2(6+N)^2}, \tag{10.71}$$

$$W^S - W^P = \frac{13 - (1+N)^2}{(4+N)^2(6+N)^2}. \tag{10.72}$$

If the population size of region A, a, is large, social welfare in region A (region B) under Case S is greater (smaller) than that in Case P. If $N > -1 + \sqrt{13} \approx 2.606$, implying that the market is close to competitive, social welfare in the country is larger in Case P. This result is similar to that of De Fraja and Delbono (1989).

If the regions are symmetric, $W^S - W^P = W_A^S - W_A^P = W_B^S - W_B^P$: the sign of $W^S - W^P$ is the same as those of $W_A^S - W_A^P$ and $W_B^S - W_B^P$. However, if the regions are asymmetric, this does not necessarily hold. For example, when we set $a = 0.6$, $\varepsilon = 0.8$, $n_A = 4$, and $n_B = 4$, we obtain $W^S - W^P \approx -0.0024$, $W_A^S - W_A^P \approx -0.0097$, and $W_B^S - W_B^P \approx 0.0085$.[5] This example shows that privatization is preferable for region A and the country as a whole, although not for region B. Even if privatization is Pareto superior (inferior) at a country level, it is not always Pareto superior(inferior) at a region level. This implies that when decisions on privatization are made, both local and national governments should be involved.

The above results can be summarized as the following proposition:

[5] It should be noted that $W^S - W^P$ is not equal to the sum of $W_A^S - W_A^P$ and $W_B^S - W_B^P$ because social welfare is measured in per capita terms: $W^S - W^P = a(W_A^S - W_A^P) + (1-a)(W_B^S - W_B^P)$.

Proposition 10.2 *If the number of private firms is large, public firms should be privatized at a country level. However, whether public firms are privatized may differ at regional and country levels.*

10.5 Optimal Market Size

Although the social welfare in a Case L scenario is not the largest, if the regions are symmetric, $a = \varepsilon = 1/2$ and $n_A = n_B$, social welfare in Case L and Case S in a mixed oligopoly becomes the same. In this section, we examine the optimal market size, or population size, in each region, when both regions are asymmetric.

Maximizing the country's social welfare (10.24), with respect to a, we obtain the following optimal size:

$$a^* = \frac{2 + \varepsilon N}{4 + N}. \tag{10.73}$$

The optimal market size of the region depends on ε and N. It can be acknowledged that while an increase in ε (the shareholding ratio in region A) increases the optimal size, an increase in N (the total number of private firms) decreases (increases) the optimal size if $\varepsilon < 1/2$ $(\varepsilon > 1/2)$.

When the market size in the region is set at a^*, public firms in both regions produce the same level of output, $q_{A0}^N = q_{B0}^N = 2/(6 + N)$. Given the optimal market size, the equilibrium outcomes in Case L become the same as those in Case S.

Proposition 10.3 *Even if the public firms are owned by the national government, to achieve the equilibrium outcomes under the state ownership case, the government must divide the region at a rate of* $a^* = (2 + \varepsilon N)/(4 + N)$.

This proposition implies that the national government can internalize the social costs of overproduction and underproduction by setting an optimal market size. An optimal market size achieves product efficiency for public firms in Case L. Thus, by setting an optimal market size, the government can replicate the equilibrium outcomes in Case S. However, it should be noted that the government cannot choose the market size such that the signs of $W_A^S - W_A^P$ and $W_B^S - W_B^P$ have the same sign as $W^S - W^P$.

10.6 Conclusion

In this chapter, we examined privatization in a mixed oligopoly with two regions in a country. We considered three cases: local ownership, state ownership, and private ownership. We first showed that in a mixed oligopoly, social welfare under a coordinated equilibrium is greater than that under an uncoordinated equilibrium. This implies that public firms should coordinate to produce efficiently without any change in consumer surplus. Second, if the number of private firms is large, country-level social welfare is greater in a privatized oligopoly than in a mixed oligopoly. Country-level privatization policies are not necessarily appropriate from a regional perspective, or in other words, they should be conducted by the national government. Third, the government can replicate a coordinated equilibrium under an uncoordinated equilibrium by setting the optimal market size. Finally, we see that the optimal market size depends on the number of private firms.

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Chapter 11
Competition and Quality in a Mixed Duopolistic Long-Term Care Market

Kota Sugahara and Minoru Kunizaki

Abstract We investigate a duopolistic long-term care market with uncertainty using a Hotelling-type spatial competition model where care providers decide the quality of the care to attract patients. We deal with three types of competition structures: (i) a duopoly with private nonprofit (NP) providers, (ii) a mixed duopoly with an NP and a private for-profit (FP) provider, and (iii) a duopoly with FP providers. We show that the equilibrium levels of quality in the mixed duopolistic market are higher than those in the NP duopoly and lower than those in the FP duopoly. Furthermore, in the mixed duopolistic market, while information improvement for the revision of the reimbursement system increases the quality levels of both providers, the effect of the information improvement in helping patients choose their care provider on the quality depends on the variance of the perceived quality.

11.1 Introduction

With many aging populations worldwide, the need for long-term care (LTC) has been growing, particularly in developed countries. However, because LTC remains a relatively small sector of the economy, the further development of both a workforce and workplaces is required.[1] Furthermore, the maintenance and improvement of the quality of LTC services have also become an important policy issue because its "quality measurement lags behind developments in health care" (OECD/European Commission 2013). In response to this situation, the OECD recommends the urgent introduction of a quality assessment system in those countries that are falling behind in efforts to develop and collect indicators on the

[1] See Colombo et al. (2011), for further detail.

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quality of care services.[2] In this chapter then, we are interested in the characteristics of competition and the effect of a quality assessment system in an LTC service market by applying theoretical considerations to a healthcare market.

In the field of economics, a significant number of studies investigate quality in the healthcare market. According to Gaynor (2006), who surveyed a large quantity of theoretical and empirical analyses in this area, an analysis of quality competition in a regulated price healthcare market showed that quality is increased via the number of hospitals and the level of the regulated price. Furthermore, Gravelle and Sivey (2010) found that the effect of improved information on the quality of profit-seeking hospitals depends on the cost difference between hospitals.

In a series of analyses on quality competition in a healthcare market, the models presented by Montefiori (2005) and Sanjo (2009) are helpful in considering the characteristics of competition and the effect of a quality assessment system in an LTC service market. Montefiori (2005) built a simple model where private for-profit hospitals compete for patients by means of the quality level in a healthcare market with uncertainty and then characterizes equilibrium outcomes. Sanjo (2009) extended Montefiori's model to a mixed duopoly by considering a publicly owned hospital that seeks to maximize social welfare. That study found that the quality level of the partially privatized hospital becomes higher than that of the private hospital.

In the context of analysis of a mixed oligopoly model, it is commonly assumed that competition exists between a publicly owned provider seeking social welfare and a profit-seeking private provider (e.g., a firm or hospital).[3] However, we find from Table 11.1 that in some countries the share of the publicly owned LTC provider is clearly smaller than the share of the public hospital.[4] In contrast, the share of private nonprofit (NP) providers is notable, particularly in France and Germany.

Therefore, we assume the existence of an NP instead of a publicly owned provider in the model and distinguish the objective of the NP's behavior from that of the publicly owned providers. Because the objective of the private provider might not be the maximization of social welfare, we assume that the NP's objective is budget maximization; thus, its profit is zero.

We study the equilibrium levels of quality in three types of competition structures: (i) a duopoly with NP providers, (ii) a mixed duopoly with an NP and private for-profit (FP) provider, and (iii) a duopoly with FP providers. We obtain two main results as follows. First, the equilibrium levels of quality in the mixed duopolistic

[2]See OECD/European Commission (2013).

[3]Ishibashi and Kaneko (2008) investigated competition and quality in a mixed oligopolistic market using a more general setting than Sanjo (2009), but did not consider uncertainty.

[4]Statistics regarding hospitals in EU countries and the United States are sourced from Nolte et al. (2014). Those concerning Japan are from the Ministry of Health, Labour and Welfare (MHLW) (2013a). Statistics on LTC facilities in EU countries come from Rodrigues et al. (2012), and those on the United States are sourced from Harris-Kojetin et al. (2013). Statistics on Japanese LTC facilities are from MHLW (2013b).

Table 11.1 Distribution of hospitals and LTC facilities by ownership type

Country	Public (%)	Private nonprofit (%)	Private for profit (%)
Hospitals			
France	35	29	39
Germany	30	36	35
United States	21	58	21
Japan	18	68	14
LTC facilities			
France	23	55	22
Germany	5	55	40
United States	5	55	40
Japan	1	43	56

market are higher than those in the NP duopoly and lower than those in the FP duopoly. Second, in the mixed duopolistic market, while information improvement for the revision of the reimbursement system increases the quality levels of both providers, the effect of the information improvement in helping patients choose their care provider on quality depends on the variance of the perceived quality.

This chapter is organized as follows. In Sect. 11.2, we describe the model for our analysis. In Sect. 11.3, we characterize the equilibrium levels of quality in the different structures of competition and compare the quality levels. In Sect. 11.4 we investigate the effect of introducing a quality assessment system on the quality levels. We conclude our analysis in Sect. 11.5 with a summary of our findings.

11.2 The Model

Our model follows those presented in Montefiori (2005) and Sanjo (2009), who investigated a healthcare market while also considering uncertainty. We extend them by looking to Gravelle and Sivey (2010) and Levaggi and Montefiori (2013) to investigate an LTC market.

11.2.1 Setting and Patients

There are two LTC providers (0 and 1) competing for patients who need to be supported by LTC services by means of the quality level in Hotelling-type spatial competition, wherein patients choose which LTC facility they are treated in by comparing the expected utilities. Facility 0, where provider 0 treats patients with the quality level x_0, is exogenously located at the endpoint 0 of the linear city with the unit length. Facility 1, where provider 1 provides care services to patients with the quality level x_1, is exogenously located at endpoint 1 of the city. We assume that $x_i \in (0, \infty]$, $i = 0, 1$.

A mass of patients, normalized to one and uniformly distributed, lie on the line. The patients' residences are exogenously fixed. Patients choose a provider by comparing the utility of the various alternatives that depend on quality level and travel costs. We assume that patients can use care services without any pecuniary burden. In contrast, a patient residing at y must pay travel costs μy to be treated at facility 0 or $\mu(1-y)$ for facility 1, where μ denotes a unit travel cost.[5]

In addition to travel costs, we consider uncertainty as a cost for the patients, following Montefiori (2005) and Sanjo (2009). They used a mean-variance method to deal with uncertainty in their models. Thus, in the present study, let $\widetilde{x}_i$ denote the perceived quality, which is normally distributed around the mean $\overline{x}_i$; that is, $\widetilde{x}_i \sim N\left(\overline{x}_i,\ \sigma^2_{x_i}\right)$, where $\sigma^2_{x_i}$ is the variance of the perceived quality. Patients who can only observe the perceived quality and not the true quality choose the provider whose care services may present the higher expected utility. They do so by comparing the perceived quality of LTC services provided by providers 0 and 1. For these patients, while the mean value $\overline{x}_i$ is a positive factor of their utility, uncertainty is disliked. We approximate the expected utility of the patient who resides at y as follows:[6]

$$U_y = \begin{cases} \alpha\overline{x}_0 - (1-\beta)\varepsilon\sigma^2_{x_0} - \mu y, & \text{if receiving from provider } 0 \\ \alpha\overline{x}_1 - (1-\beta)\varepsilon\sigma^2_{x_1} - \mu(1-y), & \text{if receiving from provider } 1, \end{cases} \tag{11.1}$$

where $\alpha(>0)$ denotes the parameter of the partial utility from the expected quality, which is described as a mean value. Furthermore, $\varepsilon(>0)$ is the parameter of the approximation of the expected utility, and $\beta \in [0,\ 1]$ denotes the information parameter on uncertainty. If the introduction of a quality assessment system provides abundant information to patients to help them choose a provider, β will approach 1, and then the variance of the perceived quality will not be a concern for patients.

Similar to a Hotelling model in which the distance from the LTC facility represents the amount of demand under the assumption of the uniform distribution of patients, the demand for the quality of each care service is derived by the following steps.[7] First, suppose a patient is indifferent to providers 0 and 1, the relationship between her/his utility in receiving an LTC service from each provider is written as:

$$\alpha\overline{x}_0 - (1-\beta)\varepsilon\sigma^2_{x_0} - \mu y = \alpha\overline{x}_1 - (1-\beta)\varepsilon\sigma^2_{x_1} - \mu(1-y). \tag{11.2}$$

Then, solving Eq. (11.2) for y yields

[5]That is, while μ is a physical travel cost for a patient who lives in a rural area where there are few LTC facilities, it may denote the searching costs (to find a desirable provider) of patients in urban areas where there are numerous facilities.

[6]See the Appendix.

[7]We implicitly assume that the quantity of service use per patient is unity.

$$y = \frac{1}{2\mu}\left[\alpha\left(\bar{x}_i - \bar{x}_j\right) + (1-\beta)\varepsilon\left(\sigma^2_{x_j} - \sigma^2_{x_i}\right)\right] + \frac{1}{2},\ i \neq j. \tag{11.3}$$

Multiplying the distance y by the population density of the patients that equals 1, we derive the following demand function for each provider:

$$D^0 = \frac{1}{2\mu}\left[\alpha(\bar{x}_0 - \bar{x}_1) + (1-\beta)\varphi\right] + \frac{1}{2}\ , \tag{11.4}$$

$$D^1 = \frac{1}{2\mu}\left[\alpha(\bar{x}_1 - \bar{x}_0) - (1-\beta)\varphi\right] + \frac{1}{2}, \tag{11.5}$$

where $\varphi = \varepsilon\left(\sigma^2_{x_1} - \sigma^2_{x_0}\right)$. Equations (11.4) and (11.5) show that the demand for the care services in each facility is a probability function that depends on the disparities between the mean values $(\bar{x}_1 - \bar{x}_0)$ and between the variances $(\sigma^2_{x_1} - \sigma^2_{x_0})$ of the perceived quality.

For the moment, we assume $\varphi > 0$ because in the LTC market in Japan, private NP providers were the first to provide LTC services, with private FP firms entering the market later with the introduction of public LTC insurance. In that situation, patients might have better information about the quality of the care delivered by NP providers than that of the latecomer FP providers. In the next section, we assume provider 0 to be the NP provider and provider 1 to be the FP firm in the case of a mixed duopoly. Therefore, we assume that $\widetilde{x}_1$ is more uncertain than $\widetilde{x}_0$, that is, $\varphi > 0$.

11.2.2 Long-Term Care Providers

The provider's cost is assumed to be a function of the quality level. Although we employ one variable, the quality level, we imagine it as the consolidated index of a multidimensional vector of the quality as in Chalkley and Malcomson (1998) and Levaggi and Montefiori (2013). Levaggi and Montefiori (2013) divided the quality of care provided by a hospital into two categories: medical and nonmedical. The former improves the quality of the clinical care, and the latter, termed hotel quality, includes the number of beds per room, visiting hours, private telephones, and nurses per ward. It was found that the presence of nonmedical factors improved the patients' stays in hospital.

Therefore, we imagine that one part of x_i is related to LTC services, which is already taken into account by the reimbursement system, and that one part of x_i includes the accommodation, the comfort level of the facility, and the caregiver's kindness toward the patient. We call this aspect of x_i hospitality quality. Hence, we represent a provider's cost function as the following equation (the two parts relate to the abovementioned quality levels):

$$C_i = cx_iD^i + \frac{x_i^2}{2}, \tag{11.6}$$

where c is a constant unit cost associated with the quality of the care services. In addition to the cost related to the demand, we assume that the hospitality quality is a public good for the patients in the LTC facility and that the cost related to the hospitality quality is a quadratic function (Brekke et al. 2006; Gravelle and Sivey 2010). We have done so because to increase the friendliness of the caregivers in the facility, the manager needs not only to train the caregivers but also to improve their workplace environment. Therefore, increasing hospitality quality might be more expensive than just training caregivers.

11.2.3 Reimbursement

We employ the following reimbursement system for our analysis[8]:

$$R_i = P_iD^i + \gamma\frac{x_i^2}{2}. \tag{11.7}$$

In contrast to the setting in Montefiori (2005) and Sanjo (2009), which assumed P_i as constant, we define P_i as a simple linear function, x_i; $P_i(x_i) = px_i$, where p, assumed as $p > c$, is the unit payment related to the quality level. We assume this because, according to studies that compare alternative reimbursement systems, constant P_i means that the reimbursement is a prospective payment system. That is, in the term related to the demand, $(P_i - cx_i)D^i$, revenue does not depend on the quality level, while the cost is associated with the quality. Ellis and McGuire (1986) pointed out that this simple prospective payment system results in underprovision and recommends a mixed reimbursement system of prospective and cost-based payments to avoid this problem.[9] Therefore, we assume that the revenue related to the demand is associated with the quality.

Furthermore, we consider the second term in Eq. (11.7): $\gamma \in [0, 1]$ is an information parameter, which indicates the ability of the purchaser who manages the reimbursement system to observe the cost of the hospitality quality. The introduction of a quality assessment system is considered to contribute to the revision of the reimbursement system, ensuring a greater correspondence with the total costs of LTC services by providing the purchaser with abundant information.

[8]Because we assume that the reimbursement system is fully financed by general tax without copayments or insurance premiums for simplicity, our analysis is based on a partial equilibrium model.

[9]Strictly speaking, they discuss this from the aspect of quantity not quality. However, their comments can be applied to the problem of quality choice (Levaggi 2005).

We simply describe the effect of the introduction of the assessment system as an increase in γ.

11.3 Equilibrium

In this section, we characterize the quality level in the three types of duopolistic markets: (i) both providers are private NP providers (an NN duopoly); (ii) provider 0 is an NP provider and provider 1 is a private FP firm (a mixed duopoly with superscript NF); and (iii) both providers are FP providers (an FF duopoly).

11.3.1 Objectives and Reaction Function

The profit of provider i is given by $\pi_i = R_i - C_i$. Thus, the profit of each provider is represented as follows:

$$\pi_0 = (p-c)x_0\left(\frac{\alpha(\bar{x}_0 - \bar{x}_1) + (1-\beta)\varphi}{2\mu} + \frac{1}{2}\right) + (\gamma - 1)\frac{x_0^2}{2}, \tag{11.8}$$

$$\pi_1 = (p-c)x_1\left(\frac{\alpha(\bar{x}_1 - \bar{x}_0) - (1-\beta)\varphi}{2\mu} + \frac{1}{2}\right) + (\gamma - 1)\frac{x_1^2}{2}. \tag{11.9}$$

As discussed in Sect. 11.1, the NP provider's objective is that of budget maximization because we imagine an LTC market in which there are few publicly owned providers seeking to maximize social welfare. The NP provider is assumed to choose x_i to achieve $\pi_i = R_i - C_i = 0$. In contrast, the FP provider is assumed to choose the quality level to maximize its profit. Following Montefiori (2005) and Sanjo (2009), we assume that the mean value of the perceived quality is equal to the true quality to obtain a reasonable solution.

From the rule of budget maximization, the quality of the care delivered by each NP provider is written as the following reaction function to the quality of the rival's care:

$$x_0^N = \Lambda(\alpha x_1 + \Phi), \tag{11.10}$$

$$x_1^N = \Lambda(\alpha x_0 + \Psi), \tag{11.11}$$

where $\Lambda = \frac{p-c}{(\gamma-1)\mu+(p-c)\alpha}$, $\Phi = -\mu - (1-\beta)\varphi$, and $\Psi = -\mu + (1-\beta)\varphi$. To obtain reasonable equilibrium solutions in the next subsection, we also assume that $\Phi < 0$ and $\Psi < 0$ for any β and φ and that $0 \leq \gamma < 1 - \frac{(p-c)\alpha}{2\mu}$. The former assumption of a negative value on the intercept of the reaction function means that the travel cost is sufficiently large. The latter assumption on the range of γ means that the purchaser will not be able to perfectly observe the cost related to the hospitality quality, even if a quality assessment system is introduced. In addition, this assumption derives the condition $2 < \alpha\Lambda$, which assures a strategic complement structure in any duopoly.

Then, the quality of the care delivered by each FP provider is written as the following reaction function from the first-order condition for profit maximization:

$$x_0^F = \frac{1}{2}\Lambda(\alpha x_1 + \Phi), \tag{11.12}$$

$$x_1^F = \frac{1}{2}\Lambda(\alpha x_0 + \Psi). \tag{11.13}$$

We recognize from Eqs. (11.10, 11.11, 11.12, and 11.13) that the NP provider is more aggressive than the FP provider. The reason why the provider's reaction depends on the difference of objective can be explained as follows. The NP provider tries to choose the highest quality available between alternatives to balance the total revenues and costs, taking the rival's quality level as given. In contrast, the FP provider chooses a quality level to balance the marginal revenue and marginal cost, also taking the rival's quality as given. When a rival's quality level increases, the damage to NP provider's balance of its account, denoted by $-\frac{\alpha(p-c)x_i}{2}dx_j$, $i \neq j$, is larger than that of the FP provider, denoted by $-\frac{\alpha(p-c)}{2}dx_j$. Therefore, the NP provider reacts against the marginal increase in the rival's quality more sensitively than the FP provider to recover the balance of its account. Thus, the reaction function of the NP is more elastic than that of the FP provider.

11.3.2 Equilibrium Levels of Quality in the Three Cases

From Eqs. (11.10) and (11.11), we derive the quality level of each provider in the case of an NN duopoly as follows:

$$x_0^{NN} = -\frac{\Lambda(\Phi + \Psi\alpha\Lambda)}{(\alpha\Lambda - 1)(\alpha\Lambda + 1)}, \tag{11.14}$$

$$x_1^{NN} = -\frac{\Lambda(\Psi + \Phi\alpha\Lambda)}{(\alpha\Lambda - 1)(\alpha\Lambda + 1)}. \tag{11.15}$$

Next, Eqs. (11.10) and (11.13) derive each quality level in the case of a mixed duopoly:

$$x_0^{NF} = -\frac{\Lambda(2\Phi + \Psi\alpha\Lambda)}{(\alpha\Lambda - \sqrt{2})(\alpha\Lambda + \sqrt{2})}, \tag{11.16}$$

$$x_1^{NF} = -\frac{\Lambda(\Psi + \Phi\alpha\Lambda)}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}. \tag{11.17}$$

Finally, Eqs. (11.12) and (11.13) derive each quality level in the case of FF duopoly as follows:

$$x_0^{FF} = -\frac{\Lambda(2\Phi + \Psi\alpha\Lambda)}{(\alpha\Lambda - 2)(\alpha\Lambda + 2)}, \tag{11.18}$$

$$x_1^{FF} = -\frac{\Lambda(2\Psi + \Phi\alpha\Lambda)}{(\alpha\Lambda - 2)(\alpha\Lambda + 2)}. \tag{11.19}$$

Here we compare the levels of the quality between the different types of duopolies. Comparing Eqs. (11.14), (11.16), and (11.18), we derive $x_0^{NN} < x_0^{NF} < x_0^{FF}$. Then, comparing Eqs. (11.15), (11.17), and (11.19), we derive $x_1^{NN} < x_1^{NF} < x_1^{FF}$.

Based on the above explanation of the reaction function of the provider to different objectives and because the reaction function of the NP provider is more elastic than that of the FP provider, the equilibrium level of the quality is lowest in the case of an NN duopoly. In contrast, the existence of an FP provider in the market calms the competition somewhat. Thus, we obtain Proposition 11.1.

Proposition 11.1 *In the long-term care market where duopolistic providers compete for patients by choosing the quality level of care, the equilibrium levels of quality in the case of a mixed duopoly are higher than those in a duopoly with private nonprofit providers and are lower than those in a duopoly with private for-profit providers.*

Proposition 11.1 seeks to determine whether the quality levels in the case of an NP duopoly and FP duopoly are higher, or lower, than those of a monopoly. We first obtain the quality levels of the private for-profit monopoly as those are easily derived from the first-order conditions to maximize a combined profit, $\pi_0 + \pi_1$. They are represented as

$$x_0^{FMON} = -\frac{\Lambda(\Phi + \Psi\alpha\Lambda)}{2(\alpha\Lambda - 2)(\alpha\Lambda + 2)}, \tag{11.20}$$

$$x_1^{FMON} = -\frac{\Lambda(\Psi + \Phi\alpha\Lambda)}{2(\alpha\Lambda - 2)(\alpha\Lambda + 2)}. \tag{11.21}$$

Comparing Eqs. (11.18) and (11.19) with (11.20) and (11.21), we obtain $x_0^{FMON} < x_0^{FF}$ and $x_1^{FMON} < x_1^{FF}$. Because the for-profit monopoly does not have to compete for patients, its administrators intend to minimize costs. Hence, the quality level in a monopoly is lower than that in a duopolistic market.

Because it is relatively complicated to derive the level of quality in a private nonprofit monopoly, we ignore the disparity of the variance for a simple comparison, that is, $\varphi = 0$ is assumed. Under this assumption, Eqs. (11.14) and (11.15) are rewritten as

$$x_0^{NN} = x_1^{NN} = -\frac{p-c}{\gamma - 1}. \tag{11.22}$$

However, because the nonprofit monopoly also has no rival and faces all the demand denoted by unity, its profit is written as

$$\Pi = (p-c)(x_0 + x_1) + \frac{1}{2}(\gamma - 1)\left(x_0^2 + x_1^2\right). \tag{11.23}$$

Taking $x_0 = x_1 = x^{NMON}$ into account, the quality level that satisfies the rule of budget maximization on Eq. (11.23) is derived as the following equation:

$$x^{NMON} = -\frac{2(p-c)}{\gamma - 1}. \tag{11.24}$$

Comparing Eqs. (11.22) and (11.24), we recognize a downward pressure of competition on the quality level of the NP provider's care.[10] Therefore, we obtain Corollary 11.1.

Corollary 11.1 *Although the introduction of competition into a private for-profit market brings higher levels of quality, competition between nonprofit providers reduces the quality level below that of a monopoly.*

Next, we compare the quality levels between different providers in a certain competition. From Eqs. (11.14) and (11.15), the difference in the quality in the case of an NN duopoly is written as

$$x_0^{NN} - x_1^{NN} = -\frac{2\Lambda}{\alpha\Lambda + 1}(1-\beta)\varphi. \tag{11.25}$$

Equations (11.18) and (11.19) derive the difference in the quality in an FF duopoly:

$$x_0^{FF} - x_1^{FF} = -\frac{2\Lambda}{\alpha\Lambda + 2}(1-\beta)\varphi. \tag{11.26}$$

From Eqs. (11.25) and (11.26), we recognize that in a duopoly with the same type of providers, it is the difference in the variance of the perceived quality between providers that determines which provider has the highest level of quality.

[10]Incidentally, x^{NMON} represents the highest level of the alternatives we considered. However, because we ignore the patient's social burden to finance the reimbursement system, we cannot conclude that a nonprofit monopoly is the most desirable market structure for LTC from a social welfare perspective.

In contrast, from Eqs. (11.16) and (11.17), the difference in the quality level in a mixed duopoly is written as

$$x_0^{NF} - x_1^{NF} = \frac{\Lambda}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}[\mu - (1-\beta)\varphi(2\alpha\Lambda - 3)]. \tag{11.27}$$

Because we assume that $2 < \alpha\Lambda$, the difference depends on the sign of the term in square brackets, that is, $\mu - (1-\beta)\varphi(2\alpha\Lambda - 3) \lesseqgtr 0 \Rightarrow x_0^{NF} \lesseqgtr x_1^{NF}$. From these findings, we obtain Proposition 11.2.

Proposition 11.2 *The difference in the quality level is characterized as follows:* (i) *in a duopoly with the same type of providers,* $\varphi \gtreqless 0 \Rightarrow x_0 \lesseqgtr x_1$*, and* (ii) *a mixed duopoly,* $\mu - (1-\beta)\varphi(2\alpha\Lambda - 3) \lesseqgtr 0 \Rightarrow x_0 \lesseqgtr x_1$*, and* $x_0 > x_1$ *if* $\varphi = 0$.

Because $\varphi = \varepsilon\left(\sigma_{x_1}^2 - \sigma_{x_0}^2\right)$, Proposition 11.2(i) means that the provider with higher uncertainty, who is initially disliked by patients, tries to choose higher quality than its rival to overcome this disadvantage in a duopoly with the same type of providers. In contrast, in a mixed duopoly, the quality level of the care delivered by the FP firm (provider 1) is lower than that of the NP provider (provider 0), even if the uncertainty of 1's quality is equal to or somewhat higher than the uncertainty of 0's quality. The reason why the result of the mixed duopoly is different from the other types of duopolies is the difference in the behaviors of the providers. That is, the NP provider offers a more sensitive reaction. Proposition 11.2(ii) means that the impact of provider 0's behavior (as an aggressive NP provider) on the equilibrium is greater than that of provider 1 who tries to overcome the disadvantage caused by the higher level of uncertainty.

11.4 The Effect of Improving Information

We consider the effect of the introduction of a quality assessment system on the quality level. Because our main interest is the effect on a mixed duopolistic LTC market, we focus on an NF duopoly. The equilibrium levels of the quality in this market are as follows:

$$x_0^{NF} = -\frac{\Lambda(2\Phi + \alpha\Psi)}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}, \; x_1^{NF} = -\frac{\Lambda(\Psi + \alpha\Phi)}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}.$$

First, we consider that the introduction of a quality assessment system assists patients to choose a care provider via improving information about the variance of the perceived quality. The effect of a marginal increase in β is represented by the following equations:

$$\frac{\partial x_0}{\partial \beta} = \frac{\Lambda(\alpha\Lambda - 2)\varphi}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}, \tag{11.28}$$

$$\frac{\partial x_1}{\partial \beta} = \frac{-\Lambda(\alpha\Lambda - 1)\varphi}{\left(\alpha\Lambda - \sqrt{2}\right)\left(\alpha\Lambda + \sqrt{2}\right)}. \tag{11.29}$$

Thus, we obtain $\frac{\partial x_0}{\partial \beta} \gtreqless 0$ and $\frac{\partial x_1}{\partial \beta} \lesseqgtr 0$ under $\varphi \gtreqless 0$.

As discussed in Sect. 11.3, because uncertainty is a disadvantage for providers, a provider with a high level of uncertainty is motivated to choose a higher level of quality to attract patients who initially dislike her/him. However, the introduction of a quality assessment system for patients, paradoxically, removes this incentive to overcome the disadvantage by reducing uncertainty. Therefore, if $\mu - (1-\beta)\varphi(2\alpha\Lambda - 3) > 0$ and $\varphi > 0$ are satisfied and thus $x_0 > x_1$, the introduction of a quality assessment system for patients increases the disparity between quality levels in a mixed duopolistic market.

We now consider that the introduction of a quality assessment system incentivizes the purchaser to revise the reimbursement system to correspond to costs related to both the care service quality and the hospitality quality. The effect of a marginal increase in γ is represented by the following equations:

$$\frac{\partial x_0}{\partial \gamma} = \frac{2[(2\Psi + \Phi\alpha\Lambda)\alpha\Lambda + 2\Phi]}{\left(\alpha^2\Lambda^2 - 2\right)^2}\left(\frac{-\mu\Lambda^2}{p-c}\right) > 0, \tag{11.30}$$

$$\frac{\partial x_1}{\partial \gamma} = \frac{(4\Phi + \Psi\alpha\Lambda)\alpha\Lambda + 2\Psi}{\left(\alpha^2\Lambda^2 - 2\right)^2}\left(\frac{-\mu\Lambda^2}{p-c}\right) > 0. \tag{11.31}$$

The signs of Eqs. (11.30) and (11.31) follow these assumptions: $2 < \alpha\Lambda$, $\Phi < 0$, and $\Psi < 0$. Because we derive $\frac{\partial x_0}{\partial \gamma} < \frac{\partial x_1}{\partial \gamma}$ when $\varphi = 0$, the introduction of a quality assessment system for the purchaser's revisions reduces the disparity between quality levels in a mixed duopolistic market when there is no difference in the variance of the perceived quality.

Summarizing the above discussions, we obtain Proposition 11.3.

Proposition 11.3 *In a mixed duopolistic market where the relationship $x_0 > x_1$ exists, the introduction of a quality assessment system for patients might increase the disparity between quality levels. In contrast, the introduction of a quality assessment system for the purchaser possibly reduces the disparity with increases in both quality levels, x_0 and x_1.*

In an LTC market with a duopolistic competition structure and uncertainty, there is the concern that the introduction of a quality assessment system for patients removes the incentive for a provider with higher uncertainty to seek a higher level of quality. However, the introduction of a quality assessment system for the purchaser to revise the reimbursement system increases the quality levels of the care services by both providers. Therefore, from the simple viewpoint of the quality level, it is more desirable that a quality assessment system is only utilized for purchasers to revise their reimbursement system.

11.5 Concluding Remarks

Ensuring and improving the quality of LTC services is an important policy issue in most countries with an aging population. Thus, the OCED has recommended introducing a quality assessment system. In this chapter, we investigated the impact of introducing such a system on the quality of LTC services in a market with uncertainty. Using Hotelling-type spatial competition, where care providers decide the quality of the care to attract patients, we investigated a duopolistic LTC market. In contrast to existing literature on mixed oligopoly models, we assumed the existence of private NP providers instead of publicly owned providers to ensure the model was appropriate for the LTC market in a wide number of countries.

We examined three types of competition structures: (i) a duopoly with private NP providers, (ii) a mixed duopoly with an NP and a private FP provider, and (iii) a duopoly with FP providers. We obtained two main results. First, the equilibrium levels of quality in the mixed duopolistic market are higher than those in the NP duopoly and lower than those in the FP duopoly. Second, in the mixed duopolistic market, while information improvement for the revision of the reimbursement system increases the quality levels of both providers, the effect of an information improvement in choosing a care provider based on quality depends on the variance of the perceived quality.

These findings lead to the following implications. First, the entry of private FP providers into the LTC market is required to improve the quality level of LTC services. Second, a quality assessment system should be introduced to revise the reimbursement system to ensure correspondence to costs related not only to care service quality but also to hospitality quality.

Despite our contributions to this topic, some unresolved issues remain. The first is the finance of the reimbursement system. In the real world, an increase in the expenditure of public or private LTC insurance is a serious problem. To consider this issue, our model should include the burden of an insurance premium or tax financing reimbursement. The second point is the assumption regarding the objective of the NP provider. Levaggi and Montefiori (2013) considered alternative objectives for a public hospital, such as market share, equality treatment, and reputation. Therefore, it is possible to assume alternative objectives (other than budget maximization) for the NP provider and to investigate the equilibrium level of quality in different situations. These issues can be addressed in future research.

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Appendix

Following Montefiori (2005), we denote the utility of a representative patient as a general function form of the perceived quality and the distance and define it quasi-concave, that is,

$$u = u(\widetilde{x}_i,\ y),\ u' > 0,\ u'' < 0. \tag{A11.1}$$

We then represent the condition for the patient to choose provider i, by using the expected utility form:

$$E[u(\widetilde{x}_i,\ y)] \geq E\big[u\big(\widetilde{x}_j,\ y\big)\big],\ i \neq j. \tag{A11.2}$$

When we assume that the perceived quality $\widetilde{x}_i$ is very close to the expected quality $\overline{x}_i = E[\,\widetilde{x}_i]$ for every state of the world, we can use the following Taylor's approximation as follows:

$$u(\widetilde{x}_i,\ y) \approx u(\overline{x}_i,\ y) + (\widetilde{x}_i - \overline{x}_i)u'_{\tilde{x}_i}(\overline{x}_i) + (\widetilde{x}_i - \overline{x}_i)^2 \frac{u''_{\tilde{x}_i}(\overline{x}_i)}{2}. \tag{A11.3}$$

Finally, using expectations and denoting $E\Big[(\widetilde{x}_i - \overline{x}_i)^2\Big]$ by $\sigma^2_{x_i}$, we obtain

$$E[u(\widetilde{x}_i,\ y)] \approx u(\overline{x}_i,\ y) + \sigma^2_{x_i} \frac{u''_{\tilde{x}_i}(\overline{x}_i)}{2}. \tag{A11.4}$$

Denoting $\frac{u''_{\tilde{x}_i}(\overline{x}_i)}{2}$ by $-\varepsilon$ and adding the term travel cost, we can represent the utility of the patient as Eq. (11.1).

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Chapter 12
Privatization with a CSR Private Firm

Akihito Itano

Abstract The purpose of this chapter is to analyze how the existence of a corporate social responsibility (CSR) private firm influences the privatization of a public firm in a mixed duopoly model. Furthermore, we compare these results with those generated from the competition between a public firm and a pure private firm, which have been the subject of many discussions. We show that replacing a pure private firm with a CSR private firm increases the consumer surplus, but lowers the privatization ratio of the public firm. This result occurs because the public firm cannot distinguish the output stemming from profit maximization from that related to the CSR considerations of the private firm under a typical Cournot competitive environment.

12.1 Introduction

Since De Fraja and Delbono (1989) first dealt with the privatization of public firms, many economists have addressed this subject. A number of recent studies have examined a new theme in existing economic theory, that is, competition between private firms (instead of public firms) that engage in corporate social responsibility (CSR) and pure private firms. The concept of CSR states that while private firms seek profits, they are also responsible for any effects their activities may have on society. Therefore, in the operation of their business and when generating profits, they must consider all stakeholders including consumers, shareholders, and communities. Thus, CSR private firms decide their output to maximize consumer surplus as well as their profit. A number of previous studies have examined CSR private firms (Goering 2008; Lambertini and Tampieri 2010; Kopel and Brand 2012). Following on from these studies, this chapter investigates how the existence of a CSR private firm influences the privatization of a public firm. We then compare these results with those from competition between a public firm and a pure private firm, which have been widely studied. More specifically, is it possible to substitute a CSR private firm for a pubic firm?

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As CSR-related issues are generally discussed in the field of business management, the main analytic methodology has been the empirical investigation of correlations between corporate social performance and corporate financial performance: can the CSR activity of the firms enhance their financial performance as measured by such business indicators as ROE or ROA? In the early 2000s, the theory of "strategic" CSR was first proposed by Baron (2001) and McWilliams and Siegel (2001) and differentiated from "basic" CSR. In contrast to a firm's basic CSR activity (e.g., corporate philanthropy), private firms implemented profit-maximizing CSR activities by considering a wide range of costs and benefits when investing in socially responsible projects. For instance, the development and launch of hybrid cars or electronic cars by major automobile manufacturers are just one example of strategic CSR. We also introduce this proposition into our discussion and analyze the influence on the privatization of public firms. That is, we claim that it is strategic CSR firms themselves that decide the degree of CSR activity to maximize their profits. This is contrast to basic CSR firms, which take it as given, as if following "invisible" social requests.

From our discussion below, we conclude that replacing a pure private firm with a CSR private firm increases the consumer surplus, but decreases the privatization ratio of the public firm. This decrease in the privatization ratio seems contrary to intuition. Because the philosophy of a CSR private firm is similar to that of a public firm, when there is a CSR private firm in the economy, the government is likely to proceed with the privatization of the public firm with the view that a CSR private firm can perform the role of the public firm. This contradictory result occurs because the public firm cannot distinguish the output led from profit maximization from that related to the CSR considerations of the private firm under typical Cournot competition.

This chapter is organized as follows. In the next section, we present a basic model. In Sect. 12.3, we construct Cournot competition between a public firm and a basic CSR private firm and investigate how an optimal privatization policy can change in response to the extent of the CSR considerations of the private firm. In Sect. 12.4, we focus on a strategic CSR private firm instead of a basic CSR private firm. Finally, in Sect. 12.5, we offer some concluding comments and suggestions for future research.

12.2 The Model

We consider a mixed duopoly economy constituted by a public firm and a CSR private firm. The public firm is indexed by firm $i = 0$, and the CSR private firm is indexed by firm $i = 1$. Furthermore, q_i denotes firm i's output. Each firm produces a homogeneous good, and the inverse demand function is given by $p = p(Q) = a - Q$. Total demand Q can be expressed as the sum of each firm's outputs in equilibrium as follows:

$$Q = q_0 + q_1. \tag{12.1}$$

We assume that each firm's cost function is identical and represents $c_i = \frac{k}{2}{q_i}^2$. For analytical simplicity, we set $k = 1$. Therefore, a firm's profit can be written as $\pi_i = p(Q)q_i - \frac{1}{2}{q_1}^2$.

In the above setting, social welfare can be expressed as

$$W = aQ - \frac{1}{2}Q^2 - \frac{1}{2}{q_0}^2 - \frac{1}{2}{q_1}^2. \tag{12.2}$$

The public firm's objective function is given by the weighted average of its profit and social welfare, following Matsumura (1998). Thus,

$$V_0 = \theta\pi_0 + (1 - \theta)W, \tag{12.3}$$

where θ $(0 \leq \theta \leq 1)$ represents the privatization ratio of the public firm.

In contrast, the CSR private firm not only seeks its profit but is also concerned with the consumer surplus to some degree. The specific objective function is similar to the public firm's one and is assumed to be the weighted average of its profit and social welfare excluding the other firm's profit:

$$\begin{aligned} V_1 &= (1 - \phi)\pi_1 + \phi\,(\pi_1 + CS) \\ &= \pi_1 + \phi\, CS \\ &= (a - Q)q_1 - \frac{1}{2}{q_1}^2 + \phi\left[\left(aQ - \frac{1}{2}Q^2\right) - (a - Q)Q\right], \end{aligned} \tag{12.4}$$

where ϕ $(0 \leq \phi \leq 1)$ represents the degree of CSR considerations.

12.3 Basic CSR Private Firm and the Privatization of a Public Firm

In this section, we derive the optimal privatization ratio of a public firm with a basic CSR private firm, named here as firm 1B. The structure of the game in this section is constituted by two stages. In the first stage, the government decides the privatization ratio to maximize social welfare. In the second stage, based on the government-determined privatization ratio, the two firms choose their output in Cournot competition.

As usual, we solve this game by backward induction. In the second stage, each firm decides its own output, taking the other firm's output as given. From the public firm's first-order condition, $\frac{\partial V_0}{\partial q_0} = 0$,, we obtain its reaction function as follows:

$$q_0 = \frac{a - q_{1B}}{2 + \theta}. \tag{12.5}$$

In the same way, we obtain the basic CSR private firm's reaction function from the first-order condition, $\frac{\partial V_{1B}}{\partial q_{1B}} = 0$:

$$q_{1B} = \frac{a - (1 - \phi)q_0}{3 - \phi}. \tag{12.6}$$

As we can see in Fig. 12.1, because the slopes of both reaction functions are negative, we find that there is a strategic substitutional relationship between the public firm and the CSR private firm. We also find by illustrating the above two reaction functions that the output of the public firm decreases and that of the basic CSR private firm increases. This occurs because the CSR private firm puts more weight on the consumer surplus, which is indicated by the upward shift of the reaction function of the CSR private firm in Fig. 12.1.

When the public firm and the basic CSR firm implement Cournot competition on their outputs, each firm's output and the total output can be obtained as follows:

$$q_0^{*}(\theta, \phi) = \frac{(2 - \phi)a}{2 + (3 - \phi)(1 + \theta)}, \tag{12.7}$$

$$q_{1B}^{*}(\theta, \phi) = \frac{(1 + \theta + \phi)a}{2 + (3 - \phi)(1 + \theta)}, \tag{12.8}$$

$$Q^{*}(\theta, \phi) = \frac{(3 + \theta)a}{2 + (3 - \phi)(1 + \theta)}. \tag{12.9}$$

Now we look the government's decision regarding the optimal privatization policy to maximize social welfare in the first stage of the game. Social welfare in this instance can be expressed as

$$\begin{aligned} W^{*}(\theta, \phi) &= aQ - \frac{1}{2}Q^2 - \frac{1}{2}{q_0}^2 - \frac{1}{2}{q_{1B}}^2 \\ &= \frac{\left(8 + 10\theta - 2\phi - 5\theta\phi + 2\theta^2 - \theta^2\phi - \phi^2\right)a^2}{[2 + (3 - \phi)(1 + \theta)]^2}. \end{aligned} \tag{12.10}$$

By differentiating (12.10) by θ and setting it to zero, we obtain the optimal privatization ratio as the function of ϕ as follows:

$$\theta^{*}(\phi) = -\frac{(2\phi - 1)(\phi - 1)}{3\phi - 5}. \tag{12.11}$$

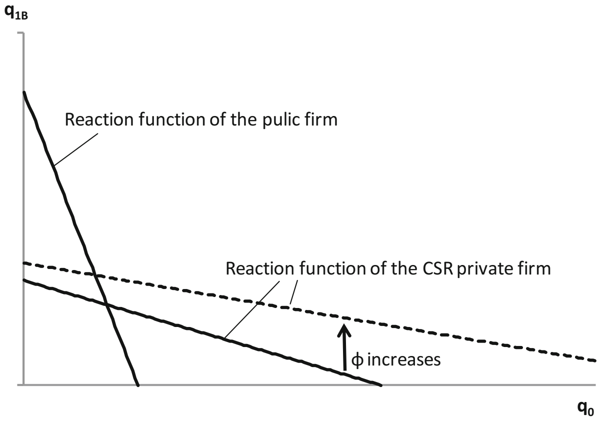

Fig. 12.1 Reaction functions of the public firm and the CSR private firm

We can confirm that the solution θ^* of (12.11) satisfies the second-order condition by substituting it into the second derivative of equation (12.10) with regard to θ.

We can illustrate the relationship between θ and ϕ facing the public firm. The case of $\phi = 0$ means an economy wherein a public firm and a pure private firm compete. Figure 12.2 shows that as the basic CSR private firm's concern for the consumer surplus increases, the optimal privatization ratio of a public firm decreases. In addition, because θ becomes negative when $1/2 \leq \phi \leq 1$, the government chooses full nationalization as a corner solution for the optimal policy.

By substituting Eq. (12.11) into Eqs. (12.7), (12.8), and (12.9), we obtain each firm's output and the total output as the function of ϕ:

$$q_0^*(\phi) = \frac{(5 - 3\phi)a}{2(\phi^2 - 4\phi + 7)}, \tag{12.12}$$

$$q_{1B}^*(\phi) = \frac{(3 + \phi)a}{2(\phi^2 - 4\phi + 7)}, \tag{12.13}$$

$$Q^*(\phi) = \frac{(4 - \phi)a}{\phi^2 - 4\phi + 7}. \tag{12.14}$$

The effect of the CSR private firm's CSR considerations on each firm's output and total demand is determined within $0 \leq \phi \leq 1$ as follows:

$$\frac{dq_0^*}{d\phi} = \frac{2(3\phi^2 - 10\phi - 1)a}{4(\phi^2 - 4\phi + 7)^2} < 0, \tag{12.15}$$

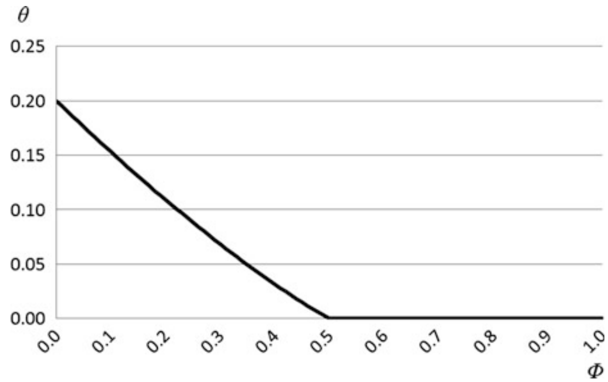

Fig. 12.2 Optimal privatization ratio of the public firm

$$\frac{dq_{1B}{}^{*}}{d\phi} = \frac{-2(\phi^2 + 6\phi - 19)a}{4(\phi^2 - 4\phi + 7)^2} > 0, \tag{12.16}$$

$$\frac{dQ^{*}}{d\phi} = \frac{(\phi^2 - 8\phi + 9)a}{(\phi^2 - 4\phi + 7)^2} > 0. \tag{12.17}$$

Finally, each firm's profit can be obtained as

$$\pi_0{}^{*}(\phi) = \frac{(5 - 3\phi)(4\phi^2 - 9\phi + 7)a^2}{8(\phi^2 - 4\phi + 7)^2}, \tag{12.18}$$

$$\pi_{1B}{}^{*}(\phi) = \frac{(9 - 4\phi)(1 - \phi)(3 + \phi)a}{8(\phi^2 - 4\phi + 7)^2}. \tag{12.19}$$

To numerically demonstrate the effect of the CSR private firm's CSR considerations on each firm's profit, we make a calibration by setting $a = 10$. The results are shown in Table 12.1.

From the above calibration analyses, when the degree of CSR considerations increases, we get the given results. First, while the output of a public firm decreases, that of a basic CSR private firm increases. In sum, total demand increases, which leads the consumer surplus, $Q^2/2$, to increase. Second, the profit of a public firm decreases, but the profit of a CSR private firm at first increases slightly and then decreases. Third, total welfare is maximized at $\phi = 1/2$. This is because the negative effect on each firm's profit exceeds the positive effect on the consumer surplus.

By this calibration, we clarify how a greater consideration of CSR results in the government implementing a weaker policy for the privatization (higher nationalization) of public firms. Furthermore, a greater concern for consumer surplus results in a decrease in output by a public firm, and this leads to a decrease in the consumer surplus. In this case, the government should choose a higher nationalization policy from a social welfare viewpoint.

Table 12.1 Effect of CSR considerations

ϕ	θ^*	$q_0{}^*$	$q_{1B}{}^*$	Q^*	CS^*	$\pi_0{}^*$	$\pi_{1B}{}^*$	W^*
0.00	0.200	3.571	2.143	5.714	16.327	8.929	6.888	32.143
0.04	0.181	3.566	2.222	5.788	16.751	8.662	6.890	32.302
0.50	0.000	3.333	3.333	6.667	22.222	5.556	5.556	33.333
0.70	0.000	3.023	3.953	6.977	24.337	4.570	4.137	33.045
1.00	0.000	2.500	5.000	7.500	28.125	3.125	0.000	31.250

12.4 Strategic CSR Private Firm and the Privatization of a Public Firm

We now consider an economy in which a strategic CSR private firm, instead of a basic CSR private firm, competes with a public firm. As noted in Sect. 12.1, a strategic CSR firm itself can choose the degree of CSR activity, whereas a basic CSR firm takes it as given. Regarding the stages of the game, the second stage is the same as in the case of the basic CSR private firm, but the first stage of this game is different in that the strategic CSR private firm decides its level of concern for consumer surplus at the same time that the government decides the optimal privatization policy. Based on each firm's output and the total output under Cournot competition, the profit of a strategic CSR firm, labeled here as firm 1S, is obtained as

$$\begin{aligned}\pi_{1S}{}^*(\theta,\phi) &= p\left(Q^*(\theta,\phi)\right)q_{1S}{}^*(\theta,\phi) - \frac{1}{2}\left(q_{1S}{}^*(\theta,\phi)\right)^2 \\ &= \frac{\left(3+6\theta-2\theta\phi+3\theta^2-2\theta^2\phi-3\phi^2-2\theta\phi^2\right)a^2}{2[2+(3-\phi)(1+\theta)]^2}.\end{aligned} \tag{12.20}$$

By differentiating (12.20) by ϕ and setting it to zero, we obtain the strategic CSR private firm's optimal degree of concern for consumer surplus as follows:

$$\phi^*(\theta) = \frac{1+\theta}{\theta^2+5\theta+5}. \tag{12.21}$$

We can confirm that the solution ϕ^* of (12.21) satisfies the second-order condition by substituting it into the second derivative of Eq. (12.20) with regard to ϕ.

Figure 12.3 illustrates the above relationship between ϕ and θ, as faced by the strategic CSR private firm. The degree of concern for consumer surplus ranges from 2/11 to 1/5 within $0 \leq \theta \leq 1$. There is also a negative relationship between ϕ and θ.

Finally, we see the equilibrium privatization ratio of the public firm and the equilibrium degree of the CSR considerations of the strategic CSR private firm. As it is difficult to obtain the equilibrium values, we give a numerical example in Fig. 12.4 by plotting the values of n. In other words, we plot Figs. 12.2 and 12.3 in

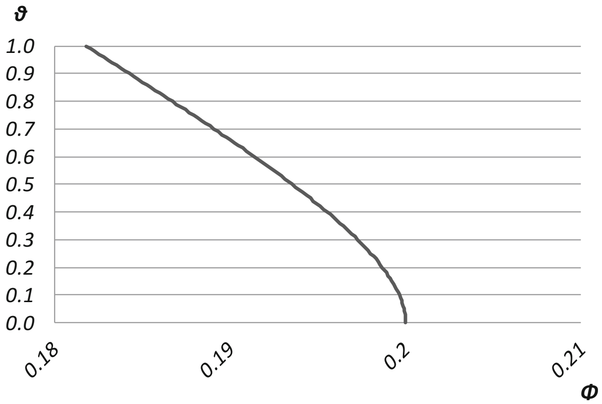

Fig. 12.3 Strategic CSR private firm's optimal degree of concern for consumer surplus

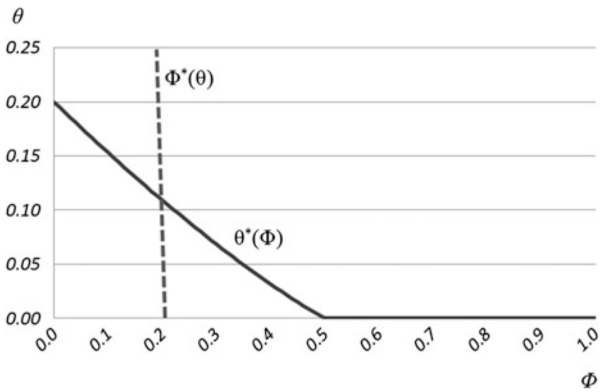

Fig. 12.4 Equilibrium for the privatization ratio of the public firm and the strategic CSR private firm's degree of concern for consumer surplus

the same figure, taking ϕ as the horizontal axis and θ as the vertical axis. As the figure shows, there is a unique interior solution.

Additionally, together with the calibration result in Sect. 12.3, we find that this economy in equilibrium can attain larger social welfare as well as total output and consumer surplus because the equilibrium degree of the CSR considerations of the strategic CSR private firm is within $\phi < 1/2$. However, the government chooses a weaker privatization policy for the public firm compared with a pure private firm, which is not concerned about CSR, that is, $\theta^* < 1/5$.

12.5 Conclusion

We found that replacing a pure private firm with a CSR private firm increases the consumer surplus, but decreases the privatization ratio of the public firm. This decrease of the privatization ratio seems to be contrary to intuition. Because the philosophy in a CSR private firm is similar to that in a public firm, when there is a CSR private firm in the economy, the government is likely to proceed with the privatization of the public firm because the CSR private firm can fulfill the role of the public firm. This contradictory result occurs because the public firm cannot distinguish output stemming from profit maximization from that related to the CSR considerations of the private firm under typical Cournot competition.

This current study could be extended by considering a scenario where the government captures CSR activity. For example, Mansakis et al. (2013) discussed CSR-certified institutions, acknowledging that CSR activities should be accepted by stakeholders to ensure that strategic CSR firms engage in such activities. In addition, it might be possible to reconsider the game structure in the case of a strategic CSR firm. For example, although we obtained the above results under a situation where a public firm and a strategic CSR private firm simultaneously decide their degree of consideration for social welfare (consumer surplus), it is also natural to assume that the government decides its privatization policy by knowing in advance the level of concern that a strategic CSR private firm has for the consumer surplus.

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Chapter 13
Market Expansion by Advertising and a Mixed Oligopoly

Minoru Kunizaki and Mitsuyoshi Yanagihara

Abstract We introduce a market-expanding measure, advertising, into the model of a mixed oligopoly and show how advertising affects the levels of production for both public and private firms. We also investigate the advertising level of these firms under a mixed oligopoly and after the privatization of the public firm. Through this analysis, we clarify the critical role of the public firm in expanding market demand. The public firm increases its level of advertising because it acknowledges the expanding effect on the behavior of private firms.

13.1 Introduction

In this chapter, we introduce into the oligopolistic model the activity of firms to expand the market or demand size. The focus of this chapter is on advertising and we attempt to portray its realistic role. We investigate how advertising affects the economy in both a mixed oligopoly and a pure private oligopoly.

Theories on mixed oligopolies have been developed in various directions, as the topics treated in this book show. One direction is the incorporation of advertising. In Japan, advertising activity by public firms, as well as private firms, has been prevalent. For instance, until the 1980s, the Japanese National Railways (JNR) aired television commercials and posted billboard advertisements in many stations to encourage consumers to travel by rail. JNR also created advertisements for more historic cities such as Kyoto and Nara to evoke feelings of nostalgia. Furthermore, private railway companies created similar advertisements. Today, three companies that operate rail routes between Osaka and Kyoto (i.e., Hankyu, Keihan, and JR, the former JNR) have engaged in advertising campaigns to promote their trains.

This chapter has three main focuses. It is understood that the advertisements of one railway company can induce people to use the services of another company because people's objective is simply to travel from A to B. That is, the

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advertisements of one company can expand its own demand as well as the market itself. Furthermore, if a public firm exists in the market, then it necessarily behaves differently from private firms. From the viewpoint of private firms, they can benefit from the advertisements of other firms, especially those of the public firm. Thus, private firms and the public firm might concentrate on (i.e., specialize in) either their output level of products and services or advertising. This is the first focus of this analysis.

Our second focus is on the effect of the number of (private) firms on outcomes, specifically output levels and the level of advertising. When the number of firms increases in the general framework of an oligopoly (including a mixed oligopoly), because the market approaches perfect competition, the output level of each firm decreases. In our framework, however, when firms (especially the public firm) engage in advertising, it is not clear whether this outcome holds or not. In addition, how does advertising change when the number of firms increases?

The third focus is on the change in the output levels and advertising expenditure when the public firm is privatized and the economy moves from a mixed oligopoly to a pure private oligopoly. In general, the output level of each private firm increases. We need to clarify these changes in a scenario where firms engage in advertising.

There are few studies that consider advertising to expand market demand in a mixed oligopoly (or duopoly).[1] The exceptions are Matsumura and Sunada (2013) and Han and Ogawa (2012). Matsumura and Sunada (2013) considered advertising competition in a mixed oligopoly, but their main focus was on second-best outcomes. Furthermore, they investigated misleading advertising as developed by Glaeser and Ujhelyi (2010), which does not affect the consumer surplus but does affect the production costs of firms.[2] Han and Ogawa (2012) examined demand-boosting advertising and investigated the level of privatization in accordance with the reaction of consumers to advertising in a mixed duopoly. They clarified that because consumers have a significant reaction to advertising, the level of privatization should be lower. Their result implies that the effect of advertising should be considered, whether or not the public firm is privatized.

This chapter follows the framework presented by Han and Ogawa (2012); however, there are three clear differences in motivation. The first is that they treat advertising expenditure in the same way as the production of the commodity. The cost functions for advertising and those for production take a quadratic form and are independent. In our model, although the production cost is linear in the amount of commodity itself, the advertising cost increases. In addition, these costs are

[1] In addition to competition in the supply of production, we consider advertising, which expands demand. In contrast, there exists another strand of research in which R&D investment (affecting the cost or productivity) is taken into account. For example, see Delbono and Denicoló (1993), Nishimori and Ogawa (2002), Haruna and Goel (2015), and Zhang and Zhong (2015).

[2] Matsumura and Sunada (2013) also considered the situation where the public firm and private firms play a two-stage game: after determining the level of advertising, firms choose the level of supply. In our framework, they simultaneously determine these levels.

dependent, which illustrates how advertising works in a mixed oligopolistic market. The second difference stems from the fact that our interest lies mainly with the effect of the (full) privatization of a public firm. In contrast, Han and Ogawa (2012) focus on the partial privatization of a public firm. In other words, we aim to investigate the role of the public firm in maximizing social welfare via advertising, and Han and Ogawa (2012) consider how the degree of privatization should be set from the viewpoint of social welfare maximization. Third, we consider n private firms, which makes it possible to analyze the difference in the number of firms or the size of the market. This is in contrast to Han and Ogawa (2012) who only considered one private firm.

In this study, we obtained the following novel results. First, in a mixed oligopoly, the number of (private) firms determines whether the output level of the public firm is larger than that of the private firms. That is, when the number of private firms is comparatively large, the output level of the private firms is larger than that of the public firm, and vice versa. In contrast, the level of advertising of the public firm is always larger than that of the private firms. This result reflects the crucial role of the public firm to expand the market via advertising. Second, when the number of private firms increases, the output level of the private firms, as well as the level of advertising, also increases. However, after the privatization of the public firm, the opposite occurs. Whether the output level of the public firm in a mixed oligopoly increases depends on the cost and demand factors. Third, although the advertising level of private firms increases and that of the public firm decreases, the total level of advertising decreases after privatization. This result also indicates that privatization weakens the key role of the public firm to expand market demand.

The reminder of this chapter is as follows. Section 13.2 presents the mixed oligopoly model. Section 13.3 considers the case where a public firm is privatized and compares outcomes in a mixed oligopoly and a pure private oligopoly, and Sect. 13.4 concludes the chapter.

13.2 Mixed Oligopoly Model

There are $n + 1$ firms, which produce a homogenous commodity using the same inputs and production technologies. When they supply the market, they can use the mean to increase demand (or equivalently, expand the market) via their advertising expenditure. This issue will be discussed in greater detail below. In this chapter, we first consider the case where firm 0 is assumed to be a public firm, with an objective to maximize social welfare, and the other n firms are assumed to be private firms, seeking to maximize their own profit.

13.2.1 Setting

As evident in the standard settings of previous studies, the inverse demand function that the firms face is assumed to be linear, and its slope is -1. In addition, we assume that advertising expenditure expands the market size. That is, when *each* firm increases its advertising expenditure by one unit, the intercept of the vertical axis and the (*market*) inverse demand curve shift upward by $0 < \beta < 1$ unit. Then the inverse demand function is given by

$$p = 1 - \sum_{i=0}^{n+1} q_i + \beta \sum_{i=0}^{n} a_i, \qquad i = 0, 1, 2, \cdots n, \tag{13.1}$$

where p, q_i, and a_i represent the price of the commodity, the output level of the commodity, and the advertising by firm i, respectively.

When firms produce goods, they should pay for the costs of its input and advertising expenditure. The total cost function of the firms is assumed to be

$$c_i = (c + \gamma a_i)q_i + \frac{\varepsilon (a_i)^2}{2}, \quad i = 0, 1, 2, \cdots n, \tag{13.2}$$

where c is a constant of the marginal cost of production. In addition, the marginal cost of production also linearly depends on advertising with a coefficient of γ. Therefore, the greater the number of firms that produce goods in the market, the higher the level of advertising. Then, the total cost represents any increase in advertising. It should be noted that in contrast to Han and Ogawa (2012), the cost of production and the advertising expenditure are asymmetrically incorporated into the cost function.[3,4] In addition, for analytical convenience in the qualitative analysis in the latter part of this chapter, we set ε as 1.

Regarding the parameters relating to the above cost function, we make the following assumption.

[3] The role of advertising is considered in much the same way in both Han and Ogawa (2012) and the present study. When goods are supplied to the market, advertising (i.e., its expenditure) is required to expand the market (demand). Furthermore, this expansion effect benefits the firm *and* other firms. On this point, this type of advertising can be acknowledged as a public good. In addition, it can also be recognized as the "supply" of goods.

[4] The marginal costs of production differ in Han and Ogawa (2012) and the present study. Han and Ogawa (2012) assumed that the marginal cost of production, as well as the marginal expenditure for advertising, is linear in the amount of production (i.e., the cost function takes a quadratic form). In contrast, in the present study, we consider the marginal cost of production as the combination of the constant term and the level of advertising.

Assumption 13.1 *Production*

$$0 < \gamma < \beta < 1 \text{ and } 0 < c < 1.$$

This assumption means that the expansion effect of advertising expenditure is larger than the cost of advertising. In other words, the net benefit of advertising is positive. Regarding the advertising cost, as usual, the marginal cost is smaller than the marginal revenue even when advertising expenditure is 0.

Finally, the profit of the firms can be expressed as

$$\begin{aligned}\pi_i &\equiv pq_i - C_i \\ &= \left(1 - \sum\nolimits_{i=0}^{n} q_i + \beta \sum\nolimits_{i=0}^{n} a_i\right) q_i - (c + \gamma a_i) q_i - \frac{(a_i)^2}{2}, i = 0, 1, 2, \cdots n.\end{aligned} \tag{13.3}$$

Private firms maximize the above profit (13.3) by choosing the output levels and advertising expenditure, given the levels determined by the other firms. The public firm maximizes social welfare (consisting not only of its own profit but also the profits of the other private firms) and the consumer surplus. Therefore, social welfare can be written as follows:

$$W \equiv \sum\nolimits_{i=0}^{n} \pi_i + \frac{\left(1 + \beta \Sigma_{i=0}^{n} a_i - p\right)^2}{2}. \tag{13.4}$$

Furthermore, the private and public firms are in Cournot–Nash competition.

13.2.2 Mixed Oligopoly

As mentioned above, the private firms maximize their own profit (13.3), and the public firm seeks to maximize social welfare (13.4), given the activities of the other firms. Under a mixed oligopoly, the set of the first-order conditions can be given as follows:

$$c + q^r(1 + n) + q^u + a^r(\gamma - n\beta) - 1 - a^u\beta = 0, \tag{13.5}$$

$$-a^r + q^r(\beta - \gamma) = 0, \tag{13.6}$$

$$1 - c - nq^r - q^u + a^u(\beta - \gamma) + a^r n\beta = 0, \tag{13.7}$$

$$-a^u + nq^r\beta + q^u(\beta - \gamma) = 0, \tag{13.8}$$

where the superscripts r and u represent the variables of the private and public firms, respectively. Note we obtain the above conditions on the assumption that the (private) firms are identical.

It is helpful to explain the properties of the Cournot–Nash equilibrium in the model by investigating the firms' reaction functions. In the above first-order conditions, (13.5) and (13.6) give the reaction functions for the output levels of the private firms:

$$c + q^u - a^u\beta - 1 - \left[n\beta^2 + \gamma^2 - (1+n)(1+\beta\gamma)\right]q^r = 0, \tag{13.9}$$

and (13.7) and (13.8) give the reaction functions for the output levels of the public firm:

$$1 + q^u\left[(\beta-\gamma)^2 - 1\right] + n\left[a^r\beta + q^r\left(\beta^2 - \beta\gamma - 1\right)\right] - c = 0. \tag{13.10}$$

With q^r as the horizontal axis and q^u as the vertical axis, the slopes of (13.9) and (13.10) are $n\beta^2 + \gamma^2 - (1+n)(1+\beta\gamma) < 0$ and $\frac{n[1-\beta(\beta-\gamma)]}{(\beta-\gamma)^2-1} < 0$, and both are negative. Furthermore, the former minus the latter gives $\frac{1+(\beta-\gamma)\left\{n\beta^3-(1+n)\beta+\left[2+n-(1+2n)\beta^2\right]\gamma+(2+n)\beta\gamma^2-\gamma^3\right\}}{(\beta-\gamma)^2-1}$, whose denominator is negative but the numerator is indeterminate. To guarantee the stability of the Cournot–Nash equilibrium in strategic substitutes with respect to the outputs, we assume that this is negative (or that the numerator is positive).

Similarly, the reaction functions of the private and public firms for advertising level can be obtained as follows:

$$(c + q^u - a^u\beta - 1)(\beta-\gamma) + a^r(1 + n - n\beta^2 + \beta\gamma + n\beta\gamma - \gamma^2) = 0, \tag{13.11}$$

$$[(\beta-\gamma)^2 - 1]a^u + n\beta(\beta-\gamma)a^r + \gamma(c + nq^r - 1) + \beta(1-c) = 0. \tag{13.12}$$

Following the same procedure, when taking a^r as the horizontal axis and a^u as the vertical axis, the slopes of (13.11) and (13.12) are $\frac{1+n-n\beta^2+\beta\gamma+n\beta\gamma-\gamma^2}{\beta(\beta-\gamma)} > 0$ and $\frac{n\beta(\beta-\gamma)}{(\beta-\gamma)^2-1} > 0$, and both are positive. The former minus the latter is $\frac{1+(\beta-\gamma)[\beta+(\beta^2-2)\gamma-2\beta\gamma^2+\gamma^3]+n(\beta-\gamma)\{-\gamma+\beta[2+(\beta-\gamma)\gamma]\}-n}{-\beta(\beta-\gamma)[1-(\beta-\gamma)^2]}$, and the denominator is negative, but the numerator is indeterminate. We also assume here that this is positive (or that the numerator is negative). This assumption then guarantees the stability of the Cournot–Nash equilibrium in strategic complements with respect to advertising.

In sum, we propose the following assumption.

Assumption 13.2 *Stability conditions*

(i) Strategic substitute for the output of the good:

$$1 + (\beta-\gamma)\left\{n\beta^3 - (1+n)\beta + \left[2 + n - (1+2n)\beta^2\right]\gamma + (2+n)\beta\gamma^2 - \gamma^3\right\} > 0.$$

(ii) Strategic complement for advertising expenditure:

$$1+(\beta-\gamma)[\beta-(2-\beta^2)\gamma-2\beta\gamma^2+\gamma^3]+n(\beta-\gamma)\{-\gamma+\beta[2+(\beta-\gamma)\gamma]\}-n<0.$$

Assumptions 13.1 and 13.2 give the following lemma.

Lemma 13.1 *Under Assumptions 13.1 and 13.2, in a mixed oligopoly:*

(i) *The Cournot–Nash equilibrium is stable.*
(ii) *q^r and q^u are strategic substitutes.*
(iii) *a^r and a^u are strategic complements.*

By solving the set of equations for the first-order conditions of (13.5), (13.6), (13.7) and (13.8), the output level of the good and the private firm's level of advertising can be obtained as

$$q^r=\frac{(1-c)(\beta-\gamma)\gamma}{\phi}, \tag{13.13}$$

$$a^r=\frac{(1-c)(\beta-\gamma)^2\gamma}{\phi}, \tag{13.14}$$

where
$\phi\equiv 1-\beta^2+\beta\left[3-(1+n)\beta^2\right]\gamma-\left[2+n-(3+2n)\beta^2\right]\gamma^2-(3+n)\beta\gamma^3+\gamma^4.$
Similarly, those of the public firm can be obtained as

$$q^u=\frac{(1-c)[1+(\beta-\gamma)\gamma-n\beta\gamma]}{\phi}, \tag{13.15}$$

$$a^u=\frac{(1-c)(\beta-\gamma)[1+(\beta-\gamma)\gamma]}{\phi}. \tag{13.16}$$

It should be noted that from Assumption 13.1, $\phi>0$ must hold in (13.16). In addition, because the output level of the public firm, (13.15), should be positive, $1+(\beta-\gamma)\gamma-n\beta\gamma>0$ must hold. Therefore, we put forward the following assumption.

Assumption 13.3 Output level and advertising are positive if (i) $\phi>0$ and (ii) $1+(\beta-\gamma)\gamma-n\beta\gamma>0$.
It should be noted that Assumption 13.3(ii) determines the upper bound of the number of private firms, $\overline{n}\equiv\frac{1+(\beta-\gamma)\gamma}{\beta\gamma}>n$. From (13.13), (13.14), 13.15) and (13.16), we can see the relationships between the advertising and the output level of each firm, the advertising of the private and public firms, and the output levels of these firms. These relationships are shown in Lemma 13.2.

Lemma 13.2 *In a mixed oligopoly*:

(i) $a^r = (\beta - \gamma)q^r$, and $a^u = \frac{1+(\beta-\gamma)\gamma-n\beta\gamma}{(\beta-\gamma)[1+(\beta-\gamma)\gamma]} q^u$.

(ii) $q^r = \frac{(\beta-\gamma)\gamma}{1+(\beta-\gamma)\gamma-n\beta\gamma} q^u$, and $a^r = \frac{(\beta-\gamma)\gamma}{1+(\beta-\gamma)\gamma} a^u$.

From Lemma 13.2, we have the following proposition.

Proposition 13.1 *In a mixed oligopoly*:

(i) *If* $\frac{1}{\beta\gamma} < n < \overline{n}$ (i.e., *n is relatively large*), *then* $q^r > q^u$,

and if $1 < n < \frac{1}{\beta\gamma}$ (i.e., *n is relatively small*), *then* $q^r < q^u$.

(ii) $a^r < a^u$ *holds*.

Proposition 13.1(ii) can be easily interpreted. As the public firm recognizes its overall effect on the market (expansion), it spends more on advertising than private firms. This means that advertising, especially by the public firm, increases the commodity output of not only the public firm but also the other private firms. Based on this finding, Proposition 1(i) provides two important implications. First, whether advertising increases the output level of the public firm depends on the number of private firms, n. When n is large, private firms produce more than the public firm because the public firm spends more on advertising for a larger market. In contrast, when n is small, as the public firm's advertising expenditure is relatively low, private firms should advertise more. This is at the expense of output, and therefore the supply of the public firm is higher than that of each private firm. This result is in contrast to that given by De Fraja and Delbono (1989). They stated that the output of the public firm is always larger than that of the private firm because the public firm has an incentive to increase the consumer surplus by setting the marginal cost equal to the marginal benefit of the consumer. In our model, in addition to controlling the output level, the public firm (as well as private firms) should care about their advertising expenditure. It is not necessary to allocate resources to produce more goods than the private firms.

Next, we investigate how the number of private firms (or equivalently, the size of the market) affects the above outcomes. The effects on the above output levels and advertising expenditure can be calculated as follows:

$$\frac{\partial q^r}{\partial n} = \frac{(1-c)(\beta-\gamma)\gamma^2\left[\beta(\beta-\gamma)^2+\gamma\right]}{\phi^2} > 0, \tag{13.17}$$

$$\frac{\partial a^r}{\partial n} = \frac{(1-c)(\beta-\gamma)^2\gamma^2\left[\beta(\beta-\gamma)^2+\gamma\right]}{\phi^2} > 0, \tag{13.18}$$

$$\frac{\partial q^u}{\partial n} = \frac{(1-c)(\beta-\gamma)\gamma[2\beta(\beta-\gamma)-1][1+(\beta-\gamma)\gamma]}{\phi^2}, \tag{13.19}$$

$$\frac{\partial a^u}{\partial n} = \frac{(1-c)(\beta-\gamma)\gamma\left[\beta(\beta-\gamma)^2+\gamma\right][1+(\beta-\gamma)\gamma]}{\phi^2} > 0. \tag{13.20}$$

From these results, we can obtain the following proposition.

Proposition 13.2 *In a mixed oligopoly, as the number of private firms increases*:

(i) *Both the output level and the advertising level of the private firms increase.*

(ii) *The advertising level of the public firm increases.*

(iii) *The output level of the public firm increases* (*decreases*) *if* $2\beta(\beta-\gamma)-1 > (<)0$.

Similar to Proposition 13.1, Proposition 13.2 means that advertising expands the market so that the output level of one private firm increases. This is in contrast to that usually seen in a general mixed oligopoly model (e.g., De Fraja and Delbono 1989). In our model, when the number of private firms increases, the public firm, as well as the private firms, increases their level of advertising, as shown in Proposition 13.2(ii). This expands the market demand, and therefore, the output level of both the private and public firms increase, as long as $2\beta(\beta-\gamma)-1>0$.

Proposition 13.2(iii) implies whether the output level of a public firm increases is indeterminate. Furthermore, if the net benefit of advertising, $\beta-\gamma$, is large, then the output level of the public firm increases, and vice versa. This can be interpreted as follows. When $\beta-\gamma$ is large, the public firm tends to expand the market by increasing its advertising taking social welfare (including the profit of the public firms) into consideration. This leads the public firm to increase its output level even if the number of private firms is large.

Taking the sum of the levels of output and the advertising expenditure of each firm, we can obtain the total amount of output and advertising as follows:

$$Q^r \equiv nq^r = \frac{n(1-c)(\beta-\gamma)\gamma}{\phi}, \tag{13.21}$$

$$A^r \equiv na^r = \frac{(1-c)n(\beta-\gamma)^2\gamma}{\phi}, \tag{13.22}$$

$$Q \equiv q^u + nq^r = \frac{(1-c)[1+\beta\gamma-(1+n)\gamma^2]}{\phi}, \tag{13.23}$$

$$A \equiv a^u + na^r = \frac{(1-c)(\beta-\gamma)[1+(1+n)(\beta-\gamma)\gamma]}{\phi}, \tag{13.24}$$

where Q^r and A^r are the total amount of output and advertising expenditure of the private firms and Q and A are those of all firms. Following the same procedure for each firm, an increase in the number of private firms affects the total output levels and advertising as follows:

$$\frac{\partial Q^r}{\partial n} = \frac{(1-c)(1-\beta+\gamma)(\beta-\gamma)(1+\beta-\gamma)\gamma[1+(\beta-\gamma)\gamma]}{\phi^2} > 0, \tag{13.25}$$

$$\frac{\partial A^r}{\partial n} = \frac{(1-c)(1-\beta+\gamma)(\beta-\gamma)^2(1+\beta-\gamma)\gamma[1+(\beta-\gamma)\gamma]}{\phi^2} > 0, \tag{13.26}$$

$$\frac{\partial Q}{\partial n} = \frac{(1-c)(\beta-\gamma)^2\gamma\left(\beta+\gamma+\beta^2\gamma-\gamma^3\right)}{\phi^2} > 0, \tag{13.27}$$

$$\frac{\partial A}{\partial n} = \frac{(1-c)(\beta-\gamma)\gamma[1+(\beta-\gamma)\gamma]\left[\beta+(\beta-\gamma)^2\gamma\right]}{\phi^2} > 0. \tag{13.28}$$

We can directly obtain the results of (13.25), (13.26), and (13.28) from (13.17), (13.18), and (13.20). That is, the total output of the private firms and the advertising level of the private firms and all firms increase when the number of private firms increases. In addition, (13.27) means that, as we have seen in (13.19), although the output level of the public firm decreases as the number of private firms increases, the total output increases.

Corollary 13.1 *In a mixed oligopoly, as the number of private firms increases, the total output level and the level advertising of the private firms and those of all firms increase.*

Finally, we focus on the profits and social welfare. The profits of private and public firms can be obtained as follows:

$$\pi^r = \frac{(1-c)^2\left[2-(\beta-\gamma)^2\right](\beta-\gamma)^2\gamma^2}{2\phi^2} > 0, \tag{13.29}$$

$$\pi^u = -\frac{(1-c)^2(\beta-\gamma)^2[1+(\beta-\gamma)\gamma]^2}{2\phi^2} < 0. \tag{13.30}$$

From (13.29), it is natural that the private firms return positive profits; however, from (13.30), the profit of the public firm is negative, even if there is no fixed cost in our model. As $0 < c < 1$, Assumption 13.1 implies that the marginal cost of production itself can be below the market price; however, the marginal cost also depends on advertising. Therefore, as long as the public firm has a role in expanding the market via advertising, its profit becomes negative.

Proposition 13.3 *In a mixed oligopoly with advertising, the profit of private firms is positive, but that of a public firm is negative.*

Then, when the number of private firms increases, the effects on the profits of both the private and public firms can be obtained as follows:

$$\frac{\partial \pi^r}{\partial n} = \frac{-(1-c)^2\left[2-(\beta-\gamma)^2\right](\beta-\gamma)^2\gamma^3\left[\beta(\beta-\gamma)^2+\gamma\right]}{\phi^3} < 0, \tag{13.31}$$

$$\frac{\partial \pi^u}{\partial n} = \frac{-(1-c)^2(\beta-\gamma)^2\gamma\left[\beta(\beta-\gamma)^2+\gamma\right][1+(\beta-\gamma)\gamma]^2}{\phi^3} < 0. \tag{13.32}$$

These results can be summarized as the following proposition.

Proposition 13.4 *In a mixed oligopoly with advertising, the profits of both private and public firms decrease as the number of private firms increases.*

From Proposition 13.2, each private firm increases its output level when the number of private firms increases, and, as Corollary 13.1 implies, the market size also becomes larger because the level of advertising increases. However, this expansion brings about an increase in the cost of advertising (as a quadratic cost function), and the cost of production also increases because of the increase in advertising. Therefore, the profit decreases.

Finally, the level of social welfare and the effect of the increase in the number of private firms on social welfare can be obtained as follows:

$$\begin{aligned} W &= \left(2\phi^2\right)^{-1}(1-c)^2\left\{1-\beta^2-2\beta\left(\beta^2-2\right)\gamma-\left[3+2n-(7+2n)\beta^2+(1+n)\beta^4\right]\gamma^2\right.\\ &\left.+2\left[2(1+n)\beta^2-4-3n\right]\beta\gamma^3+(1+n)\left(3+n-6\beta^2\right)\gamma^4+4(1+n)\beta\gamma^5-(1+n)\gamma^6\right\}, \end{aligned} \tag{13.33}$$

$$\begin{aligned} \frac{\partial W}{\partial n} &= -(2\phi^3)^{-1}(1-c)^2(\beta-\gamma)^2\gamma\{2\beta(\beta^2-1)+(\beta^2-2)(1+3\beta^2)\gamma\\ &+\beta[\beta^4-2-7\beta^2+n(2-2\beta^2+\beta^4)]\gamma^2+[5+(1+7n)\beta^2-(3+4n)\beta^4]\gamma^3\\ &+\beta[7-4n+2(1+3n)\beta^2]\gamma^4-[4+n+2(2n-1)\beta^2]\gamma^5+(n-3)\beta\gamma^6+\gamma^7\}. \end{aligned} \tag{13.34}$$

As the expression of the level of social welfare is complex, the effect of the increase in the number of private firms is indecisive. In other words, it is possible that social welfare decreases as the number of private firms increases.

13.3 Privatization

13.3.1 Pure Private Oligopoly

In this section, we investigate the privatization of the public firm. In this case, all the firms maximize their own profit and the market becomes purely oligopolistic. Then, the output level and the advertising level of each firm can be obtained as follows:

$$\hat{q}_i = \frac{1-c}{\psi}, \quad i = 0, 1, \cdots, n, \tag{13.35}$$

$$\hat{a}_i = \frac{(1-c)(\beta-\gamma)}{\psi}, \quad i = 0, 1, \cdots, n, \tag{13.36}$$

where $\psi \equiv 2 - (\beta-\gamma)^2 + n[1-\beta(\beta-\gamma)] > 0$ and the variables with hats represent those in a pure private oligopoly. The total output level and advertising level can be easily obtained by multiplying (13.35) and (13.36) by n, that is, $\hat{Q} = n\hat{q}_i$ and $\hat{A} = n\hat{a}_i$.

Then, the effect of an increase in the number of firms on the above variables can be calculated as follows:

$$\frac{\partial \hat{q}_i}{\partial n} = -\frac{(1-c)[1-\beta(\beta-\gamma)]}{\psi^2} < 0, \tag{13.37}$$

$$\frac{\partial \hat{a}_i}{\partial n} = -\frac{(1-c)(\beta-\gamma)[1-\beta(\beta-\gamma)]}{\psi^2} < 0. \tag{13.38}$$

From these results, we obtain the following proposition.

Proposition 13.5 *In a pure private oligopoly, as the number of firms increases, both the output level and the advertising level of the private firms decrease.*

Previous studies have shown that the output of each firm decreases as the number of firms increases. This occurs because the market conditions are approaching perfect competition. In our setting, it is not clear whether such a tendency can be seen because the firms buy advertising to expand the market. Even so, both the output level and advertising level of each firm decrease. This implies that in a pure private oligopoly, the decline in oligopolistic power resulting in anincrease in the number of firms dominates the effect of market expansion by an increase in advertising. This can be also attributed to the cost structure of the firm that exhibits this quadratic form.

The level of profit and the effect of the increase in the number of firms on profit can be obtained as follows:

$$\hat{\pi}_i = \frac{(1-c)^2\left[2-(\beta-\gamma)^2\right]}{2\psi^2}, \tag{13.39}$$

$$\frac{\partial \hat{\pi}_i}{\partial n} = -\frac{(1-c)^2\left[2-(\beta-\gamma)^2\right][1-\beta(\beta-\gamma)]}{\psi^3} < 0. \tag{13.40}$$

From (13.40), as the number of firms increases, the profit of each firm decreases, which is generally true in a pure private oligopoly model without advertising.

Finally, the level of social welfare and the effect of the increase in the number of firms on social welfare can be calculated as follows:

$$\hat{W} = \frac{(1-c)^2(1+n)\left[3+n-(\beta-\gamma)^2\right]}{2\psi^2}, \tag{13.41}$$

$$\begin{aligned} \frac{\partial \hat{W}}{\partial n} = \frac{(1-c)^2}{2\psi^3}\langle 2+(\beta-\gamma)\Big\{ &\beta[2-\beta^2+n(3-\beta^2)] \\ &+(4+n+\beta^2+2n\beta^2)\gamma-(n-1)\beta\gamma^2-\gamma^3\Big\}\rangle > 0. \end{aligned} \tag{13.42}$$

Summarizing these results, we obtain the following corollary.

Corollary 13.2 *In a pure private oligopoly, as the number of firms increases:*

(i) *The profit decreases.*
(ii) *Social welfare increases.*

These results have already been seen in typical oligopoly models. As we pointed out in Proposition 13.4, the profit of both private and public firms decreases. However, the production level increases in our mixed oligopoly with advertising when the number of firms increases. Furthermore, in a pure private oligopoly, social welfare increases; however, in our mixed oligopoly, it does not. This fact implies that advertising clearly expands demand and increases the consumer surplus, but represents a greater cost for firms when the number of firms increases. Therefore, advertising brings benefits to consumers but might also harm the profits of firms.

13.3.2 Comparison

Finally, we compare the outcomes in a mixed oligopoly with those in a pure private oligopoly. First, we present the difference between the levels of output for the two cases as

$$q^r - \hat{q}_i = \frac{(1-c)\left(\beta^2 - \beta\gamma + n\beta\gamma - 1\right)}{\psi\phi} < 0, \tag{13.43}$$

$$q^u - \hat{q}_i = \frac{(1-c)}{\psi\phi}\left(1 + n - n\beta^2 + \beta\gamma - n^2\beta\gamma + n\beta^3\gamma \right.$$
$$\left. + n^2\beta^3\gamma - \gamma^2 - 2n\beta^2\gamma^2 - n^2\beta^2\gamma^2 + n\beta\gamma^3\right), \tag{13.44}$$

$$Q - \hat{Q} = \frac{(1-c)}{\psi\phi}\left(1 + \beta\gamma - n\beta\gamma + n\beta^3\gamma + n^2\beta^3\gamma - \gamma^2 \right.$$
$$\left. -2n\beta^2\gamma^2 - n^2\beta^2\gamma^2 + n\beta\gamma^3\right). \tag{13.45}$$

From (13.43), we can see that the output of the private firm is larger for a pure private oligopoly than for a mixed oligopoly. This result has been acknowledged in previous studies. However, (13.44) means that it is not possible to determine whether the output levels of the public firm are larger than that of the privatized (public) firm, which is positive in previous studies. As we have seen in Proposition 13.1, this depends on the cost and demand factors and can be attributed to advertising. This result also means that it is not possible to determine whether (13.45) is positive or negative.

Next, we compare advertising for the two cases:

$$a^r - \hat{a}_i = \frac{(1-c)(\beta-\gamma)\left(-1 + \beta^2 - \beta\gamma + n\beta\gamma\right)}{\psi\phi} < 0, \tag{13.46}$$

$$a^u - \hat{a}_i = \frac{(1-c)(\beta-\gamma)\left(-1 - n + n\beta^2 - \beta\gamma - 2n\beta\gamma + \gamma^2\right)}{\psi\phi} > 0, \tag{13.47}$$

$$A - \hat{A} = \frac{(1-c)(\beta-\gamma)(1 + \beta\gamma + n\beta\gamma + n^2\beta\gamma - \gamma^2)}{\psi\phi} > 0. \tag{13.48}$$

In contrast to the above comparison for output levels, the signs from (13.46), (13.47) and (13.48) are all determinate. It is interesting to note that the advertising level of private firms increases after privatization. This occurs because private firms should increase advertising after the privatization of a public firm, which had previously advertised aggressively to maximize social welfare (13.46). Therefore, after privatization, none of the firms consider social welfare (or consumer surplus), and the total level of advertising decreases.[5]

To summarize, we obtain the following proposition.

[5] We were not able to obtain clear results for a comparison of profit levels and social welfare levels.

Proposition 13.6 *When a public firm is privatized:*

(i) *The output level of private firms increases, but the change in that of privatized (public) firms is ambiguous. Therefore, the change in the total output level is ambiguous.*
(ii) *The advertising level of private firms increases and that of public firms decreases. The total advertising expenditure decreases as the decrease in the role of the public firm to expand the market size dominates the increase in the advertising of private firms.*

13.4 Concluding Remarks

This chapter considered advertising as a means of expanding the market or demand size in two oligopolistic models: a mixed oligopoly and a pure private oligopoly. We also investigated how the output level and advertising change when the number of private firms increases in a mixed oligopoly and pure private oligopoly and how these change when a public firm is privatized. In contrast with the results of previous studies on mixed oligopolies, we showed that in a mixed oligopoly, when the number of private firms is relatively large (small), the output level of the private firms is larger (smaller) than that of the public firm. Regarding the advertising level of the public firm, it is always larger than that of private firms. Furthermore, when the number of private firms increases, the output level of the private firms, as well as their advertising level, increases in a mixed oligopoly. Finally, although the advertising level of private firms increases and that of the public firm decreases, the total advertising level decreases after privatization. These results are unique in our study because advertising, especially that of the public firm, contributes to expand the market.

Although our results provide new insights, our analysis does have some limitations. First, we did not fully analyze the effects of privatization on profit and social welfare. Further investigation should determine how these levels are affected by a change in the number of firms and by the privatization of public firms. This will clarify whether the public firm should be privatized in accordance with the strength of the market expansion under advertising or the market structure (e.g., the number of firms). Regarding this issue, it would be helpful to provide a numerical example. Second, a graphical explanation, as given in Chap. 2 of this book, might be useful. This would help readers to intuitively understand how advertising alters market demand and the behavior of firms.

Throughout this study, we have acknowledged the crucial role played by public firms in using advertising to expand the market. In this way, the public firm is a social welfare maximizer. It can induce private firms to supply more products to increase the consumer surplus. It can be argued that this is one of the key roles of the public firm.

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