Chapter 13
Reappraisal of Constraints on R-parity Violation from EDM at the Leading Order

13.1 Analysis and Results

To obtain upper limits on RPV couplings, it suffices to divide the experimental upper bounds on the EDMs of electron, neutron, ^{205}Tl and ^{199}Hg atoms by the coefficients of Table 12.3. If several EDM experimental data give upper limits for one RPV bilinears, we take the tightest limit. We will see that in some cases, the upper limits are changed from previous analyses, due to the use of our correct formula for the Barr-Zee type contribution derived in Chap. 11.

Type 1

The first type is the leptonic bilinears $\lambda_{311}\lambda^*_{322}$ and $\lambda_{211}\lambda^*_{233}$ which contribute exclusively to the electron EDM via the Barr-Zee type diagram [see Eq. (11.26)]. The electron EDM is actually strongly constrained by the result of the EDM experiment using YbF molecules and the current limit is $d_e < 1.05 \times 10^{-27} e$ cm [1]. From the calculation of the RPV contribution above, we can give the upper limits shown in Table 13.1 [we have also shown the limits given to the semi-leptonic RPV bilinears which contribute to the electron EDM (type 2)]. The limits, which are from the EDM of ^{205}Tl, a paramagnetic atom sensitive to the electron EDM, are also displayed for comparison.

Type 2

The next type is the semi-leptonic RPV bilinears (which involve the electron) of the form $\lambda_{i11}\lambda'^*_{ikk}$ ($i = 2, 3$ and $k = 1, 2, 3$). They can be constrained by the P, CP-odd e-N interactions, as was first pointed by Herczeg [4]. These interactions are constrained via the electron Barr-Zee type graph, and also by the P, CP-odd electron-nucleon (e-N) interactions. The P, CP-odd e-N interactions [see Eq. (8.3)]

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Table 13.1 Upper limits on the absolute value of combinations of RPV couplings ($i = 2, 3$) via the electron EDM

	$\lvert\mathrm{Im}(\lambda_{322}\lambda^*_{311})\rvert$	$\lvert\mathrm{Im}(\lambda_{233}\lambda^*_{211})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i22}\lambda^*_{i11})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i33}\lambda^*_{i11})\rvert$
Limits to RPV from YbF molecule				
Upper limits ($m_{\tilde{\nu}} = 1$ TeV)	2.1×10^{-3}	1.9×10^{-4}	6.6×10^{-3}	2.9×10^{-4}
Upper limits ($m_{\tilde{\nu}} = 5$ TeV)	4.5×10^{-2}	3.7×10^{-3}	1.4×10^{-1}	5.4×10^{-3}
Upper limits ($m_{\tilde{\nu}} = 100$ GeV)	2.9×10^{-5}	3.3×10^{-6}	9.2×10^{-4}	5.9×10^{-6}
Formulae of Ref. [2] ($m_{\tilde{\nu}} = 100$ GeV)	1.9×10^{-6}	3.3×10^{-7}	6.0×10^{-6}	6.6×10^{-7}
Limits to RPV from ^{205}Tl EDM				
Upper limits ($m_{\tilde{\nu}} = 1$ TeV)	3.1×10^{-3}	2.8×10^{-4}	9.7×10^{-3}	4.2×10^{-4}
Upper limits ($m_{\tilde{\nu}} = 5$ TeV)	6.6×10^{-2}	5.5×10^{-3}	2.1×10^{-1}	8.2×10^{-3}
Upper limits ($m_{\tilde{\nu}} = 100$ GeV)	4.4×10^{-5}	4.9×10^{-6}	1.4×10^{-4}	8.6×10^{-6}
Formulae of Ref. [2] ($m_{\tilde{\nu}} = 100$ GeV)	2.8×10^{-6}	4.8×10^{-7}	8.7×10^{-6}	9.7×10^{-7}

The upper table shows limits given from the experimental analysis of the YbF molecule. The lower table shows limits deduced from the EDM of ^{205}Tl atom [3] using the relation (8.31)

can be given by multiplying the P, CP-odd electron-quark interaction of Eq. (11.31) with the quark contents of nucleon. In this work, we use the scalar and pseudoscalar contents of the nucleon derived in Sect. 6.1 [see Eqs. (6.12), (6.13), (6.17), (6.21) and (6.23)] [5–7]. The EDM of atoms is a very sensitive probe of the P, CP-odd e-N interactions. The type 2 RPV bilinears receive the tightest upper bounds from the EDM of ^{199}Hg atom [8]. The dependence of the EDM of ^{199}Hg atom on the P, CP-odd e-N interactions are given in Eq. (8.32) [9, 10]. The upper limits are shown in Table 13.2.

We see that the EDM of the ^{199}Hg atom gives a very tight constraints on the CP phases between RPV couplings. We must be careful for $\mathrm{Im}(\lambda_{i11}\lambda'^*_{i33})$, since the Barr-Zee graph contributing to electron EDM can give also tight constraints on these same RPV couplings. The previous analysis pointed that the Barr-Zee graph can give tighter constraints [4], but the formula used in this analysis was not correct [11]. By using Eq. (11.26), we see that the EDM of ^{199}Hg atom can give through the P, CP-odd electron-nucleon tighter constraints than that from the electron EDM using the YbF molecule experiment.

Type 3

For the type 3, RPV bilinears can be constrained by the Barr-Zee type diagram of the d-quark EDM with heavy fermions in the loop. The d-quark EDM contributes to the nucleon EDM. The strongest limit on the type 3 RPV bilinears is given by the

Table 13.2 Upper limits on the absolute values of combinations of RPV couplings from the EDM of ^{199}Hg atom via the P, CP-odd electron-nucleon interactions

RPV couplings	$\lvert\mathrm{Im}(\lambda_{i11}\lambda'^*_{i11})\rvert$	$\lvert\mathrm{Im}(\lambda_{i11}\lambda'^*_{i22})\rvert$	$\lvert\mathrm{Im}(\lambda_{i11}\lambda'^*_{i33})\rvert$
Upper limits	$7.9 \times 10^{-10}[m_{\tilde{\nu}}]^2$	$3.6 \times 10^{-8}[m_{\tilde{\nu}}]^2$	$1.8 \times 10^{-6}[m_{\tilde{\nu}}]^2$

$i = 2, 3$. $[m_{\tilde{\nu}}]$ is the mass of sneutrino in unit of 100 GeV. Upper limits were given independently

Table 13.3 Upper limits on the absolute value of combinations of RPV couplings from neutron EDM via the Barr-Zee type graph of the d quark. Here $i = 2, 3$

RPV couplings	$\lvert\mathrm{Im}(\lambda_{i22}\lambda'^{*}_{i11})\rvert$	$\lvert\mathrm{Im}(\lambda_{i33}\lambda'^{*}_{i11})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i22}\lambda'^{*}_{i11})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i33}\lambda'^{*}_{i11})\rvert$
Upper limits ($m_{\tilde{\nu}} = 1$ TeV)	1.3×10^{-1}	1.2×10^{-2}	4.2×10^{-1}	1.8×10^{-2}
Upper limits ($m_{\tilde{\nu}} = 5$ TeV)	2.8	2.3×10^{-1}	8.7	3.3×10^{-1}
Upper limits ($m_{\tilde{\nu}} = 100$ GeV)	1.7×10^{-3}	2.0×10^{-4}	5.5×10^{-3}	3.5×10^{-4}
Formulae of Ref. [2] ($m_{\tilde{\nu}} = 100$ GeV)	1.1×10^{-4}	1.9×10^{-5}	3.5×10^{-4}	3.9×10^{-5}

neutron EDM experimental data [12]. The corresponding upper bounds are shown in Table 13.3 (we have also shown hadronic type bilinears which contribute also to the electromagnetic Barr-Zee type graphs of the d-quark). We have used the relation (6.83) to obtain the dependence of the neutron EDM on the d-quark EDM.

The limits given here by the single neutron EDM are rather moderate. One of the simple explanation is the absence of some large enhancement mechanism like the electron EDM contribution to the paramagnetic atoms. Nevertheless, these semi-leptonic RPV bilinears can only be constrained efficiently with the neutron EDM. They can also be constrained from the atomic EDM, but the nucleon EDM contribution suffers from Schiff's screening which weakens their effect.

Type 4

The fourth type is the hadronic RPV bilinears ($\lambda'_{ijj}\lambda'^{*}_{ikk}$, $i, j, k = 1, 2, 3$ with $j \neq k$). These can be strongly constrained by the purely hadronic EDMs and the Schiff moments of diamagnetic atoms. The elementary level processes involve the Barr-Zee type chromo-EDM (11.30) and the P, CP-odd 4-quark interaction (11.31). These two processes contribute to the nucleon EDM and the P, CP-odd pion-nucleon interactions, and their relations are given by Eqs. (6.81), (6.82), (6.83) and (6.84). The tightest limits on type 4 RPV bilinears are given by the neutron [12] and the ^{199}Hg EDM [8] experimental data. The relation between the above P, CP-odd hadron level interactions and the ^{199}Hg EDM is given by Eqs. (7.56) [13] and (8.32) [9]. For the dependence of the ^{199}Hg Schiff moment, we have used the average of Table 7.1. We have shown in Table 13.4 the upper limits to the hadronic RPV bilinears from the neutron and ^{199}Hg EDM. We can see the tightness of the constraints by comparing the

Table 13.4 Upper limits on the absolute value of combinations of hadronic RPV bilinears from neutron EDM and ^{199}Hg atomic EDM ($i = 1, 2, 3$)

RPV couplings	$\lvert\mathrm{Im}(\lambda'_{i11}\lambda'^{*}_{i22})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i11}\lambda'^{*}_{i33})\rvert$	$\lvert\mathrm{Im}(\lambda'_{i22}\lambda'^{*}_{i33})\rvert$
Upper limits ($m_{\tilde{\nu}} = 1$ TeV)	3.1×10^{-4}	1.1×10^{-5}	1.2×10^{-5}
Upper limits ($m_{\tilde{\nu}} = 5$ TeV)	6.5×10^{-3}	2.1×10^{-4}	2.2×10^{-4}
Upper limits ($m_{\tilde{\nu}} = 100$ GeV)	4.3×10^{-6}	2.3×10^{-7}	2.4×10^{-7}
Formulae of Ref. [2] ($m_{\tilde{\nu}} = 100$ GeV)	2.5×10^{-7}	2.6×10^{-8}	2.7×10^{-8}

limits to the same RPV bilinears given via electromagnetic Barr-Zee (see Table 13.3), which are of 3 to 4 orders larger in magnitudes. This large ratio can be explained by the suppression of the electromagnetic coupling in the Barr-Zee contribution. The ratio between Eqs. (11.30) and (11.26) is suppressed both by the electromagnetic coupling α_{em} and by the square of the d-type quark charge, which renders the chromo-EDM contribution to be important. The neutron and ^{199}Hg EDMs gave close upper limits to the RPV bilinears, except for $\mathrm{Im}(\lambda'_{i22}\lambda'^{*}_{i33})$. This is because this bilinear are from the chromo-EDM of the strange quark. In our analysis, the strange contribution enters only via the nucleon EDM. As the nucleon EDM contribution is suppressed by Schiff's screening for the ^{199}Hg atom, the strange contribution is relatively suppressed. Our result shows the importance of the strangeness, but this result, together with the u and d contributions, suffers from large theoretical uncertainties, including the uncertainty of the quark mass, value of the quark condensates and also the calculation of the Schiff moment for the ^{199}Hg EDM. Taking all these topics into account, the result may change even by orders of magnitude. The result shows nevertheless the large sensitivity of the neutron and ^{199}Hg EDMs against the purely hadronic RPV bilinears. We should also add some discussions concerning the relative size between the chromo-EDM and the P, CP-odd 4-quark interaction. In this case also, the correct size of the chromo-EDM is smaller than that used in previous analyses [11]. In the present case however, the quantitative comparison of the two contributions is meaningless because of the large theoretical uncertainty of the hadronic and nuclear level calculations.

Type 5

The type 5 RPV bilinears are those constrained by the muon EDM and include $\mathrm{Im}(\lambda_{122}\lambda^{*}_{133})$, $\mathrm{Im}(\lambda_{i22}\lambda'^{*}_{i22})$, $\mathrm{Im}(\lambda_{i22}\lambda'^{*}_{i33})$. These bilinears contribute only via the Barr-Zee type diagram of the muon EDM [see Eq. (11.26)]. The muon EDM is particular in the sense that at the leading order, its dependence on RPV bilinear has no overlap with other available observables. This means that the type 5 RPV bilinears can be constrained only by muon EDM. Unfortunately, constraints from the muon EDM experimental data cannot be discussed, since the present experimental sensitivity is too weak to give any limit on RPV interactions.

Type 6

The last type of RPV bilinears, $\lambda_{i33}\lambda'^{*}_{i22}$ and $\lambda_{i33}\lambda'^{*}_{i33}$ cannot be probed by the currently available EDM experimental data, and nor by any projected experiments. One possibility to probe them is the tau EDM, which may be technically difficult.

Table 13.5 Upper limits on the absolute value of the combinations of RPV bilinears from presently available experimental data. Results were given for $m_{\mathrm{SUSY}} = 1\mathrm{TeV}$

RPV couplings		Limits from other exp.	Limits from EDMs	EDM observables used
Type 1	$\lvert\mathrm{Im}(\lambda_{311}\lambda^*_{322})\rvert$	0.15	2.1×10^{-3}	YbF molecule [1]
	$\lvert\mathrm{Im}(\lambda_{211}\lambda^*_{233})\rvert$	0.25	1.9×10^{-4}	YbF molecule [1]
Type 2	$\lvert\mathrm{Im}(\lambda_{211}\lambda'^*_{211})\rvert$	2.9×10^{-2}	7.9×10^{-8}	^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda_{311}\lambda'^*_{311})\rvert$	3.6×10^{-3}	7.9×10^{-8}	^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda_{211}\lambda'^*_{222})\rvert$	2.9×10^{-2}	3.6×10^{-6}	^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda_{311}\lambda'^*_{322})\rvert$	1.7×10^{-2}	3.6×10^{-6}	^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda_{211}\lambda'^*_{233})\rvert$	0.70	1.8×10^{-4}	^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda_{311}\lambda'^*_{333})\rvert$	0.36	1.8×10^{-4}	^{199}Hg EDM [8]
Type 3	$\lvert\mathrm{Im}(\lambda_{122}\lambda'^*_{111})\rvert$	4.4×10^{-2}	1.2×10^{-1}	neutron EDM [12]
	$\lvert\mathrm{Im}(\lambda_{322}\lambda'^*_{311})\rvert$	6.0×10^{-3}	1.2×10^{-1}	neutron EDM [12]
	$\lvert\mathrm{Im}(\lambda_{133}\lambda'^*_{111})\rvert$	2.6×10^{-2}	1.1×10^{-2}	neutron EDM [12]
	$\lvert\mathrm{Im}(\lambda_{233}\lambda'^*_{211})\rvert$	2.9×10^{-2}	1.1×10^{-2}	neutron EDM [12]
Type 4	$\lvert\mathrm{Im}(\lambda'_{111}\lambda'^*_{122})\rvert$	5.0×10^{-3}	3.1×10^{-4}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{211}\lambda'^*_{222})\rvert$	3.2×10^{-3}	3.1×10^{-4}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{311}\lambda'^*_{322})\rvert$	6.8×10^{-4}	3.1×10^{-4}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{111}\lambda'^*_{133})\rvert$	3.1×10^{-4}	1.1×10^{-5}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{211}\lambda'^*_{233})\rvert$	8.0×10^{-2}	1.1×10^{-5}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{311}\lambda'^*_{333})\rvert$	1.4×10^{-2}	1.1×10^{-5}	neutron EDM [12] or ^{199}Hg EDM [8]
	$\lvert\mathrm{Im}(\lambda'_{122}\lambda'^*_{133})\rvert$	2.0×10^{-4}	1.2×10^{-5}	neutron EDM [12]
	$\lvert\mathrm{Im}(\lambda'_{222}\lambda'^*_{233})\rvert$	8.0×10^{-2}	1.2×10^{-5}	neutron EDM [12]
	$\lvert\mathrm{Im}(\lambda'_{322}\lambda'^*_{333})\rvert$	6.8×10^{-2}	1.2×10^{-5}	neutron EDM [12]
Type 5	$\lvert\mathrm{Im}(\lambda_{122}\lambda^*_{133})\rvert$	0.15	–	muon EDM
	$\lvert\mathrm{Im}(\lambda_{122}\lambda'^*_{122})\rvert$	2.9×10^{-2}	–	muon EDM
	$\lvert\mathrm{Im}(\lambda_{322}\lambda'^*_{322})\rvert$	2.9×10^{-2}	–	muon EDM
	$\lvert\mathrm{Im}(\lambda_{122}\lambda'^*_{133})\rvert$	1.8×10^{-3}	–	muon EDM
	$\lvert\mathrm{Im}(\lambda_{322}\lambda'^*_{333})\rvert$	0.60	–	muon EDM
Type 6	$\lvert\mathrm{Im}(\lambda_{133}\lambda'^*_{122})\rvert$	1.7×10^{-2}	–	–
	$\lvert\mathrm{Im}(\lambda_{233}\lambda'^*_{222})\rvert$	2.9×10^{-2}	–	–
	$\lvert\mathrm{Im}(\lambda_{133}\lambda'^*_{133})\rvert$	1.0×10^{-3}	–	–
	$\lvert\mathrm{Im}(\lambda_{233}\lambda'^*_{233})\rvert$	0.70	–	–

13.2 Summary

In this chapter, we have discussed the contributions of the R-parity violating (RPV) interactions to the EDM of the neutron, ^{205}Tl, ^{199}Hg atoms, and YbF molecule, and have given limits to the RPV interactions by assuming the dominance of single bilinear of RPV couplings. In doing this, we have calculated the correct contribution of the two-loop level Barr-Zee type EDM diagram, and the result has given one order of magnitude looser limits to products of RPV couplings. This point merits some attention, because the relative size between the contribution from the P, CP-odd 4-fermion interaction is changed, and the phenomenological analysis was altered for the P, CP-odd electron-nucleon interactions. We have also classified the bilinears of RPV couplings into 6 types, and presented their characteristics in detail. This has made clear the interrelations between the RPV couplings and the EDM observables. We give in Table 13.5 the summary of the currently available experimental upper limits to all RPV bilinears considered so far. We show also limits on RPV couplings provided by other experiments.

We note once again that these limits were derived by assuming that only one bilinear of RPV couplings is dominant in each analysis. The phenomenological analysis with the consideration of the whole RPV parameter space should also be done, as was done in the analysis of the EDM constraints within R-parity conserving supersymmetry by Ellis et al. [14]. This is the subject of the Chap. 14.

References

1. J.J. Hudson et al., Nature **473**, 493 (2011)
2. D. Chang, W.-F. Chang, M. Frank, W.-Y. Keung, Phys. Rev. D **62**, 095002 (2000)
3. B.C. Regan et al., Phys. Rev. Lett. **88**, 071805 (2002)
4. P. Herczeg, Phys. Rev. D **61**, 095010 (2000)
5. J.F. Donoghue, C.R. Nappi, Phys. Lett. B **168**, 105 (1986)
6. K. Takeda, S. Aoki, S. Hashimoto, T. Kaneko, T. Onogi, N. Yamada, JLQCD Collaboration, in *PoS LATTICE2010*, 160 (2010) [arXiv:1012.1907 [hep-lat]]
7. T.P. Cheng, L.F. Li, Phys. Rev. Lett. **62**, 1441 (1989)
8. W.C. Griffith et al., Phys. Rev. Lett. **102**, 101601 (2009)
9. V.A. Dzuba, V.V. Flambaum, S.G. Porsev, Phys. Rev. A **80**, 032120 (2009)
10. V.V. Flambaum, I.B. Khriplovich, Zh. Eksp. Teor. Fiz. **89**, 1505 (1985)
11. N. Yamanaka, T. Sato, T. Kubota, Phys. Rev. D **85**, 117701 (2012)
12. C.A. Baker et al., Phys. Rev. Lett. **97**, 131801 (2006)
13. S. Ban, J. Dobaczewski, J. Engel, A. Shukla, Phys. Rev. C **82**, 015501 (2010)
14. J. Ellis, J.S. Lee, A. Pilaftsis, JHEP **1010**, 049 (2010)
15. A.R. Zhitnitsky, Phys. Rev. D **55**, 3006 (1997)
16. H. Ohki et al., JLQCD Collaboration. Phys. Rev. D **78**, 054502 (2008)
17. K. Takeda et al., Phys. Rev. D **83**, 114506 (2011)
18. V.V. Flambaum, I.B. Khriplovich, Sov. Phys. JETP **62**, 872 (1985)
19. J. Ellis, J.S. Lee, A. Pilaftsis, JHEP **1102**, 045 (2011)
20. J. Ellis, J.S. Lee, A. Pilaftsis, arXiv:1009.1151 [math.OC]