

Chapter 11

Assessing the Quality of the Management of Degree Programs by Latent Class Analysis

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11.1 Introduction

In the evaluation of university quality, questionnaires with multi-item scales (Likert type) are often used in order to measure specific characteristics which are known to be relevant for the evaluation. The joint distribution of multiple responses provides a complete information in order to attach an overall measure of perceived quality to each student.

The aim of this chapter is to point out classes (clusters) of students (cases) who share a homogenous perception of the quality of the management of their degree programs and to highlight profiles of responses which define each of the identified classes. Latent Class Analysis (LCA) is the modeling approach applied in order to sort out latent classes of observations from a multi-way table of polytomous variables. The ranking of classes has been made using an overall measure of dissimilarity between distributions. The procedure has been used in order to propose a composite indicator of the quality level of the degree program.

This chapter is divided into 5 sections. In Sect. 11.2 the process of building composite indicators of quality of services is discussed and some of the main critical steps are highlighted. In Sect. 11.3 the LCA approach is described. In Sect. 11.4 the proposed method is applied to data on university course evaluation. Section 11.5 provides some final remarks.

11.2 Building up a Composite Indicator

To make clearer and faster comparisons and to highlight possible critical aspects the evaluation process needs practical tools which allow to sort out objects and units (teachers, tutors, courses, facilities, etc.). These tools are usually composite indicators which summarize evaluations expressed by respondents to different indicator

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variables. The process of building up composite indicators is characterized by a high level of arbitrariness in the definition of many critical components [6]:

1. the *indicator variables* adopted in order to operationalize the attribute;
2. the *transformations* applied in order to re-scale the set of indicator variables;
3. the *weighting scheme* selected in order to discriminate the relevance of each of the re-scaled indicator variables;
4. the *merging function* used in order to summarize multiple indicators in a single statement.

The final results are strongly influenced by researchers' choices and no definitive solutions have been so far proposed in the literature. The use of a modeling approach, especially in the explorative phase, may support and validate researchers' decisions concerning transformations, merging functions, scaling methods, weighting schemes, etc. Most of the statistical models used for measuring unobservable variables throughout indirect indicators are known as Latent Variables Models (Structural Equations Models, Item Response Models, Latent Class Analysis, Classical Scaling Methods, Partial Least Squares Regression etc. [2, 9, 10, 14, 16, 18]) and their use has widely increased in the last decade [4–7, 15, 17].

11.2.1 A Measure of the Perceived Quality of a University Service

In this work, it is assumed by hypothesis that the unobservable attribute *quality of a degree program* is measured indirectly by classifying respondents into groups which are homogeneous in terms of the perceived level of satisfaction of their members. The intensity of the attribute owned by each class needs to be assessed. Specifically, the work focuses on the following steps:

- to set up a statistical approach which allows to sort out mutually exclusive groups (classes) of students characterized by a different perception of the *quality of the management* (QM) of their degree program;
- to sketch the profile of each class (cluster) on the basis of the intensity of the latent attribute;
- to sort classes on the basis of the intensity of the attribute as perceived by students (from the *lowest* to the *highest* intensity);
- to rank degree programs on the basis of the distributions of students across classes.

The approach followed uses tools provided by LCA in order to spot out mutually exclusive classes of students. Each latent class groups together students who share the same perceived level of the quality of the managing of their degree program. Cases are classified into clusters on the basis of posterior probabilities estimated directly from students' response patterns to the items of the questionnaire. Next, classes are sorted moving from a measure of distance between distributions.

11.3 Methodological Issues

LCA aims to identify a number R of categorical classes which clusters observations characterized by a different intensity of the latent variable θ – which is supposed to be categorical – moving from individual responses to a set of categorical indicator variables (i.e. moving from the cross classification of J polytomous indicators). The model assumes that any dependency across responses provided to manifest indicators is explained “by a single unobserved ‘latent’ categorical variable” [12] θ which takes categories $\theta_r (\theta_1, \dots, \theta_R)$. Responses to manifest indicators are independent conditional upon the values of the latent variable

$$\pi_{y_1 \dots y_J | \theta_r} = P(Y_1 = y_1 | \theta_r) \dots P(Y_J = y_J | \theta_r). \tag{1}$$

In order to simplify the notation in the following we denote θ_r just with r . More specifically, by indicating with Y_{ijk} the indicator variable which assumes value 1 if student i ($i = 1, \dots, n$) selects category (outcome) k ($k = 1, \dots, K$ the categories) of item j ($j = 1, \dots, J$, the manifest indicators), the probability that an individual i in class r of the latent variable θ has a particular response pattern is given by

$$f(Y_i; \pi_r) = \prod_{j=1}^J \prod_{k=1}^K (\pi_{rjk})^{Y_{ijk}} \tag{2}$$

where π_{rjk} is the probability that an observation in latent class r provides the k outcome to item j . The model maximizes the log-likelihood

$$\sum_{i=1}^N \ln \sum_{r=1}^R p_r \prod_{j=1}^J \prod_{k=1}^K (\pi_{rjk})^{Y_{ijk}} \tag{3}$$

with respect to π_{rjk} and p_r . The latter is the probability to belong to each of the r classes.

The `poLCA` package in R-language [12] uses the algorithm *EM* (expectation-maximization). Moving from the estimates of \hat{p}_r and $\hat{\pi}_{rjk}$, the posterior probability that a unit which provides a particular set of responses belongs to a specific class is calculated using Bayes’ theorem

$$\hat{P}(r | Y_i) = \frac{\hat{p}_r f(Y_i; \hat{\pi}_r)}{\sum_{r=1}^R \hat{\pi}_r f(Y_i; \hat{\pi}_r)}. \tag{4}$$

The algorithm starts using (4) (setting initial guesses for the parameters $\hat{\pi}_{rjk}$ and \hat{p}_r) in order to estimate the posterior probability that an individual belongs to a class conditional upon the observed pattern of responses on the J items.

In the second step the log-likelihood function with the updated values of $\hat{\pi}_{rjk}$ and \hat{p}_r is maximized. The two steps are automatically iterated until the convergence

in the log-likelihood is reached. The prediction of the latent class memberships can be improved by using the information available on unit characteristics. The model with covariates is known as Latent Regression Class Model [1, 13].

11.3.1 Sorting Latent Classes

Classes have been sorted moving from the vector of estimated parameters $\hat{\pi}_{rj}$. It has been assessed how much the observed item response probability of each item was dissimilar from the expected item response probability of a hypothetical class of “Zero Satisfied Students” (ZSS). The latter is an extreme distribution with all observations clustered in the first response category.

Different approaches can be used in order to compare two distributions [7, 15] in terms of distance. The dissimilarity index [11] for ordered variables has been adopted. Indicating with F_A and F_B the cumulated probability distributions of two categorical ordered variables “A” and “B” with K categories, the dissimilarity between distributions can be assessed by

$$Z' = \sum_{k=1}^{K-1} |F_{A_k} - F_{B_k}|, \quad (5)$$

the maximum values the index can assume is equal to $K - 1$. Thus the relative index z' is

$$z' = \frac{1}{K - 1} \sum_{k=1}^{K-1} |F_{A_k} - F_{B_k}|; \quad (6)$$

it varies between [0,1] (the value is 0 when the two distributions are similar). As a proxy of the overall level of satisfaction of the class the average value of the dissimilarity index calculated on the entire set of items has been used. Classes have been ranked in a *continuum* according to the values of this indicator.

11.4 The Application

11.4.1 The Data

The application deals with data on the evaluation of university courses gathered at the Faculty of Economics of the University of Palermo in 2004–2005. The evaluation form used in the survey is divided in separated sections in which students provide information on biographical details, university career, and students’ assessments on several aspects of university courses (facilities, lecture programs, teaching). The second column of Table 11.1 shows the distribution of the evaluation forms

Table 11.1 Evaluations collected per degree program

Degree	# Evaluations	%
A	32	1.70
B	44	2.33
C	109	5.78
D	297	15.74
E	245	12.98
F	344	18.23
G	816	43.24
Total	1887	100.00

according to which degree program (DP) the course belongs to. The number of questionnaires collected by DP varies from 32 to 816.

In the main section of the evaluation form items take the form of questions (propositions) to which the student is invited to attest how much she/he agrees. All items in the main sections are measured on a four categories Likert scale: *Definitely No* (DN), *More No than Yes* (MN), *More Yes than No* (MY), *Definitely Yes* (DY). This work looks over the joint distribution of 5 items devoted to collect students' opinions on the management of the degree scheme. We selected items concerning the evaluation of general management aspects (QM): the coordination among courses (I_1), the overall workload during the term (I_2), the scheduled hours of the lectures (I_3), the overall organization (I_4), the facilities of the classroom (I_5). The internal consistency reliability of the scale has been assessed using the *Cronbach's α* [8] coefficient, which signals the degree of the internal homogeneity of the selected indicator variables (if they are measuring or not the same dimension of the underlying variable). The coefficient assumes values between 0 and 1, the closer is the value to 1, the higher is the internal reliability of the scale. The *Cronbach's α* has been calculated for the whole scale (0.709) and removing each of the 5 items (Table 11.2). The moderately-high level of the coefficient is consistent with the assumption that the selected indicators load on the same dimension.

Table 11.3 exhibits the rate of responses for each category of the 5 ordinal indicators. The bulk of the responses is in the categories "MN" and "MY": "MY" is always the modal and median category with a rate of observations between 30.45 and 41.57%.

Table 11.2 *Cronbach's α* coefficients

Items	<i>Cronbach's α</i>
'Are you satisfied about ...	
I_1 ... the degree of coordination among courses'	Without I_1 0.733
I_2 ... the overall workload during a term'	Without I_2 0.779
I_3 ... the scheduled hours of lectures'	Without I_3 0.658
I_4 ... the overall organization'	Without I_4 0.665
I_5 ... the facilities in the classroom'	Without I_5 0.669
Whole scale	0.709

Table 11.3 Percentage of responses in each category

Item	Def. no	Mod. no	Mod. yes	Def. yes
I_1	15.21	23.72	39.11	21.96
I_2	16.86	24.96	41.57	16.61
I_3	18.39	35.37	36.89	9.34
I_4	16.67	23.80	37.12	22.42
I_5	27.17	27.57	30.45	14.81

11.4.2 The Analysis

The LCA approach is adopted in an exploratory way. The analysis starts by increasing the number of latent classes moving from a complete independent model with just one latent class. Models with a different number of latent classes are compared in terms of BIC or AIC; the first is recommended for basic latent class models [13]. The LCA measures of goodness of fit are displayed in Table 11.4. Moving from M_2 to M_4 both measures recommend an increase in the number of latent classes, whereas moving from M_4 to M_6 the AIC decreases and the BIC increases making not straightforward the choice between the two models. The procedure has been applied for the 4 and 5 classes models and results have been compared in terms of their readability and power to highlight classes of observations which are characterized by a different intensity of the latent attribute “satisfaction”. Analysis in the following refers to the 5 classes model (M_5); it exhibits the greatest distance in the *continuum* between scores assigned to extreme classes (*unsatisfied, satisfied*). The LCA model has been estimated several times in order to avoid local maxima. Moreover, just to list classes in the output according to an ordering criteria the model has been run again applying the function `poLCA.reorder` implemented in the package `poLCA` [13].

The item response probability conditional upon the latent class memberships ($\hat{\pi}_{rjk}$) are reported in Table 11.5. We can observe that C_1 contains students who have the highest probability to score “DN” for all items, thus it represents the class of the “*unsatisfied students*”. C_4 groups those students who prevalently score the category “MY” and it is labeled the class of “*moderately satisfied students*”. Students in class 5 have a probability which spans between 0.43 and 0.83 to answer “DY”; thus the class represents the cluster of “*satisfied students*”. It seems to be sensible to sort out

Table 11.4 Measures of goodness of fit

#Classes	Model	#Par.s	Measures of goodness of fit		
6	M_6	95	AIC: 23357.9	BIC: 23884.4	Dev: 929.9
5	M_5	79	AIC: 23375.4	BIC: 23813.2	Dev: 979.4
4	M_4	63	AIC: 23419.6	BIC: 23768.8	Dev: 1055.7
3	M_3	47	AIC: 23519.7	BIC: 23780.2	Dev: 1187.8
2	M_2	31	AIC: 23969.9	BIC: 24141.7	Dev: 1669.9
1	M_1	15	AIC: 24881.7	BIC: 24964.9	Dev: 2613.8

Table 11.5 Item response probability conditional upon latent class memberships

Item	$\hat{\pi}_{rjDN}$	$\hat{\pi}_{rjMN}$	$\hat{\pi}_{rjMY}$	$\hat{\pi}_{rjDY}$
Class 1				
I_1	0.32	0.16	0.30	0.22
I_2	0.80	0.11	0.06	0.03
I_3	0.73	0.19	0.04	0.04
I_4	0.72	0.17	0.09	0.03
I_5	0.76	0.14	0.06	0.04
Class 2				
I_1	0.09	0.25	0.50	0.15
I_2	0.20	0.54	0.25	0.00
I_3	0.24	0.68	0.08	0.00
I_4	0.17	0.53	0.29	0.01
I_5	0.37	0.39	0.24	0.01
Class 3				
I_1	0.24	0.19	0.29	0.27
I_2	0.12	0.25	0.41	0.22
I_3	0.19	0.36	0.39	0.05
I_4	0.13	0.13	0.29	0.46
I_5	0.25	0.22	0.27	0.26
Class 4				
I_1	0.05	0.29	0.49	0.17
I_2	0.01	0.15	0.75	0.08
I_3	0.00	0.28	0.67	0.05
I_4	0.02	0.20	0.68	0.09
I_5	0.12	0.32	0.47	0.10
Class 5				
I_1	0.12	0.19	0.26	0.43
I_2	0.00	0.02	0.15	0.83
I_3	0.00	0.00	0.25	0.74
I_4	0.02	0.01	0.20	0.78
I_5	0.13	0.10	0.31	0.46

classes from the *least satisfied* to the *most satisfied* $C_1 < C_4 < C_5$. No clear sorting order appears for classes C_2 and C_3 .

The predicted class memberships by modal posterior probability for the 5 class model are equal to $\bar{p}_r(0.104, 0.261, 0.232, 0.316, 0.085)$. The closeness of predicted and estimated shares of class memberships $\hat{p}_r(0.111, 0.232, 0.269, 0.307, 0.081)$ is a further measure of the goodness of fit of the selected model [13].

The criteria used to sort out the latent classes (i.e., to locate them in a *continuum*) and to set the order in the previously mentioned `poLCA.reorder` function has been described in Sect. 11.3.1. Denoting with \hat{P}_{rjk} the cumulated distribution of $\hat{\pi}_{rjk}$ and with P_{ZSSjk} the cumulated distribution of p_{ZSSjk} [$P_{ZSSjk} = (1, 1, 1, 1)$] for the class of ZSS, the dissimilarity index has been calculated for each of the J item in each of the R classes

$$z'_{rj} = \frac{1}{K-1} \sum_{k=1}^K |P_{rjk} - P_{ZSSjk}|. \quad (7)$$

The average value of z'_{rj} calculated for each class (\bar{z}'_r) is used to sort classes in a *continuum* and to handle them in further analysis as ordered categories.

$$\bar{z}'_r = \frac{1}{J} \sum_{j=1}^J z'_{rj}. \quad (8)$$

The ranking of the classes, from the “least satisfied” – least likely to score a higher category – to the “most satisfied” – most likely to score a higher category – is: $C_1 < C_2 < C_3 < C_4 < C_5$. Table 11.6 shows the value of z' for the five items in each latent class. Almost all items show an increasing value of z'_{rj} moving from class 1 to class 5 (with the exception of item I_4 which decreases from class C_3 and C_4).

The last two rows in Table 11.6 show the value of \bar{z}'_r and their standard deviations within each class. The *within* class variability of z'_{rj} can be seen as a measure of reliability of \bar{z}'_r . The *between class* variability of z'_{rj} (0.045) explains 81% of the *total* variability of z'_{rj} (0.055). Scores attached to each class in order to locate it in the continuum are \bar{z}'_r (0.195, 0.375, 0.550, 0.589, 0.824). From the values of \bar{z}'_r arises that class 3 and class 4 identify students with a similar perception of the QM. A deeper look to the item response probability of both classes shows that students in the first class discriminate more (the modal category varies across items); nevertheless students in class 4 have a higher probability to provide outcomes in the category “MY”.

In order to validate the use of \bar{z}'_r as overall index of satisfaction of the class, the dissimilarity index across pairs of estimated item response probabilities within each class has been calculated. Results depicted in Table 11.7 show that the dissimilarity of the distributions of the items within each class is quite low and the highest value observed is 0.38 for class 2. The predicted class membership probabilities highlight that the 10.5% of surveyed students are *unsatisfied*, the 31.6% are *moderately satisfied* and the 8.5% belong to the class of *satisfied students*.

Table 11.6 A comparison across classes using z' values (M_5)

Item	$C_1: z'_{1j}$	$C_2: z'_{2j}$	$C_3: z'_{3j}$	$C_4: z'_{4j}$	$C_5: z'_{5j}$
I_1	0.474	0.572	0.531	0.592	0.664
I_2	0.102	0.353	0.579	0.636	0.935
I_3	0.127	0.279	0.438	0.585	0.912
I_4	0.143	0.378	0.689	0.615	0.911
I_5	0.128	0.293	0.514	0.515	0.701
\bar{z}'_r	0.195	0.375	0.550	0.589	0.824
$sd(z')$	0.018	0.010	0.006	0.002	0.013

The procedure adopted to score classes has been replicated for the 4 classes model in order to assess the sensibility of the scoring method to the number of classes. By applying the same procedure the vector of score attached to classes is equal to \hat{z}_r (0.233, 0.375, 0.570, 0.724); the range of variation of \bar{z}'_r is smaller but contiguous categories differentiate more across students with a different perception of the overall quality. However, as a consequence of the narrower range of variation of \bar{z}'_r the rate of variability in \bar{z}'_r explained by the *between* classes variability is slightly lower (78.5%) with respect to the 5 classes model. Results in terms of scores assigned to latent classes are summarized in Table 11.8. The rate of students in each class assigned by modal posterior probability \bar{p}_r (0.1298, 0.239, 0.4822, 0.1489) shows that the choice of a simpler model changes the distributions of students across classes with a greater clustering in the extreme ones. Thus even if the 4 classes

Table 11.7 Matrix of dissimilarity between pairs of items in each class

Item	I_1	I_2	I_3	I_4	I_5
Class 1					
I_1	0.00	0.32	0.29	0.27	0.29
I_2	0.32	0.00	0.06	0.06	0.03
I_3	0.29	0.06	0.00	0.03	0.04
I_4	0.27	0.06	0.03	0.00	0.04
I_5	0.29	0.03	0.04	0.04	0.00
Class 2					
I_1	0.00	0.27	0.38	0.24	0.28
I_2	0.27	0.00	0.12	0.03	0.12
I_3	0.38	0.12	0.00	0.15	0.19
I_4	0.24	0.03	0.15	0.00	0.13
I_5	0.28	0.12	0.19	0.13	0.00
Class 3					
I_1	0.00	0.11	0.18	0.12	0.02
I_2	0.11	0.00	0.12	0.16	0.11
I_3	0.18	0.12	0.00	0.27	0.18
I_4	0.12	0.16	0.27	0.00	0.14
I_5	0.02	0.11	0.18	0.14	0.00
Class 4					
I_1	0.00	0.18	0.12	0.13	0.06
I_2	0.18	0.00	0.09	0.05	0.19
I_3	0.12	0.09	0.00	0.06	0.13
I_4	0.13	0.05	0.06	0.00	0.14
I_5	0.06	0.19	0.13	0.14	0.00
Class 5					
I_1	0.00	0.27	0.21	0.23	0.06
I_2	0.27	0.00	0.07	0.04	0.25
I_3	0.21	0.07	0.00	0.04	0.19
I_4	0.23	0.04	0.04	0.00	0.21
I_5	0.06	0.25	0.19	0.21	0.00

Table 11.8 A comparison across classes using z' values (M_4)

Item	$C_1: z'_{1j}$	$C_2: z'_{2j}$	$C_3: z'_{3j}$	$C_4: z'_{4j}$
I_1	0.470	0.571	0.574	0.603
I_2	0.156	0.347	0.614	0.800
I_3	0.153	0.267	0.545	0.685
I_4	0.204	0.398	0.612	0.883
I_5	0.181	0.292	0.505	0.648
\bar{z}'	0.233	0.375	0.570	0.724
$sd(z')$	0.014	0.012	0.002	0.011

model would be straightforwardly adopted in an analysis aimed to identify clusters of students, it would led to a lost of useful information if the final aim is to summarize results in a synthetic indicator.

A composite indicator of students' perceived quality of the management of the degree programs at faculty level is obtained as a linear combination of the scores assigned to each latent class \bar{z}'_r weighted for the of rate of students \bar{p}_r

$$IS = \sum_{r=1}^R \bar{p}_r \bar{z}'_r. \tag{9}$$

In the following the sensibility of the composite indicator to the choice of the number of classes as been assessed. For the 5 classes model the value of the indicator at faculty level is equal to $IS = 0.503$; it is calculated as a combination of the system of weights $\bar{p}_r(0.105, 0.261, 0.232, 0.316, 0.085)$ with the scores $\bar{z}'_r(0.195, 0.375, 0.550, 0.589, 0.824)$. The composite indicator calculated for model 4 shows a similar value ($IS = 0.502$).

A comparison among the 7 degree programs of the faculty has been made moving from the rate of students in the five latent classes. The indicator IS_{DP} has been applied to the seven degree programs weighting the rate of observations in each class with \bar{z}'_r . Results are provided in the last columns of Tables 11.9 and 11.10. Table 11.9 shows that the main differences in the distributions of students across the five classes are observed in the rate of cases in the extreme categories: the lowest rates of *unsatisfied students* are observed for degree programs "A", "F" and "C", while the highest rates of *satisfied students* are observed for degree programs "D",

Table 11.9 Frequencies of observations in each class by degree program (M_5)

Degree program	$C_1: \bar{p}_1$	$C_2: \bar{p}_2$	$C_3: \bar{p}_3$	$C_4: \bar{p}_4$	$C_5: \bar{p}_5$	IS_{DP}
G	0.126	0.319	0.208	0.276	0.071	0.479
E	0.121	0.253	0.246	0.306	0.074	0.495
B	0.091	0.227	0.250	0.318	0.114	0.521
A	0.031	0.312	0.250	0.312	0.094	0.522
D	0.102	0.184	0.269	0.318	0.127	0.529
F	0.064	0.195	0.270	0.387	0.084	0.531
C	0.064	0.239	0.156	0.422	0.119	0.534
Faculty	0.105	0.261	0.232	0.316	0.085	0.503

Table 11.10 Frequencies of observations in each class by degree program (M_4)

Degree program	$C_1: \bar{p}_1$	$C_2: \bar{p}_2$	$C_3: \bar{p}_3$	$C_4: \bar{p}_4$	IS_{DP}
G	0.151	0.286	0.435	0.129	0.483
E	0.152	0.236	0.495	0.118	0.491
A	0.062	0.312	0.469	0.156	0.512
B	0.114	0.205	0.477	0.205	0.523
D	0.131	0.171	0.494	0.204	0.524
C	0.101	0.211	0.495	0.193	0.524
F	0.078	0.186	0.573	0.163	0.532
Faculty	0.130	0.239	0.482	0.149	0.502

“C” and “B”. The main evidence comparing the seven distributions is that the degree programs have a high rate of students in the class of *moderately satisfied*. Results of M_4 are consistent with those obtained for M_5 . The small variability across the IS_{DP} values could be partially explained considering that the degree programs belong to the same faculty and thus they largely share a common management.

11.5 Final Remarks

The chapter provides a method to rank degree schemes moving from students’ joint response pattern to a set of ordered indicators. The LCA approach allows to skip the problem of choosing *transformation functions* and *weighting schemes* in order to summarize multiple indicators into a single measure.

Further researches are still in progress in order to: (a) improve the prediction of students’ class membership by taking into account students’ characteristics; (b) assess the sensitivity of the approach to the method adopted to rank latent classes; (c) validate the method on other data sets concerning the evaluation of course management; (d) explore the potentiality of the modeling approach on longitudinal studies. Furthermore LCA [1, 3, 12] classifies students on the bases of the posterior probability (modal probability) that a unit which provides a specific response pattern belongs to a specific class. In the modal assignment the variability in students’ probability to belong to each class is completely ignored. This means that in a four classes model a student with a pattern of response as 0.24, 0.25, 0.25, 0.26 would be deterministically placed in class 4. A simulation analysis could be carried out in order to assess the sensibility of the overall index to criteria of assignment of the units which relies on the variability of the probability vector.

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