Chapter 6
Hadron Level Calculation

The first many-body physics relevant to the evaluation of the electric dipole moment (EDM) is the effects on the hadron level. In this chapter we present the calculations of the contributions from the leading P, CP-odd hadronic mechanisms. The processes considered are the nucleon EDM, P, CP-odd nucleon-nucleon interaction, P, CP-odd electron-nucleon interaction due to the P, CP-odd quark level operators, the quark EDM, quark chromo-EDM, gluon chromo-EDM (i.e. Weinberg operator), θ-term, P, CP-odd 4-quark interaction, and the P, CP-odd electron-quark interaction. The schematic dependences of the hadronic scale operators on the quark level operators are shown in Fig. 6.1.

The P, CP-odd lagrangian on the hadron level

$$\mathscr{L}_{\text{hadron}} = \mathscr{L}_{\text{edm}} + \mathscr{L}_{\pi NN} + \mathscr{L}_{eN}, \tag{6.1}$$

with

- the nucleon EDM

$$\mathscr{L}_{\text{edm}} = -\frac{i}{2} \sum_{N=p,n} d_N \bar{N} \sigma_{\mu\nu} \gamma_5 N F^{\mu\nu}, \tag{6.2}$$

- the P, CP-odd pion-nucleon interaction

$$\mathscr{L}_{\pi NN} = \sum_{N=p,n} \sum_{a=1}^{3} \Big[\bar{g}^{(0)}_{\pi NN} \bar{N} \tau^a N \pi^a + \bar{g}^{(1)}_{\pi NN} \bar{N} N \pi^0 + \bar{g}^{(2)}_{\pi NN} (\bar{N} \tau^a N \pi^a - 3\bar{N} \tau^3 N \pi^0) \Big], \tag{6.3}$$

where a denotes the isospin index, and

- the P, CP-odd electron-nucleon interaction

$$\mathscr{L}_{eN} = -\frac{G_F}{\sqrt{2}} \sum_{N=p,n} \Big[C_N^{\text{SP}} \bar{N} N \, \bar{e} i \gamma_5 e + C_N^{\text{PS}} \bar{N} i \gamma_5 N \, \bar{e} e + \frac{1}{2} C_N^{\text{T}} \epsilon^{\mu\nu\rho\sigma} \bar{N} \sigma_{\mu\nu} N \, \bar{e} \sigma_{\rho\sigma} e \Big]. \tag{6.4}$$

N. Yamanaka, *Analysis of the Electric Dipole Moment in the R-parity Violating Supersymmetric Standard Model*, Springer Theses,
DOI: 10.1007/978-4-431-54544-6_6, © Springer Japan 2014

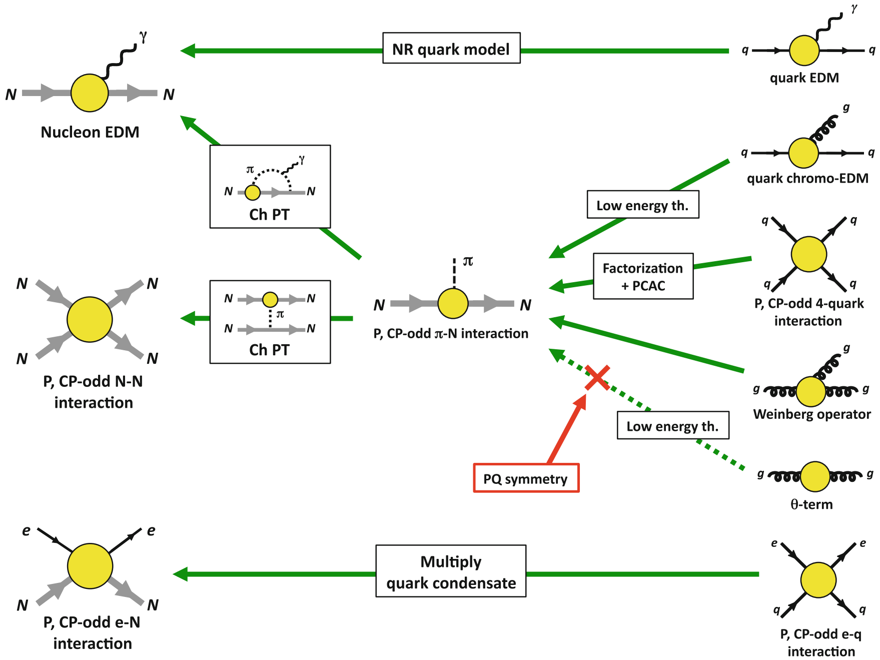

Fig. 6.1 Detailed flow diagram of the dependence of the hadron level P, CP-odd processes on leading quark level P, CP-odd operators. The hadron level processes are shown in the *left side* and the quark level operators in the *right side*. The Weinberg operator contribution is not discussed in our analysis

The P, CP-odd nucleon-nucleon interaction receives the leading contribution from one-pion exchange as shown in the flow chart of Fig. 6.1. It can be obtained by combining the above P, CP-odd pion-nucleon interaction $\mathscr{L}_{\pi NN}$ with the standard P, CP-even pion-nucleon interaction ($\mathscr{L} = g_{\pi NN} \bar{N} i \gamma_5 \tau N \cdot \pi$).

P, CP-odd quark level operators include the θ-term, quark EDM, quark chromo-EDM, four-fermion interaction and the Weinberg operator (gluon chromo-EDM). These operators with the lowest mass dimension, give the leading contribution to the hadron level P and CP violations. The leading quark level P, CP-odd interactions are the followings:

- θ-term:

$$\mathscr{L}_\theta = \frac{g_s^2}{64\pi^2} \bar{\theta}\, \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}, \tag{6.5}$$

- quark EDM:

$$\mathscr{L}_{\text{qEDM}} = -\frac{i}{2} \sum_{q=u,d,s} d_q\, \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}, \tag{6.6}$$

- quark chromo-EDM:

$$\mathscr{L}_{\text{cEDM}} = -\frac{i}{2} \sum_{q=u,d,s} d_q^c \, g_s \bar{q} \sigma_{\mu\nu} t_a \gamma_5 q G_a^{\mu\nu}, \tag{6.7}$$

- P, CP-odd 4-quark interactions:

$$\mathscr{L}_{4\text{q}} = \frac{G_F}{\sqrt{2}} \sum_{q,q\prime=u,d,s,c,b} \left[C_{qq'} (\bar{q}q)(\bar{q}\prime i\gamma_5 q\prime) + \frac{1}{2} C'_{qq'} \epsilon^{\mu\nu\rho\sigma} (\bar{q}\sigma_{\mu\nu}q)(\bar{q}\prime\sigma_{\rho\sigma}q\prime) \right], \tag{6.8}$$

- P, CP-odd electron-quark interactions:

$$\mathscr{L}_{eq} = -\frac{G_F}{\sqrt{2}} \sum_{q=u,d,s,c,b} \left[C_{eq}^{\text{SP}} \bar{q}q\bar{e}i\gamma_5 e + C_{eq}^{\text{PS}} \bar{q}i\gamma_5 q\bar{e}e + \frac{1}{2} C_{eq}^{\text{T}} \varepsilon^{\mu\nu\rho\sigma} \bar{q}\sigma_{\mu\nu}q\bar{e}\sigma_{\rho\sigma}e \right], \tag{6.9}$$

- Weinberg operator [1]:

$$\mathscr{L}_w = \frac{1}{6} w \frac{G_F}{\sqrt{2}} f^{abc} \varepsilon^{\alpha\beta\gamma\delta} G^a_{\mu\alpha} G^b_{\beta\gamma} G^{\mu,c}_{\delta}, \tag{6.10}$$

where f^{abc} is the $SU(3)$ structure constant of the Lie algebra. (The Weinberg operator will not be discussed in this thesis, since it is not relevant in our analysis.)

To obtain the P, CP-odd hadron level interactions, we need the results from model calculations which need many inputs, such as hadron matrix elements, quark masses, etc. Strong interaction processes should *in principle* be calculated within the framework of the Quantum chromodymanics (QCD). However, we still do not have fully analytic non-perturbative methods to calculate hadron level processes starting from quark level interactions. One way of calculating them in QCD is the Lattice QCD simulation, which consists of numerical analysis using Montecarlo techniques in discretized space-time, but not many data for hadron matrix elements needed in EDM analysis are currently available, and their calculations remain one of the important homework. On the side of model calculations, many results are available. Hadron matrix elements needed in EDM analysis were essentially evaluated using non-relativistic quark models, low energy theorems, chiral techniques and QCD sum rules [2]. In this thesis, we mainly focus on the chiral approach using low energy theorems for the evaluation of hadron matrix elements. The first reason of this choice is that many calculations of hadron matrix elements are available. Since different models have different source of errors, it is very important to use the same model for every hadron matrix elements needed in the analysis. The second reason is the systematic estimation of the precision of hadron matrix elements. We will also use the results from *ab initio* lattice QCD calculations if available. In the following, we first examine the quark contents of nucleon which are needed in many subsequent analyses. We will then briefly review the method using low energy theorems which provides us with the relation between quark level P, CP-odd operators and the P, CP-odd meson-baryon couplings. After obtaining meson-baryon couplings, we will discuss the calculation of nucleon EDMs using the chiral method. The Peccei-Quinn mechanism is then reviewed. We finally summarize the results.

6.1 Quark Contents of the Nucleon

In the evaluation of the hadron level effective P, CP-odd interactions, we often need the quark contents of the nucleon with suitable Lorentz structure. (for example, the P, CP-odd electron-nucleon (e-N) interactions $C_N^{\rm SP} \bar N N\, \bar e i\gamma_5 e = \sum_q C_{eq}^{\rm SP} \langle N|\bar q q|N\rangle\, \bar q q\, \bar e i\gamma_5 e$.) Explicitly, we must calculate $\langle N|\bar q q|N\rangle$, $\langle N|\bar q i\gamma_5 q|N\rangle$ and $\langle N|\bar q \sigma^{\mu\nu} q|N\rangle$ ($N = p, n\,; q = u, d, s, c, b$). We begin first with the evaluation of the scalar content of nucleon $\langle N|\bar q q|N\rangle$. The physical meaning of these matrix elements is the fraction of the quark mass over the nucleon mass. In the classic phenomenological approach using SU(3) symmetry and breaking with baryon mass splitting [3, 4], and the u, d quark content of the nucleon (the so-called σ term, $\sigma \equiv \frac{m_u+m_d}{2}\langle p|\bar u u + \bar d d|p\rangle = 55 \sim 75\,{\rm MeV}$ [5]), we obtain the following values:

$$\langle p|\bar u u|p\rangle = 7.7, \tag{6.11}$$

$$\langle p|\bar d d|p\rangle = 6.9, \tag{6.12}$$

$$\langle p|\bar s s|p\rangle = 4.2, \tag{6.13}$$

where we have equated the proton-neutron mass splitting $\langle p|\bar u u - \bar d d|p\rangle = \frac{m_p^0 - m_n^0}{m_u - m_d}$, the Ξ-Λ mass splitting $\langle p|\bar u u + \bar d d - 2\bar s s|p\rangle = 3\frac{m_\Xi - m_\Lambda}{m_s}$ and the σ-term above. The recent values of the quark and baryon masses have been used, with $m_u \approx 2.5\,{\rm MeV}$, $m_d \approx 4.9\,{\rm MeV}$, $m_s \approx 100\,{\rm MeV}$, $m_p^0 - m_n^0 = -2.05$ MeV (nucleon masses without electromagnetic contribution), $m_\Xi = 1321$ MeV and $m_\Lambda = 1116$ MeV [6–8]. The quark contents of the nucleon were also evaluated in lattice QCD [9–13]. The u and d quark contents are in agreement with lattice QCD simulations, and the result shows that the chiral expansion in u and d quark masses works well. The strange quark content of the nucleon merits a short discussion. The earlier quenched lattice QCD simulations for $\langle N|\bar s s|N\rangle$ showed agreement with the classic value (Eq. 6.13). However, recent lattice analyses with dynamical quarks indicate that the strange content of the nucleon is suppressed about one order compared with the classic values. This can be understood by the suppression of disconnected quark loop (sea quark) contribution. In this analysis, we adopt the results of the lattice QCD analyses with dynamical quarks [10–13]. Calculations present a small result with

$$\langle p|\bar s s|p\rangle = \langle N|\bar s s|N\rangle \approx 0.1\ , \tag{6.14}$$

derived with $m_s = 100\,\text{MeV}$. This disagreement shows that the chiral expansion in terms of strange quark mass is rather difficult. The result of Eq. (6.13) was from using the first order expansion of the strange quark mass. However, higher order corrections can be important since m_s is far from the chiral limit. For these reasons, we will use the result of lattice QCD calculations ($\langle p|\bar{s}s|p\rangle \approx 0.1$) [10–13] in this thesis.

The charm and bottom quark contributions can be calculated by using the heavy quark expansion [3, 4]. The leading order contribution of the heavy quark condensate is given by

$$\langle N|\bar{Q}Q|N\rangle \approx -\left\langle N\left|\frac{\alpha_s}{12\pi m_Q} G_{\mu\nu,a}G_a^{\mu\nu}\right|N\right\rangle + O\left(\frac{1}{m_Q^2}\right), \tag{6.15}$$

where Q stands for heavy quark. By neglecting higher order terms of $O(1/m_Q^2)$, we obtain

$$\langle N|\bar{c}c|N\rangle \approx 3\times 10^{-2}, \tag{6.16}$$

$$\langle N|\bar{b}b|N\rangle \approx 1\times 10^{-2}, \tag{6.17}$$

where the quark masses $m_c = 1.3\,\text{GeV}$ and $m_b = 4.2\,\text{GeV}$ were used [6–8].

We now calculate the matrix element $\langle N|\bar{q}i\gamma_5 q|N\rangle$ by using PCAC and axial anomaly [14] to get

$$\langle p|\bar{q}i\gamma_5 q|p\rangle = \frac{m_N}{m_q}\left(\Delta q' + \frac{\alpha_s}{2\pi}\Delta g\right), \tag{6.18}$$

where $\Delta q'$ is the fraction of the axial vector current of the quark q in the proton, Δg is defined by $\langle p|\mathrm{Tr}\, G_{\mu\nu}\tilde{G}^{\mu\nu}|p\rangle = -2m_N\Delta g\bar{u}_p i\gamma_5 u_p$ [14], where $G_{\mu\nu}$ is the gluon field strength and $\tilde{G}_{\mu\nu}$ its dual. We use $\Delta u' = 0.82$, $\Delta d' = -0.44$, $\Delta s' = -0.11$ [15, 16], $(\alpha_s/2\pi)\Delta g = -0.16$ [14] and the recent values of quark masses cited previously. This gives

$$\langle p|\bar{u}i\gamma_5 u|p\rangle = 248, \tag{6.19}$$

$$\langle p|\bar{d}i\gamma_5 d|p\rangle = -115, \tag{6.20}$$

$$\langle p|\bar{s}i\gamma_5 s|p\rangle = -2.5. \tag{6.21}$$

The pseudoscalar condensate of the bottom quark can also be calculated in heavy quark expansion [3, 4]. The leading order contribution is given by

$$\langle N|\bar{Q}i\gamma_5 Q|N\rangle \approx -\left\langle N\left|\frac{\alpha_s}{8\pi m_Q} G_{\mu\nu,a}\tilde{G}_a^{\mu\nu}\right|N\right\rangle + O\left(\frac{1}{m_Q^2}\right). \tag{6.22}$$

This gives

$$\langle N|\,\bar{c}i\gamma_5 c\,|N\rangle \approx -9\times 10^{-2},$$
$$\langle N|\,\bar{b}i\gamma_5 b\,|N\rangle \approx -3\times 10^{-2}. \tag{6.23}$$

Again, higher order corrections were neglected.

The calculation of the tensor matrix element was done in quenched lattice QCD with the Wilson quark [17]. The result is

$$\langle p|\bar{u}\sigma^{\mu\nu}u|p\rangle = (0.839\pm 0.060)\,\bar{p}\sigma^{\mu\nu}p,$$
$$\langle p|\bar{d}\sigma^{\mu\nu}d|p\rangle = -(0.231\pm 0.055)\,\bar{p}\sigma^{\mu\nu}p,$$
$$\langle p|\bar{s}\sigma^{\mu\nu}s|p\rangle = -(0.046\pm 0.034)\,\bar{p}\sigma^{\mu\nu}p. \tag{6.24}$$

Here the strange quark tensor charge receives purely disconnected contributions. All these results suffer from a large finite volume effect, and it is difficult to determine the strange quark tensor charge.

6.2 The PCAC Techniques and the P, CP-Odd Meson-Baryon Interactions

We now explain several calculational techniques. The first one is the *PCAC technique*. The quark sector of QCD at a few hundred of MeV is known to have approximately chiral flavor $SU(3)_L\times SU(3)_R\times U(1)_V$ symmetry. Quarks have small masses, which explicitly break the axial $SU(3)$ symmetry, but this symmetry breaking is quite small compared to the scale of confinement, so an approximate relation for axial currents is possible. This is the *partially conserved axial vector current* (PCAC) relation. One of the important consequence of PCAC is the soft-pion theorem:

$$\langle \pi^c B_a|\,O_i\,|\,B_b\rangle \approx \frac{i}{f_\pi}\langle B_a|\,[O_i,(J_5^c)_0]\,|\,B_b\rangle, \tag{6.25}$$

where B is an $SU(3)$ octet baryon state and $(J_5^c)_0$ the axial charge. The baryon field and the Nambu-Goldstone boson (meson) field π^c are defined as

$$\pi^c = M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta^0}{\sqrt{6}} \end{pmatrix}, \tag{6.26}$$

and

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}. \tag{6.27}$$

The right-hand side of the PCAC relation receives corrections from meson rescattering. Since we are interested in very low energy meson exchange, this contribution can be ignored. This relation is part of the low energy theorem, and is often said to be model independent. It can be used to reduce the P and CP violating hadron matrix elements with external meson to the P and CP-even hadron matrix elements without external meson, and so it will be used here to calculate the P, CP-odd meson-baryon couplings (for example, $\bar{g}^{(0)}_{\pi NN}$, $\bar{g}^{(1)}_{\pi NN}$, $\bar{g}^{(2)}_{\pi NN}$).

An important example is the calculation of the θ-term contribution to the neutron EDM [18]. The θ-term (Eq. 6.5) is related via the chiral anomaly to the CP-odd quark mass $\mathscr{L}_{\mathrm{CPVM}} = -\bar{\theta} m_* (\bar{u} i\gamma_5 u + \bar{d} i\gamma_5 d + \bar{s} i\gamma_5 s)$ (with $m_* \equiv \frac{m_u m_d}{m_u + m_d}$), and the parameter $\bar{\theta}$ can be transferred to each other. By applying the PCAC relation to $\langle N\pi^a | \mathscr{L}_{\mathrm{CPVM}} \,|\, N\rangle$, we obtain

$$\bar{g}^{(0)}_{\pi NN}(\bar{\theta}) = \frac{\bar{\theta} m_*}{f_\pi} \langle N | \, \bar{q}\tau^3 q \,|\, N\rangle \approx 0.015\bar{\theta} \,, \tag{6.28}$$

where $q = u, d$. The hadron matrix elements $\langle p | \, \bar{u}u \,|\, p\rangle$ and $\langle p | \, \bar{d}d \,|\, p\rangle$ were derived from the proton and neutron mass splitting (see previous chapter). The strange quark contribution was neglected since it is of the order of m_*/m_s.

Another important example of calculation using the PCAC relation is the calculation of the chromo-EDM contribution to the meson-baryon couplings. The discussion below follows the paper of Hisano and Shimizu [19, 20]. After using the PCAC relation, we obtain the following contribution to the P, CP-odd pion-nucleon couplings:

$$\langle B_a \pi^c | \mathscr{L}_{\mathrm{cEDM}} | B_b \rangle = \sum_{q,q'} \frac{1}{f_\pi} d^c_q \langle B_a | \, \bar{q} g_s G^{\mu\nu}_a \sigma_{\mu\nu} t_a T_c q \,|\, B_b\rangle \,, \tag{6.29}$$

where T_c is the generator of the flavor $SU(3)$ symmetry, and t_a the one of the color $SU(3)$. The values of these condensates are not calculable with chiral techniques. For the evaluation of the mixed condensate in nucleon $\langle B_a | \, \bar{q}' g_s G^{\mu\nu}_a \sigma_{\mu\nu} t_a T_c q \,|\, B_b\rangle$, we use the relation based on QCD sum rules [21–23] with the saturation of the lightest 0^{++} state and the low energy theorem [24] adopted in Refs. [3, 4, 25–27] to write

$$\langle B_a | \, \bar{q} g_s G^{\mu\nu}_a \sigma_{\mu\nu} t_a T_c q \,|\, B_b\rangle \approx \frac{5}{3} m_0^2 \langle B_a | \, \bar{q} T_c q \,|\, B_b\rangle, \tag{6.30}$$

where $m_0^2 \equiv \langle 0 | \, \bar{q} g_s G^{\mu\nu}_a \sigma_{\mu\nu} t_a q \,|\, 0\rangle / \langle 0 | \, \bar{q} q \,|\, 0\rangle \approx 0.8\,(\mathrm{GeV})^2$ [21–23, 28, 29]. For the derivation, see Appendix C.

The result of the θ-term and chromo-EDM contributions can then be regrouped to get the following expression:

$$\langle B_a \pi^c | \mathcal{L}_{\theta+\mathrm{cEDM}} | B_b \rangle \approx \sum_q \frac{1}{f_\pi} \left(2\alpha_q m_q + \frac{5}{3} d_q^c m_0^2 \right) \langle B_a |\, \bar{q} T_c q \mid B_b \rangle$$
$$= \sum_q \frac{1}{f_\pi} \langle B_a |\, \bar{q} \{ T_c, A \} q \mid B_b \rangle, \tag{6.31}$$

where $\sum_q \alpha_q = \bar{\theta}$. $A = \mathrm{diag}(A_u, A_d, A_s)$ is the flavor $SU(3)$ breaking effect, with components

$$A_q = \alpha_q m_q + \frac{5}{6} m_0^2 d_q^c \,. \tag{6.32}$$

If we assume that the Peccei-Quinn symmetry and the axion mechanism hold, the θ-term contribution will become unphysical. We can then expect the term with α_q to vanish. However, it is known that the chromo-EDM can induce the θ-term even in the presence of the axion mechanism [30, 31]. The induced chromo-EDM contribution is

$$\alpha_q = -\frac{m_0^2}{2} \frac{d_q^c}{m_q}, \tag{6.33}$$

so that

$$A_q = -\frac{m_0^2}{2} d_q^c + \frac{5}{6} m_0^2 d_q^c = \frac{m_0^2}{3} d_q^c \approx 0.27 d_q^c \,\mathrm{GeV}. \tag{6.34}$$

We will see the derivation of the contribution of the chromo-EDM to the induced θ-term under the axion mechanism later in Sect. 6.5.

By considering the flavor $SU(3)$ breaking matrix element of Eq. (6.31), we obtain the following P, CP-odd meson-baryon interaction:

$$\mathcal{L}_{CPV} = \frac{1}{\sqrt{2} f_\pi} \big[\langle p | \bar{s}s - \bar{d}d | p \rangle \mathrm{Tr}(\bar{B} B \{M, A\})$$
$$+ \langle p | \bar{u}u - \bar{d}d | p \rangle \mathrm{Tr}(\bar{B} \{M, A\} B)$$
$$+ 2 \langle p | \bar{d}d | p \rangle \mathrm{Tr}(AM) \mathrm{Tr}(\bar{B} B) \big]. \tag{6.35}$$

The relevant P, CP-odd meson-baryon lagrangian is

$$\mathcal{L}_{CPV} = \frac{\langle p | \bar{s}s - \bar{d}d | p \rangle}{\sqrt{2} f_\pi} \left[(A_d + A_s) \bar{\Sigma}^+ p \bar{K}^0 + (A_u + A_s) \left(\frac{\bar{\Sigma}^0}{\sqrt{2}} + \frac{\bar{\Lambda}}{\sqrt{6}} \right) p K^- \right.$$
$$\left. + (A_u + A_s) \bar{\Sigma}^- n K^- + (\mathrm{h.c.}) - \frac{4}{\sqrt{6}} A_s \bar{p} p \eta^0 \right]$$

$$
\begin{aligned}
&+\frac{\langle p|\bar{u}u-\bar{d}d|p\rangle}{\sqrt{2}f_\pi}\Bigg[(A_u+A_d)\bar{p}n\pi^+ - \frac{2}{\sqrt{6}}(A_u+A_s)\bar{p}\Lambda K^+ + (\text{h.c.}) \\
&\qquad\qquad + 2A_u\bar{p}p\left(\frac{\pi^0}{\sqrt{2}}+\frac{\eta^0}{\sqrt{6}}\right) - \frac{2}{\sqrt{2}}A_d\bar{n}n\pi^0\Bigg] \\
&+\frac{2\langle p|\bar{d}d|p\rangle}{\sqrt{2}f_\pi}\left(\frac{A_u-A_d}{\sqrt{2}}\pi^0 + \frac{A_u+A_d-2A_s}{\sqrt{6}}\eta^0\right)(\bar{p}p+\bar{n}n),
\end{aligned}
\tag{6.36}
$$

where +(h.c.) means that we add the hermitian conjugates of the terms in its left side.

From this we can derive the P, CP-odd pion-nucleon interactions:

$$
\mathscr{L}_{\pi NN} = \frac{1}{2f_\pi}(A_u+A_d)\langle p|\bar{u}u-\bar{d}d|p\rangle\bar{N}\tau^a N\pi^a + \frac{1}{2f_\pi}(A_u-A_d)\langle p|\bar{u}u+\bar{d}d|p\rangle)\bar{N}N\pi^0.
\tag{6.37}
$$

The P, CP-odd pion-nucleon couplings can be explicitly written as

$$
\bar{g}^{(0)}_{\pi NN}(\bar{\theta}, d^c_q) = \frac{1}{2f_\pi}(A_u+A_d)\langle p|\bar{u}u-\bar{d}d|p\rangle,
\tag{6.38}
$$

$$
\bar{g}^{(1)}_{\pi NN}(\bar{\theta}, d^c_q) = \frac{1}{2f_\pi}(A_u-A_d)\langle p|\bar{u}u+\bar{d}d|p\rangle.
\tag{6.39}
$$

For the case of P, CP-odd 4-quark interaction, the vacuum factorization approximation can be used. Combined with the PCAC method, we get the following relation:

$$
\begin{aligned}
\bar{g}^{(1)}_{\pi NN}(C_{qd}) &= \sum_{q=u,d,s,c,b} \langle\pi^0 N|C_{qd}\frac{G_F}{\sqrt{2}}\,\bar{q}q\,\bar{d}i\gamma_5 d|N\rangle \\
&\approx \sum_{q=u,d,s,c,b} C_{qd}\frac{G_F}{\sqrt{2}}\langle\pi^0|\bar{d}i\gamma_5 d|0\rangle\langle N|\,\bar{q}q|N\rangle \\
&= -\frac{F_\pi m_\pi^2 G_F}{2\sqrt{2}m_d}\sum_{q=u,d,s,c,b} C_{qd}\langle N|\,\bar{q}q|N\rangle.
\end{aligned}
\tag{6.40}
$$

We should note that in this discussion, we have not considered the vacuum condensation of the neutral mesons, which, combined with the CP-even pion-nucleon interaction, can generate an additional contribution to the CP-odd pion-nucleon interaction. This effect may be important as it contributes to the same chiral order as the CP-odd pion-nucleon coupling calculated in this section (see Fig. 3 of Ref. [2] and the corresponding discussion).

6.3 Nucleon EDMs with Chiral Method

Let us move to the evaluation of the nucleon EDM. The one-loop term in the expansion of the nucleon EDM has "chiral logarithm" $\ln(m_\pi/\Lambda)$ (Λ is some hadron scale cutoff). This term appears to be independent of the chiral model chosen [32, 33]. The idea is to consider this term as the leading term in the chiral expansion. The expansion of the nucleon EDM in the chiral perturbation theory has also divergent terms at the one-loop level, which give the renormalization of the low energy constant of the tree level and have to be determined for accurate calculation. In the present case we consider the leading chiral logarithmic terms to be the dominant contribution, although the theoretical uncertainty is as large as order one. The improvement of the accuracy of the nucleon EDM calculation is an important subject which must be treated in the near future. We will return to this subject briefly in the final summary. With the above assumption, the chiral contribution to the nucleon EDM can be evaluated using the following chiral lagrangian.

$$\begin{aligned}\mathscr{L}_{mBB} \approx & - g_{\pi NN}\bar{p}i\gamma_5 p\pi^0 - \sqrt{2}g_{\pi NN}(\bar{p}i\gamma_5 n\pi^+ + \text{h.c.}) - g_{\eta NN}\bar{p}i\gamma_5 p\eta^0 \\ & + g_{K\Lambda N}(\bar{p}i\gamma_5\Lambda K^+ + \text{h.c.}) \\ & - g_{K\Sigma N}(\bar{\Sigma}^+ i\gamma_5 p\bar{K}^0 + \bar{\Sigma}^- i\gamma_5 nK^- + \frac{1}{\sqrt{2}}\bar{\Sigma}^0 i\gamma_5 pK^- + \text{h.c.})\,,\end{aligned} \tag{6.41}$$

where $g_{\pi NN} = \frac{m_N}{f_\pi}(D+F) \approx 12.6$, $g_{\eta NN} = \frac{m_N}{\sqrt{3}f_\pi}(3F-D) \approx 3.0$, $g_{K\Lambda N} = \frac{m_N+m_\Lambda}{2\sqrt{3}f_\pi}(D+3F) \approx 6.4$ and $g_{K\Sigma N} = \frac{m_N+m_\Sigma}{\sqrt{2}f_\pi}(D-F) \approx 6.0$, with $D = 0.81$ and $F = 0.44$. In deriving these values, we neglected the isospin splitting. The detail of the derivation is explained in Appendix D.

The P, CP-odd meson-baryon interactions contribute to the single nucleon EDM through one-loop diagrams as shown in Figs. 6.2 and 6.3.

The calculation of the one-loop contribution of the nucleon EDM to the leading chiral logarithm gives the following results:

$$d_n = \frac{e}{8\pi^2 f_\pi^2}(A_u x_u^{(n)} + A_d x_d^{(n)} + A_s x_s^{(n)}), \tag{6.42}$$

$$d_p = \frac{e}{8\pi^2 f_\pi^2}(A_u x_u^{(p)} + A_d x_d^{(p)} + A_s x_s^{(p)}), \tag{6.43}$$

with

$$\begin{aligned}x_u^{(n)} &= (D+F)\langle p|\bar{u}u - \bar{d}d|p\rangle \log\frac{m_N}{m_\pi} + (D-F)\langle p|\bar{d}d - \bar{s}s|p\rangle \log\frac{m_\Sigma}{m_K} \approx 4.1, \\ x_d^{(n)} &= (D+F)\langle p|\bar{u}u - \bar{d}d|p\rangle \log\frac{m_N}{m_\pi} \approx 1.9, \\ x_s^{(n)} &= (D-F)\langle p|\bar{d}d - \bar{s}s|p\rangle \log\frac{m_\Sigma}{m_K} \approx 2.2,\end{aligned}$$

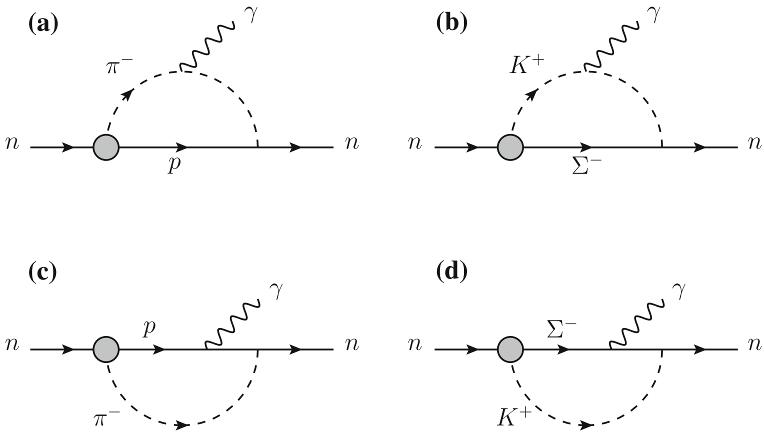

Fig. 6.2 Meson-loop contribution to the neutron EDM. The *grey blob* denotes the P, CP-odd meson-baryon coupling. Graphs (**b**) and (**d**) are strangeness contribution

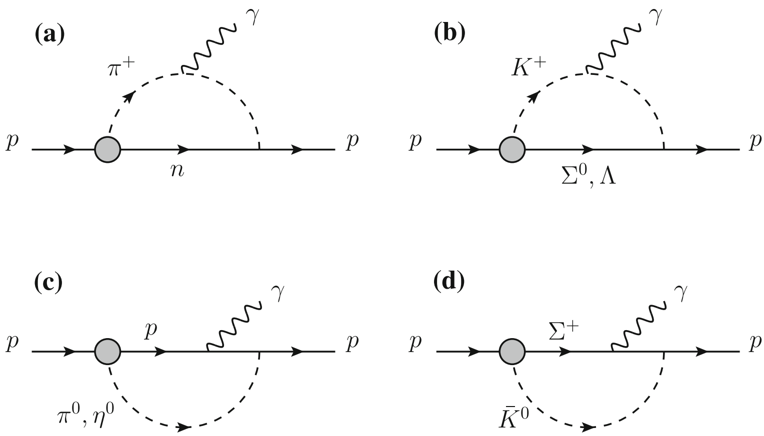

Fig. 6.3 Meson-loop contribution to the proton EDM. The *grey blob* denotes the P, CP-odd meson-baryon coupling. Graphs (**b**) and (**d**) are strangeness contribution

$$\begin{aligned}
x_u^{(p)} &= (D+F)\langle p|\bar{d}d - \bar{u}u|p\rangle \log\frac{m_N}{m_\pi} + \frac{D-F}{2}\langle p|\bar{d}d - \bar{s}s|p\rangle \log\frac{m_\Sigma}{m_K} \\
&\qquad + \frac{D+3F}{6}\langle p|\bar{d}d + \bar{s}s - 2\bar{u}u|p\rangle \log\frac{m_\Lambda}{m_K} \approx -3.2, \\
x_d^{(p)} &= (D+F)\langle p|\bar{d}d - \bar{u}u|p\rangle \log\frac{m_N}{m_\pi} \approx -1.9, \\
x_s^{(p)} &= \frac{D-F}{2}\langle p|\bar{d}d - \bar{s}s|p\rangle \log\frac{m_\Sigma}{m_K} + \frac{D+3F}{6}\langle p|\bar{d}d + \bar{s}s - 2\bar{u}u|p\rangle \log\frac{m_\Lambda}{m_K} \approx -1.3.
\end{aligned} \tag{6.44}$$

We also write down the case without strangeness contribution (K, Λ and Σ neglected) for comparison.

$$x_u'^{(n)} = x_d'^{(n)} = -x_u'^{(p)} = -x_d'^{(p)} = (D+F)\langle p|\bar{u}u - \bar{d}d|p\rangle \log \frac{m_N}{m_\pi} \approx 1.9. \quad (6.45)$$

In this case, we see that the proton and neutron EDMs have the same size and opposite sign. From Eqs. (6.38) and (6.42), we can derive the well-known formula of the neutron EDM generated by the isoscalar P, CP-odd pion-nucleon interaction in the leading chiral logarithm approximation:

$$d_n(\bar{g}^{(0)}_{\pi NN}) \approx \frac{e g_{\pi NN} \bar{g}^{(0)}_{\pi NN}}{4\pi^2 m_N} \ln \frac{m_N}{m_\pi}. \quad (6.46)$$

The detailed derivation of the above formula is given in Appendix D. By using Eq. (6.28), we obtain the dependence of the neutron EDM on the θ-term [18]:

$$d_n(\bar{\theta}) \approx \bar{\theta} \frac{e g_{\pi NN} m_*}{4\pi^2 m_N f_\pi} \langle p|\bar{u}u - \bar{d}d|p\rangle \ln \frac{m_N}{m_\pi} \approx 2 \times 10^{-16} \bar{\theta}\, e\,\text{cm}. \quad (6.47)$$

We see that the neutron EDM is very sensitive to the θ-term.

6.4 Quark EDM Contribution to the Nucleon EDM

The next topic is the derivation of the nucleon EDM from the quark EDM. The CP-odd electromagnetic form factor of the nucleon is given by

$$\left\langle N(p-q) \middle| j_\mu^{(\text{em})} \middle| N(p) \right\rangle_{\text{CPV}} = -\frac{g_2(q^2)}{2m_N} \bar{u}_N(p-q)\sigma_{\mu\nu}q^\nu \gamma_5 u_N(p), \quad (6.48)$$

where u_N is the nucleon spinor and q^ν is the momentum transfer. The EDM of the nucleon d_N is defined by the limit of zero momentum transfer of the CP-odd nucleon form factor:

$$\frac{d_N}{e} \equiv \lim_{q^2 \to 0} \frac{g_2(q^2)}{2m_N}. \quad (6.49)$$

In the presence of the quark EDM operator, the leading contribution to the CP-odd nucleon matrix element is

$$\left\langle N(p-q) \middle| j_\mu^{(\text{em})} \middle| N(p) \right\rangle_{\text{CPV}} = \sum_{q=u,d,s} -\frac{1}{e} \left\langle N(p-q) \middle| d_q \bar{q}\sigma_{\mu\nu}q^\nu \gamma_5 q \middle| N(p) \right\rangle, \quad (6.50)$$

where d_q is the quark EDM. By taking the limit $q^\nu \to 0$ and the nucleon on-shell after combining Eqs. (6.48) and (6.50), we see that the momentum transfer q^ν can be factorized. The dependence of the nucleon EDM on the quark EDM is therefore related to the tensor charge by

$$d_N \bar{u}_N \sigma_{\mu\nu} u_N = d_q \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle. \tag{6.51}$$

We can thus use the result of the lattice QCD calculation of the quark tensor charge in nucleon (see Eq. (6.24)) to give the dependence of the nucleon EDM on the quark EDMs. The QCD sum rules calculation gives also a consistent result within the theoretical uncertainty [34].

It should be noted that the non-relativistic reduction of the tensor current gives the spin density. Such a situation is realized in the non-relativistic constituent quark model, which assumes that the nucleon is formed of three massive constituent quarks with confining inter-quark potential and small spin/isospin dependent interactions. The constituent quark model is known to work well in the hadron spectroscopy. The tensor charge of the constituent quark in nucleon (or the quark EDM contribution to the nucleon EDM) can be calculated with the $SU(2)$ algebra, and is given as

$$d_n(d_u, d_d) = \frac{4}{3} d_d - \frac{1}{3} d_u, \tag{6.52}$$

$$d_p(d_u, d_d) = \frac{4}{3} d_u - \frac{1}{3} d_d, \tag{6.53}$$

with d_u and d_d the EDMs of the u and d quarks, respectively. For the detailed derivation, see Appendix E. We see that the non-relativistic constituent quark model predicts a larger EDM than the lattice QCD result (6.24).

In watching this discrepancy, two sources of deviation can naïvely be inferred. The first possibility is the dressing of the bare quark tensor charge (or the bare quark EDM) by gluons, and the second one is the spin-dependent bound state effect. If we assume that the non-relativistic quark model works well, this discrepancy should originate in the gluon dressing of the tensor vertex of the quark.

The difference between the constituent quark model prediction and the lattice QCD result is of $O(1)$. As the hadronic level evaluation of other CP-odd quark level operators involves much more theoretical uncertainties, we do not need to be very sensitive on this discrepancy. In this thesis, we will use the results of the non-relativistic constituent quark model (6.52) and (6.53) for simplicity.

6.5 Theta Term and Strong CP Problem

In QCD, there are many distinct classes of gluon field configurations which are not connected by continuous gauge transformations. It is actually possible to assign to each of them a distinct topological charge called *winding number*. There are also gauge transformations which can shift the winding number. This so-called "large gauge transformation" can be generated with the following function [35]

$$\Lambda_1(\mathbf{x}) = \frac{\mathbf{x}^2 - d^2}{\mathbf{x^2} + d^2} + \frac{2id\mathbf{x}\cdot\tau}{\mathbf{x^2} + d^2}, \tag{6.54}$$

where $\mathbf{x}$ and τ are respectively the spatial and internal $SU(2)$ subgroup gauge coordinates, and d an arbitrary parameter. This gauge transformation has an impressive characteristic, it actually mixes the external and internal coordinates. To obtain gauge transformations which change the winding number arbitrarily, we simply have to gauge transform the gluon field using the gauge function $\Lambda_n(\mathbf{x}) = [\Lambda_\mathbf{1}(\mathbf{x})]^\mathbf{n}$. To obtain a fully gauge invariant vacuum, we have to sum up all classes of vacuum states with different winding number. The following coherent superposition of states $|\, m\,\rangle$ with winding number m satisfies the large gauge invariance:

$$|\,\theta\,\rangle = \sum_m e^{-im\theta}|\,m\,\rangle, \tag{6.55}$$

with θ an arbitrary real parameter which specifies the QCD vacuum. This is the *θ-vacuum*. Under gauge transformation, the θ-vacuum behaves as

$$U_n|\,\theta\,\rangle = e^{in\theta}|\,\theta\,\rangle, \tag{6.56}$$

where U_n is the large gauge transformation associated with the gauge function Λ_n. With non-zero θ, we can reexpress the action by adding a new term. The generic amplitude with QCD θ-vacuum can be written as

$$\langle\,\theta\,|\,X\,|\,\theta\,\rangle = \sum_{m_+,\,m_-} e^{i(m_+ - m_-)\theta}\langle\,m_+\,|\,X\,|\,m_-\rangle, \tag{6.57}$$

where X is some operator. The shift of the winding number $(m_+ - m_-)$ between asymptotic initial and final states can be measured by the topological charge operator $\int \tilde{G}G = \frac{g_s^2}{64\pi^2}\int d^4x\,\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma}$, as $\langle m_-|\int\tilde{G}G\,|\,m_+\rangle = m_+ - m_-$. Applying it to the generic amplitudes, we obtain

$$\langle\,\theta\,|\,X\,|\,\theta\,\rangle = \sum_{m_+,\,m_-}\langle\,m_+\,|\,Xe^{i\theta\int\tilde{G}G}\,|\,m_-\,\rangle = \langle\,\theta = 0\,|\,Xe^{i\theta\int\tilde{G}G}\,|\,\theta = 0\,\rangle. \tag{6.58}$$

The above expression means that we can translate the general dynamics in the θ-vacuum by the dynamics in $\theta = 0$ vacuum with a new term $S_\theta = \theta\int\tilde{G}G$ in the action. This term is called the θ-term (see Eq. (6.5)), and has an important status in the physics of CP violation. The θ-term is a total derivative, so it should not be relevant in perturbation theory, but it is actually not invariant under the "large gauge transformation", and can have an important role at the non-perturbative level.

Moreover, the existence of finite θ-term induces a sizable P and CP violation, as we have seen previously.

We will now see that the θ-term is related to the chiral transformation. The θ-term has the same form as the anomaly term relevant in the Ward identity of the axial (or chiral) $U(1)_A$ current. The 4-divergence of the axial $U(1)_A$ current can be written as

$$\partial_\mu j_5^\mu = 2\sum_q^{N_f} m_q \bar{q} i\gamma_5 q + \frac{N_f g_s^2}{16\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}, \tag{6.59}$$

where $\tilde{G}_a^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^a$, $j_5^\mu \equiv \sum_q^{N_f} \bar{q}\gamma^\mu\gamma_5 q$ and N_f is the number of flavors for quarks. This is the concrete expression which describes the anomaly effect. The physical meaning of the anomaly is the breakdown of the classical level symmetry at the quantum level. Let us define the following "modified" chiral charge:

$$\tilde{Q}_5 = \int d^3x \left\{ j_5^0 - K^0 \right\}, \tag{6.60}$$

where K^μ is defined such that $\partial_\mu K^\mu \equiv 2N_f \tilde{G}G$. The chiral transformation with this modified chiral charge is conserved for massless quarks. Note that this chiral charge is not gauge invariant. When we apply the gauge transformation U_n which changes the winding number by n, we have

$$U_n \tilde{Q}_5 U_n^{-1} = n(\tilde{Q}_5 - 2N_f). \tag{6.61}$$

This means that the θ-vacuum is modified by chiral rotations. The large gauge transformation of the chirally rotated θ-vacuum (with angle α) is

$$U_n e^{i\alpha\tilde{Q}_5}|\,\theta\,\rangle = U_n e^{i\alpha\tilde{Q}_5} U_n^{-1} U_n |\,\theta\,\rangle = e^{in(\theta-2N_f\alpha)} e^{i\alpha\tilde{Q}_5}|\,\theta\,\rangle = e^{in(\theta-2N_f\alpha)}|\,\theta - 2N_f\alpha\rangle. \tag{6.62}$$

The chiral phase α and the parameter θ of the θ-vacuum can actually be transferred to each other.

Let us now see the chiral transformation of quarks. The bare quark mass matrix is in general not diagonal and may have complex phase. We assume that the quark mass term has the following form

$$\mathscr{L}_m = -\bar{q}'_L \hat{m}' q'_R - \bar{q}'_R \hat{m}'^\dagger q'_L, \tag{6.63}$$

where $q'_{L/R}$ and $\hat{m}'$ are the bare quark and its mass matrix, respectively, with all considered flavors. As the physical mass matrix $\hat{m}$ should be real to have no tachyonic quarks, the chiral rotation of the basis $q'_{L/R} = e^{\mp i\alpha} U_{L/R} q_{L/R}$, where $U_{L/R}$ is the $SU(N_f)$ unitary matrix diagonalizing $\hat{m}'$, is needed to eliminate the axial $U(1)_A$ phase α as

$$\mathscr{L}_m = -e^{2i\alpha}\bar{q}_L U_L^\dagger \hat{m}' U_R q_R - e^{-2i\alpha}\bar{q}_R U_R^\dagger \hat{m}'^\dagger U_L q_L = -\bar{q}_L \hat{m} q_R - \bar{q}_R \hat{m}^\dagger q_L, \tag{6.64}$$

such that $\hat{m}$ is diagonal and $\mathrm{Im}[\det \hat{m}] = \mathrm{Im}[\det(e^{2i\alpha}\hat{m}')] = 0 \Leftrightarrow 2N_f\alpha = -\arg(\det \hat{m}')$. The total relevant θ-term is then

$$\bar{\theta} = \theta + \arg(\det \hat{m}'). \tag{6.65}$$

The physical effect of the $\bar{\theta}$ can be calculated with its contribution entirely transferred to the "P, CP-odd mass" $\eta\bar{q}i\gamma_5 q$ via the chiral rotation. The quark mass lagrangian can thus be expressed as

$$\mathscr{L}_m = -\bar{q}_L \hat{M} q_R - \bar{q}_R \hat{M}^\dagger q_L = -\sum_q^{N_f} m_q \bar{q}q - \eta \sum_q^{N_f} \bar{q}i\gamma_5 q, \tag{6.66}$$

where $\hat{M}$ is the quark mass matrix with the entire $\bar{\theta}$ contribution transferred, and η the corresponding P, CP-odd quark mass. The parameter η should not depend on the quark flavor since the effect of $\bar{\theta}$ was transferred via axial $U(1)_A$ transformation. From this, it is evident that the P, CP-odd effects are suppressed for heavier quarks. For QCD with 3 quark flavors, η satisfies the following relation

$$\bar{\theta} = \arg(\det \hat{M}) = \arg[(m_u + i\eta)(m_d + i\eta)(m_s + i\eta)]. \tag{6.67}$$

We thus have for small $\bar{\theta}$

$$\eta \approx \bar{\theta}\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \approx \bar{\theta}\frac{m_u m_d}{m_u + m_d} \equiv \bar{\theta} m_*, \tag{6.68}$$

where the second equality is the approximation for heavy strange quark mass. We obtain finally the following replacement between the θ-term and the P, CP-odd quark mass:

$$\bar{\theta}\frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} \leftrightarrow -\bar{\theta} m_* \sum_q^{N_f} \bar{q}i\gamma_5 q. \tag{6.69}$$

This replacement now gives us the possibility to calculate the hadron matrix element of the P, CP-odd pion-nucleon vertex with the PCAC techniques, as described in previous sections. It is therefore possible to estimate the physical contribution to the observables such as the neutron EDM. We have seen that the θ-term of the QCD was strongly constrained by EDM experimental data. Crewther et al. have shown that the neutron EDM experimental data constrain the parameter $\bar{\theta}$ to be less than 1 part of 10^{10} [18]. This remarkable fine-tuning can also be seen in many experimental data:

- Neutron EDM:
  From the previous analysis [18] (see Eq. (6.47)) and the experimental upper limit on neutron EDM ($d_n < 2.9 \times 10^{-26} e\,\mathrm{cm}$) [36], we obtain

$$|\bar{\theta}| < 1 \times 10^{-10}. \tag{6.70}$$

- Decay of η meson [18, 37]:

$$\mathrm{Br}(\eta \to \pi^+\pi^-) = 1.8 \times 10^2 \bar{\theta}. \tag{6.71}$$

With the experimental data $\mathrm{Br}(\eta \to \pi^+\pi^-) < 1.3 \times 10^{-5}\mathrm{eV}$ [38], we obtain

$$|\bar{\theta}| < 2.7 \times 10^{-4}. \tag{6.72}$$

- EDMs of diamagnetic atoms [39, 40]:

Like the neutron EDM, these puts also via P, CP-odd pion-nucleon interactions severe constraints on the θ-parameter.
From the experimental data of the EDM of ^{199}Hg atom we have [39]

$$|\bar{\theta}| < 3 \times 10^{-10}. \tag{6.73}$$

From the experimental data of the EDM of ^{129}Xe atom we have [40]

$$|\bar{\theta}| < 7 \times 10^{-7}. \tag{6.74}$$

All these results present strong arguments for the absence of physical contribution of the strong CP lagrangian. This is in contrast to the physical contribution of the anomalous $\int \tilde{G}G$ contribution to the heavy η' meson, which is believed to be the solution to the $U(1)_A$ problem [41–44]. This problem is called the *Strong CP problem*. These data, although leaving a little possibility of miraculous accidental cancellation, make us think of some mechanism that renders the parameter $\bar{\theta}$ to be unphysical.

One possible resolution to this problem was proposed by Peccei and Quinn, by introducing a new field coupled to the Strong CP lagrangian, the *axion* [45]. Its lagrangian (together with the Strong CP lagrangian) is given as follows:

$$\mathscr{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \left(\bar{\theta} + \frac{a(x)}{f_a}\right) \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a}, \tag{6.75}$$

where $a(x)$ is the axion field. This field is assumed to be the pseudo-Nambu-Goldstone boson of some chiral $U(1)_{\mathrm{PQ}}$ symmetry spontaneously broken at some high energy scale f_a. From this lagrangian, we see that if a vacuum expectation value $\langle a \rangle = -f_a \bar{\theta}$ is developed by the axion, the Strong CP lagrangian becomes irrelevant in the dynamics. We will see that this is exactly what happens. Let us give the following effective lagrangian

$$\mathscr{L}_a^{\mathrm{eff}} = \frac{1}{2}\partial_\mu a \partial^\mu a - K_1 \left(\bar{\theta} + \frac{a}{f_a}\right) - \frac{1}{2} K \left(\bar{\theta} + \frac{a}{f_a}\right)^2 + \cdots . \tag{6.76}$$

Coefficients K and K_1 can be determined by the calculation of the correlators involving the topological charge $G\tilde{G}$. First, we consider the case with only $\mathscr{L}_a + \mathscr{L}_{\text{QCD}}$. In this case, K_1 vanishes since there are no way to annihilate the odd number of axions. The coefficient K is called the topological susceptibility, and can be obtained by calculating the following correlator [41–44, 46, 47]

$$\begin{aligned} K &= -i \lim_{k\to 0} \int d^4x\, e^{ik(x-y)} \left\langle 0 \left| T\left\{ \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a}(x) \frac{\alpha_s}{8\pi} G^b_{\rho\sigma} \tilde{G}^{\rho\sigma,b}(y) \right\} \right| 0 \right\rangle \\ &\approx -m_* \langle 0|\bar{q}q|0\rangle + O(m_*^2), \end{aligned} \tag{6.77}$$

where $\langle 0|\bar{q}q|0\rangle \equiv -f_\pi^2 m_\pi^2/(m_u + m_d) \simeq -(0.280\,\text{MeV})^3$ is a negative number. For the derivation of Eq. (6.77), see Appendix C. We will then obtain a system which dynamically chooses the vacuum such that $\bar{\theta} + \langle a\rangle/f_a = 0$, which eliminates the effect of the Strong CP term. This is one scenario which solves the Strong CP problem, called the *axion mechanism*. This spontaneous choice of the vacuum will of course give massive excitations around $\langle a\rangle = -f_a\bar{\theta}$ with the mass of $\frac{1}{f_a}\sqrt{-m_*\langle 0|\bar{q}q|0\rangle}$. The search for this axion particle gives null result at the present time, and the axion is thought to be very light, constraining the scale of the $U(1)_{\text{PQ}}$ symmetry breaking to be $f_a > 10^{10}$ GeV.

We now consider the linear term of the axion potential with K_1 (the second term of the right-hand side of Eq. (6.76)). This term is generated by the correlation between the topological charge $G\tilde{G}$ and the P, CP-odd operator O_{CP}. The coefficient K_1 is given as follows [30, 31]:

$$\begin{aligned} K_1(O_{CP}) &= -i \lim_{k\to 0} \int d^4x e^{ik(x-y)} \left\langle 0 \left| T\left\{ \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a}(x) O_{CP}(y) \right\} \right| 0 \right\rangle \\ &= -i \int d^4x \left\{ -\frac{1}{2N_f} \delta(x_0 - y_0) \langle 0|[\, j_5^0(x),\, O_{CP}(y)\,]|0\rangle \right. \\ &\qquad \left. -\frac{1}{N_f} \sum_q \langle 0|\, T\{\, m_q \bar{q} i\gamma_5 q(x) O_{CP}(y)\,\}|0\rangle \right\}. \end{aligned} \tag{6.78}$$

For instance, the chromo-EDM contribution ($O_{CP} = -\frac{i}{2} d^c_q \bar{q} g_s \sigma^{\mu\nu} G^a_{\mu\nu} t_a \gamma_5 q$) can be written as

$$K_1 = -\frac{m_*}{2} \sum_{q=u,d,s} \frac{d^c_q}{m_q} \langle 0|\bar{q} g_s \sigma^{\mu\nu} G^a_{\mu\nu} t_a q|0\rangle. \tag{6.79}$$

The derivation of K_1 for the chromo-EDM is presented in Appendix C. The presence of the linear term with K_1 is important since the minimum of the axion potential receives a shift, and thus generates an induced θ-term $\theta_{\text{ind}} = -K_1(O_{CP})/K$ [30, 31]. For the chromo-EDM, this gives the following induced θ-term:

$$\theta_{\text{ind}} = -\frac{m_0^2}{2} \sum_{q=u,d,s} \frac{d_q^c}{m_q}. \tag{6.80}$$

This result has been used in Sect. 6.2.

For the Weinberg operator, the first term of the second equality of Eq. (6.78) gives no contribution, since it involves no quark field operators. The second term is also small, since isospin violation must occur to generate a pion from the isoscalar Weinberg operator in the intermediate state. The θ-term induced by the Weinberg operator is therefore suppressed by at least a factor of light quark mass.

6.6 Summary of Hadron Level Calculation

Here we summarize the dependences of the hadron level P, CP-odd interactions on P, CP-odd quark level operators. These results will be used in our discussion.

- P, CP-odd πNN interactions:

$$\bar{g}_{\pi NN}^{(0)} = 5.9 \times 10^{13} \frac{d_u^c + d_d^c}{\text{cm}} \tag{6.81}$$

$$\bar{g}_{\pi NN}^{(1)} = 1.0 \times 10^{15} \frac{d_u^c - d_d^c}{\text{cm}} - \sum_{q'=u,d,s,c,b} C_{q'd} \frac{F_\pi m_\pi^2 G_F}{2\sqrt{2} m_d} \langle N|\bar{q}' q'|N\rangle \tag{6.82}$$

- Neutron EDM:

$$d_n = \frac{4}{3} d_d - \frac{1}{3} d_u + 4.9 e d_u^c + 2.3 e d_d^c + 2.6 e d_s^c, \tag{6.83}$$

- Proton EDM:

$$d_p = -\frac{1}{3} d_d + \frac{4}{3} d_u - 3.8 e d_u^c - 2.3 e d_d^c - 1.6 e d_s^c, \tag{6.84}$$

- P, CP-odd electron-nucleon interactions:

$$C_p^{\text{SP}} = \sum_{q=u,d,s,c,b} C_{eq}^{\text{SP}} \langle p|\bar{q} q|p\rangle \tag{6.85}$$

$$C_n^{\text{SP}} = \sum_{q=u,d,s,c,b} C_{eq}^{\text{SP}} \langle n|\bar{q} q|n\rangle \tag{6.86}$$

$$C_p^{\text{PS}} = \sum_{q=u,d,s,c,b} C_{eq}^{\text{PS}} \langle p|\bar{q} i\gamma_5 q|p\rangle \tag{6.87}$$

$$C_n^{\text{PS}} = \sum_{q=u,d,s,c,b} C_{eq}^{\text{PS}} \langle n|\bar{q} i\gamma_5 q|n\rangle \tag{6.88}$$

References

1. S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989)
2. M. Pospelov, A. Ritz, Ann. Phys. **318**, 119 (2005)
3. J.F. Donoghue, C.R. Nappi, Phys. Lett. B **168**, 105 (1986)
4. A.R. Zhitnitsky, Phys. Rev. D **55**, 3006 (1997)
5. J. Gasser, H. Leutwyler, M.E. Sainio, Phys. Lett. B **253**, 252 (1991)
6. K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010)
7. K. Nakamura et al. (Particle Data Group), Partial update for the 2012 edition (URL: http://pdg.lbl.gov) (2011)
8. J. Beringer et al. (Particle Data Group), Phys. Rev. D **86**, 010001 (2012)
9. R.D. Young, A.W. Thomas, Nucl. Phys. A **844**, 266 (2010)
10. K. Takeda, S. Aoki, S. Hashimoto, T. Kaneko, T. Onogi, N. Yamada (JLQCD Collaboration), PoS **LATTICE2010**, 160 (2010) [arXiv:1012.1907 [hep-lat]]
11. H. Ohki et al. (JLQCD Collaboration), Phys. Rev. D **78**, 054502 (2008)
12. K. Takeda et al., Phys. Rev. D **83**, 114506 (2011)
13. G.S. Bali et al. (QCDSF Collaboration), Phys. Rev. D **85**, 054502 (2012)
14. T.P. Cheng, L.F. Li, Phys. Rev. Lett. **62**, 1441 (1989)
15. A. Airapetian et al. (HERMES Collaboration), Phys. Rev. D **75**, 012007 (2007)
16. J.R. Ellis, M. Karliner, arXiv:hep-ph/9601280
17. S. Aoki, M. Doui, T. Hatsuda, Y. Kuramashi, Phys. Rev. D **56**, 433 (1997)
18. R.J. Crewther, P. Di Vecchia, G. Veneziano, E. Witten, Phys. Lett. B **88**, 123 (1979)
19. J. Hisano, Y. Shimizu, Phys. Lett. B **581**, 224 (2004)
20. J. Hisano, Y. Shimizu, Phys. Rev. D **70**, 093001 (2004)
21. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385 (1979)
22. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 448 (1979)
23. L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rept. **127**, 1 (1985)
24. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **191**, 301 (1981)
25. T. Falk, K.A. Olive, M. Pospelov, R. Roiban, Nucl. Phys. B **560**, 3 (1999)
26. V.M. Khatsymovsky, I.B. Khriplovich, A.R. Zhitnitsky, Z. Phys. C **36**, 455 (1987)
27. V.M. Khatsymovsky, I.B. Khriplovich, A.S. Yelkhovsky, Ann. Phys. **186**, 1 (1987)
28. V.M. Belyaev, B.L. Ioffe, Zh Eksp Teor. Fiz. **83**, 876 (1982)
29. V.M. Belyaev, B.L. Ioffe, Sov. Phys. JETP **56**, 493 (1982)
30. I. Bigi, N.G. Uraltsev, Sov. Phys. JETP **100**, 198 (1991)
31. I. Bigi, N.G. Uraltsev, Nucl. Phys. B **353**, 321 (1991)
32. A. Pich, E. de Rafael, Nucl. Phys. B **367**, 313 (1991)
33. B. Borasoy, Phys. Rev. D **61**, 114017 (2000)
34. M. Pospelov, A. Ritz, Phys. Rev. D **63**, 073015 (2001)
35. R. Jackiw, C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976)
36. C.A. Baker et al., Phys. Rev. Lett. **97**, 131801 (2006)
37. K. Kawarabayashi, N. Ohta, Nucl. Phys. B **175**, 477 (1980)
38. F. Ambrosino et al. (KLOE Collaboration), Phys. Lett. B **606**, 276 (2005)
39. W.C. Griffith et al., Phys. Rev. Lett. **102**, 101601 (2009)
40. M.A. Rosenberry et al., Phys. Rev. Lett. **86**, 22 (2001)
41. R.J. Crewther, Phys. Lett. B **70**, 349 (1977)
42. E. Witten, Nucl. Phys. B **156**, 269 (1979)
43. G. Veneziano, Nucl. Phys. B **159**, 213 (1979)
44. S. Coleman, *Aspect of Symmetry* (Cambridge University Press, Cambridge, 1988)
45. R.D. Peccei, H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977)
46. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **166**, 493 (1980)
47. P. Di Vecchia, G. Veneziano, Nucl. Phys. B **171**, 253 (1980)