

# Chapter 4

## Neoclassical Economic Growth and Public Policy

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### 4.1 Introduction

Most of economic agents evaluate the current economic situation with the economic growth rate, especially the growth rate of gross domestic product (GDP).<sup>1</sup> Every countries face the different economic growth rate such that the economic growth rates of G7 countries are shown in Fig. 4.1 from 1980 to 2014. The fluctuation of economic growth rate in each country is not similar. Why are the economic growth rate in each country different?

For the analysis of that, we develop many economic growth theories.<sup>2</sup> Solow [15], Cass [5] and others develop the neoclassical growth model.<sup>3</sup> At the steady state (long-run market equilibrium) on the neoclassical growth model, the economic growth rate is shown by the exogenous parameters such as the growth rate of

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<sup>1</sup>If we denote  $GDP_t$  the economic growth rate of year  $t$ , we can show the economic growth rate at year  $t$ ,  $\gamma_t$  as

$$\gamma_t(\%) = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}} \times 100.$$

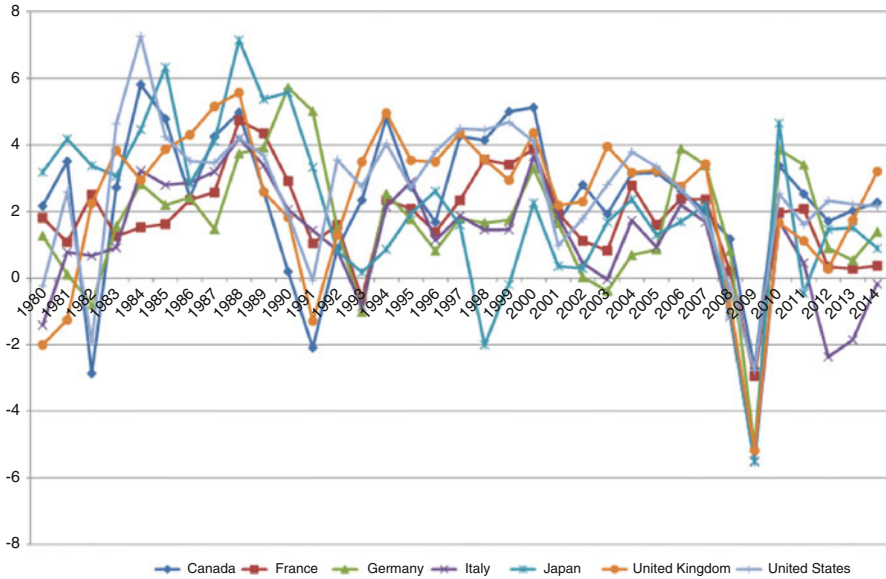
<sup>2</sup>For the text book on this issue, we have Intriligator [10], Barro and Sala-i-Martin [4], Acemoglu [1], Aghion and Howitt [2] and others.

<sup>3</sup>The representative growth model other than neoclassical growth model is Harrod [8] and Domar [7].

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**Fig. 4.1** Economic growth rate (Source: [www.imf.org/external/pubs/ft/weo/2014/02/weodata/index.aspx](http://www.imf.org/external/pubs/ft/weo/2014/02/weodata/index.aspx))

population and/or the technological growth rate. The growth rates in the steady state are not affected by the economic policy, which affect the economic variables and social welfare. We discuss how economic policy do not influence growth rate. On the other hand, Romer [13], Rebelo [12], and Barro [3] develop the economic model where economic policy affects also economic growth rate in the steady state, as we will discuss this issue next chapter. Based on their models, we can discuss the effects of policy on growth rate. In this chapter, to discuss the policy effects in economic growth model, we show the basic neoclassical growth model.<sup>4</sup> Furthermore, introducing the intergenerational resource allocation into the model, we also show the overlapping generations model developed by Samuelson [14] and Diamond [6].

## 4.2 Neoclassical Growth Model

In this section, we discuss the neoclassical growth model without consumers' optimal behaviour.<sup>5</sup> In this model, we suppose the closed economy where the aggregate output is produced by the aggregative physical capital and the aggregative

<sup>4</sup>This chapter is based on Omori [11].

<sup>5</sup>You can see the more detail discussion at Intriligator [10].

labour. These inputs are substitutable and the marginal productivity of each input is positive but decreasing. Then, the aggregative production function is shown by

$$Y = F(K, L), \quad (4.1)$$

where  $Y$  is the aggregative output,  $K$  is the aggregative capital and  $L$  is the aggregative labour. In this function, we assume the constant return to scale and the homogeneity of function. When the output per capita is  $y = \frac{Y}{L}$  and the capital per capita is  $k = \frac{K}{L}$ , the production function per capita is given by

$$y = f(k). \quad (4.2)$$

Following Inada [9], the production function is satisfied with the Inada conditions as follows,

$$\lim_{k \rightarrow 0} f'(k) = \infty \quad \text{and} \quad \lim_{k \rightarrow \infty} f'(k) = 0.$$

The equilibrium condition of good market is

$$Y = C + I,$$

where  $C$  is the aggregative consumption and  $I$  is the aggregative investment. As we denote  $c$  and  $i$  by the consumption per capita and the investment per capita, respectively, the equilibrium condition of good market per capita is

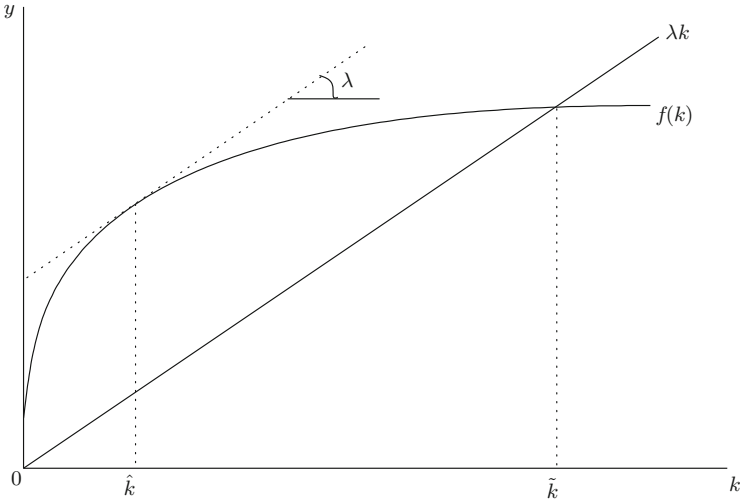
$$y = c + i. \quad (4.3)$$

As the capital accumulation can be differentiated with respect to time,  $t$ , the aggregative investment is

$$I = \dot{K} + \mu K. \quad (4.4)$$

where  $\dot{K}$  is the variable which is the differentiation with respect to time,  $t$  and the  $\mu$  is the constant positive depreciation rate. When the growth rate of population,  $\frac{\dot{L}}{L}$  is shown by  $n$  which is the nonnegative parameter and  $\dot{k}$  is the per capita variable which is the differentiation with respect to time, the investment per capita is shown by

$$i = \dot{k} + (\mu + n)k = \dot{k} + \lambda k, \quad (4.5)$$



**Fig. 4.2** The neoclassical growth equation

where  $\lambda = \mu + n$ .<sup>6</sup> Using (4.2), (4.3) and (4.5), “The neoclassical growth equation” is derived as

$$f(k) = c + \lambda k + \dot{k}. \tag{4.6}$$

Figure 4.2 shows the relationship between  $f(k)$  and  $\lambda k$ . The difference between  $f(k)$  and  $\lambda k$  is  $c + \dot{k}$  as shown in Fig. 4.3.

At  $\hat{k}$ ,  $c + \dot{k}$  is maximised but, at  $\tilde{k}$ ,  $c + \dot{k}$  is zero. We suppose one solution of this differentiation equation, that is  $\dot{k} = 0$ . At  $\hat{k}$ , if  $\dot{k}$  is zero, the consumption is maximised and the economy is in the “The Golden path”. Because the slope of  $f(k)$  and  $\lambda k$  is the same at  $\hat{k}$  as shown in Fig. 4.2, the following condition is satisfied with

$$f'(k) = \lambda = \mu + n. \tag{4.7}$$

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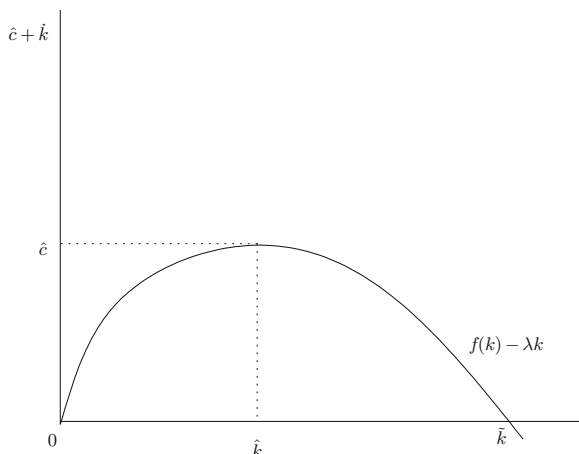
<sup>6</sup>Divided (4.4) by  $L$ ,

$$i = \frac{I}{L} = \frac{\dot{K}}{L} + \mu \frac{K}{L} = \frac{\dot{K}}{L} + \mu k.$$

Furthermore, differentiate the capital per capita,  $k$  with respect to time,  $t$ ,

$$\dot{k} = \frac{d(\frac{K}{L})}{dt} = \frac{\dot{K}}{L} - \frac{\dot{L}K}{L^2} = \frac{\dot{K}}{L} - nk.$$

From these equations, we can show (4.5).

**Fig. 4.3**  $c + \dot{k}$ 

This condition is called the “Golden Rule,” because, on this condition, the capital makes consumption maximise. However, this equilibrium is not stable as the capital is decreasing when  $\hat{k} > k$ .

### 4.3 Neoclassical Optimal Growth Model

In this section, introducing the consumers’ optimal behaviour into the neoclassical growth model, we show the neoclassical optimal growth model. We suppose one representative consumer in the economy. We assume his(her) instantaneous utility as

$$U = U(c), \quad \frac{dU(c)}{dc} = U'(c) > 0 \quad \text{and} \quad \frac{d^2U(c)}{dc^2} = U''(c) < 0.$$

This utility function is assumed to be the following conditions.

$$\lim_{c \rightarrow 0} U'(c) = \infty \quad \text{and} \quad \lim_{c \rightarrow \infty} U'(c) = 0.$$

The elasticity of marginal utility is assumed by

$$\theta(c) = -c \frac{U''(c)}{U'(c)}. \quad (4.8)$$

We note that the marginal utility is positive for the positive consumption.

The representative consumer chooses the sequence of consumption to maximise the lifetime utility subject to the budget constraint. When the positive discount rate

is  $\rho$ , the optimal behaviour for the representative consumer is formulized as

$$\begin{aligned} \max_c \int_0^{\infty} U(c)e^{-\rho t} dt, \\ \text{s.t. } \dot{k} = f(k) - c - \lambda k. \end{aligned}$$

The current value Hamiltonian is shown by

$$H = U(c) + q(f(k) - c - \lambda k), \quad (4.9)$$

where  $q$  is the shadow price of capital.<sup>7</sup> Based on the maximum principle, the first order condition for  $c$  is

$$\frac{\partial H}{\partial c} = U'(c) - q = 0. \quad (4.10)$$

This condition shows that consumer chooses the sequence of  $c$  to equal the marginal utility of consumption with shadow price of capital,  $q$ . The time path of  $q$  is satisfied with

$$\dot{q} = \rho q - \frac{\partial H}{\partial k} = (\rho - f'(k) + \lambda) q. \quad (4.11)$$

The transversality condition is

$$\lim_{t \rightarrow \infty} qk = 0.$$

Therefore, we can derive the time path of consumption as

$$\dot{c} = \frac{1}{\theta(c)} [f'(k) - \lambda - \rho] c.$$

Finally, this economy is summarised with the following dynamical system,

$$\dot{c} = \frac{1}{\theta(c)} [f'(k) - \lambda - \rho] c, \quad (4.12)$$

and

$$\dot{k} = f(k) - c - \lambda k. \quad (4.13)$$

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<sup>7</sup>See Intriligator [10] for the way to solve the intertemporal decision.

We suppose the one special solution of this dynamical system is  $\hat{c} = \hat{k} = 0$ .  $\dot{c} = \dot{k} = 0$  means that the consumption and the capital per capita is constant in the equilibrium over time. From  $\dot{c} = 0$  on (4.12), we can derive the following condition as

$$f'(k^*) = \lambda + \rho. \quad (4.14)$$

and  $\dot{k} = 0$  on (4.13) makes us the following equation,

$$c^* = f(k^*) - \lambda k^*. \quad (4.15)$$

In the equilibrium, to satisfy with (4.14) and (4.15),  $k^*$  and  $c^*$  uniquely exist. These are also satisfied with the following condition,

$$0 < c^* < f(k^*).$$

This equilibrium is in the steady state (steady growth path).

Next, we consider the economic growth rate in the steady state. In the steady state,  $\dot{k} = 0$  gives us the following equation,

$$\dot{k} = \frac{d\left(\frac{K}{L}\right)}{dt} = \frac{K}{L} \left( \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \right) = 0.$$

Similarly,  $\dot{c} = 0$  gives

$$\dot{c} = \frac{d\left(\frac{C}{L}\right)}{dt} = \frac{C}{L} \left( \frac{\dot{C}}{C} - \frac{\dot{L}}{L} \right) = 0.$$

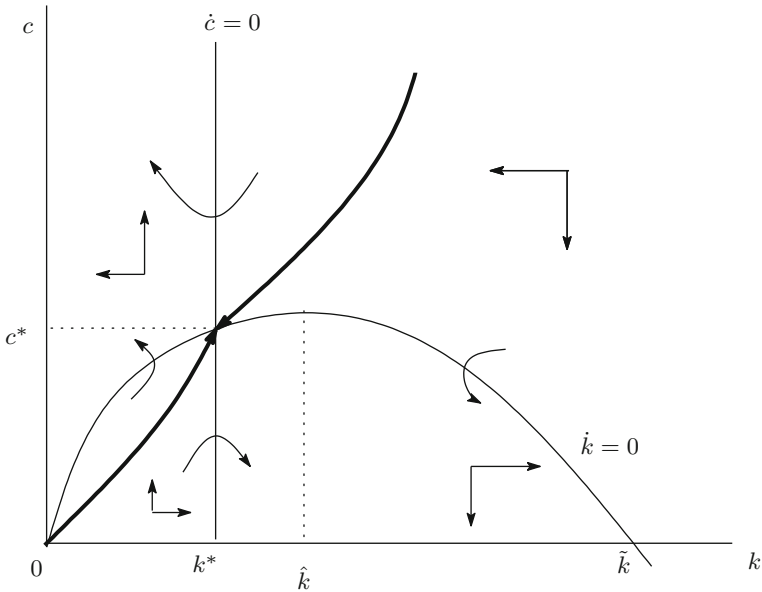
Therefore,

$$\frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{L}}{L} = n.$$

The economic growth rate on the steady state in the neoclassical optimal growth model is shown by the exogenous parameter,  $n$ , which is the growth rate of population. Because the consumption and capital per capita is constant in the steady state, the aggregative consumption ( $C = cL$ ), the aggregative capital ( $K = kL$ ) and the aggregative output ( $Y = yL$ ) grows at the rate of  $n$ .

We discuss the dynamical system (4.12) and (4.13) with the phase diagram on Fig. 4.4.

From (4.12), we can show  $f'(k) = \lambda + \rho$  in the case of  $\dot{c} = 0$  and the following relationships.



**Fig. 4.4** Phase diagram

If

$$f'(k) \begin{bmatrix} > \\ = \\ < \end{bmatrix} \lambda + \rho, \text{ then } \dot{c} \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0.$$

In other words, if

$$k \begin{bmatrix} < \\ = \\ > \end{bmatrix} k^*, \text{ then } \dot{c} \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0.$$

When  $k < k^*$ ,  $c$  increases but, when  $k > k^*$ ,  $c$  decreases.

On the other hand, from (4.13), we can show the following condition. If

$$f(k) - \lambda k \begin{bmatrix} > \\ = \\ < \end{bmatrix} c, \text{ then } \dot{k} \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0.$$

When  $c$  is in the upper area of  $\dot{k}$  line,  $k$  decreases but, when  $c$  is in the lower area of  $\dot{k}$  line,  $k$  increases.



Therefore, in Fig. 4.4, the two bold lines toward the cross point of  $(c, k)$  are on the optimal growth path.

The local stability of solutions to the dynamical system can be determined by the characteristic roots of the coefficients with linearization of system at the neighbourhood of equilibrium. That is,

$$\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} \cong \begin{bmatrix} 0 & E \\ -1 & \rho \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \end{bmatrix},$$

where  $E = \frac{c^* f''(k^*)}{\sigma(c^*)}$ . As we denote the characteristic root by  $\epsilon$ , the characteristic equation is

$$\epsilon^2 - \rho\epsilon + E = 0,$$

and the roots are

$$\epsilon = \frac{\rho \pm \sqrt{\rho^2 - 4E}}{2}.$$

Because the roots are real and opposite in sign, the equilibrium of system is a saddle point. Historically given the capital per capita,  $k_0$ , the consumers chooses the consumption and the capital for such  $k_0$  and the economy goes to the equilibrium as shown in Fig. 4.4. As the bold path is in the optimal growth path,  $k$  and  $c$  should keep in the equilibrium over time.

## 4.4 Technological Progress

Above discussed, the per capita variables are constant in the steady state because the technology is assumed to be constant return to scale and the same production function is assumed over time. However, if we take into consideration the technological progress, the per capita variables would be changed. In this subsection, we examine the neoclassical optimal growth model with the technological progress. We define the technological progress per effective unit labour as

$$M \equiv VL,$$

where  $V$  is the technological progress (level). The technological progress is supposed to be the labour augmenting progress,  $v = \frac{\dot{V}}{V}$ . Based on this technological progress, we can rewrite the aggregative production function as

$$Y = F(K, M).$$

Therefore, the dynamical system can be rewritten as

$$\dot{c}_M = \frac{1}{\theta(c)} [f'(k_M) - \nu - \lambda - \rho] c_M, \quad (4.16)$$

and

$$\dot{k}_M = f(k_M) - c_M - (\nu + \lambda) k_M, \quad (4.17)$$

where  $c_M = \frac{C}{M}$  and  $k_M = \frac{K}{M}$ .

One special solution of this dynamical system is  $\dot{c}_M = \dot{k}_M = 0$ . The optimal growth path is the cross point of  $\dot{c}_M = 0$  and  $\dot{k}_M = 0$ . Although  $c$  is constant in the steady state without technological progress,  $c_M$  is constant in the steady state with technological progress. The aggregative consumption is shown by  $C = c_M VL$  and the economic growth rate in steady state is the sum of population growth rate and the technological progress rate.<sup>8</sup> However, these rate are exogenous parameters and economic policy can not affect them in the steady state of neoclassical optimal growth model.

## 4.5 Neoclassical Economic Growth and Public Policy

The economic policy does not affect the economic growth rate in steady state. To clarify these effects, in this section, introducing public policy into neoclassical optimal growth model, we examine the effects of public policy on economic growth rate in the steady state.

We suppose a government which collects the wage income tax for the productive public goods. Then, we suppose the aggregative production function including

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<sup>8</sup>In the steady state, from  $\dot{c}_M = 0$ ,

$$\dot{c}_M = \frac{d\left(\frac{C}{VL}\right)}{dt} = \frac{C}{VL} \left( \frac{\dot{C}}{C} - \frac{\dot{V}}{V} - \frac{\dot{L}}{L} \right) = 0.$$

Similarly from  $\dot{k}_M = 0$ ,

$$\dot{k}_M = \frac{d\left(\frac{K}{VL}\right)}{dt} = \frac{K}{VL} \left( \frac{\dot{K}}{K} - \frac{\dot{V}}{V} - \frac{\dot{L}}{L} \right) = 0.$$

Then,

$$\frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{\dot{V}}{V} + \frac{\dot{L}}{L} = \nu + n.$$

publicly provided goods as

$$Y = F(K, G, L), \quad (4.18)$$

where  $G$  is the aggregative but not accumulated public goods. This production function is also assumed to be homogeneous function. The output per capita is shown by

$$y = f(k, g),$$

where  $g$  is the public goods per capita,  $g = \frac{G}{L}$ . When the wage income tax rate is  $\tau$ , the budget constraint per capita is

$$\dot{k} = (1 - \tau)f(k, g) - c - \lambda k. \quad (4.19)$$

An instantaneous government budget constrain is given by

$$\tau f(k, g) = g.$$

Therefore, the representative consumer's optimal behaviour is formulized as

$$\begin{aligned} \max_c \quad & \int_0^{\infty} U(c)e^{-\rho t} dt, \\ \text{s.t.} \quad & \dot{k} = (1 - \tau)f(k, g) - c - \lambda k. \end{aligned}$$

The current value Hamiltonian is

$$H = U(c) + q((1 - \tau)f(k, g) - c - \lambda k), \quad (4.20)$$

where  $q$  is the shadow price of capital and the first order condition is given by

$$\frac{\partial H}{\partial c} = U'(c) - q = 0. \quad (4.21)$$

The shadow price,  $q$  is satisfied with the following condition,

$$\dot{q} = \rho q - \frac{\partial H}{\partial k} = (\rho - (1 - \tau)f'_k + \lambda) q,$$

where  $f'_k = \frac{\partial f(k, g)}{\partial k}$ . The transversality condition is

$$\lim_{t \rightarrow \infty} qk = 0.$$

Therefore, the consumption per capita is derived from these conditions,

$$\dot{c} = \frac{1}{\theta(c)} [(1 - \tau)f'_k - \lambda - \rho] c.$$

We can summarised the dynamical system in this model as

$$\dot{c} = \frac{1}{\theta(c)} [(1 - \tau)f'_k - \lambda - \rho] c, \quad (4.22)$$

and

$$\dot{k} = (1 - \tau)f(k, g) - c - \lambda k. \quad (4.23)$$

The special solution of this system is  $\dot{c} = \dot{k} = 0$ . Without technological progress, the aggregative consumption ( $C = cL$ ), the aggregative capital ( $K = kL$ ) and the aggregative output ( $Y = yL$ ) grows at the rate of  $n$ . In the neoclassical optimal growth model, public policy affects the variables per capita and the aggregative variables but does not affect the economic growth rate in steady state.

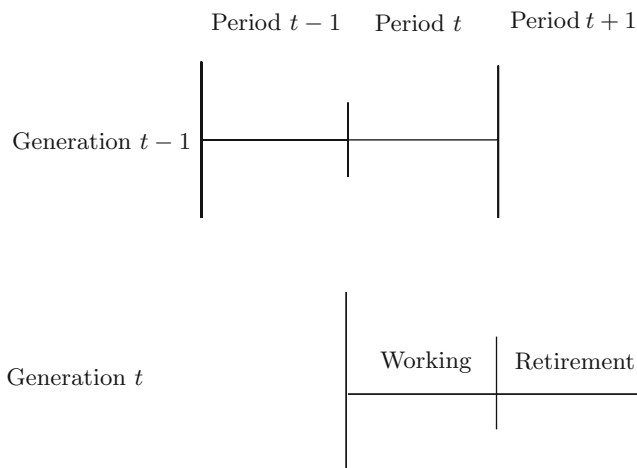
## 4.6 Overlapping Generations Model

Until this section, we suppose that time is continuous and the consumer lives infinitely. However, we can not show the intertemporal consumers' behaviour, especially saving behaviour in above models. We can also suppose that time is discrete and the consumer lives for some periods. At one period, the multiple generations of consumer are supposed to exist in the economy. Samuelson [14] and Diamond [6] develop the overlapping generations model which includes these consumers' consideration. In this section, we discuss this model.

In this model, the consumers live for two periods, the working period and the retirement period, in a closed one good economy. The economy grows at the growth rate of population,  $n$ . When the population of working generation at period  $t$  is assumed to be  $L_t$  and that at period  $t + 1$  is  $L_{t+1}$ , the relationship between them is shown by  $L_{t+1} = (1 + n)L_t$ . We assume away the depreciation rate of capital. As shown in Fig. 4.5, at period  $t$ , there exists two generations, the working generation and the retirement generation. We call the working generation at period  $t$  as generation  $t$ .

The utility for the representative consumer of generation  $t$  is derived from the consumptions at each period. That is,

$$U^t = U^t(c_t^t, c_{t+1}^t), \quad (4.24)$$



**Fig. 4.5** Overlapping generations model

where  $c_t^t$  is the consumption of the working period for generation  $t$  at period  $t$  and  $c_{t+1}^t$  is the consumption of the retirement period for generation  $t$  at period  $t+1$ .

The generation  $t$  in the first period of their lives, the working generation, supply their labour inelastically to firms. They divide their wage income,  $w_t$ , between current consumption,  $c_t^t$  and saving for consumption when retirement,  $s_t$ . The budget constraint for the working period of representative generation  $t$  is

$$w_t = c_t^t + s_t. \quad (4.25)$$

The generation  $t$  in the second (final) period of their lives, the retirement, consume their accumulated savings and interest. We assume no bequests. The budget constraint for the retirement period of generation  $t$  is given by

$$c_{t+1}^t = (1 + r_{t+1}) s_t, \quad (4.26)$$

where  $r_{t+1}$  is the interest rate at  $t+1$ .

Given  $w_t$  and  $r_{t+1}$ , a representative agent of generation  $t$  chooses  $c_t^t$  and  $c_{t+1}^t$  to maximise his utility (4.24) subject to budget constraints (4.25) and (4.26). We can show optimal condition as

$$\frac{U_t^t}{U_{t+1}^t} = 1 + r_{t+1},$$

where  $U_t^t = \frac{\partial U^t}{\partial c_t^t}$  and  $U_{t+1}^t = \frac{\partial U^t}{\partial c_{t+1}^t}$ .

Based on this condition, the saving function per capita for generation  $t$  is shown as

$$s_t = s_t(w_t, 1 + r_{t+1}). \quad (4.27)$$

Similar to the neoclassical growth model, firms produce output with capital and labour.<sup>9</sup> The aggregate production function is expressed as follows,

$$Y_t = F(K_t, L_t),$$

where  $Y_t$  is the aggregate output at period  $t$ ,  $K_t$  the aggregate capital,  $L_t$  the aggregate labour at  $t$ . The production technology exhibits a constant return to scale, and the marginal productivity of each input is positive and decreasing. We define  $y_t \equiv \frac{Y_t}{L_t}$  and  $k_t \equiv \frac{K_t}{L_t}$ . The output per the working generation can be rewritten:

$$y_t = f(k_t).$$

Firms behave perfect competitively to maximise their profit. From the first-order conditions for profit maximisation, factor prices are derived as follows:

$$w_t = f(k_t) - k_t f'(k_t), \quad (4.28)$$

and

$$r_t = f'(k_t). \quad (4.29)$$

As the working generation supply their labour inelastically to firms, we consider the capital market for the equilibrium. The equilibrium condition of capital market and Walras' Law makes the goods market be in equilibrium. The equilibrium condition of capital market is

$$(1 + n) k_{t+1} = s_t(w_t, 1 + r_{t+1}). \quad (4.30)$$

This economy is summarised to the following dynamic equation,

$$(1 + n) k_{t+1} = s_t(w_t(k_t), 1 + r_{t+1}(k_{t+1})). \quad (4.31)$$

When  $k_t$  is equal to  $k_{t+1}$ , the economy is in the steady state.

Next, we examine the stability of equilibrium. From (4.31), in the neighbourhood of equilibrium, we can show the relationship between  $k_{t+1}$  and  $k_t$  as

$$\frac{dk_{t+1}}{dk_t} = \frac{-s_w(k_t) k_t f''(k_t)}{1 + n - s_r(k_{t+1}) f''(k_{t+1})},$$

where  $s_w = \frac{\partial s_t}{\partial w_t}$  and  $s_r = \frac{\partial s_t}{\partial r_t}$ .

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<sup>9</sup>The overlapping generations model is developed based on the neoclassical growth model.

When the following condition is satisfied, the equilibrium is stable. If

$$\left| \frac{-s_w(k^*) k^* f''(k^*)}{1+n-s_r(k^*) f''(k^*)} \right| < 1, \quad (4.32)$$

the equilibrium is stable.

Let the aggregative capital at period  $t+1$  be  $K_{t+1} = k_{t+1}L_{t+1}$  and that at period  $t$  be  $K_t = k_tL_t$ . As  $k_t = k_{t+1}$  and  $L_{t+1} = (1+n)L_t$  in steady state, the economic growth rate is shown as

$$\frac{K_{t+1}}{K_t} = \frac{k_{t+1}L_{t+1}}{k_tL_t} = (1+n). \quad (4.33)$$

Even in the overlapping generations model with the neoclassical production function and the explicit saving behaviour, the economic growth rate is shown by the exogenous parameter and the economic policy does not affect the growth rate in steady state.

## 4.7 Concluding Remarks

Finally, although the economic growth rate fluctuates in each country as in Fig. 4.1, assuming the constant return to scale on aggregative production function in the neoclassical (optimal) economic growth model, the economic growth rate on such model is shown by the exogenous parameters, the growth rate of population and/or technological progress rate. On the model, the economic policy can not affect the economic growth rate. However, as discussed next chapter, if we assume different type of production function, we could examine the effects of economic policy on economic growth rate in steady state.

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