2 Kinematics

In this chapter, the foundation for path planning and navigation of a wheel-drive robot is given. First the basic formulas, which allow to describe the motion of the vehicle in a 3D environment, are introduced. Then the solution to the kinematics problem considering different types of wheels fixed at specific positions on the vehicle platform is described. At the end of the chapter, geometric kinematics solutions for typical wheel-driven robotic platforms are presented.

2.1 Basics

In the following section, a short introduction to the kinematics calculation (pose and velocity) is given and applied to a standard wheel-driven robot. Figure 2.1 shows two successive positions of a vehicle in a 3D environment. The pose of the robot is represented by a frame through the kinematic center of the robot (see figure 2.2). Path planning can be reduced to finding a navigation strategy which transforms the starting frame to the next frame. In a 2D scenario (e. g. driving in an office environment) a transfer vector (x,y,α) is determined which describes the displacement in x- and y-direction as well as the orientation represented as rotation around the z-axis. In the next section it is shown how – based on the rotational speed of the vehicle wheels – the velocity of the kinematic center can be calculated. Through the integration of the velocity vector over time, the pose change is determined.

In general the pose of an arbitrary object in the Cartesian space can be described as a six-tuple $(x,y,z,\alpha,\beta,\gamma)$. The position vector ${}^{O}\vec{u}$ in an object frame O can be presented in base frame coordinates ${}^{B}\vec{u}$ by

$$ {}^{B}\vec{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + {}^{B}_{O}R(\alpha,\beta,\gamma)\,{}^{O}\vec{u} \tag{2.1} $$

with $(x,y,z)^T$ being the translational vector between the origin of the two frames and R the respective (combined) rotation matrix.

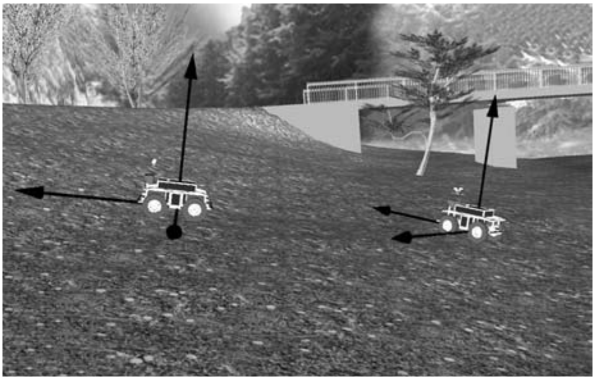

Figure 2.1 Transformation of robot coordinate systems

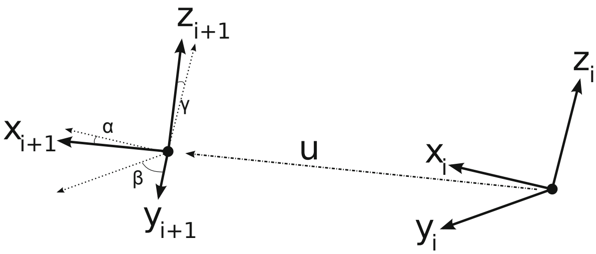

Figure 2.2 Schematics depicting the coordinate systems in figure 2.1

Another possibility to express the same as above are homogeneous transformation matrices. Those 4×4 matrices (for 3D space) are composed as shown below:

$$\begin{bmatrix} R & u \\ P & s \end{bmatrix} \tag{2.2}$$

with

R**:** 3×3 rotation matrix

u**:** position vector $u = (u_x,u_y,u_z)^T$

P**:** perspective transformation (in general $P = (0,0,0)$)

s**:** scaling factor (in general $s = 1$)

Matrix R usually is a combination of several rotations around the elementary axis. The rotation matrix $R_x(\alpha)$ describes a rotation around the x-axis of an arbitrary coordinate system via the angle α. The other two required matrices are defined the same way.

$$R_z(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \tag{2.4}$$

$$R_y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{2.5}$$

There are two possible ways of expressing linked rotations: The roll, pitch, yaw system and Euler angles. The former implies the linked rotations are performed around the fixed axes of the coordinate system while the latter uses variable rotation axes. A linked rotation around, for instance, the z-axis (angle α) then x-axis (angle β) and finally z-axis (angle γ) can be expressed as:

$$\text{Euler:} \qquad R_s = R_z(\alpha) \cdot R_x(\beta) \cdot R_{z'}(\gamma) \tag{2.6}$$

$$\text{Roll-pitch-yaw:} \qquad R_s = R_z(\alpha) \cdot R_y(\beta) \cdot R_x(\gamma) \tag{2.7}$$

As one can see, the axes in the latter case remain fixed while in the first case rotations along "new" axes are performed. In case of terrestrial robotics the roll, pitch, yaw system is selected to transfer the frames into each other. An illustration of the concept can be found in figure 2.3.

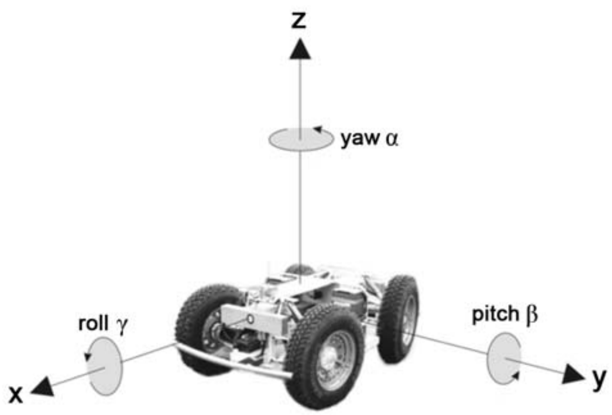

Figure 2.3 Roll, pitch, yaw: rotation along fixed coordinate axes

The system originates from the field of aviation. The difference to terrestrial robotics is the yaw-axis, which is changed from facing down to up for usability reasons.

Since an arbitrary orientation in 3D space can be achieved using only three rotations, the resulting linked rotation R_s can be expressed as presented in equation 2.8. For writing and reading convenience the abbreviations sα for $\sin(\alpha)$ and respectively cα for $\cos(\alpha)$ will be used in the following equations:

$$R_s = \begin{bmatrix} \mathrm{c}\alpha\mathrm{c}\beta & \mathrm{c}\alpha\mathrm{s}\beta\mathrm{s}\gamma - \mathrm{s}\alpha\mathrm{c}\gamma & \mathrm{c}\alpha\mathrm{s}\beta\mathrm{c}\gamma + \mathrm{s}\alpha\mathrm{s}\gamma \\ \mathrm{s}\alpha\mathrm{c}\beta & \mathrm{s}\alpha\mathrm{s}\beta\mathrm{s}\gamma + \mathrm{c}\alpha\mathrm{c}\gamma & \mathrm{s}\alpha\mathrm{s}\beta\mathrm{c}\gamma - \mathrm{c}\alpha\mathrm{s}\gamma \\ -\mathrm{s}\beta & \mathrm{c}\beta\mathrm{s}\gamma & \mathrm{c}\beta\mathrm{c}\gamma \end{bmatrix} \tag{2.8}$$

The velocity vectors can be calculated similar to the transformation of the pose from one frame to another. Suppose there is a linear velocity vector ${}^{B}\vec{v}_Q$ of an arbitrary point Q presented in frame B which should be transformed to frame A. This transformation can be calculated as

$${}^{A}\vec{v}_Q = {}^{A}_{B}R\,{}^{B}\vec{v}_Q \tag{2.9}$$

If the origin of frame B has also a linear velocity relative to frame A then

$${}^{A}\vec{v}_Q = {}^{A}\vec{v}_{OB} + {}^{A}_{B}R\,{}^{B}\vec{v}_Q \tag{2.10}$$

If in addition point Q is rotating around an arbitrary axis with the rotational velocity ${}^{A}\Omega_B$ then the linear velocity can be calculated with

$${}^{A}\vec{v}_Q = {}^{A}\vec{v}_{OB} + {}^{A}_{B}R\,{}^{B}\vec{v}_Q + {}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}Q \tag{2.11}$$

A rotational vector ${}^{B}\vec{\omega}$ related to frame B can be transferred to frame A with

$${}^{A}\vec{\omega} = {}^{A}_{B}R\,{}^{B}\vec{\omega} \tag{2.12}$$

If there are several segments which are connected to each other by rotational joints or prismatic joints, the rotational and the linear velocity can be calculated step by step for each segment starting from the base frame. The rotational velocity ${}^{i+1}\omega_{i+1}$ and the linear velocity ${}^{i+1}\vec{v}_{i+1}$ due to frame $i+1$ can be determined as:

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}e_{z_{i+1}} \tag{2.13}$$

$${}^{i+1}\vec{v}_{i+1} = {}^{i+1}_{i}R\left({}^{i}\vec{v}_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right) \tag{2.14}$$

with ${}^{i}P_{i+1}$ the vector in direction of the segment, i, ω_i the rotational velocity and θ_i the rotation of segment i around the elementary z-axis. It has to be taken into consideration that ${}^{i+1}_{i}R$ is the inverse of the orientation transformation ${}^{i}_{i+1}R$ from frame i to $i+1$. Because ${}^{i}_{i+1}R$ is an orthogonal matrix its inverse ${}^{i}_{i+1}R^{-1} = {}^{i+1}_{i}R$ is just the transposed matrix ${}^{i}_{i+1}R^{T}$ (see equation 2.15).

$$ {}^{i+1}_{i}R(\theta) = {}^{i}_{i+1}R^{-1}(\theta) = {}^{i}_{i+1}R^{T}(\theta) = \begin{pmatrix} \mathrm{c}\alpha & \mathrm{s}\alpha & x_w \\ -\mathrm{s}\alpha & \mathrm{c}\alpha & y_w \\ 0 & 0 & 1 \end{pmatrix} \tag{2.15} $$

2.2 Wheel kinematics

In the following, the stepwise calculation of the linear and the rotational velocity will be applied to wheeled vehicles operating in a 2D environment. The question to be answered is how the rotational velocity of each wheel can be determined, as the kinematic center is moved with a linear velocity $\dot{x}$ due to the x-axis, $\dot{y}$ due to the y-axis and $\dot{\theta}$ the rotational velocity around the z-axis. Also the inverse of this problem should be determined. Therefore, the basic wheel types shown in figure 2.4 will be considered.

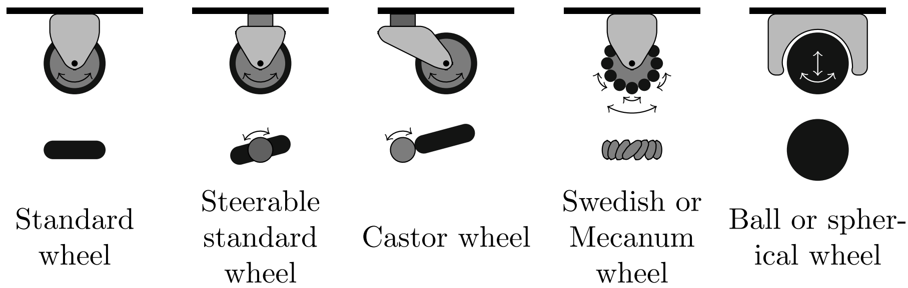

Figure 2.4 The basic wheel-types

It is obvious that for the standard and steerable standard wheel there should be no sliding orthogonal to the wheel plane (the velocity must be zero). The linear speed of each of these wheels in rolling direction can be calculated with $r\dot{\psi}$ (r is the radius of the wheel and $\dot{\psi}$ is the rotational speed of the wheel). This formula could also be applied for the rolling speed of the castor wheel and the spherical wheel. If d is the offset between the wheel axis and the vertical axis of rotation, the linear velocity orthogonal to the wheel plane is the rotational velocity around the vertical axis times the length of the offset $(-d_c\dot{\beta})$.

In case of the Swedish or Mecanum wheel, in which passive rollers are mounted in an angle γ (γ is normally 45° or 90°) on the perimeter of the main wheel, the linear velocity in rolling direction is $r\dot{\psi}\cos\gamma$. Because the Swedish or Mecanum wheel is able to move in an omnidirectional way, the speed orthogonal to the wheel plane can be calculated as $r\dot{\psi}\sin\gamma + r_{pr}\dot{\psi}_{pr}$ (with r_{pr} the radius and $\dot{\psi}_{pr}$ the rotational speed of the roller).

To determine the velocity of a wheel due to the velocity vector $\vec{v} = (\dot{x},\dot{y},\dot{\theta})^T$ of the kinematic center, one must first define the robot coordinate frame, which has its origin in the kinematic center of the vehicle (see figure 2.5). By definition, $\alpha = 0$ if the normal vector of the wheel plane is located on the x-axis and has the same orientation. β determines the angle between the straight line through the kinematic center and the fixing point of the wheel and the y-axis of the wheel frame. Parameter d is the distance from the kinematic center to the fixing point of the wheel on the chassis. The wheel coordinate system has its x-axis in the rolling direction and the y-axis as the normal to the wheel plane. The linear speed of the wheel is in the direction of the x-axis of the wheel frame. This parameter definition can be used for all wheel types. In case of the Swedish or Mecanum wheel an additional parameter γ has to be introduced, which describes the angle between the x-axis of the wheel and rolling axis of the rollers.

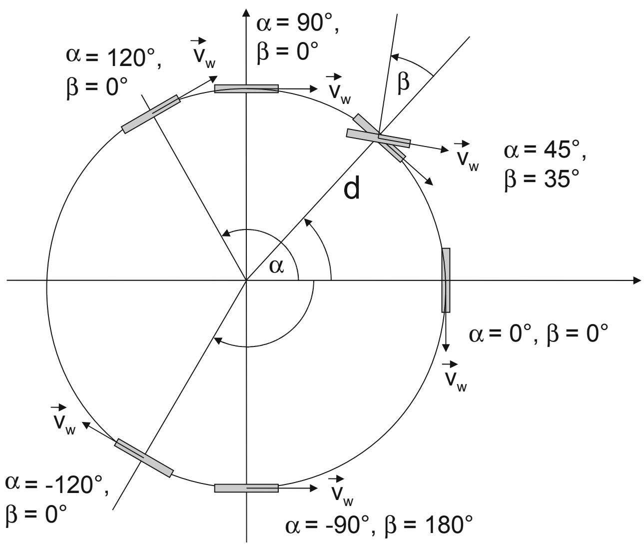

Figure 2.5 The parameters for solving the kinematic problem for different wheel positions

Supposing the velocity vector $\vec{v} = (\dot{x},\dot{y},\dot{\theta})^T$ of the kinematic center is given, equation 2.13 and equation 2.14 should be applied to calculate the linear velocity of a standard wheel. Thus, we receive ${}^1\omega_1 = (0,0,\dot{\theta})^T$ and ${}^1(\vec{v})_1 = (\dot{x},\dot{y},0)^T$. Because there is no additional rotational speed, all ${}^i\omega_i = (0,0,\dot{\theta})^T$ (see equation 2.13). A stepwise application of equation 2.14 will deliver the following ${}^i(\vec{v})_i$.

After the rotation around the z-axis with angle α, ${}^2\vec{v}_2$ is calculated as:

$$ {}^2\vec{v}_2 = \begin{pmatrix} \mathrm{c}\alpha\dot{x} + \mathrm{s}\alpha\dot{y} \\ -\mathrm{s}\alpha\dot{x} + \mathrm{c}\alpha\dot{y} \\ 0 \end{pmatrix} \tag{2.16} $$

Due to the translation d, ${}^3\vec{v}_3$ is:

$$ {}^3\vec{v}_3 = \begin{pmatrix} \mathrm{c}\alpha\dot{x} + \mathrm{s}\alpha\dot{y} \\ -\mathrm{s}\alpha\dot{x} + \mathrm{c}\alpha\dot{y} + d\dot{\theta} \\ 0 \end{pmatrix} \tag{2.17} $$

The last rotation around the z-axis with angle $\beta - 90°$ transfers the x-axis of the last frame to the rolling direction of the wheel. For the calculation of ${}^4\vec{v}_4$ in the next equation, $\sin(\beta - 90°) = -\cos(\beta)$ and $\cos(\beta - 90°) = \sin(\beta)$ is used.

$$ {}^4\vec{v}_4 = \begin{pmatrix} \mathrm{s}(\alpha+\beta)\dot{x} - \mathrm{c}(\alpha+\beta)\dot{y} - \mathrm{c}\beta d\dot{\theta} \\ \mathrm{c}(\alpha+\beta)\dot{x} + \mathrm{s}(\alpha+\beta)\dot{y} + \mathrm{s}\beta d\dot{\theta} \\ 0 \end{pmatrix} \tag{2.18} $$

The last step is to equalize the linear velocity vector of the standard wheel to that of equation 2.18.

$$ \begin{pmatrix} \mathrm{s}(\alpha+\beta) & -\mathrm{c}(\alpha+\beta) & -\mathrm{c}\beta d \\ \mathrm{c}(\alpha+\beta) & \mathrm{s}(\alpha+\beta) & \mathrm{s}\beta d \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r\dot{\psi} \\ 0 \\ 0 \end{pmatrix} \tag{2.19} $$

In case of the steerable standard wheel, equation 2.19 can be used in the same way, if the fixed angle β is replaced by a function $\beta(t)$. This equation could also be applied to the spherical wheel (because of the forces which affect the wheel and change $\beta(t)$, only a linear velocity in the rolling direction exists). If the wheel is a castor wheel, the y-component of the velocity vector is depending on the angular velocity $\dot{\beta}$ and the length of the rod (see equation 2.20).

$$\begin{pmatrix} \mathrm{s}(\alpha+\beta) & -\mathrm{c}(\alpha+\beta) & -\mathrm{c}\beta d \\ \mathrm{c}(\alpha+\beta) & \mathrm{s}(\alpha+\beta) & \mathrm{s}\beta d \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r\dot{\psi} \\ -d_c\dot{\beta} \\ 0 \end{pmatrix} \tag{2.20}$$

The Swedish or Mecanum wheel is able to move in an omnidirectional way. Therefore, lateral movement of the wheel should be possible and can be calculated with equation 2.21.

$$\begin{pmatrix} \mathrm{s}(\alpha+\beta+\gamma) & -\mathrm{c}(\alpha+\beta+\gamma) & -\mathrm{c}(\beta+\gamma)d \\ \mathrm{c}(\alpha+\beta+\gamma) & \mathrm{s}(\alpha+\beta+\gamma) & \mathrm{s}(\beta+\gamma)d \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r\dot{\psi}\cos\gamma \\ r\dot{\psi}\sin\gamma + r_{pr}\dot{\psi}_{pr} \\ 0 \end{pmatrix} \tag{2.21}$$

2.2.1 Kinematics of a differential drive vehicle

To calculate the kinematics of a differential drive, vehicle first the wheel types used have to be determined. This type of robot has two fixed standard wheels which are mounted on one axis. The kinematic center is located in the middle of the axis; the distance between the kinematic center and each wheel should be d. To solve the kinematics problem the coordinate system to define the parameters must be specified. The origin of this frame lies on the kinematic center. One solution for modelling the wheel configuration is to place the wheels on the y-axis of the coordinate frame. As shown in figure 2.5, the parameters are $\alpha_l = 90°$,$\beta_l = 0°$,$\alpha_r = -90°$,$\beta_r = 180°$. An example of a differential drive robot is MARVIN, the mobile vehicle of the University of Kaiserslautern (see figure 2.6).

Based on equation 2.19 one obtains for the left and the right wheel:

$$\begin{aligned} \mathrm{s}(\alpha_l+\beta_l)\dot{x} - \mathrm{c}(\alpha_l+\beta_l)\dot{y} - \mathrm{c}\beta_l d\dot{\theta} &= r_l\dot{\psi}_l \\ \mathrm{c}(\alpha_l+\beta_l)\dot{x} + \mathrm{s}(\alpha_l+\beta_l)\dot{y} + \mathrm{s}\beta_l d\dot{\theta} &= 0 \\ \mathrm{s}(\alpha_r+\beta_r)\dot{x} - \mathrm{c}(\alpha_r+\beta_r)\dot{y} - \mathrm{c}\beta_r d\dot{\theta} &= r_r\dot{\psi}_r \\ \mathrm{c}(\alpha_r+\alpha_r)\dot{x} + \mathrm{s}(\alpha_r+\beta_r)\dot{y} + \mathrm{s}\beta_r d\dot{\theta} &= 0 \end{aligned} \tag{2.22}$$

If the above mentioned parameters are inserted, the following equation will result:

$$\begin{aligned} \dot{x} - d\dot{\theta} &= r_l\dot{\psi}_l \\ \dot{y} &= 0 \\ \dot{x} + d\dot{\theta} &= r_r\dot{\psi}_r \\ \dot{y} &= 0 \end{aligned} \tag{2.23}$$

After solving the equation system on receives:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(r_l\dot{\psi}_l + r_r\dot{\psi}_r) \\ 0 \\ \frac{1}{2d}(-r_l\dot{\psi}_l + r_r\dot{\psi}_r) \end{pmatrix} \tag{2.24}$$

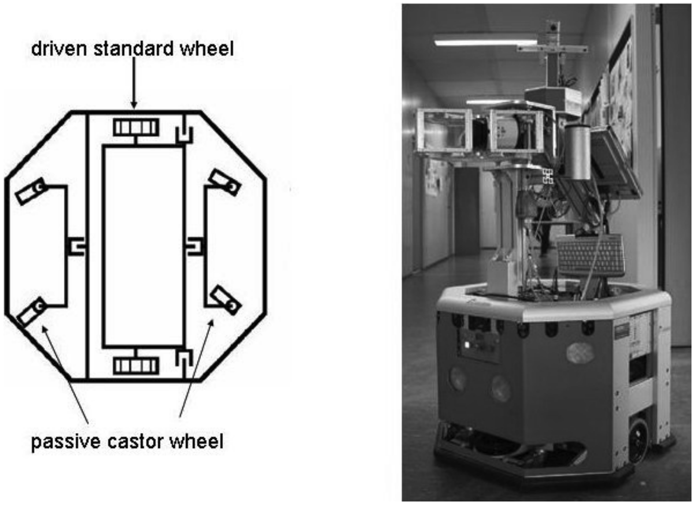

Figure 2.6 The autonomous vehicle MARVIN of the University of Kaiserslautern

2.2.2 Kinematics of an omnidirectional vehicle

To increase the mobility of a vehicle, an omnidirectional drive can be used. The climbing robot CROMSCI (see figure 2.7) of the University of Kaiserslautern, for example, is equipped with such a drive, in which 3 steerable standard wheels are mounted with an angle displacement of 120° between them (see figure 2.8). To set up the kinematics equations one can model the wheel configuration as shown in figure 2.5 with the kinematic center in the middle of the robot. The front wheel is located on the x-axis, the two rear wheels have a displacement to the front wheel of ±120°. As shown in figure 2.5 $\alpha_1 = 0°$,$\alpha_2 = 120°$,$\alpha_3 = -120°$. $\beta_{1,2,3}$ are the control parameters to determine the direction of the vehicle movements. The parameter d describes the distance between the wheel's contact point and the robot center.

For the navigation of CROMSCI it is necessary to calculate based on the desired linear and rotational velocities of the kinematic center $(\dot{x},\dot{y},\dot{\theta})^T$, the single wheel velocities and the orientations $(r_1\dot{\psi}_1,r_2\dot{\psi}_2,r_3\dot{\psi}_3,\beta_1,\beta_2,\beta_3)$.

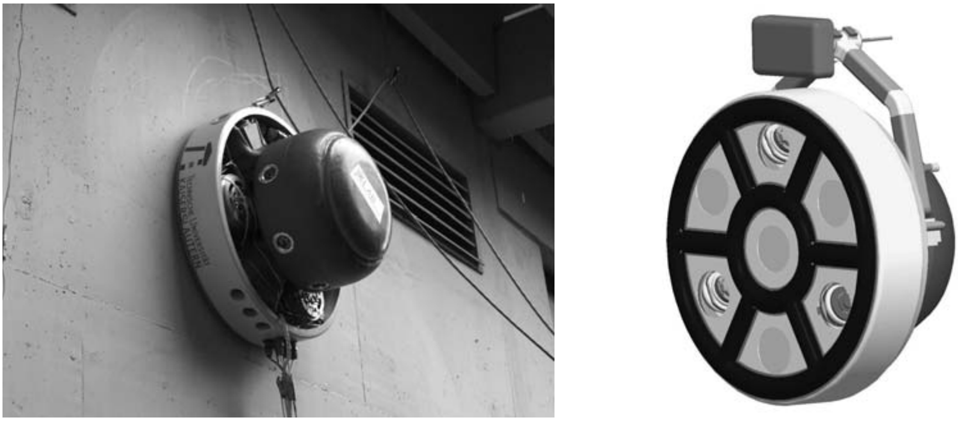

Figure 2.7 The climbing robot CROMSCI (left) and the wheel settings (right)

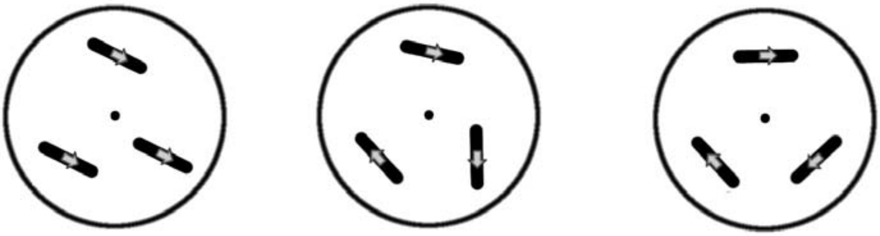

Figure 2.8 Typical orientations of the 3 steerable wheels of an omnidirectional vehicle

Applying equation 2.19 for each wheel leads to the following equation systems:

$$
\begin{aligned}
&\mathrm{s}(\alpha_1+\beta_1)\dot{x}-\mathrm{c}(\alpha_1+\beta_1)\dot{y}-d\cdot\mathrm{c}(\beta_1)\dot{\theta}=r_1\dot{\psi}_1\\
&\mathrm{s}(\alpha_2+\beta_2)\dot{x}-\mathrm{c}(\alpha_2+\beta_2)\dot{y}-d\cdot\mathrm{c}(\beta_2)\dot{\theta}=r_2\dot{\psi}_2\\
&\mathrm{s}(\alpha_3+\beta_3)\dot{x}-\mathrm{c}(\alpha_3+\beta_3)\dot{y}-d\cdot\mathrm{c}(\beta_3)\dot{\theta}=r_3\dot{\psi}_3\\
&\mathrm{c}(\alpha_1+\beta_1)\dot{x}+\mathrm{s}(\alpha_1+\beta_1)\dot{y}+d\cdot\mathrm{s}(\beta_1)\dot{\theta}=0\\
&\mathrm{c}(\alpha_2+\beta_2)\dot{x}+\mathrm{s}(\alpha_2+\beta_2)\dot{y}+d\cdot\mathrm{s}(\beta_2)\dot{\theta}=0\\
&\mathrm{c}(\alpha_3+\beta_3)\dot{x}+\mathrm{s}(\alpha_3+\beta_3)\dot{y}+d\cdot\mathrm{s}(\beta_3)\dot{\theta}=0
\end{aligned}
\qquad (2.25)
$$

Based on the last 3 equations of 2.25, the steering angles β_i,$i = 1,2,3$ are determined:

$$\begin{aligned}
&\mathrm{c}(\alpha_i + \beta_i) \cdot \dot{x} + s(\alpha_i + \beta_i) \cdot \dot{y} + d \cdot \mathrm{s}(\beta_i) \cdot \dot{\theta} = 0 \\
\Rightarrow &\mathrm{c}(\alpha_i) \cdot \mathrm{c}(\beta_i) \cdot \dot{x} - \mathrm{s}(\alpha_i) \cdot \mathrm{s}(\beta_i) \cdot \dot{x} \\
&\quad + \mathrm{s}(\alpha_i) \cdot \mathrm{c}(\beta_i) \cdot \dot{y} + \mathrm{c}(\alpha_i) \cdot \mathrm{s}(\beta_i) \cdot \dot{y} + d \cdot \mathrm{s}(\beta_i) \cdot \dot{\theta} = 0 \\
\Rightarrow &\mathrm{c}(\beta_i) \cdot (\mathrm{c}(\alpha_i) \cdot \dot{x} + \mathrm{s}(\alpha_i) \cdot \dot{y}) = \mathrm{s}(\beta_i) \cdot (\mathrm{s}(\alpha_i) \cdot \dot{x} - \mathrm{c}(\alpha_i) \cdot \dot{y} - d \cdot \dot{\theta}) \\
\Rightarrow &\tan(\beta_i) = \frac{\mathrm{s}(\beta_i)}{\mathrm{c}(\beta_i)} = \frac{\mathrm{c}(\alpha_i) \cdot \dot{x} + \mathrm{s}(\alpha_i) \cdot \dot{y}}{\mathrm{s}(\alpha_i) \cdot \dot{x} - \mathrm{c}(\alpha_i) \cdot \dot{y} - d \cdot \dot{\theta}} \\
\Rightarrow &\beta_i = \mathrm{atan2}\Big((\mathrm{c}(\alpha_i) \cdot \dot{x} + \mathrm{s}(\alpha_i) \cdot \dot{y}),(\mathrm{s}(\alpha_i) \cdot \dot{x} - \mathrm{c}(\alpha_i) \cdot \dot{y} - d \cdot \dot{\theta})\Big)
\end{aligned} \tag{2.26}$$

From equation 2.25, the angular velocity of the wheel $\dot{\psi}_i$ can be calculated using β_i:

$$\dot{\psi}_i = \frac{1}{r_i}\Big(\mathrm{s}(\alpha_i + \beta_i)\dot{x} - \mathrm{c}(\alpha_i + \beta_i)\dot{y} - d \cdot \mathrm{c}(\beta_i)\dot{\theta}\Big) \tag{2.27}$$

2.2.3 Kinematics of a vehicle with Mecanum wheels

Another drive system suited for an omnidirectional vehicle are Mecanum wheels. Those are convex cylinders arranged in a 45° angle relative to the wheel plane. Two pairs of independently driven Mecanum wheels are sufficient to enable omnidirectional movement. The orientation of the rollers of the wheels lying on a common diagonal axis is equal. The rollers of the other two wheels are oriented in the opposite direction. In figure 2.9 (right side) 4 typical movements of the vehicle are shown. If all wheels move with the same velocity in the same direction, the robot drives straight ahead. The machine will turn if the right and left wheels move in opposite direction with the same velocity. A sideward motion is possible if the neighboring wheels move in opposite direction with the same velocity. A diagonal motion results if the two wheels on the diagonal move in the same direction with the same velocity. This type of drive was applied for the vehicles PRIAMOS of Prof. Dillmann's research group at the University of Karlsruhe (see figure 2.10).

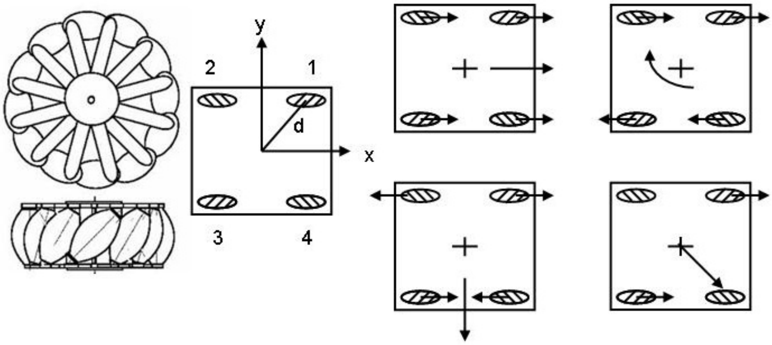

Figure 2.9 Schematic configuration of a Mecanum wheel

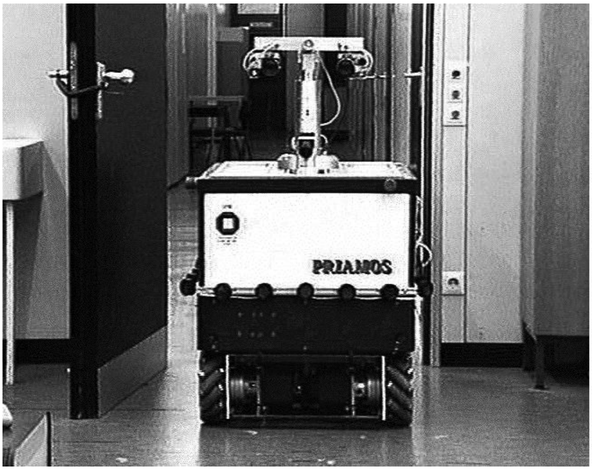

Figure 2.10 The mobile robot PRIAMOS of the University of Karlsruhe driven by Mecanum wheels [DKWW95] (courtesy of Prof. Dillmann, TH Karlsruhe)

To set up the kinematic equation, the parameters (α,β,γ) for each wheel must be determined. The order of the wheels is shown in figure 2.9. The parameters for four wheels are:

$$
\begin{aligned}
\alpha_1 &= 45^\circ, & \beta_1 &= 45^\circ, & \gamma_1 &= -45^\circ \\
\alpha_2 &= 135^\circ, & \beta_2 &= -45^\circ, & \gamma_2 &= 45^\circ \\
\alpha_3 &= -135^\circ, & \beta_3 &= 225^\circ, & \gamma_3 &= -45^\circ \\
\alpha_4 &= -45^\circ, & \beta_4 &= 135^\circ, & \gamma_4 &= 45^\circ
\end{aligned}
$$

Using equation 2.21, and supposing all driven wheels have the same radius r, the same distance d from the kinematic center and the above mentioned parameters for α,β,γ are inserted, we receive:

$$\mathrm{s}(45°)\dot{x} - \mathrm{c}(45°)\dot{y} - d\dot{\theta} = r \cdot \mathrm{c}(-45°)\dot{\psi}_1 \tag{2.28}$$

$$\mathrm{s}(135°)\dot{x} - \mathrm{c}(135°)\dot{y} - d\dot{\theta} = r \cdot \mathrm{c}(45°)\dot{\psi}_2 \tag{2.29}$$

$$\mathrm{s}(45°)\dot{x} - \mathrm{c}(45°)\dot{y} - d\dot{\theta} = r \cdot \mathrm{c}(-45°)\dot{\psi}_3 \tag{2.30}$$

$$\mathrm{s}(135°)\dot{x} - \mathrm{c}(135°)\dot{y} - d\dot{\theta} = r \cdot \mathrm{c}(45°)\dot{\psi}_4 \tag{2.31}$$

Based on this equation system, the velocities $\dot{x}$,$\dot{y}$,$\dot{\theta}$ of the kinematic center can be calculated:

$$\begin{aligned} \dot{x} &= \frac{r}{4}(\dot{\psi}_1 + \dot{\psi}_2 + \dot{\psi}_3 + \dot{\psi}_4) \\ \dot{y} &= \frac{r}{4}(-\dot{\psi}_1 + \dot{\psi}_2 - \dot{\psi}_3 + \dot{\psi}_4) \\ \dot{\theta} &= \frac{r}{d\sqrt{2}}(\dot{\psi}_1 - \dot{\psi}_2 - \dot{\psi}_3 + \dot{\psi}_4) \end{aligned} \tag{2.32}$$

The velocity vector of the kinematic center can be determined as:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \frac{r_{\text{wheel}}}{4} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ C & -C & -C & C \end{pmatrix} \cdot \begin{pmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \\ \dot{\psi}_4 \end{pmatrix} \tag{2.33}$$

with $C = \frac{2\sqrt{2}}{d}$. In order to assume the absence of slip the following must hold: $\dot{\psi}_4 = \dot{\psi}_1 + \dot{\psi}_2 - \dot{\psi}_3$

2.2.4 Pose calculation based on velocities

Using equation 2.34 and introducing the time interval Δt, the incremental paths can be determined for 2D navigation as:

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ \omega \end{pmatrix} \cdot \Delta t \tag{2.34}$$

Assuming the velocity $v = (v_x(t), v_y(t))_A$ given in a robot fixed coordinate frame (x_A, y_A), angular velocity $\omega(t)$ and the robot pose (x, y, θ) in the world coordinate system given, one can compute the trajectory as:

$$\theta(t) = \int_0^t \omega(\tau)d\tau + \theta_0 \tag{2.35}$$

$$x(t) = \int_0^t \dot{x}(\tau)d\tau + x_0 \tag{2.36}$$

$$y(t) = \int_0^t \dot{y}(\tau)d\tau + y_0 \tag{2.37}$$

Vehicle velocities are $\dot{x}$ due to the x-axis, $\dot{y}$ due to the y-axis and $\omega = \dot{\theta}$ around z-axis.

2.3 Geometrical solution for vehicle kinematics

As presented above, the drive kinematics can be calculated by using the wheel parameters. Nevertheless, geometrical solutions are commonly used if the robot is moving only in 2D space. That is because it is much easier to calculate and comprehend. In the following, some standard vehicle concepts well known from literature will be presented and the kinematics problem will be solved geometrically.

2.3.1 Differential drive

A differential drive setup consists of two independently driven wheels and thus only circular arc trajectories are possible. Therefore, two special cases occur: $R = 0$ and $R = \infty$. The former results in a rotation on the spot while the latter results in a straight route. Hence differential drives possess two independent DOF. Midway between the driven wheels the kinematic center is situated. An exemplary robot system based on differential drive is the service robot ARTOS developed at the RRLab at the University of Kaiserslautern (see figure 2.11).

Given the single wheel velocities v_l and v_r of the left respectively right wheel and a time step Δt, the length of the driven ways for each wheel and the kinematic center Δs_m can be calculated (see figure 2.12):

$$\begin{aligned}\Delta s_l &= v_l \cdot \Delta t \\ \Delta s_r &= v_r \cdot \Delta t \\ \Delta s_m &= (\Delta s_l + \Delta s_r)/2\end{aligned} \tag{2.38}$$

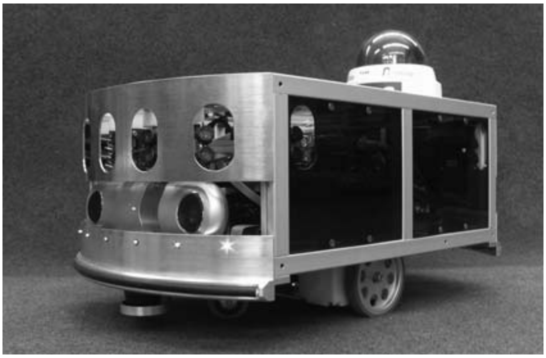

Figure 2.11 Differential drive robot ARTOS of the University of Kaiserslautern

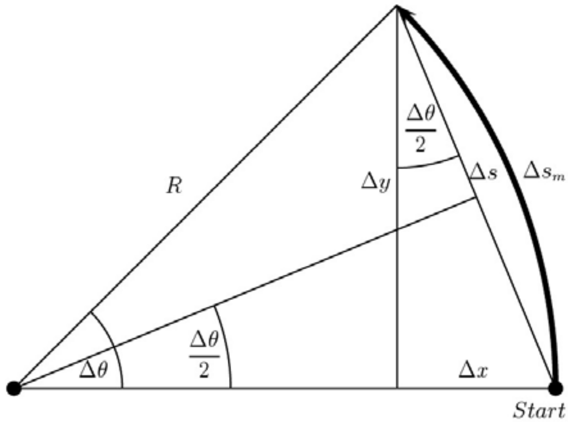

Figure 2.12 Geometrical solution of the differential drive kinematic

Based on this and the distance d between a wheel and the kinematic center, the radius R can be derived:

$$R = d \cdot \frac{\Delta s_r + \Delta s_l}{\Delta s_r - \Delta s_l} \tag{2.39}$$

Keep in mind that we drive a left curve. Otherwise the radius will be negative (curve to the right). The change in orientation can be calculated using

$$\Delta\theta = \frac{\Delta s_m}{r_m} \tag{2.40}$$

$$= \frac{\Delta s_r - \Delta s_l}{2 \cdot d} \tag{2.41}$$

For calculating translation changes one need the length of

$$\Delta s = 2 \cdot r_m \cdot \sin\left(\frac{\Delta\theta}{2}\right) \tag{2.42}$$

Based on this and the robot's orientation θ_0 at the starting position, the final changes can be derived:

$$\begin{aligned}\Delta x &= \Delta s \cdot \cos\left(\frac{\Delta\theta}{2} + \theta_0\right) \\ \Delta y &= \Delta s \cdot \sin\left(\frac{\Delta\theta}{2} + \theta_0\right)\end{aligned} \tag{2.43}$$

2.3.2 Tricycle drive

This very common setup is based on a three wheel concept, see figure 2.13. The steerable front wheel is driven while the two wheels in the back are free-wheeling. Again this simply results in circular arc trajectories, but this time the minimum radius is bigger than zero. Therefore, the robot is unable to turn on the spot. The kinematics can be derived in an analogue way to the one of Ackermann steering that will be presented in the next paragraph.

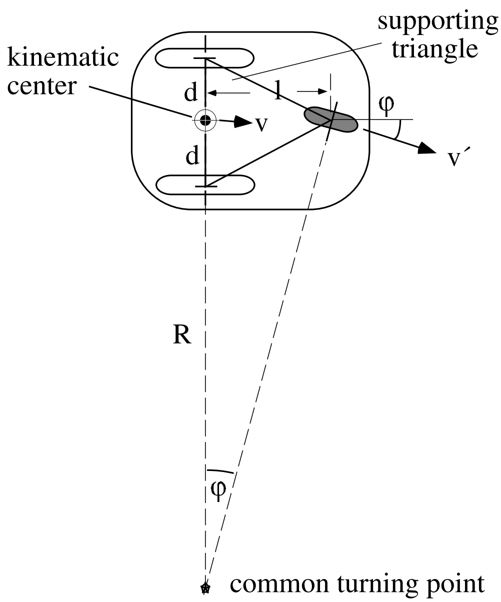

Figure 2.13 Tricycle kinematics

Suppose the steering angle φ and the linear velocity v' of the driven wheel are given, one can calculate the Radius R and the velocity v of the kinematic center with wheel distance d and axis distance l:

$$\begin{aligned} R &= l \cdot \cot \varphi \\ v' &= \frac{v}{\cos \varphi} \end{aligned} \tag{2.44}$$

These two values can be used to derive the velocities of both rear wheels:

$$\begin{aligned} v_l &= v \cdot \frac{R+d}{R} \\ v_r &= v \cdot \frac{R-d}{R} \end{aligned}$$

Based on these values the vehicles kinematics as shown before in section 2.3.1 can be calculated.

2.3.3 Ackermann steering

This specific type of drive system is mostly found in the field of automotive applications. It consists of a fixed axle and another one connecting the parallel steered wheels. In case the driven wheels are connected to the fixed axle, a differential has to be included in the setup in order to allow for curved trajectories. If the steered wheels are driven the differential is obsolete. Ackermann drive setups possess three degrees of freedom, however they are not independent. The control of an Ackermann steering is complex, as all car owners might already have experienced themselves. Nevertheless, those setups find various application in the field of robotics, for instance in the commercial outdoor platform RobuCar. The desired drive speed is denoted v_D while v_{RR} and v_{LR} denote the rear left and right wheel speed. v_{RF}, v_{LF} denote the respective speeds for the front axles while l denotes the length of the vehicle and d the distance between wheel and kinematic center.

Based on the introduces parameters and steering angle φ one can derive the circle's radius

$$R = \frac{l}{\tan \varphi} \tag{2.45}$$

and finally the four wheel velocities

$$
\begin{aligned}
v_{LR} &= \frac{(R-d)\cdot v_D}{R} \\
v_{RR} &= \frac{(R+d)\cdot v_D}{R} \\
v_{LF} &= \frac{\sqrt{(R-d)^2+l^2}\cdot|\tan\varphi|}{l}\cdot v_D \\
v_{RF} &= \frac{\sqrt{(R+d)^2+l^2}\cdot|\tan\varphi|}{l}\cdot v_D
\end{aligned}
\tag{2.46}
$$

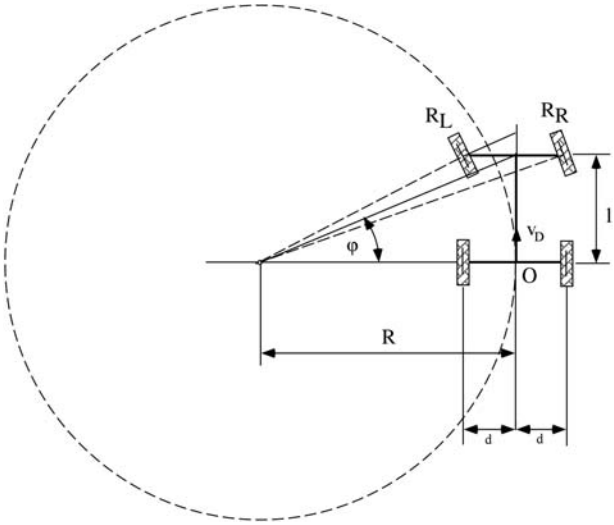

Figure 2.14 Kinematics of Ackermann steering

2.3.4 Double Ackermann steering

In a double Ackermann steering both axles are steerable, see figure 2.15. It is obvious that such a setup is kinematically even more complex and problematic than the Ackermann setup already discussed. When a curve is steered, two rotation points of the robot motion will occur. This yields slip of the single wheels. This way positioning the robot becomes quite complicated. However, the advantages are a smaller turning circle as well as the possibility of sideward motions in case both axles are steered in parallel. Especially in off-road applications (e. g. robot RAVON in figure 2.16), the errors of this configuration are lower than those of the interaction between vehicle and terrain.

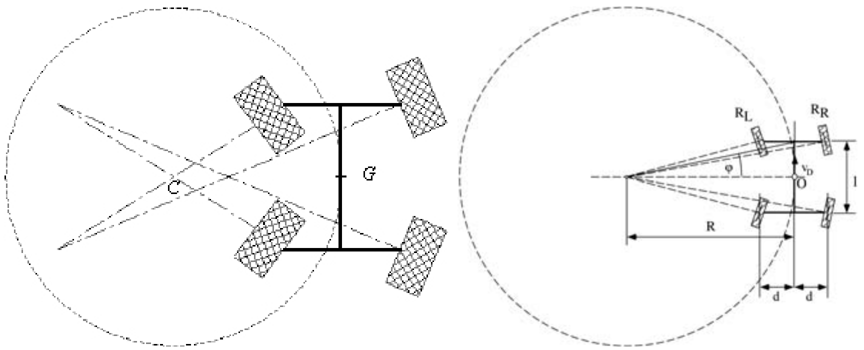

Figure 2.15 Double Ackermann steering with slip (left) and with common pivot point (right)

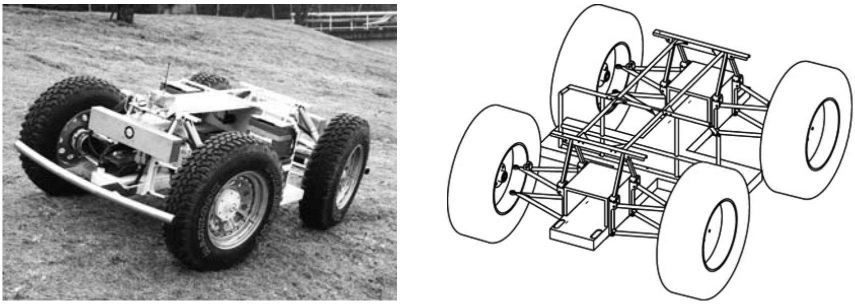

Figure 2.16 Robot RAVON of University of Kaiserslautern at an early stage

Using known length l, distance d, desired velocity v_D and steering angle φ as already presented in section 2.3.3, one can calculated the radius R and wheel velocities:

$$
\begin{aligned}
R &= \frac{l}{\tan\varphi} \\
v_{LR} &= \frac{\sqrt{\left(\frac{R}{2} - d\right)^2 + \frac{l}{4}^2} \cdot 2|\tan\varphi|}{l} \cdot v_D \\
v_{RR} &= \frac{\sqrt{\left(\frac{R}{2} + d\right)^2 + \frac{l}{4}^2} \cdot 2|\tan\varphi|}{l} \cdot v_D \\
v_{LF} &= \frac{\sqrt{\left(\frac{R}{2} - d\right)^2 + \frac{l}{4}^2} \cdot 2|\tan\varphi|}{l} \cdot v_D \\
v_{RF} &= \frac{\sqrt{\left(\frac{R}{2} + d\right)^2 + \frac{l}{4}^2} \cdot 2|\tan\varphi|}{l} \cdot v_D
\end{aligned}
\qquad (2.47)
$$

2.3.5 Synchro drive

The main feature of this type of drive system is that all wheels are equally steered, see figure 2.17. The minimum amount of motors required is two: the first one drives the wheels by either a chain or a belt, the second one is responsible for controlling the steering angle. Thus, all wheels are always rotating equally fast and are facing the same way. A vehicle equipped with such a drive setup is able to reach any given point in a plane but is limited to two DOF since it cannot rotate. This is of special importance for the layout of the sensors since they will not necessarily face the direction of the movement. An example for a robot that relies on such a drive setup is the industrial service robot Viper [GRD98].

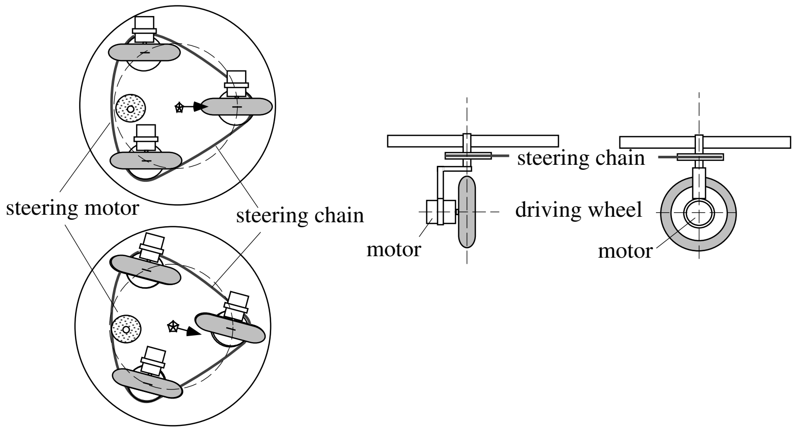

Figure 2.17 Schematics of a basic synchro drive

The deduction of the drive kinematics is straightforward because only basic trigonometry is applied to derive the solution. Let Δs denote the length of the travelled path of the driven wheels while φ is the steering angle. Thus we receive

$$\begin{aligned} \Delta x &= \Delta s \cdot \cos\varphi \\ \Delta y &= \Delta s \cdot \sin\varphi \end{aligned} \tag{2.48}$$

If used the other way around, the above equation will determine the desired parameters:

$$\begin{aligned} \Delta s &= \sqrt{\Delta x^2 + \Delta y^2} \\ \varphi &= \arctan \frac{\Delta y}{\Delta x} \end{aligned} \tag{2.49}$$

As already mentioned the synchro drive is unable to perform rotations. This, however, implies that $\Delta\varphi = 0 = const$ holds true!

2.3.6 Omnidrive

An omnidrive system consists of a minimum of two independently steered wheels with one or more free-wheeling passive wheels serving as supporting wheels. Thus the vehicle is able to move in a plane with three DOF.

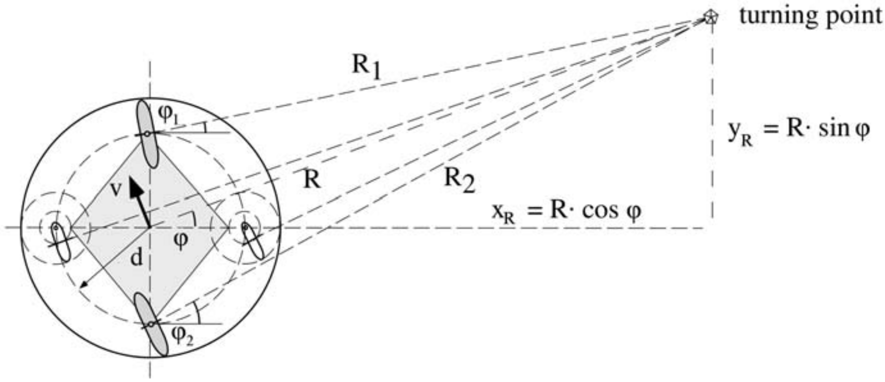

Figure 2.18 Omnidrive kinematics

Motion radius R, steering angle φ, linear velocity v of the kinematic center and wheel distance d are given. To calculate the single wheel orientations and velocities that are needed, the following helping distances can be used:

$$\begin{aligned} x_R &= R \cdot \cos\varphi \\ y_R &= R \cdot \sin\varphi \end{aligned} \tag{2.50}$$

Based on this one can calculate the wheel parameters of the first

$$\begin{aligned} \varphi_1 &= \arctan \frac{y_R - d}{x_R} \\ R_1 &= R \cdot \frac{\cos\varphi}{\cos\varphi_1} \\ v_1 &= v \cdot \frac{R_1}{R} \end{aligned} \tag{2.51}$$

and of the second wheel:

$$\begin{aligned} \varphi_2 &= \arctan \frac{y_R + d}{x_R} \\ R_2 &= R \cdot \frac{\cos\varphi}{\cos\varphi_2} \\ v_2 &= v \cdot \frac{R_2}{R} \end{aligned} \tag{2.52}$$

Regarding a generalized omnidrive one needs different parameters which describe the kinematic setup. At the distance d from the kinematic center of a vehicle there is a wheel at coordinates (x_i,y_i) with radius r. At an orientation φ with respect to the x-axis the vehicle drives at a velocity v and has an angular speed of ω as shown in figure 2.19.

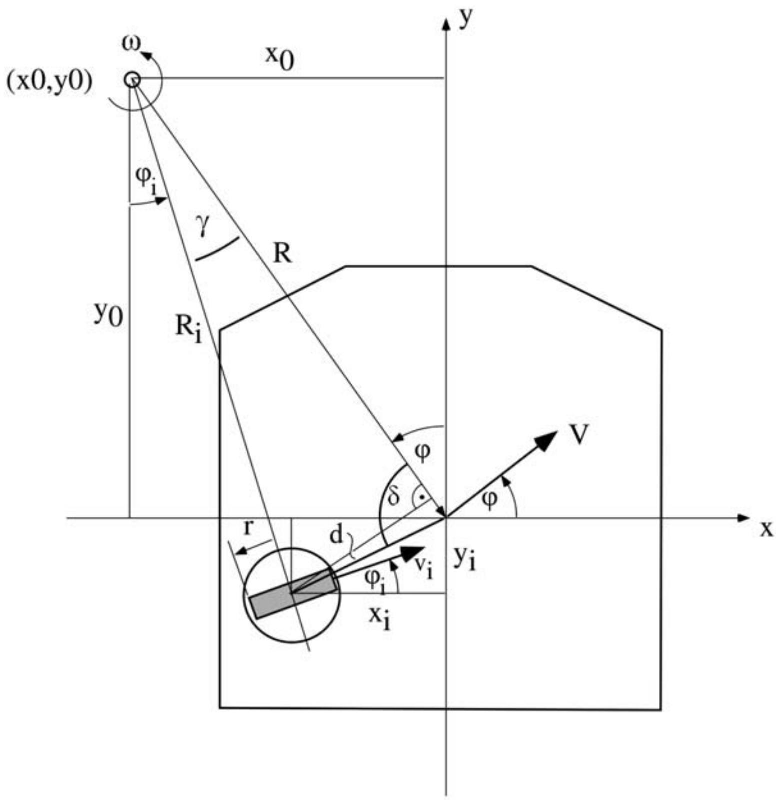

Figure 2.19 Generalized omnidrive

Given the linear velocity v, angular velocity ω, angle φ and the coordinates (x_i,y_i) of a driving wheel number i one can calculate the pivot point with coordinates (x_0,y_0) and the motion radius R:

$$\begin{aligned} R &= \frac{v}{\omega} \\ x_0 &= -R\sin\varphi \\ y_0 &= R\cos\varphi \end{aligned} \tag{2.53}$$

By using these values the steering angle φ_i and the angular velocity ψ_i of wheel i can be derived:

$$\begin{aligned} R_i &= \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \\ \psi_i &= \omega \frac{R_i}{r} \\ \varphi_i &= \sin^{-1}\left(\frac{x_0 - x_i}{R_i}\right) \end{aligned} \tag{2.54}$$

Driving straight on means $R \to \infty \Longrightarrow \varphi_i = \varphi \;;\; \psi_i = v/r$.

2.4 Applying mobile robot kinematics

With the kinematic models obtained above, it is now possible to solve simple navigation tasks. These models are the basis for any mobile robot application. The first problem to be solved is localization. Based on the angular velocities of the wheels and the kinematic parameters of the vehicle, one can determine the robot pose on a 2D plane. The resulting pose is only a first estimation because of slip effects of the wheels.

In figure 2.20, a typical scenario is presented in which a differential drive robot is supposed to move along a square of 10 m × 10 m. The wheel velocities are calculated due to its kinematics and the desired path. These velocities could directly be used for the closed-loop controllers, which determine e. g. the power of the motors. The ellipses show the area in which the real position of the robot could be for each step of movement. This error is dependent on the kinematics of the vehicle. One can observe that the size of the area is increasing with movement, because of the summation of slippage error. It is also shown that the distance error is smaller than the rotational error for a differential drive robot. This results in the elliptic shape of the areas. The orientation of the different ellipses depends on the orientation of the robotic system. This so-called dead reckoning localization is sufficient for short paths but not precise enough for navigation. In chapter 4 different methods for solving the localization problem will be introduced.

The kinematics could also be used to determine the DOF (degree of freedom) of the robot. In [SN04] the DOF are separated in degree of mobility and degree of steerability. The mobility describes possible independent motions based on changes to wheel velocities which are not restricted by kinematic constraints like the sliding condition. In the case of a differential drive robot the mobility degree is 2, because a movement along the robots' y-axis is restricted. Not all three motions of a vehicle in a plane (x-direction, y-direction, rotation) are possible at the same time. The steerability indicates the number of steerable wheels, which could be independently controlled. In case of a differential drive robot the steerability is 0, in case of a tricycle it is 1. The summation of both degrees leads to the maneuverability of the robot system.

Overall, kinematics is the foundation for solving different problems like localization, navigation, or SLAM, which will be presented in the next chapters.

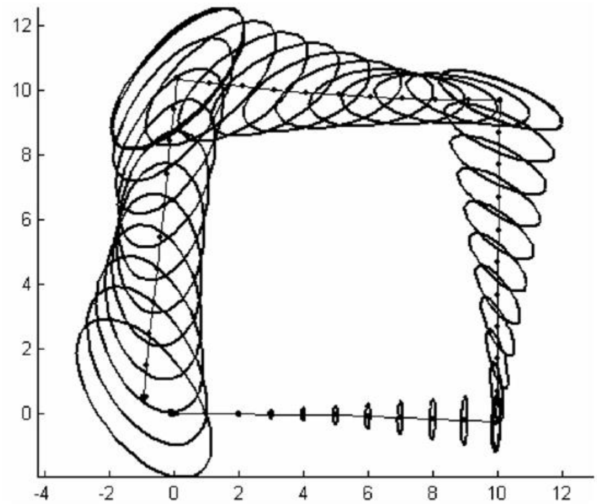

Figure 2.20 Typical error caused by odometry using a differential drive