

Flight Trajectory Optimization for Modern Jet Passenger Aircraft with Dynamic Programming

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Abstract Future Air Transportation Systems (ATS) would eventually have to treat the ever increasing demands in aviation industry. Fuel consumption and flight time could be significantly reduced by modifying the current sector based system into a more relaxed user priority based ATS. Trajectory Based Operations (TBO) is considered as one of the key technologies in this transition. This paper concentrates on a proposed flight trajectory optimization tool based on Dynamic Programming (DP), meteorological data and an aircraft performance model to obtain fuel minimum 4D-optimal flight trajectories for a single jet passenger aircraft considering the effect of wind conditions. Major drawbacks such as “Curse of Dimensionality” in DP are overcome with a unique method called “Moving Search space Dynamic Programming (MS-DP)” method. A quantitative evaluation has revealed that an average reduction of 9% in fuel consumption could be achieved with a tradeoff of flight time which exceeds at an average of 7% compared to a series of flight data measured by a commercial GPS receiver in an airliner cabin.

Keywords Aircraft performance optimization • Flight time • Fuel consumption

1 Introduction

Future Air Transportation Systems (ATS) would eventually have to treat the ever increasing air traffic, sky-rocketing fuel prices, CO₂ emission, noise pollution concerns and economic instability in the aviation industry with effective solutions

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to provide a reliable, safe and efficient service to its customers [1]. The current ATS is considered as a safe system but the aging operational procedures and ground based Air Traffic Control (ATC) systems will overrun the capacity it was originally designed for in the next couple of decades. Especially, in terminal airspaces of busy airports, aircraft are bound to follow vector control procedures to maintain a smooth air traffic flow. International flight routes such as North Pacific (NOPAC) Composite Route System (CRS) and Pacific Organized Track System (PACOTS) are relatively more relaxed than domestic flight routes and are optimized daily according to weather forecasts and air traffic conditions, which are called as User Preferred Routes (UPR). Hence, a study on benefits obtained in a futuristic domestic airspace is vital to understand the necessary solutions to meet the future demands with a cost effective and efficient approach.

Collaborative Action for Renovation of Air Traffic Systems (CARATS) is the proposed long term research plan for the improvement of Japan ATS. It has introduced 4D-Trajectory Based Operations (TBO) as one of the key technologies to realize the transformation of sector based Air Traffic Control (ATC) system into a more relaxed and efficient Air Traffic Management (ATM) system. This is a core solution in the “Free Flight” concept, which is considered as a safe flight operating capability and would let the operators to select the optimal flight path and speed in real time [1]. 4D-flight trajectory optimization is formulated as an optimal control problem to be proposed in TBO and had been extensively studied over the last few decades [2–4]. Various mathematical programming approaches such as calculus of variations and direct method have been proved to have positive impact on the optimization process [5, 6]. On the contrary, these methods possess disadvantages in providing the global optimum and predicting the computational time due to iterative calculation methods they use in the process. Therefore, a reliable numerical optimization method is a necessity to obtain optimal solutions to be applied in TBO concept. Therefore, Dynamic Programming (DP) is considered as one of the most suitable tools to evaluate the global optimum while contemplating the required constraints [7].

This research is dedicated to contribute towards a futuristic ATS with the objective of developing a 4D-flight trajectory optimization tool for a single jet passenger aircraft to minimize the fuel consumption by considering the presence of wind conditions. DP is utilized in the optimization process to obtain the global optimum and a unique calculation method is proposed in order to overcome the drawbacks of DP. A quantitative evaluation is implemented to generate optimal trajectories along one of the busiest domestic flight routes between Haneda and Fukuoka, and obtained results are compared with a series of flight data measured in airliner cabin by using a commercial GPS data logger. Air data are estimated by applying meteorological data from Japan Meteorological Agency (JMA) to assure the clarity of measured GPS data. The aircraft performance is analyzed with the application of Base of Aircraft Data (BADA) aircraft performance model by the European Organization for the Safety of Air Navigation (EUROCONTROL).

2 Optimal Trajectory Design

Generally, an aircraft’s motion is expressed by six degree of freedom (6 DOF) equations of motion. In this study, state variables are reduced by using point mass approximations and omitting the state variables which represent the aircraft’s attitude. Furthermore, the dynamics of velocity direction change is neglected while the dynamics of velocity change which is directly related to energy is considered.

2.1 Equations of Motion

An aircraft’s 3D-translational motion is reviewed in Fig. 1. The azimuth angle relative to air ψ_a , path angle relative to air γ_a and the engine thrust T are used as control variables and the equations of motion are solved with respect to the state variables of latitude ϕ , longitude θ , geo-potential altitude H and true airspeed V_{TAS} .

$$\frac{d\theta}{dt} = \frac{1}{(R_0 + H) \cos \phi} (V_{TAS} \cos \gamma_a \sin \psi_a + W_x) \tag{1}$$

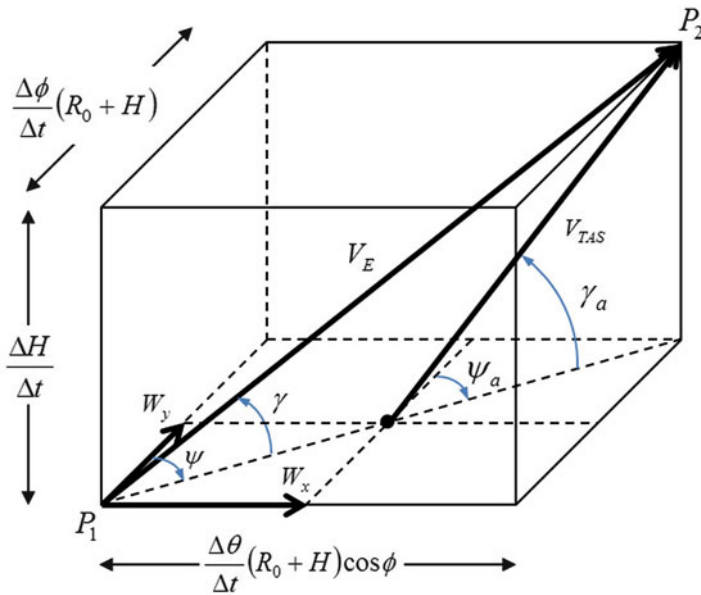


Fig. 1 Definition of point mass equations

$$\frac{d\phi}{dt} = \frac{1}{R_0 + H} (V_{TAS} \cos \gamma_a \cos \psi_a + W_y) \quad (2)$$

$$\frac{dH}{dt} = V_{TAS} \sin \gamma_a \quad (3)$$

$$m \frac{dV_E}{dt} \cos(\gamma_a - \gamma) \cos(\psi_a - \psi) = T - D - mg \sin \gamma_a \quad (4)$$

m	Aircraft mass	γ	Path angle
g	Gravity acceleration	ψ_a	Azimuth angle relative to air
R_0	Earth radius	ψ	Ground track angle
D	Aerodynamic drag	W_x	Zonal wind component
γ_a	Path angle relative to air	W_y	Meridional wind component

Southerly winds and westerly winds are considered as positive and the vertical wind component is assumed negligible. The aircraft's position is defined by latitude, longitude and geo-potential altitude and the aircraft's speed is given with true airspeed and speed V_E which is on Earth-centered Earth-fixed coordinate system. Each velocity direction is calculated by path angle and azimuth angle relative to air and, path angle and ground track angle respectively. Speed V_E is considered due to the effect of wind vector on the acceleration of aircraft. This acceleration vector is parallel to V_E and its cosine component is added to Eq. (4) to evaluate the required engine thrust which is parallel to V_{TAS} vector. Parameters relative to aerodynamic system are depicted with index a .

2.2 Meteorological Data

The Japan Meteorological Agency (JMA) provides a variety of weather data on global and domestic atmospheric conditions. These are known as Numerical Weather Prediction (NWP) models and periodically updated weather data are available for commercial use. Physical variables shown in Table 1 are provided by two main NWP models, Global Spectral Model (GSM) and Meso Scale Model (MSM) in a grid point value (GPV) format on a coordinate frame defined by longitude, latitude and barometric pressure surface. The precision of these data was reviewed by referring to an airline onboard flight data [8]. Results depict that numerical data provided by GSM model are more precise than MSM model and is suitable to be used in trajectory optimization analysis. This difference emerges according to so called *position error* which occurs in weather prediction by MSM model due to its fine mesh resolution [9]. Hence, GSM model for Japan region was applied in this study and its fundamental properties are given in Table 1.

Table 1 GSM Japan region model fundamental properties [10]

Article	Model properties
Data format	GRIB2
Initial time value	00, 06, 12, 18 (UTC)
Region	Longitude: 120–150°, latitude: 20–50°
Resolution	Longitude direction: 0.25° Latitude direction: 0.2°
Barometric pressure levels	12 levels {mean sea level 100 hPa}
Physical variables	Geopotential altitude, temperature, vertical flow, relative humidity, wind velocity

Table 2 Operational limitations for performance optimization

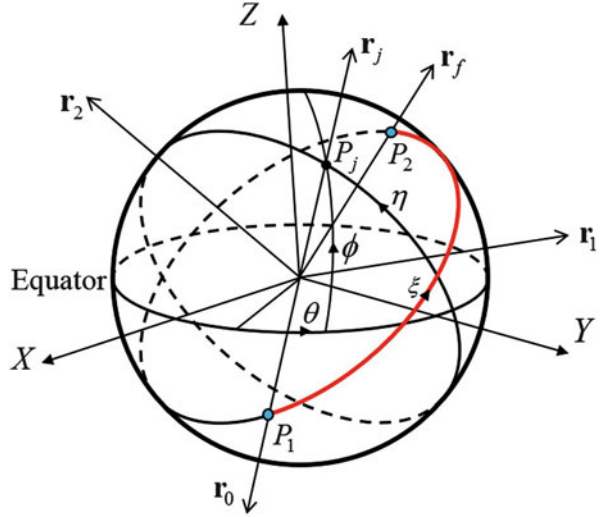
Limitation	Value
Maximum operating calibrated airspeed	310 kt
Stall speed (for reference mass)	149 kt
Minimum operating calibrated airspeed	Stall speed × 1.3 kt
Maximum operating Mach number	0.84
Maximum operating altitude	43,100 ft

2.3 Aircraft Performance Model

Generally, a model capable of providing sufficient and precise data is essential to discuss an aircraft’s performance, especially to conjecture the fuel consumption at a specific flight condition. Base of Aircraft Data (BADA) model 3.9 developed and maintained by European Organization for the Safety of Air Navigation (EUROCONTROL) is applied in the proposed optimization model to evaluate the aircraft’s performance [11]. The operational limitations provided in BADA model for the subjected aircraft are given in Table 2 and are used to sustain the validity of obtained solutions.

A review on the variations in thrust, fuel consumption, airspeed and other parameters is a necessity when performing a dynamic optimization for a complete flight profile. Models which are defined in the BADA model are used to estimate these variations. The aerodynamic model provides useful equations to calculate the lift and drag coefficients and the fuel consumption model provides unique computational algorithms to obtain the fuel flow for each phase of flight. The nominal fuel flow is a function of true airspeed and the minimum fuel flow is a function of altitude which applies for idle thrust conditions. The thrust model provides the maximum thrust available at each flight phase. According to a numerical analysis we could understand that the numerical error of evaluating the fuel consumption is ±5% compared to an airliner onboard flight data [12]. Therefore, the same model was applied to calculate the fuel consumption of both reference trajectories and corresponding optimal trajectories to reduce the effect by this error.

Fig. 2 Transition from Earth-centered Earth-fixed coordinate system to polar coordinate system



2.4 Dynamic Programming Approach

Generally, a steady or quasi-steady flight of an aircraft is assumed by categorizing its flight into climb, cruise and descent phases in order to evaluate the performance optimization. In case we assume that prognostic wind data is in possession, it creates a further necessity of dynamic optimization to achieve a flight trajectory which could minimize the fuel consumption. Hence we apply the DP method in our optimization analysis because of its unique qualities provided in [13].

2.4.1 Application of DP

State variables ϕ and θ are transformed to ξ and η in the Polar coordinate system and they represent downrange angle and cross range angle respectively as shown in Fig. 2. This provides a feasible calculation platform for our proposed optimization model.

The downrange angle which transits monotonously with time, depicts the progress angle from initial point P_1 along the Great Circle Route (GCR), which is the shortest aerial path between the initial point P_1 and terminal point P_2 . The cross range angle provides the lateral deviation from GCR at an arbitrary point P_j . Unit vectors \mathbf{r}_0 and \mathbf{r}_f are along the initial and terminal points respectively and unit vectors \mathbf{r}_1 and \mathbf{r}_2 are used in the coordinate system transformation process. Then the three state variables H , η and calibrated airspeed V_{CAS} are discretized and applied in the optimization model. Note that state variable V_{TAS} is transformed to V_{CAS} and is expressed as V .

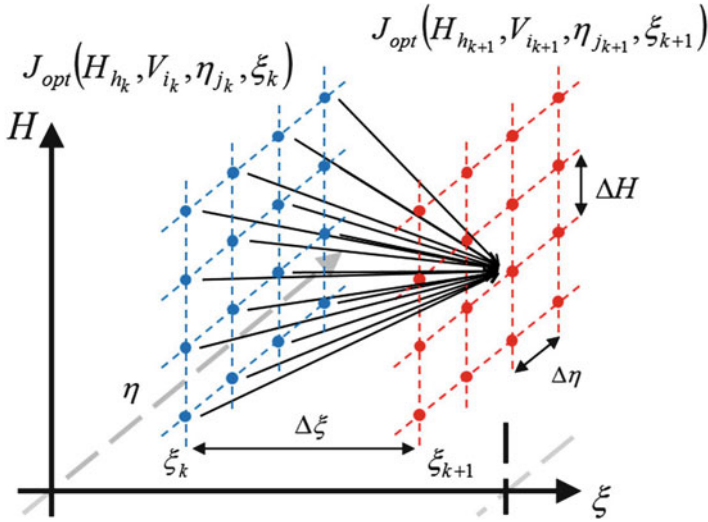


Fig. 3 DP logic on trajectory transition (dimension of velocity is omitted to simplify the image)

2.4.2 Performance Index

The total fuel consumption is defined as the performance index J , which is the integral of fuel flow FF during the flight time $t_f - t_0$ as shown in Eq. (5).

$$J = \int_{t_0}^{t_f} FF(t) dt \tag{5}$$

Then the optimal performance index J_{opt} is derived with H , V , η and ξ which could be expressed as $J_{opt}(H, V, \eta, \xi)$. The global optimum is obtained by solving the combinatorial optimization problem for all transitions between the grid points. According to DP algorithm, the principle of optimality is defined as follows:

$$J_{opt}(H_{h_{k+1}}, V_{i_{k+1}}, \eta_{j_{k+1}}, \xi_{k+1}) = \min_{\substack{h_k \rightarrow h_{k+1} \\ i_k \rightarrow i_{k+1} \\ j_k \rightarrow j_{k+1}}} [(H_{h_k}, V_{i_k}, \eta_{j_k}, \xi_k) + \Delta J] \tag{6}$$

$$\Delta J = FF \Big|_{\xi_k}^{\xi_{k+1}} \Delta t \tag{7}$$

The increment of performance index for each transition is derived by ΔJ where the fuel flow is assumed to be constant in each transition. Figure 3 depicts the DP logic on trajectory transition in the state space grid.

Although DP possesses various advantages as an optimization method, it also has several major drawbacks [13, 14]. To reduce these effects we have utilized a unique method called “Moving Search space Dynamic Programming (MS-DP)”, constructed by integrating the concept of gradient method and DP. Since the number of calculations is proportional to the square of number of grid points on the plane of independent variable, it could be drastically reduced if the search area in state space is limited. In this method, optimization calculations are implemented in a partial space around a reference trajectory to reduce the computational time. If a solution is obtained in the partial search space, a new search space is generated around the solution and optimization calculation continues until the solution is converged, where the new solution is identical to the previous reference trajectory. The validity of this unique calculation method is thoroughly discussed in [13].

2.5 GPS Data Analysis

A series of flight data were used in this study as reference data to understand the benefits obtained by the proposed optimization tool. A commercial GPS data logger was used inside the cabin of an airborne aircraft (after departure warnings are turned off and before landing warnings are turned on) to measure the aircraft’s 3D-position (latitude, longitude and geometrical altitude) and ground speed. An analysis was implemented to confirm the validity of these data before utilizing them as reference data in the optimization process [15].

2.5.1 Lateral Deviation from Airway

The maximum deviation from the airway was recorded only at a magnitude of several dozen meters [15] and large deviations were noticed near the waypoints due to the performed maneuvers in order to maintain its predetermined flight course and heading angle change procedures.

Figure 4 illustrates the quantitative evaluation on the deviation of a series of flight routes from its corresponding airways. The bias error from the radius of fuselage is considered in this analysis for clarity because the data are measured at the airliner cabin window seat.

2.5.2 Estimation of Air Data

Pressure altitude and Mach number are estimated with the application of weather data to understand the current flight procedures. Figure 5 resembles not only a quantitative evaluation of the deviation from predetermined pressure altitude but also the accuracy of weather prediction data. Also, from the BADA model we are aware that the subjected aircraft generally performs at a cruising speed of

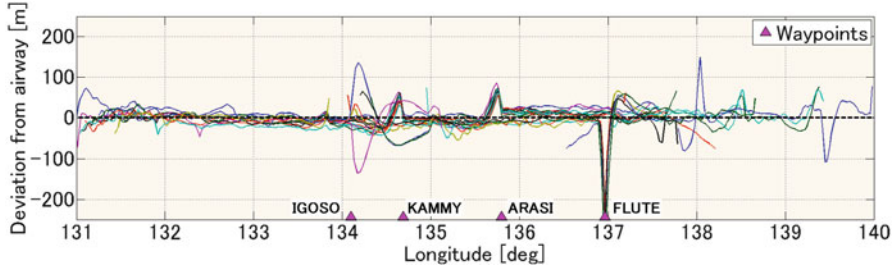


Fig. 4 Quantitative evaluation for lateral deviation from airway

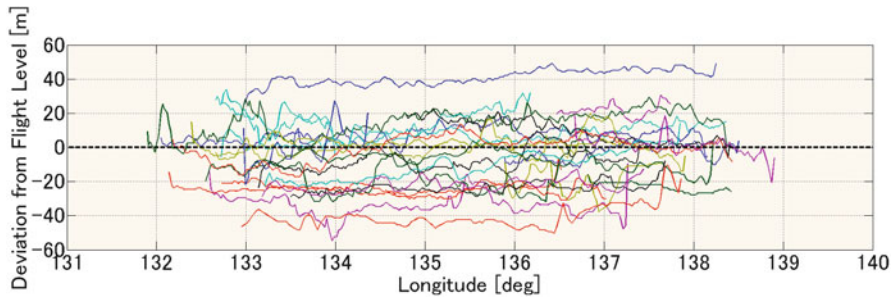


Fig. 5 Quantitative evaluation for deviation from cruising pressure altitude

Mach 0.84. Yet a quantitative evaluation for the deviation of Mach number could not be achieved because we were not aware of the precise Mach number setting for each flight case [15].

From the obtained results we could confirm that GPS measured flight data are sufficiently reliable to be used as reference data. We could also confirm that present day passenger aircraft are,

- Capable of maintaining flight profiles with high precision along the airway and the cruising pressure altitude according to provided onboard flight data on its Flight Management Systems (FMS).
- Bound to follow vector control procedures before entering the terminal airspace of busy airports in order to adjust the arrival sequence.

3 Results

A total of 14 flights are analyzed with seven flights each from Haneda (RJTT) to Fukuoka (RJFF) and Fukuoka (RJFF) to Haneda (RJTT) which are shown in Table 3. The estimated flight profile in the optimization process for each flight case is given in Table 4. Initial and terminal values of altitude and calibrated airspeed are set to be identical at each flight case to perform a fair evaluation.

Table 3 Reference GPS data

Flight case (FC)	Year/month/day	Departure/destination	Data record time (JST)	
			Initial time	Final time
01	2011/6/18	RJFF/RJTT	14:02	15:20
02	2011/6/19	RJTT/RJFF	20:00	21:48
03	2011/6/26	RJTT/RJFF	19:58	21:28
04	2011/7/31	RJTT/RJFF	19:59	21:30
05	2011/8/28	RJTT/RJFF	18:50	20:31
06	2011/9/02	RJFF/RJTT	09:53	11:30
07	2011/9/24	RJFF/RJTT	10:57	12:20
08	2011/10/01	RJFF/RJTT	15:04	16:22
09	2011/10/02	RJTT/RJFF	19:54	21:41
10	2011/10/08	RJFF/RJTT	16:11	17:29
11	2011/10/10	RJTT/RJFF	11:22	12:59
12	2011/10/15	RJFF/RJTT	16:04	17:42
13	2011/10/17	RJTT/RJFF	10:20	12:00
14	2011/10/21	RJFF/RJTT	9:55	11:20

Table 4 Estimated flight profile

Variable	Description
Flight route	RJTT: 35.5483°N 139.9078°E RJFF: 33.8903°N 130.3317°E
Down range	Resolution: 31 grid points
Altitude	Minimum: 3,000 m, maximum: 13,000 m Resolution: 101 grid points
Calibrated airspeed	Minimum: 100 ms ⁻¹ , maximum: 160 ms ⁻¹ Resolution: 61 grid points
Cross range	2.5° on each side of Great Circle Route Resolution: 101 grid points

3.1 Quantitative Evaluation

This section discusses the results obtained 4D-flight trajectory optimization tool. They are compared with the previously discussed flight data and achieved benefits are reviewed. Figure 6 depicts the total reference data used in the analysis and data plots in Fig. 7 illustrate the achieved efficiency in fuel consumption with respect to exceeded flight time compared to reference GPS data.

We could achieve an average of 9% for fuel consumption efficiency with exceeding the flight time with an average of 7%. The GPS logger does not provide data regarding the aircraft mass at respective measuring points. Hence, we have utilized the reference mass value of 208,700 kg as the initial mass at all cases.

The results in Fig. 7 show that the optimal routes tends to spend more flight time than the reference GPS route in order to reduce the fuel consumption. In case of flights from RJTT to RJFF, it is reasonable that most of the optimal trajectories

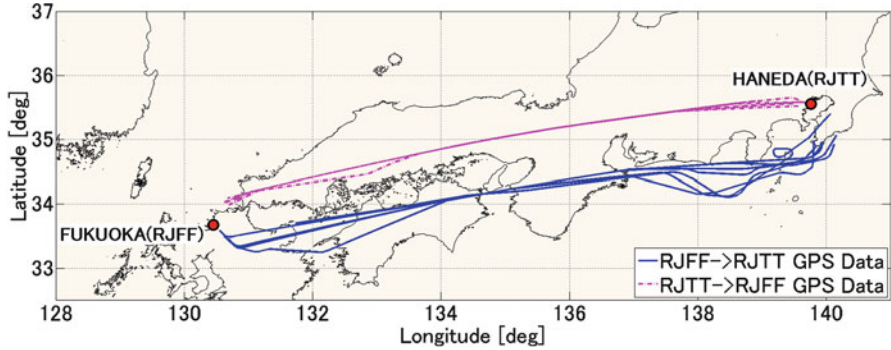


Fig. 6 GPS measured data over Haneda (RJTT) ↔ Fukuoka (RJFF) flight route

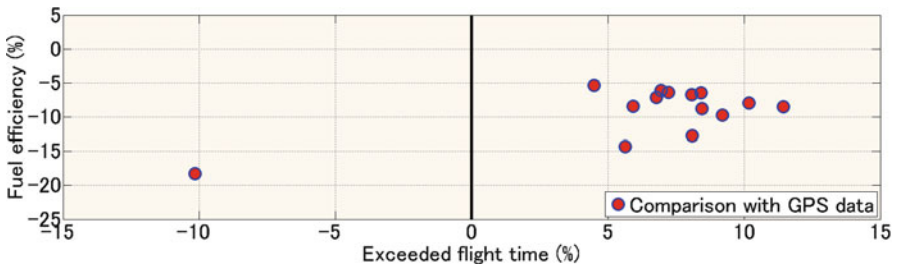


Fig. 7 Tradeoff between fuel consumption and exceeded flight time

tend to exceed flight time largely compared to their respective reference trajectories because both reference trajectory and optimal trajectory perform close to GCR. Furthermore, in flight cases from RJFF to RJTT, the flight time excess is smaller or negative. From this quantitative evaluation it could be considered that current flight plans in FMS give higher priority to save flight time than saving the amount of fuel consumed. Critical points of the optimization process are explained in detail in the next subsections with particular flight cases and obtained optimal results at each respective case.

3.1.1 Preference of Descent Speed

Subfigures a-i in Fig. 8 illustrate several parameters related to aircraft’s performance based on FC: 10. The most prominent feature of this analysis is the selection of speed and altitude as shown from Fig. 8a-d. Aircraft on the reference trajectory (refers to as *ref. aircraft*) tends to perform a longer cruise phase and a descent with steep rate. Also the *ref. aircraft* performs the cruise phase at a predetermined flight altitude and at high speeds in all phases of flight. On the contrary, aircraft on the optimal trajectory (refers to as *opt. aircraft*) performs at a higher altitude and a relatively lower airspeed. This is because aircraft achieve efficient performance

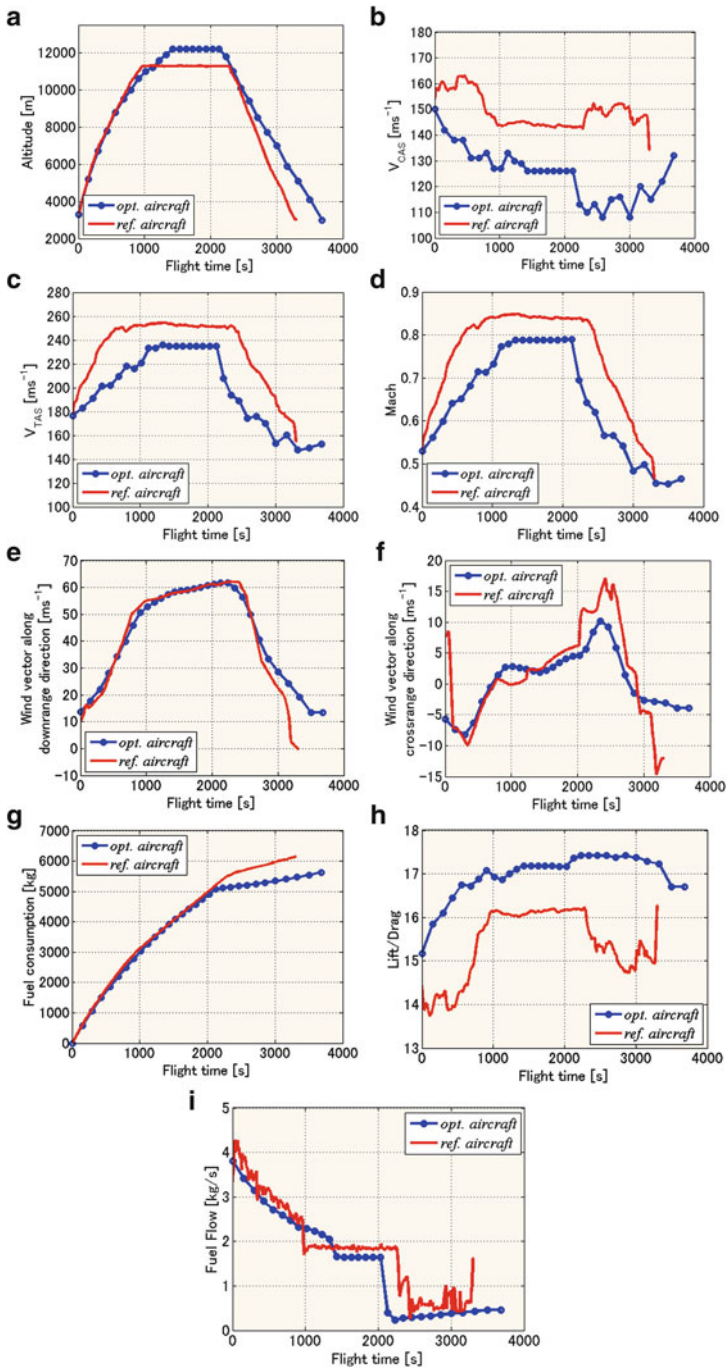


Fig. 8 Characteristics comparison for FC: 10. (a) Flight altitude. (b) Calibrated airspeed. (c) True airspeed. (d) Mach number. (e) Wind vector along downrange direction. (f) Wind vector along cross range direction. (g) Fuel consumption. (h) Lift drag ratio. (i) Fuel flow

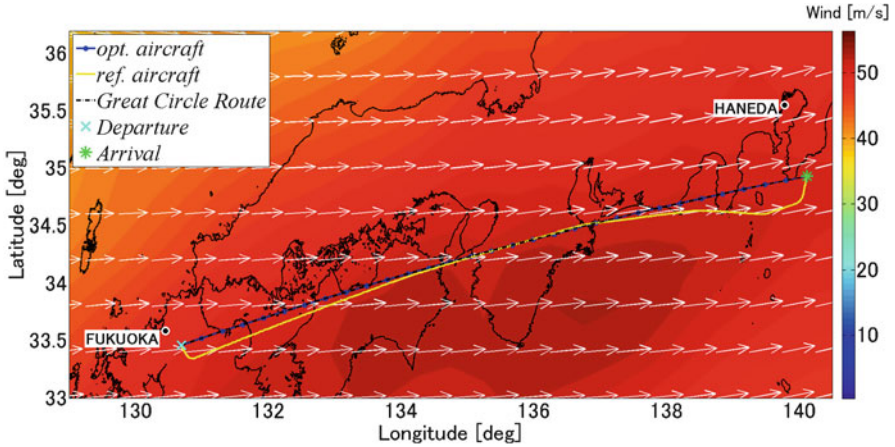


Fig. 9 Lateral flight profile comparison for FC: 10

when flying close to the service ceiling of flight envelope according to BADA model. Furthermore *opt. aircraft* uses tailwinds efficiently to reduce the aircraft speed which are depicted in Fig. 8e, f. This results a reduction of fuel consumption as illustrated in Fig. 8g and a higher lift drag ratio as shown in Fig. 8h compared to *ref. aircraft*. The reduction of fuel consumption is also visible from the fuel flow comparison in Fig. 8i. Furthermore, the aircraft applies idle thrust settings at descent phase at both cases to save unnecessary fuel burn. Slower speed and smooth descent rate results the *opt. aircraft* to spend more flight time to reach the destination. Due to the existence of strong uniform tailwinds, *opt. aircraft* tends to follow the GCR to minimize the flight range while the *ref. aircraft* is bound to vector control procedures near Haneda airport as shown in Fig. 9. The color bar represents the wind magnitude along the flight space at cruising altitude of *opt. aircraft*.

In this comparison, *ref. aircraft* performs with 6,152.6 kg of fuel consumption at a flight time of 3,305 s while the *opt. aircraft* performs with 5,630.2 kg of fuel consumption at a flight time of 3,683 s which results an efficiency of 8.5% in fuel consumption with a tradeoff of 11% in exceeding flight time.

3.1.2 Vector Control Procedures

This subsection discusses results compared to FC: 12 which the *ref. aircraft* experiences vector control and holding procedures at the terminal airspace of Haneda airport. From Fig. 10a it is visible that *ref. aircraft* had to maintain a level flight during its descent as a result of vectoring. On the other hand, the *opt. aircraft* performs a typical flight mission upon the assumption of relaxed ATC procedures. Similar to results on FC: 10, the *opt. aircraft* tends to perform at a higher altitude to obtain greater performance. Furthermore, from Fig. 10b, c it is

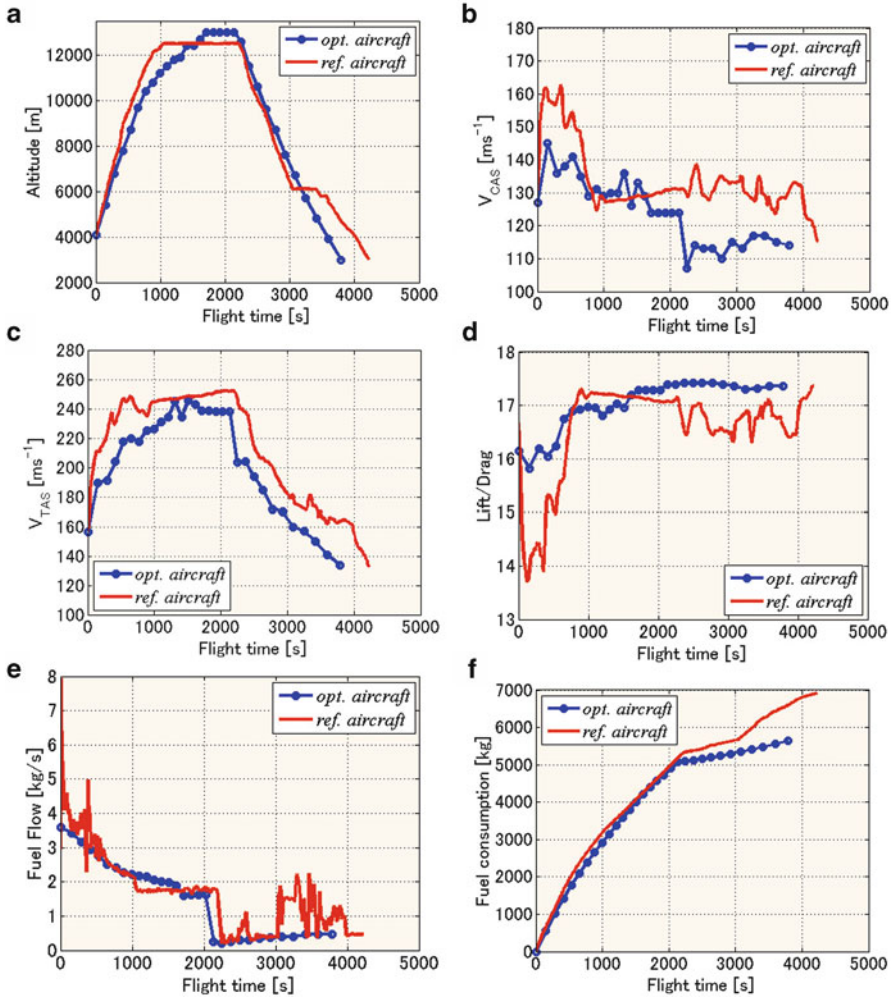


Fig. 10 Characteristics comparison for FC: 12. (a) Flight altitude. (b) Calibrated airspeed. (c) True airspeed. (d) Lift drag ratio. (e) Fuel flow. (f) Fuel consumption

understood that *ref. aircraft* climbs at a steep rate and performs a longer cruise and descent with a holding at relatively high speeds. This results a lower lift drag ratio and also additional fuel flow as depicted in Fig. 10d, e. Hence, the reduction in fuel consumption is clearly visible from Fig. 10f. Also according to current ATC procedures, aircraft approaching Haneda airport from West direction have to pass through the initial fix located at South of airport. Yet, with the assumption of ATC constraints free ATS system is applied, Fig. 11 indicates that *opt. aircraft* tends to use the advantage of tailwinds and laterally deviates from GCR which results a significant reduction of fuel consumption and flight time compared to *ref. aircraft*.

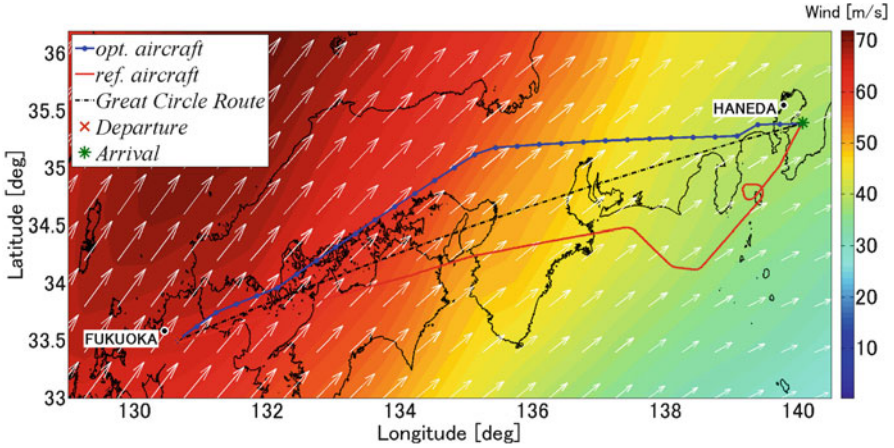


Fig. 11 Lateral flight profile comparison for FC: 12

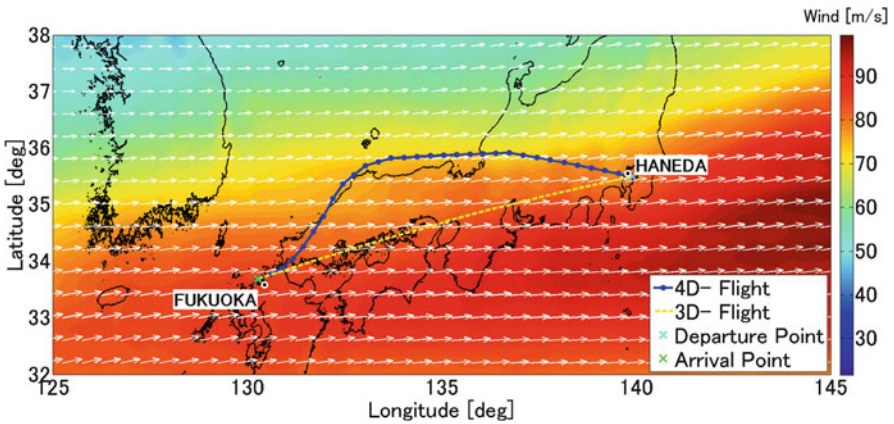


Fig. 12 Lateral flight profile at 25,000 Pa barometric pressure altitude

The *opt. aircraft* performs with 5,643.7 kg of fuel consumption at a flight time of 3,791 s while the *ref. aircraft* performs with 6,905.5 kg of fuel consumption at a flight time of 4,220 s which results an efficiency of 18.3% in fuel consumption and an efficiency of 10.1% in flight time. This flight case results the largest reduction of fuel consumption and also reduction in flight time.

3.1.3 Effect of Wind Shear

To clarify the effect of wind shear on lateral deviation of an aircraft in our proposed model, a comparative analysis was performed by creating two optimal trajectories in the presence of strong wind shear, one with the ability to laterally deviate (refers

to as 4D-flight) and the other without this ability (refers to as 3D-flight). The lateral flight profile is given in Fig. 12 with the magnitude of wind shear. According to the BADA model, aircraft achieves high performance at high altitudes. Therefore both optimal flight paths tend to perform at high altitudes close to the service ceiling. Yet, in the presence of strong headwinds in the region we could understand that the aircraft would try to pass through where the effect of headwinds is relatively small. Hence, the 4D-flight has taken the tradeoff with flight range into consideration and as a result, the flight range has become larger compared to 3D-flight but has been able to reduce the fuel consumption and flight time. This example clearly indicates that the strong wind shear could have a great impact on flight trajectory optimization to succeed future demands in an effective approach.

In this case, 3D-flight performs with 9,651.1 kg of fuel consumption at a flight time of 6,031 s while the 4D-flight performs with 9,349.9 kg of fuel consumption at a flight time of 5,948 s which results an efficiency of 3.12% in fuel consumption and 1.38% in flight time.

4 Conclusions

We conclude this study with an overview of this research followed by a discussion on obtained results and future works to be accomplished [16].

Flight trajectory optimization of a jet passenger aircraft for climb, cruise and descent phases has been formulated as an optimal control problem by considering its inherent performance characteristics by assuming no ATC constraints are engaged. The aircraft's 3D-position and airspeed were considered as state variables to implement the optimization in a provided grid space with heading angle, flight path angle and engine thrust as control variables.

Dynamic Programming method was introduced as a suitable optimization tool because of its unique qualities. Also a unique calculation method called "Moving Search Space Dynamic Programming (MS-DP)" method was proposed to overcome DP's drawbacks and demonstrated its validity through a numerical analysis [12]. Other utilized models were also checked for their validity to be applied in the optimization process.

A commercial GPS logger was used in airliner cabin to measure a series of flight data and their reliability was thoroughly checked by estimating several air data.

A quantitative evaluation was performed and its benefits were reviewed. An average of 9% fuel efficiency could be obtained with a tradeoff of 7% exceeding flight time by creating optimal trajectories for 14 flight cases between Haneda (RJTT) and Fukuoka (RJFF). In a free arrival time optimization process we could understand that the aircraft on optimized routes adjusts the Top of Descent (TOD) setting and performs the descent phase with a smoother rate compared to conventional operating procedures. It is also a fact that present ATC system bounds the aircraft through vector control procedures which results additional fuel burn and

flight time. By providing a more relaxed airspace, pilots would be able to perform efficient flight missions such as,

- Flying at minimum flight distance at the presence of uniform headwinds or tailwinds.
- Adjust the descent airspeed to maintain a high lift drag ratio as a continuous descent approach.
- Perform lateral deviations from predetermined airway to use strong tailwinds or avoid strong headwinds to reduce unnecessary fuel burn.

The constrained workspace of this study has to be emphasized, namely providing free airspace for a single passenger aircraft to perform a 4D-optimal trajectory neither considering any ATC restrictions nor constraints. However, the main objective of this research is to generate a fundamental, yet reliable optimal flight trajectory model to lay the foundation for future studies in the field of Air Traffic Management. Considering multiple aircraft influences the optimal control problem to be more complicated. By enhancing the scope of DP applications such as to analyze optimal flight trajectories with arrival time constraints [17] and Conflict Detection and Resolution (CD&R) methods for multi objective optimization strategies, one could achieve an overall optimization of the system.

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