Chapter 12
Norms and Games as Integrating Components of Social Organizations

Yasuo Nakayama

Abstract Previous chapters in this book mainly discuss individual humans from the viewpoint of psychology or neuroscience. In this chapter, we discuss how mental states and attitudes of individuals are related to the society from a philosophical viewpoint. In this discussion, we are particularly concerned with the question of what kinds of roles norms play when individuals establish social connections. Then, we discuss how various kinds of human capacities are related to each other and how people build and maintain social organizations. In the end, we see that humans are beings that have cognitive capacities to form their own lives by behaving in accordance with the accepted norms and changing the norms for better lives.

Keywords Social organizations • Collective beliefs • Social norms • Obligation • Normative systems • Action space • Gamification • Language game • Information update • Knowledge representation

12.1 What Are Norms?

What are norms? Why are norms important for explaining human behaviors? One of the main aims of this chapter is to answer these questions. Let us start with the first question. Our preliminary answer is that norms are *what normative sentences express*. Then, what are normative sentences? We distinguish between *assertive* and *normative* sentences. They both belong to declarative sentences, but express different kinds of contents.[1] Assertive sentences simply describe states or events in the world. Examples of assertive sentences are the following:

(1a) There are three tables in this room.

[1] Different kinds of speech acts may be distinguished (Seale 1979). Our view is that the distinction between assertive and normative sentences is fundamental, and speech acts can be expressed in terms of this distinction. See Nakayama (2014).

Y. Nakayama (✉)
Graduate School of Human Sciences, Osaka University, Suita, Osaka, Japan
e-mail: nakayama@hus.osaka-u.ac.jp

M. Kasaki et al. (eds.), *Cognitive Neuroscience Robotics B*,
DOI 10.1007/978-4-431-54598-9_12

(1b) A presidential election took place in the USA in 2013.

The first sentence describes a physical fact, and the second sentence does a social fact. Unlike physical facts, social facts are those the existence of which is based on collective beliefs (Searle 1995, 2010; Nakayama 2011). Collective beliefs are closely related to social norms (See Sect. 12.4 and Appendix).

There are three kinds of normative sentences. They express *obligation*, *prohibition*, and *permission*, respectively:

(2a) 'You ought to be quiet in the lecture' expresses an obligation.
(2b) 'It is forbidden to sleep in the lecture' expresses a prohibition.
(2c) 'It is permitted to ask questions anytime when you do not completely understand' expresses a permission.

It is difficult to say of a normative sentence whether it is true or false. Then, it may be more appropriate to ask whether it is collectively accepted or not. This difficulty of assigning a truth value reveals a fundamental difference between normative and assertive sentences. For an assertive sentence is, in principle, true or false.

Sentences (2a) to (2c) are not purely normative but involve certain assertive components. For example, sentence (2a) can be paraphrased as a conditional obligation in the following way:

(2a*) If you are sitting with other students in the lecture, you ought to be quiet.

The second clause expresses an obligation, but the first clause expresses the condition in which this obligation is imposed. Suppose a female student is sitting in the lecture. Then, we can check by observation whether she fulfills the obligation. If she has been quiet during the whole lecture, we say that she has fulfilled it. The fulfillment of an obligation is often observable. This suggests that facts and norms are different but closely related.

Assertive and normative sentences are not the only kinds of sentences. There are other types of sentences, such as *imperative* and *interrogative* sentences. But there is a good reason to claim that assertive and normative sentences are semantically more fundamental than other types of sentences. In fact, Nakayama (2014) shows that all types of speech acts entail assertive or normative components.

There are logical relations among three kinds of normative modalities: *obligation*, *prohibition*, and *permission*.[2] If something is *permitted*, it is not *forbidden*. And if something is *forbidden*, its negation is an *obligation*.[3] In Sect. 12.4, we construct a logical framework that describes logical properties of normative modalities, but we cannot go into full detail and only illustrate some main principles of the

[2] These normative modalities are also called *deontic modalities*.

[3] These two conditional propositions are expressed as $\mathbf{P}p \rightarrow \neg\mathbf{F}p$ and $\mathbf{F}p \rightarrow \mathbf{O}\neg p$ in the *standard deontic logic* (SDL). SDL is a modal logic proposed by von Wright. In SDL, *prohibition* and *permission* are definable through *obligation*: $\mathbf{F}p \Leftrightarrow_{\mathrm{def}} \mathbf{O}\neg p$ and $\mathbf{P}p \Leftrightarrow_{\mathrm{def}} \neg\mathbf{O}\neg p$, where $\neg$ is the sign of negation. See Åqvist (2002).

logical framework for normative systems (See Sects. 12.4, 12.5, and Appendix of this chapter).

12.2 Why Norms Are so Important? – Plays, Games, and Norms

In this section, we give the first answer to the question why norms are important for human life. The answer is that a child becomes a member of a society *through accepting common beliefs and sharing social norms*. Sharing social norms means that people in a social organization agree on what they should do and what they are forbidden to do; or in other words, that they plan what to do (individually) in accordance with social norms.

A social organization might be complex, but a game can be used as a toy model for it. Games illustrate in a simple way how social interactions work. Thus, children can learn social behaviors through playing games. In fact, some developmental psychologists, such as Jean Piaget (1896–1980) and Lev Vygotsky (1896–1934), did extensive researches on plays of children. They held that the mastery of games is crucial for learning social behaviors. For example, Piaget proposed three broad stages of play activity (Piaget 1951; Birch 1997: p. 55f):

(3a) **Mastery play** corresponds to the sensori-motor stage of development (from birth to two years, approximately). Important things at this stage are practice, control of movements, and exploration of objects through sight and touch. Children's play activity contains repetitive movements, and children enjoy them for the simple pleasure of demonstrating their developing mastery of the relevant skills.

(3b) **Symbolic play** coincides with the pre-operational stage (from two to seven years, approximately). Children employ fantasy, make believe in play, and delight in using one object to symbolize another—a chair may become a motor car, and a sheet a fashionable dress.

(3c) **Play with rules** characterizes the operational stage (from seven years onwards, approximately). Children's thought processes are developed to be more logical, and their play involves the use of rules and procedures.

This description of each stage demonstrates how plays and games in a broad sense are crucial components of the human life from its beginning. Plays usually contain some normative components; there are actions you should do and actions you must not do. You are required to make your choice and what you can choose is restricted by the rules of the play. Such normative constraints define your *action space*. The learning of correct normative behaviors through plays prepares children to behave socially in the real life. When children go to school, they learn how to deal with social norms. For example, they ought to listen to the teacher during the lesson; they ought to stay in the school until school hours are over, and so on.

Recent successful applications of gamification also indicate how attractive games are for humans. *Gamification* means the use of game thinking and game mechanics in a non-game context to engage users and solve problems (Zichermann and Cunningham 2011). Boring activities can be made more attractive by employing gamification techniques, such as giving rewards, transforming a problem into a cooperative game, or visualizing each development stage of competition in real time.

12.3 Language Games

Language game is one of the crucial ideas that characterize the philosophy of Ludwig Wittgenstein (1889–1951). There are two interesting facts about language games. First, there are *primitive* language games, i.e., those that can be played without presupposing other language games. Children can learn primitive language games directly through playing them. Primitive language games are simpler and easier to learn than non-primitive language games. Second, language games have dynamic aspects. A new language game could emerge anytime and an old one could be forgotten. There is a kind of metabolism in the system of language games.

The second paragraph of Wittgenstein's *Philosophical Investigations* (*PI*) describes an example of primitive language game:

> That philosophical concept of meaning has its place in a primitive idea of the way language functions. But one can also say that it is the idea of a language more primitive than ours.
>
> Let us imagine a language for which the description given by Augustine is right. The language is meant to serve for communication between a builder A and an assistant B. A is building with buildingstones: there are blocks, pillars, slabs and beams. B has to pass the stones, and that in the order in which A needs them. For this purpose they use a language consisting of the words "block", "pillar", "slab", "beam". A calls them out;—B brings the stone which he has learnt to bring at such-and-such a call.—Conceive this as a complete primitive language. (Wittgenstein 1958: *PI* I.§2)

This primitive language game can be interpreted as a two player game. The game consists of two main rules:

[Rule 1] The builder is permitted to call out one of the four nouns "block", "pillar", "slab", "beam", when the assistant is waiting for a call.

[Rule 2] When the builder calls out a noun, the assistant is obligated to bring a stone denoted by the noun.

The builder makes a move of calling out one of the four nouns, and the assistant makes a move of delivering an appropriate stone to the builder. The builder and the assistant alternate moves. The game continues until the builder decides to end it.

Wittgenstein also suggests how the total system of language games develops.

> Do not be troubled by the fact that languages (2) and (8) consist only of orders. If you want to say that this shows them to be incomplete, ask yourself whether our language is complete;—whether it was so before the symbolism of chemistry and the notation of the

> infinitesimal calculus were incorporated in it; for these are, so to speak, suburbs of our language. (And how many houses or streets does it take before a town begins to be a town?) Our language can be seen as an ancient city: a maze of little streets and squares, of old and new houses, and of houses with additions from various periods; and this surrounded by a multitude of new boroughs with straight regular streets and uniform houses. (Wittgenstein 1958: *PI* I.§18)

Here, Wittgenstein talks about *our language*, i.e., the language of our linguistic community. However, this description is applicable to the development of the language of one particular person as well, because she has a personal history and has learned different sorts of language games in various situations.

If we take these two paragraphs from *PI* together, we obtain a picture of how a child learns a language. She starts playing various kinds of primitive language games. When she becomes adult, she has experienced many language games. Some of them are her favorite ones, and some of them are forgotten. She plays the former every day, and she no longer plays the latter. She plays language games through her life time.

12.4 Logic for Normative Systems

An aim of this chapter is to describe social norms rigorously and systematically. For this purpose, we use *Logic for Normative Systems* (LNS) that is proposed in Nakayama (2010, 2011). LNS is initially proposed to elucidate normative inferences and clarify the notion of *normativity*. As we discussed in Sect. 12.1, norms are closely related with facts. LNS is a framework that can describe context-dependent interactions between norms and facts. At first, we begin with clarifying what normative systems are.

Nakayama (2010) defines *Logic for Normative Systems* (LNS) in a certain way, and the following is a modification of the definition proposed there.[4]

(4a) A *normative system NS* consists of *belief base T* and *normative base N*. A normative base consists of *obligation base OB* and *permission base PER*. Thus, $NS = \langle T, \langle OB, PER \rangle \rangle$.[5] Each of T, OB, and PER is a set of sentences (more precisely, a set of sentences in *First-Order Logic*[6]).

(4b) A normative system NS is consistent ⇔ *union*(NS) is consistent, where *union* $(NS) = T \cup OB \cup PER$.

[4] We use *not*, &, ⇒, and ⇔ as meta-semantic abbreviation for *not*, *and*, *if . . . then*, and *if and only if*. For more precise definition and characterization of LNS, see Appendix of this chapter. The reader may consult Nakayama (2013, 2014) as well.

[5] Formally, we can avoid the use of *PER*, because what is permitted is uniquely determined when we determine *T* and *OB*. However, in this chapter, we include *PER* into the structure of normative systems, because we allow information updates of normative systems with respect to permission, in order to take ordinary language practices into consideration.

[6] First-Order Logic is the *standard logic* that is usually used for proving mathematical theorems.

(4c) A sentence q belongs to the *belief set* of $NS \Leftrightarrow q$ follows from T.
(4d) A sentence q belongs to the *obligation set* of $NS \Leftrightarrow$ (NS is consistent & q follows from $T \cup OB$ & q does not belong to the *belief set* of NS).
(4e) A sentence q belongs to the *prohibition set* of $NS \Leftrightarrow$ the negation of q belongs to the *obligation set* of NS.
(4f) A sentence q *belongs to the permission set of* $NS \Leftrightarrow$ ($union(NS) \cup \{q\}$ is consistent & q does not belong to the *belief set* of NS).
(4g) We interpret that NS represents a normative system accepted by a person or a group at a particular time. Thus, we allocate *what a person* (or *a group*) *believes to be true* to the *belief base*, *what he believes to ought to be done* to the *obligation base*, and *what he believes to be permitted* to the *permission base*.

According to this definition, a normative system presupposes certain non-normative theories and facts. The contents of beliefs are expressed in terms of assertive sentences, and hence non-normative theories and facts are treated as beliefs in NS (see (4g)). Beliefs are separated from norms. In other words, they include no common element (see (4d) to (4f)). If you perform an action that is your obligation, then it becomes a fact and a part of your beliefs that you have performed this action. As a result, the obligation ceases to be your obligation. Beliefs and normative requirements are closely related this way. According to (4f), permission is understood in a wide sense: everything compatible with a given normative system is considered to be permissible. The *set of all actions that are permitted for you*, we call *your action space*. When you fully accept a given normative system, you would always plan your action within your current action space.

In this chapter, we use the following abbreviations (Table 12.1).

We now understand how a social organization G is connected with G's members. Suppose that $ns(G)$ is the normative system of G. Member's sharing of a normative system can be easily defined as follows, where a normative system comprises their normative bases, i.e., a belief base, an obligation base, and a permission base:

(5) All members of G share the normative system $ns(G) \Leftrightarrow$ For every member a of G, each of a's normative bases contains a corresponding normative base of G (i.e., $bel(G) \subseteq bel(a)$ & $ob(G) \subseteq ob(a)$ & $per(G) \subseteq per(a)$ & $ns(G) = \langle bel(G), \langle ob(G), per(G) \rangle \rangle$ & $ns(a) = \langle bel(a), \langle ob(a), per(a) \rangle \rangle$).

When all members of G share a normative system, they share their beliefs and norms (obligations and permissions). Only those whose normative systems are

Table 12.1 Abbreviations in LNS

Abbreviation	Meaning	How to read
$\mathbf{B}_{NS}\, p$	p belongs to the *belief set* of NS	It is *believed* in NS that p
$\mathbf{O}_{NS}\, p$	p belongs to the *obligation set* of NS	It is *obligated* in NS that p
$\mathbf{F}_{NS}\, p$	p belongs to the *prohibition set* of NS	It is *forbidden* in NS that p
$\mathbf{P}_{NS}\, p$	p belongs to the *permission set* of NS	It is *permitted* in NS that p

consistent and who believe that the normative requirements in question have not been fulfilled can share *G*'s norms (for an exact formulation, consult (A3a) to (A3d) in the Appendix). The shared normative system of *G* constitutes a part of the personal normative system of each of *G*'s members. It influences behaviors of *G*'s members, because it partially determines their action spaces.

12.5 Normative Systems in Reality

In this section, we consider two examples of normative systems and show how to represent them in LNS. LNS is designed to deal with normative systems in the ordinary life. We then choose two natural examples from our daily life.

The first example is from the Constitution of the United States (*CUS*). In principle, any law can be treated in a similar way. Now, look at Article 1, Section 1 of *CUS*:

[*CUS* Art. 1. Sect. 1] All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

This sentence can be divided into two components:

(6a) A Congress of the USA should be composed of Senate and House of Representatives.
(6b) Only the Congress of the USA has legislative Powers. That means that it is *permitted* for the Congress of the USA to make laws in the USA, while it is *forbidden* for any other organization to do so.

We can transform these sentences in order to express them in LNS. The transformation process involves two parts. First, we eliminate normative expressions from (6a) and (6b). Second, we separate permission components from prohibition components in (6b). After these operations, we obtain the following three sentences:

(7a) A Congress of the USA is composed of Senate and House of Representatives.
(7b) The Congress of the USA makes laws in the USA.
(7c) For any *x*, (if *x* is an organization that is not the Congress of the USA, then *x* does not make laws in the USA).

We put each sentence in an appropriate normative context, so that the meaning of (6a) and (6b) can be recaptured. We then obtain:

Sentences (7a) and (7c) belong to the *obligation base* of *CUS*, and (7b) belongs to the *permission base* of *CUS* (i.e., $\{(7a), (7c)\} \subseteq ob(CUS)$ & $\{(7b)\} \subseteq per(CUS)$).[7]

[7] Actually, we should translate sentences (7a), (7b), (7c) into sentences of First-Order Logic. For simplicity, we omit this translation step.

The *normative system of the Constitution of the United States* can be defined as follows: $ns(CUS) = \langle bel(CUS), \langle ob(CUS), per(CUS)\rangle\rangle$.[8] With this definition, we obtain three conclusions that roughly capture the content of (6a) and (6b)[9]:

(8a) It is an *obligation* in *ns*(*CUS*) that a Congress of the USA is composed of Senate and House of Representatives.[10]
(8b) It is *permitted* in *ns*(*CUS*) that the Congress of the USA makes laws in the USA.
(8c) It is *forbidden* in *ns*(*CUS*) that an organization other than the Congress of the USA makes laws in the USA.

The second example is taken from submission rules of a journal. We extract only a small fragment from the submission rules of *Annals of the Japan Association for Philosophy of Science*.

(9a) Manuscripts considered for publication in this journal should be original and related to philosophy of science.
(9b) Manuscripts should be written in English, German, or French, within 8000 words.

As we did for the first example, first, we omit normative expressions from the original sentences, and then we put them in the normative contexts that are indicated by the original sentences.

(10a) If author *x* considers manuscript *y* for publication in *AJAPS*, then *y* is original and related to philosophy of science.
(10b) If author *x* considers manuscript *y* for publication in *AJAPS*, then *y* is written in English, German, or French within 8000 words.
(10c) (10a) and (10b) belong to the obligation base of *ns*(*AJAPS*) (i.e., $\{(10a), (10b)\} \subseteq ob(AJAPS)$).

From this description, it follows in *ns*(*AJAPS*) that any manuscript that is considered for publication in *AJAPS* should be original, should be related to philosophy of science, and should be written in English, German, or French, within 8000 words (Formally, $\mathbf{B}_{ns(AJAPS)}$ (*m* is considered for publication in *AJAPS*) $\Rightarrow \mathbf{O}_{ns(AJAPS)}$ (*m* is original $\wedge$ *m* is related to philosophy of science $\wedge$ *m* is written in English, German, or French $\wedge$ *m* is within 8000 words)). If someone sends to *JSPS* a manuscript that violates some of the normative rules in *ns*(*AJAPS*), then it will be automatically rejected by *JAPS*.

As you have seen in this section, it is relatively easy to express ordinary normative systems in LNS.

[8] *bel*(*CUS*) means the *belief base* of *CUS*.

[9] For this derivation, please consult theorems in Appendix of this chapter.

[10] In LNS, once a Congress of the USA is built, it is no longer obligatory to construct the Congress, and its existence becomes a (social) fact.

12.6 Dynamic Normative Logic and Information Update

Dynamic Normative Logic (DNL) is proposed as an extension of LNS in Nakayama (2013). DNL is developed to describe information update of normative systems.[11] In this chapter, we use the expression $ns^2(a, t)$ to refer to the *normative state*[12] accepted by agent a in period of time t, where this agent may be an individual or a group. We use the expressions $bel^2(a, t)$, $ob^2(a, t)$, and $per^2(a, t)$ as the *belief base of agent a in time t,* the *obligation base of a in t*, and the *permission base of a in t*, respectively. Thus, $ns^2(a, t) = \langle bel^2(a, t), \langle ob^2(a, t), per^2(a, t)\rangle\rangle$.

To emphasize the *part of information that is updated*, we distinguish between the elementary theory $et(a)$ and the set $fact^2(a, t)$ (facts for a in t). In other words, the belief base $bel^2(a, t)$ is composed of mutually excluding two sets, $et(a)$ and $fact^2(a, t)$ (Formally, $bel^2(a, t) = et(a) \cup fact^2(a, t)$ & $et(a) \cap fact^2(a, t) = \emptyset$).[13] In many cases, only $fact^2(a, t)$ is updated and $ob^2(a, t)$ and $per^2(a, t)$ remain unchanged during the time interval under consideration. In such a case, we sometimes simply write: $ns^2(a, t) = \langle bel^2(a, t), \langle ob(a), per(a)\rangle\rangle$.

In DNL, we can describe information update of collective normative systems. Because we consider the change of normative states through time, we slightly modify the previous definition (5) of the sharing of normative systems as follows.

(11) All members of G share the normative state $ns^2(G, t)$ in period of time $t \Leftrightarrow$ For every member a of G, each of a's normative bases in t contains a corresponding normative base of G in t respectively (i.e., $bel^2(G, t) \subseteq bel^2(a, t)$ & $ob^2(G, t) \subseteq ob^2(a, t)$ & $per^2(G, t) \subseteq per^2(a, t)$ & $ns^2(G, t) = \langle bel^2(G, t), \langle ob^2(G, t), per^2(G, t)\rangle\rangle$ & $ns^2(a, t) = \langle bel^2(a, t), \langle ob^2(a, t), per^2(a, t)\rangle\rangle$.

Thus, the normative state of G in t represents the common part of the normative states of all members of G in t. In many cases, the elementary theory and norms remain unchanged, and only the set of facts is updated.[14] When all members of G share a normative state in t, they share their non-normative beliefs and their norms in t, to the extent that they believe in t that the normative requirement in

[11] DNL is a framework for logical analysis of social interactions. Dynamic epistemic logic is a well-known framework developed for the same purpose. See van Benthem (2011). However, his description is restricted on propositional logic and does not discuss about quantification. In Sects. 12.7 and 12.8, we will see that typical games can be defined in DNL. A game is a kind of normative system that includes information update (Nakayama 2013). A move in a play changes the previous information state.

[12] We say sometimes *normative state* instead of *normative system*, when information update plays a role. In other words, a normative state is a normative system that an agent has for a period of time. The index 2 in $ns^2(a, t)$ indicates that ns^2 is a function with two arguments. Generally, we use $ns^2(a, \tau)$, when term τ involves a temporal component.

[13] $\emptyset$ denotes the empty set, i.e., the set that contains no element.

[14] In such a case, for every time t and for every G's member a, $ob^2(a, t) = ob^2(G, t)$ & $per^2(a, t) = per^2(G, t)$.

question has not been fulfilled, provided that their normative systems are consistent (for an exact formulation, consult (A4a) to (A4d) in the Appendix).

12.7 Dynamic Normative Logic and Language Games

We can describe primitive language games in DNL. Let us take the example discussed in Sect. 12.3 again (We refer to this example as *PI§2*). The elementary theory and norms do not change, and only the set of facts is updated in this example. Thus, the elementary theory of *PI§2*, the obligation base for *PI§2*, and the permission base for *PI§2* can be expressed as *et*(*PI§2*), *ob*(*PI§2*), and *per*(*PI§2*), respectively, independently from the temporal component.

We interpret this language game as that in which a builder A creates obligations for an assistant B, and B tries to fulfill each of the obligations at a time.[15] An obligation for B consists in bringing a piece of $int_A(\alpha)$ to A, when A calls out 'α.'[16] To fulfill this obligation successfully, B has to interpret α correctly. B can successfully play this language game, only if B's interpretation of α coincides with A's interpretation of α (Formally, $int_B(\alpha) = int_A(\alpha)$). If B's interpretation is wrong, then A would reject receiving $int_B(\alpha)$. After several trials, B would learn the right interpretation of α, so that $int_B(\alpha) = int_A(\alpha)$.[17] In other words, we can say that B has learned this language game, if B always brings a piece of $int_A(\alpha)$ for any α in {"block", "pillar", "slab", "beam"}.

We use $ns^2(\{A, B\}, n)$ as the normative state of group {A, B} at game stage n, where n is an integer and $ns^2(\{A, B\}, n) = \langle et(PI§2) \cup fact^2(\{A, B\}, n), \langle ob(PI§2), per(PI§2)\rangle\rangle$. We use a function t_f to express the period of time of a game stage; $t_f(n)$ means the period of time of game stage n. For example, when this language game starts at 2014 May 23 12:00 and the first A's call takes place at 2014 May 23 12:05, $t_f(0)$ is uniquely determined as the interval [2014 May 23 12:00, 2014 May 23 12:05).

We assume that elementary theory *et*(*PI§2*) contains the sentence 'If m is smaller than n, then the time of m is earlier than the time of n' (Formally, $\{\forall m\ \forall n\ (m \leq n \rightarrow t_f(m) \leq_t t_f(n))\} \subseteq et(PI§2)$).[18] In this example, the permission base contains the sentence (12a), and the obligation base *ob*(*PI§2*) contains the sentence (12b) (i.e., *per*(*PI§2*) = {(12a)} and *ob*(*PI§2*) = {(12b)}).

---

[15] A command is a speech act that creates an obligation for the addressee. See Searle (1979), Yamada (2008), and Nakayama (2014).

[16] $int_A(\alpha)$ means A's interpretation of word α.

[17] This problem of finding the right interpretation for an uttered expression seems closely related to the *symbol grounding problem*. The latter problem is explained as follows: "abstract, arbitrary symbols such as words need to be grounded in something other than relations to more abstract arbitrary symbols if any of those symbols are to be meaningful" (Glenberg and Robertson 2000, p. 381). See also Shapiro (2011, pp. 95–98).

[18] *et*(*PI§2*) contains axioms of linear order for $\leq$ and $\leq_t$ as well.

(12a) For every word α in {"block", "pillar", "slab", "beam"} and for every game stage n, A calls out 'α' in $t_f(n+1)$, when the assistant is waiting for a call in $t_f(n)$.
(12b) For every word α in {"block", "pillar", "slab", "beam"} and for every game stage n, if A calls out 'α' in $t_f(n)$, then B brings a piece of $int_{\mathrm{A}}(\alpha)$ to A in $t_f(n+1)$.

We assume that A and B share the normative state of {A, B} during the game. This assumption is justified by the story in *PI§2*. Now, a possible development of the language game in *PI§2* can be described as follows.

> A calls out "block", when B is waiting for a call in $t_f(0)$. Then, in $ns^2(\{\mathrm{A,B}\},1)$, B is obligated to bring a piece of int_{A}(block) to A in $t_f(2)$. In accordance with this obligation, B brings a piece of int_{A}(block) to A in $t_f(2)$. A calls out "pillar", when B is waiting for a call in $t_f(2)$. Then, in $ns^2(\{\mathrm{A,B}\}, 3)$, B is obligated to bring a piece of int_{A}(pillar) to A in $t_f(4)$. In accordance with this obligation, B brings a piece of int_{A}(pillar) to A in $t_f(4)$. (The game continues in this way.)

This development can be more formally described as follows:

$fact^2(\{\mathrm{A, B}\}, 0) = \{$B is waiting for a call in $t_f(0)\}$.

$\mathbf{P}_{ns2(\{\mathrm{A,B}\},\ 0)}$ (A calls out "block" in $t_f(1)$).[19]

$fact^2(\{\mathrm{A, B}\}, 1) = fact^2(\{\mathrm{A, B}\}, 0) \cup \{$A calls out "block" in $t_f(1)\}$.

$\mathbf{O}_{ns(\{\mathrm{A,B}\},\ 1)}$ (B brings a piece of int_{A}(block) to A in $t_f(2)$).[20]

$fact^2(\{\mathrm{A, B}\}, 2) = fact^2(\{\mathrm{A, B}\}, 1) \cup \{$B brings a piece of int_{A}(block) to A in $t_f(2)$, B is waiting for a call in $t_f(2)\}$.

$\mathbf{P}_{ns2(\{\mathrm{A,B}\},\ 2)}$ (A calls out "pillar" in $t_f(3)$).

$fact^2(\{\mathrm{A, B}\}, 3) = fact^2(\{\mathrm{A, B}\}, 2) \cup \{$A calls out "pillar" in $t_f(3)\}$.

$\mathbf{O}_{ns2(\{\mathrm{A,B}\},\ 3)}$ (B brings a piece of int_{A}(pillar) to A in $t_f(4)$).

$fact^2(\{\mathrm{A, B}\}, 4) = fact^2(\{\mathrm{A, B}\}, 3) \cup \{$B brings a piece of int_{A}(pillar) to A in $t_f(4)$, B is waiting for a call in $t_f(4)\}$.

. . .

As this example indicates, DNL can formally represent not only primitive language games but also typical games, such as chess and card games.

---

[19] The consistency of $union(ns^2(\{\mathrm{A, B}\},0)) \cup \{$A calls out "block" in $t_f(1)\}$ can be shown by constructing a finite model.

[20] For the derivation of this sentence, you use theorem (A2e1) in Appendix.

12.8 Dynamic Normative Logic, Games, and Roles

In this section, we show how to describe baseball games in DNL. For this purpose, we consult the rules of *Major League Baseball* (Official Baseball Rules 2013, 2.00). The rules correspond to a normative system *ns*(*MLB*), where ⟨*et*(*MLB*), ⟨*ob*(*MLB*), *per*(*MLB*)⟩⟩. The rules are many and very complex, and so we can only describe a small fragment of them.

There are batters, a pitcher, a catcher, fielders, and umpires in a baseball game. They are distinguished in terms of the roles they play during a game.

[OBR*4.03] When the ball is put in play at the start of, or during a game, all fielders other than the catcher shall be on fair territory.

(a) The catcher shall station himself directly back of the plate. He may leave his position at any time to catch a pitch or make a play except that when the batter is being given an intentional base on balls, the catcher must stand with both feet within the lines of the catcher's box until the ball leaves the pitcher's hand.
(b) The pitcher, while in the act of delivering the ball to the batter, shall take his legal position.
(c) Except the pitcher and the catcher, any fielder may station himself anywhere in fair territory.

Notice that several roles are different in normative requirement. The differences can be reflected in the obligation base and the permission base in *ns*(*MLB*).

(13a) The catcher stations himself directly back of the plate.
(13b) The catcher leaves his position at any time to catch a pitch or make a play.
(13c) If the batter is being given an intentional base on balls, then the catcher stands with both feet within the lines of the catcher's box until the ball leaves the pitcher's hand.
(13d) If the pitcher is in the act of delivering the ball to the batter, then he takes his legal position.
(13e) Every fielder who is neither the pitcher nor the catcher stations himself anywhere in fair territory.
(13f) $\{(13a), (13c), (13d)\} \subseteq ob(MLB)$ & $\{(13b), (13e)\} \subseteq per(MLB)$.

With the help of this list of normative requirements, it is evident that a player needs to know about his own role in the baseball game in order to play it. For example, when you are registered as a catcher, if you mistake yourself as a fielder, you will station yourself in a wrong position and violate some rules of *MLB*.

Let us take as an example of information update a scene in a baseball game between teams *A* and *B*. The initial normative state can be represented as $ns^3(MLB, \{A, B\}, 0\blacktriangledown 0/3) = \langle et(MLB) \cup fact^3(MLB, \{A, B\}, 0\blacktriangledown 0/3), \langle ob(MLB), per(MLB)\rangle\rangle$. Suppose that team *A* has scored 5 runs and team *B* scored only 2 runs before the seventh inning; more simply, that *score* (A: 5, B: 2, $7\blacktriangledown 0/3$). Suppose also that an umpire now calls out a player in team *A*. Then, the score is updated and the number of outs is raised: *score* (A: 5, B: 2, $7\blacktriangledown 1/3$) (formally, $fact^3(MLB, \{A, B\}, 7\blacktriangledown 1/3) =$

$fact^3$(*MLB*, {*A*, *B*}, 7▼0/3)∪{*score* (*A*: 5, *B*: 2, 7▼1/3)}). We can describe the development of a baseball game in this way.

The game ends when the termination condition is satisfied. The precise termination condition of *MLB*-games is found in OBR.

[OBR*4.11] The score of a regulation game is the total number of runs scored by each team at the moment the game ends.

(a) The game ends when the visiting team completes its half of the ninth inning if the home team is ahead.
(b) The game ends when the ninth inning is completed, if the visiting team is ahead.
(c) If the home team scores the winning run in its half of the ninth inning (or its half of an extra inning after a tie), the game ends immediately when the winning run is scored.

These rules can be expressed as components of the elementary theory of *MLB* (Formally, {[OBR*4.11] (a) + (b) + (c)} ⊆ *et*(*MLB*)). They determine the end of *MLB*-games.

12.9 Games and Dynamic Knowledge Representation

As we have just seen, a baseball game has a rich and complex structure. In this section, we suggest that some real situations can be interpreted as games. Schank and Abelson (1977) proposed what they called the '*script*' as a useful framework for knowledge representation. They described a restaurant script from a customer's viewpoint. The restaurant script consists of four scenes, *entering*, *ordering*, *eating*, and *exiting* (these scenes may have further internal structures). Schank's theory treats every memory as episodic, i.e., as that which is organized around personal experiences. He regards a *script* as a generalized episode. For this reason, a script provides a passive and static way of knowledge representation. In contrast, DNL-representation can provide a dynamic knowledge representation of the relevant activities in a social place.[21]

Eating at a restaurant can be interpreted as a process in a game played by customers, waiters, cooks, a cashier, and an owner. These people play certain roles in a *theater* of a restaurant. In this game, the same person may sometimes play different roles in different situations. For example, a waiter becomes a cashier when a customer wants to pay the bill.

As we saw in Sect. 12.8, roles can be determined through normative requirements. For example, a restaurant customer is a person who may order a meal and has to pay for it. A waiter is a person who should ask customers what meals they

[21] Nakayama (2013) describes a small part of a restaurant scene in full detail in DNL.

want and should serve them with the meals they order. A cook is a person who should cook ordered meals. Different players of the game have different goals. A customer's goal is to eat a meal, and an owner's goal is to satisfy customers with good meals and get money from them.

As an example, we describe normative requirements for customers and waiters.

{A customer orders an available meal, A customer eats a meal she ordered} $\subseteq$ *per* (*RT*).
{A non-customer orders no meal} $\subseteq$ *ob*(*RT*).
{A customer pays the bill of all meals she ordered} $\subseteq$ *ob*(*RT*).
{A waiter asks customers about their orders, a waiter serves customers with meals they ordered when the meals have been cooked} $\subseteq$ *ob*(*RT*).

All of these descriptions can be explicitly written in DNL. We can use DNL to describe ongoing processes in a concrete situation. In a game, you may develop strategies to achieve a goal. For example, the owner might propose new menus to attract restaurant customers. In representing a game in a social place in DNL, phenomena pertaining to that game can be analyzed from different viewpoints of players.

In sum, DNL-representation is a uniform dynamic knowledge representation that is more complex and flexible than script representation. Because of this uniformity, it is easy to combine DNL-representations of activities in different contexts without giving rise to confusion.

12.10 Roles and Social Organizations

A baseball game is a kind of competition between two teams. Each team consists of more than nine members and has an internal structure that is determined by the roles of the team members. A member plays as a pitcher, a catcher, or one of seven fielders. Real social organizations have more complex structures than baseball teams, but the basic principles behind them are not very different.

Many social organizations, such as countries, cities, companies, and universities, are hierarchically structured. For example, a university consists of faculties, and each faculty divides into several departments. There are teaching staffs, office staffs, and students in each faculty. They are distinguished in terms of normative requirements pertinent to university activities.

Nakayama (2011: p. 153f) roughly characterizes social organization as follows.

(14a) [Social organization as a four-dimensional object] Social organization *O* is a four-dimensional object.[22] In other words, a social organization is an entity that has a temporal extension and an internal structure.

[22] For four-dimensionalism and mereology, see Sider (2001) and Nakayama (2009).

(14b) [Humans and artifacts as components of a social organization] A social organization is, in general, a mereological sum $G+A$, where G is a group of rational agents (usually, humans) and A is a mereological sum of certain artifacts. Thus, $O=G+A$. However, there are social organizations that are solely composed of humans. In such case: $O=G$.

(14c) [Self-knowledge of members] If social organization O exists at t and $O=G+A$ or $O=G$, then every member of G knows at t that she belongs to O.

(14d) [Member's knowledge about O's property] Suppose that $O=G+A$ and O exists at t. Then, there is at least one member in G who believes at t that for every part x of A, x belongs to O.

(14e) [Common belief about the existence of a social organization] The existence of social organization O is a common belief of group G, where $O=G+A$ or $O=G$.

(14f) [Structure of social organization and its maintenance] O is structured in the way in which the possibility of O's future existence increases. In other words, if necessary, O's structure is adjusted so that the possibility of O's future existence increases.

As we already mentioned in this section, the structure of a social organization is determined by the roles and functions of its sub-organizations and their members. The characterization (14f) suggests that the internal structure of a social organization is determined and changed so as to realize its survival.[23]

12.11 Meta-games and Meta-norms

We can analyze a collective action of a social organization as a move in a game; for example, as a move in a survival game in which several organizations aim to maintain their existence, and perform collective actions to achieve this aim (Nakayama 2011).

Nakayama (2011) also suggests that a game can be composed of several sub-games. A tournament game is an example of a meta-game, i.e., a game that involves iteration of sub-games. A win in a sub-game is interpreted as a successful move in the meta-game. A tournament game is also a survival game. To remain in a tournament game, a team has to keep winning in a series of sub-games.

A legal philosopher Herbert L. A. Hart (1907–1992) distinguished between primary rules (rules of conduct) and secondary rules (rules addressed to officials and which set out to affect the operation of primary rules) (Hart 1961). According to Hart, a legal system is the union of primary and secondary rules. There are three types of secondary rules:

(15a) [Rules of recognition] They clarify what belongs to the primary rules.

[23] There are discussions on norms, roles, and plays in sociology. See Mead (1934), Parsons (1937), Merton (1949), Goffman (1959), and Habermas (1981).

(15b) [Rules of change] They are rules for changing the primary rules.
(15c) [Rules of adjudication] They are rules that give certain people the power to judge whether the primary rules are violated.

Hart's distinction of rules roughly corresponds to the distinction between norms and meta-norms in our terms. Meta-norms are norms at the meta-level and define or change social normative systems at a lower level. They should be socially approved. Article 1, Section 1 of *CUS*, as we analyzed it in Sect. 12.5, expresses a meta-norm that defines the structure and function of the Congress of the United States.

All Japanese laws are considered to be approved by the Japanese people, when they are approved in the Japanese parliament. The *Japanese Constitution* (*JS*) is a normative system that defines when a law is considered to be approved in the Japanese parliament. Thus, *JS* is a meta-normative system. There are a lot of meta-games and meta-norms.

12.12 Concluding Remarks

We have analyzed normative systems in this chapter. Laws and regulations are typical examples of normative systems. In addition, we have seen that a game can be interpreted as a dynamic system of players whose actions are restricted by a normative system. As we suggested, many social activities can be interpreted as games. In playing a game, players choose and perform their actions within the normative constrains given by the rules of the game and the roles they play in the game. An actual person lives in various normative systems and various games. So she always has to make a decision which game and what kind of role she will play. Her choices and performances give a social organization reality. In other words, when nobody follows social norms approved in a social organization, the organization is ignored and its reality is lost.

Appendix

In this appendix, we describe the definitions and theorems mentioned in the main text in more exact manners.

The definitions (8a) to (8f) of LNS can be expressed more rigorously as follows:

(A1a) When each of T, OB, and PER is a set of sentences in *First-Order Logic*, $NS = \langle T, \langle OB, PER \rangle \rangle$ is a *normative system*.
(A1b) A normative system NS is consistent $\Leftrightarrow$ $union(NS)$ is consistent, where $union(NS) = T \cup OB \cup PER$.
(A1c) $\mathbf{B}_{NS}\, q \Leftrightarrow T \vdash q$.
(A1d) $\mathbf{O}_{NS}\, q \Leftrightarrow (consistent(union(NS))\ \&\ T \cup OB \vdash q\ \&\ not\ \mathbf{B}_{NS}\, q)$.

(A1e) $\mathbf{F}_{NS}\, q \Leftrightarrow \mathbf{O}_{NS}\, \neg q$.
(A1f) $\mathbf{P}_{NS}\, q \Leftrightarrow (consistent(union(NS) \cup \{q\})\ \&\ not\ \mathbf{B}_{NS}\, q)$.

Based on these definitions, we can easily prove the main theorems of LNS, where *NS* ($NS = \langle T, \langle OB, PER\rangle\rangle$) indicates a normative system.

(A2a) $(\mathbf{B}_{NS}\,(p \to q)\ \&\ \mathbf{B}_{NS}\, p) \Rightarrow \mathbf{B}_{NS}\, q$.
(A2b1) $(\mathbf{O}_{NS}\,(p \to q)\ \&\ \mathbf{O}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2b2) $\mathbf{F}_{NS}\, p \Leftrightarrow \mathbf{O}_{NS}\, \neg p$.
(A2b3) $\mathbf{O}_{NS}\, p \Rightarrow \mathbf{P}_{NS}\, p$.
(A2b4) $\mathbf{F}_{NS}\, p \Rightarrow not\ \mathbf{P}_{NS}\, p$.
(A2c1) $\mathbf{P}_{NS}\, p \Rightarrow not\ \mathbf{B}_{NS}\, p$.
(A2c2) $\mathbf{B}_{NS}\, p \Rightarrow (not\ \mathbf{O}_{NS}\, p\ \&\ not\ \mathbf{F}_{NS}\, p\ \&\ not\ \mathbf{P}_{NS}\, \mathrm{p})$.
(A2d1) $(\mathbf{O}_{NS}\,(p \to q)\ \&\ \mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2d2) $(\mathbf{O}_{NS}\,(p \wedge q)\ \&\ \mathrm{not}\ \mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, p$.
(A2d3) $(\mathbf{O}_{NS}\,(p \wedge q)\ \&\ \mathbf{B}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2d4) $(\mathbf{O}_{NS}\,(p \vee q)\ \&\ \mathbf{B}_{NS}\, \neg p) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2d5) $(\mathbf{O}_{NS}\,(p \vee q)\ \&\ \mathbf{F}_{NS}\, p) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2d6) $(\mathbf{O}_{NS}\, p\ \&\ not\ \mathbf{B}_{NS}\, q) \Rightarrow \mathbf{O}_{NS}\,(p \vee q)$.
(A2d7) $(\mathbf{B}_{NS}\,(p \to q)\ \&\ \mathbf{O}_{NS}\, p\ \&\ \mathbf{P}_{NS}\, q) \Rightarrow \mathbf{O}_{NS}\, q$.
(A2e1) $(\mathbf{O}_{NS}\, \forall x_{1\ldots}\, \forall x_{\mathrm{n}}\, (P(x_1, \ldots, x_{\mathrm{n}}) \to Q(x_1, \ldots, x_{\mathrm{n}}))\ \&\ \mathbf{B}_{NS}\, P(a_1, \ldots, a_{\mathrm{n}})\ \&\ not\ \mathbf{B}_{NS}\, Q(a_1, \ldots, a_{\mathrm{n}})) \Rightarrow \mathbf{O}_{NS}\, Q(a_1, \ldots, a_{\mathrm{n}})$.
(A2e2) $(\mathbf{F}_{NS}\, \exists x_{1\ldots}\, \exists x_{\mathrm{n}}\, (P(x_1, \ldots, x_{\mathrm{n}}) \wedge Q(x_1, \ldots, x_{\mathrm{n}}))\ \&\ \mathbf{B}_{NS}\, P(a_1, \ldots, a_{\mathrm{n}})\ \&\ not\ \mathbf{B}_{NS}\, \neg Q(a_1, \ldots, a_{\mathrm{n}})) \Rightarrow \mathbf{F}_{NS}\, Q(a_1, \ldots, a_{\mathrm{n}})$.

Two theorems express the relationship between the normative states of a group and those of its members:

[T1] If all members of *G* share the normative system *ns*(*G*), then the following sentences hold:

(A3a) For all $a \in G$, $(\mathbf{B}_{ns(G)}\, p \Rightarrow \mathbf{B}_{ns(a)}\, p)$.
(A3b) For all $a \in G$, $((\mathbf{O}_{ns(G)}\, p\ \&\ consistent(ns(a))\ \&\ not\ \mathbf{B}_{ns(a)}\, p) \Rightarrow \mathbf{O}_{ns(a)}\, p)$.
(A3c) For all $a \in G$, $((\mathbf{F}_{ns(G)}\, p\ \&\ consistent(ns(a))\ \&\ not\ \mathbf{B}_{ns(a)}\, p) \Rightarrow \mathbf{F}_{ns(a)}\, p)$.
(A3d) For all $a \in G$, $((\mathbf{P}_{ns(G)}\, p\ \&\ consistent(ns(a))\ \&\ not\ \mathbf{B}_{ns(a)}\, p) \Rightarrow \mathbf{P}_{ns(a)}\, p)$.

[T2] If all members of *G* share the normative state $ns^2(G, t)$ at *t*, then the following sentences hold:

(A4a) For all time *t* and for all $a \in G$, $(\mathbf{B}_{ns2(G,t)}\, p \Rightarrow \mathbf{B}_{ns2(a,t)}\, p)$.
(A4b) For all time *t* and for all $a \in G$, $((\mathbf{O}_{ns2(G,t)}\, p\ \&\ consistent(ns^2(a, t))\ \&\ not\ \mathbf{B}_{ns2(a,\ t)}\, p) \Rightarrow \mathbf{O}_{ns2(a,\ t)}\, p)$.
(A4b) For all time *t* and for all $a \in G$, $((\mathbf{F}_{ns2(G,t)}\, p\ \&\ consistent(ns^2(a, t))\ \&\ not\ \mathbf{B}_{ns2(a,\ t)}\, p) \Rightarrow \mathbf{F}_{ns2(a,\ t)}\, p)$.
(A4b) For all time *t* and for all $a \in G$, $((\mathbf{P}_{ns2(G,t)}\, p\ \&\ consistent(ns^2(a, t))\ \&\ not\ \mathbf{B}_{ns2(a,\ t)}\, p) \Rightarrow \mathbf{P}_{ns2(a,\ t)}\, p)$.

T1 and T2 indicate the problem of consistency for a personal normative system. This problem is quite interesting in view of our real life experience. Suppose that you belong to groups G_1 and G_2 whose normative systems are mutually inconsistent, and that all members of G_1 and all members of G_2 share their normative systems $ns(G_1)$ and $ns(G_2)$. Then, your normative system becomes inconsistent, and, according to (A1d), (A1e), and (A1f), all of your normative requirements disappear. This is a case of social dilemma that many of us face in the real life.[24]

Exercises

Discuss the following problems:

1. What is a *normative system*?
2. What is *Logic for Normative Systems*?
3. What is *Dynamic Normative Logic*?
4. How can normative powers be explained?
5. How can roles be explained?
6. How are individuals and social organizations related?

References

Åqvist, L.: Deontic logic. In: Gabbay, D., Guenthner, F. (eds.) Handbook of Philosophical Logic, vol. 8, pp. 147–264. Kluwer Academic Pub, Dordrecht (2002)

Birch, A.: Developmental Psychology: From Infancy to Adulthood, 2nd edn. Macmillan Press, London (1997)

Glenberg, A., Robertson, D.: Symbol grounding and meaning: a comparison of high-dimensional and embodied theories of meaning. J. Mem. Lang. **43**, 379–401 (2000)

Goffman, E.: The Presentation of Self in Everyday Life. University of Edinburgh Social Sciences Research Centre, Edinburgh (1959)

Habermas, J.: Theorie des kommunikativen Handelns, Bde, 1-2. Suhrkamp, Frankfurt am Main (1981)

Hart, H.L.A.: The Concept of Law. Oxford University Press, Oxford (1961)

Mead, G.H.: Mind, Self, and Society. Edited by C. W. Morris. University of Chicago Press, Chicago (1934)

Merton, R.K.: Social Theory and Social Structure: Toward the Codification of Theory and Research. Free Press, New York (1949)

Nakayama, Y.: The Construction of Contemporary Nominalism: Its Application to Philosophy of Nominalism. Syunjyu-sha, Tokyo (2009), in Japanese

Nakayama, Y.: Logical framework for normative systems. In: SOCREAL 2010: Proceedings of the 2nd International Workshop on Philosophy and Ethics of Social Reality, 27–28 March 2010, Hokkaido University, Sapporo, Japan, pp. 19–24 (2010)

Nakayama, Y.: Norms and Games: An Introduction to the Philosophy of Society. Keiso shobo, Tokyo (2011), in Japanese

[24] This kind of social dilemmas is discussed in Nakayama (2011) chap. 7 sect. 3.

Nakayama, Y.: Dynamic normative logic and information update. In: Yamada, T. (ed.) SOCREAL 2013: 3rd International Workshop on Philosophy and Ethics of Social Reality, Abstracts, Hokkaido University, Sapporo, Japan, pp. 23–27 (2013)

Nakayama, Y.: Speech Acts, Normative Systems, and Local Information Update. In: Y. I. Nakano, et al. (eds.) New Frontiers in Artificial Intelligence, LNAI 8417, Springer, pp. 98–114 (2014)

Parsons, T.: The Structure of Social Actions. Free Press, New York (1937/1967)

Piaget, J.: Play, Dreams and Imitation in Childhood. Routledge and Kegan Paul, London (1951)

Schank, R.C., Abelson, R.P.: Scripts, Plans, Goals and Understanding: An Inquiry into Human Knowledge Structures. Lawrence Erlbaum Associates, Hillsdale (1977)

Searle, J.R.: Expression and Meaning – Studies in the Theory of Speech Acts. Cambridge University Press, Cambridge (1979)

Searle, J.R.: The Construction of Social Reality. Free Press, New York (1995)

Searle, J.R.: Making the Social World: The Structure of Human Civilization. Oxford University Press, Oxford (2010)

Shapiro, L.: Embodied Cognition. Routledge, London (2011)

Sider, T.: Four-Dimensionalism: An Ontology of Persistence and Time. Oxford University Press, Oxford (2001)

van Benthem, J.: Logical Dynamics of Information and Interaction. Cambridge University Press, Cambridge (2011)

Wittgenstein, L.: Philosophical Investigations. Translated by G. E. M. Anscomb, 2nd edn. Basil Blackwell, Oxford (1958)

Yamada, T.: Logical dynamics of some speech acts that affect obligations and preferences. Synthese **165**(2), 295–315 (2008)

Zichermann, G., Cunningham, C.: Gamification by Design. O'Reilly Media, Cambridge, MA (2011)