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Geometer's Angle

Dynamic Root Rectangles Part Two: The Root-Two Rectangle and Design Applications

Abstract. "Dynamic symmetry" is the name given by Jay Hambidge for the proportioning principle that appears in "root rectangles" where a single incommensurable ratio persists through endless spatial divisions. In Part One of a continuing series [Fletcher 2007], we explored the relative characteristics of root-two, -three, -four, and -five systems of proportion and became familiar with diagonals, reciprocals, complementary areas, and other components. In Part Two we consider the "application of areas" to root-two rectangles and other techniques for composing dynamic space plans.

Introduction

"Dynamic symmetry" is the term given by the early-twentieth-century artist and scholar Jay Hambidge for the system of incommensurable ratios that appear in root rectangles and that replicate through endless spatial divisions, while expressing the relationship between one level of scale and the next.[1] In contrast to passive or static symmetry, which Hambidge associates with the radial subdivisions that characterize regular geometric figures or crystals, dynamic symmetry governs the spiral-like growth exhibited in plants and shells. Hambidge observes dynamic symmetry in ancient Egyptian bas reliefs and Classical Greek pottery and temple plans.[2]

The root-two rectangle can be formed from the side and hypotenuse of a 45°-45°-90° triangle. In this article we explore the dynamic properties of this elementary quadrilateral shape and consider possibilities for design applications.

Review: Root-Rectangles in Series

When a square divides into diminishing root rectangles or when root rectangles expand from a square, elements of dynamic symmetry become apparent. Fig. 1 illustrates how expanding root rectangles develop from a square. The diagonal of the preceding square or rectangle equals the long side of the succeeding four-sided figure.[3] The short side of each root rectangle is 1. The long sides progress in the series $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$… .

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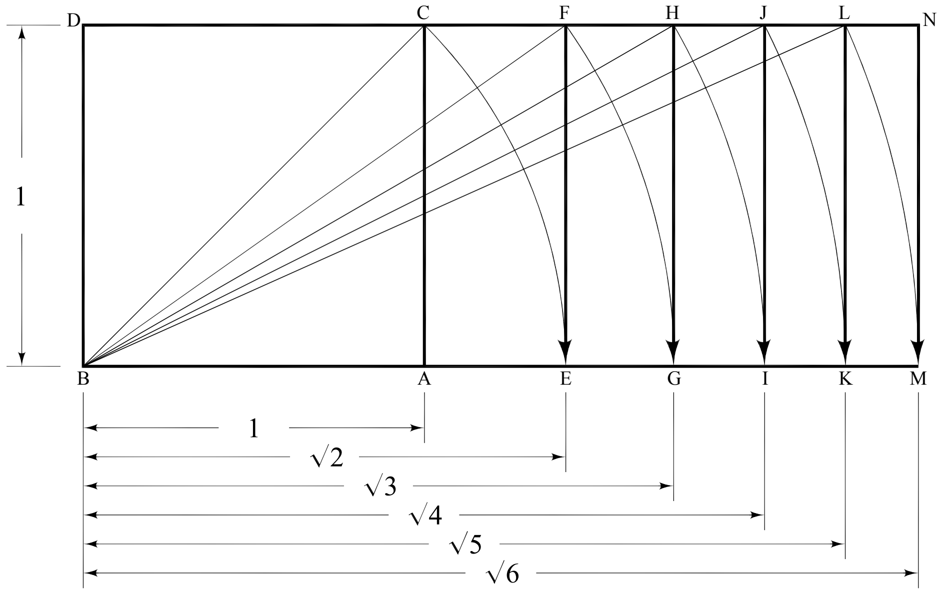


DB : BA :: 1 : 1
DB : BE :: 1 : √2
DB : BG :: 1 : √3
DB : BI :: 1 : √4
DB : BK :: 1 : √5
DB : BM :: 1 : √6

Fig. 1

Another method for obtaining root rectangles is from the radii of arcs that increase in whole number increments.[4]

- Draw a horizontal baseline AB equal in length to one unit.
- From point A, draw an indefinite line perpendicular to line AB that is slightly longer in length.
- Place the compass point at A. Draw a quarter-arc of radius AB that intersects line AB at point B and the indefinite vertical line at point C.
- Place the compass point at B. Draw a quarter-arc (or one slightly longer) of the same radius, as shown.
- Place the compass point at C. Draw a quarter-arc (or one slightly longer) of the same radius, as shown.
- Locate point D, where the two quarter-arcs (taken from points B and C) intersect.
- Place the compass point at D. Draw a quarter-arc of the same radius that intersects the indefinite vertical line at point C and line AB at point B (fig. 2).

* * *

- Connect points D, B, A, and C.

The result is a square (DBAC) of side 1 (fig. 3).

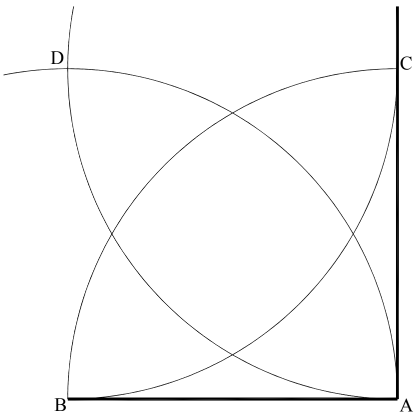


Fig. 2

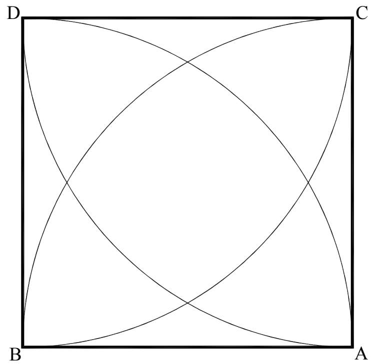


Fig. 3

- Add a length of ½ unit to line DB, so that the line DM equals 1.5.
- Place the compass point at M. Draw an arc of radius MD that intersects the extension of line BA at point E.
- From point E, draw a line perpendicular to line EB that intersects the extension of line DC at point F.
- Connect points D, B, E, and F.

The result is a root-two rectangle (DBEF) with short and long sides of 1 and $\sqrt{2}$.

- Add a length of ½ unit to line DM, so that the line DN equals 2.0.
- Place the compass point at N. Draw an arc of radius ND that intersects the extension of line BE at point G.
- From point G, draw a line perpendicular to line GB that intersects the extension of line DF at point H.
- Connect points D, B, G, and H.

The result is a root-three rectangle (DBGH) with short and long sides of 1 and $\sqrt{3}$.

- Add a length of ½ unit to line DN, so that the line DO equals 2.5.
- Place the compass point at O. Draw an arc of radius OD that intersects the extension of line BG at point I.
- From point I, draw a line perpendicular to line IB that intersects the extension of line DH at point J.
- Connect points D, B, I, and J.

The result is a root-four rectangle (DBIJ) with short and long sides of 1 and $\sqrt{4}$.

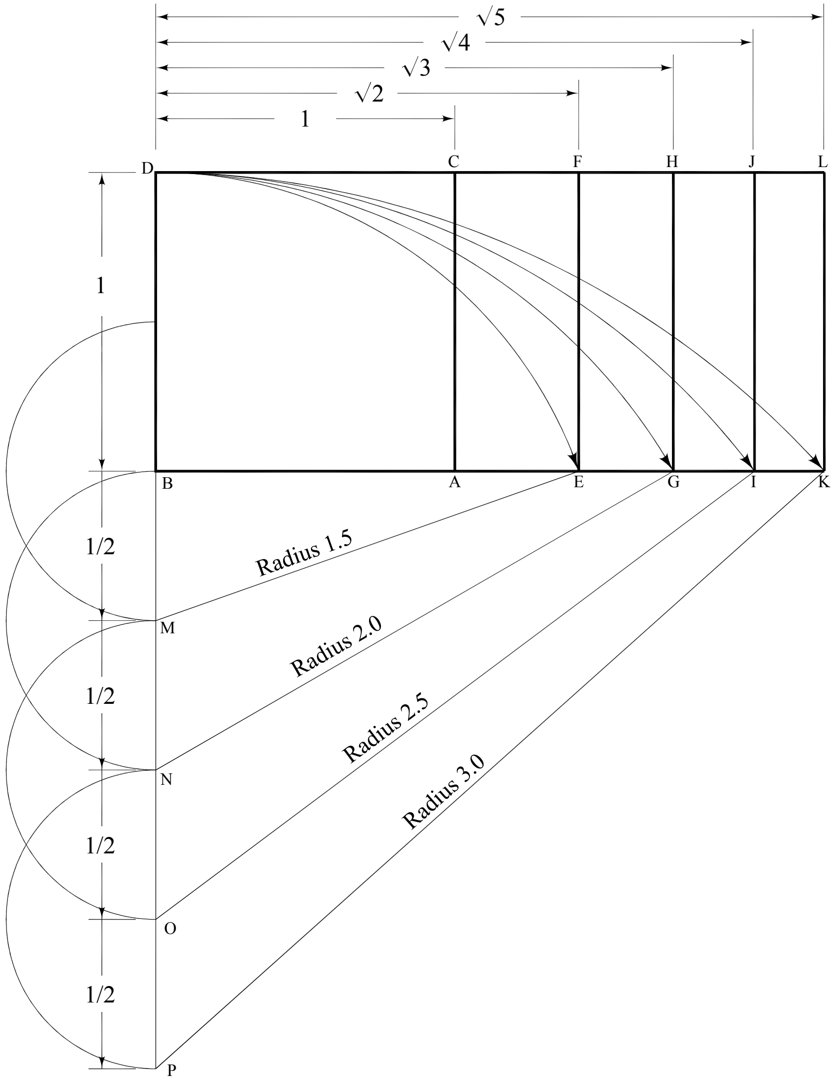
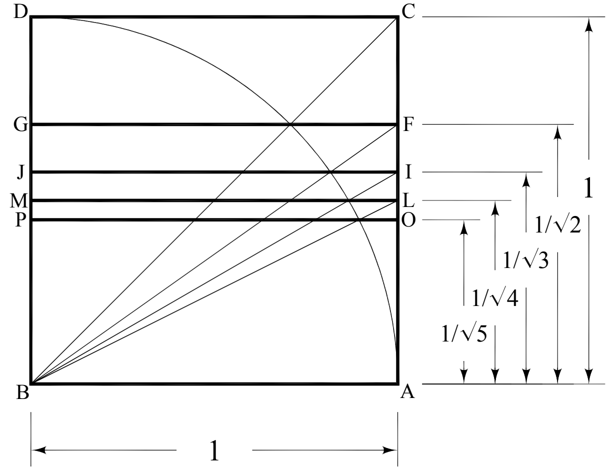


Fig. 4

- Add a length of ½ unit to line DO, so that the line DP equals 3.0.
- Place the compass point at P. Draw an arc of radius PD that intersects the extension of line BI at point K.
- From point K, draw a line perpendicular to line KB that intersects the extension of line DJ at point L.
- Connect points D, B, K, and L.

The result is a root-five rectangle (DBKL) with short and long sides of 1 and √5. The process can continue infinitely (fig. 4).

Fig. 5 illustrates how diminishing root rectangles develop from a quarter-arc that is drawn within a square.[5] The long side of each root rectangle is 1. The short sides progress in the series 1/√2, 1/√3, 1/√4, 1/√5… .



DB : BA :: 1 : 1
GB : BA :: 1 : √2
JB : BA :: 1 : √3
MB : BA :: 1 : √4
PB : BA :: 1 : √5

Fig. 5

Review: Diagonal, Reciprocal, and Complementary Patterns

At the heart of Hambidge's system is the fact that root rectangles produce reciprocals of the same proportion when the diagonal of the major rectangle and the diagonal of its reciprocal intersect at right angles.[6] The diagonals divide into vectors of equiangular distance apart, progressing in spiral-like fashion, while locating endless spatial divisions in continued proportion. The middle of any three adjacent vectors is the mean proportional or geometric mean of the other two.

A root-two rectangle divides into two reciprocals in the ratio 1 : √2. The area of each reciprocal is one-half the area of the whole (fig. 6a).

A root-three rectangle divides into three reciprocals in the ratio 1 : √3. The area of each reciprocal is one-third the area of the whole (fig. 6b).

A root-four rectangle divides into four reciprocals in the ratio 1 : √4. The area of each reciprocal is one-fourth the area of the whole (fig. 6c).

A root-five rectangle divides into five reciprocals in the ratio 1 : √5. The area of each reciprocal is one-fifth the area of the whole (fig. 6d).[7]

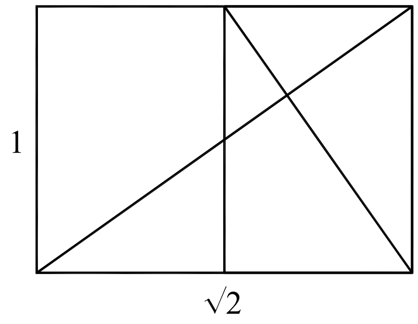


Fig. 6a

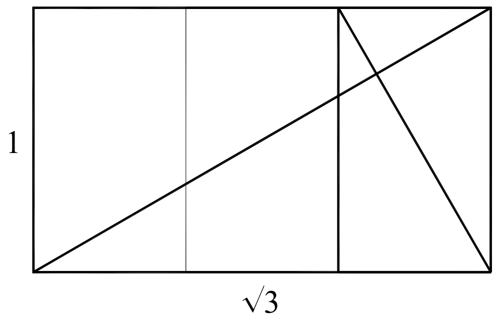


Fig. 6b

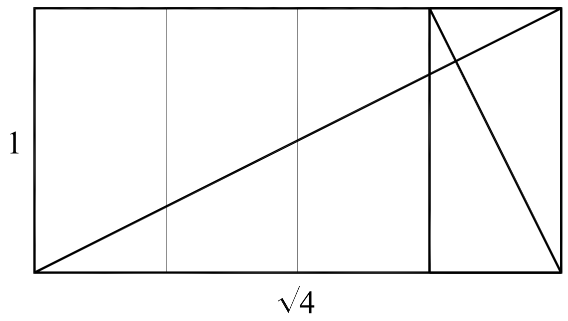


Fig. 6c

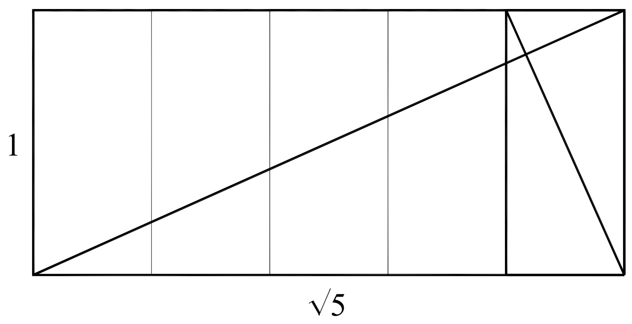


Fig. 6d

How to divide a root rectangle into reciprocals

For this demonstration we use the root-two rectangle, but the principle applies to all rectangles of dynamic proportions.

- Draw a square (DBAC) of side 1 (repeat figures 2 and 3).

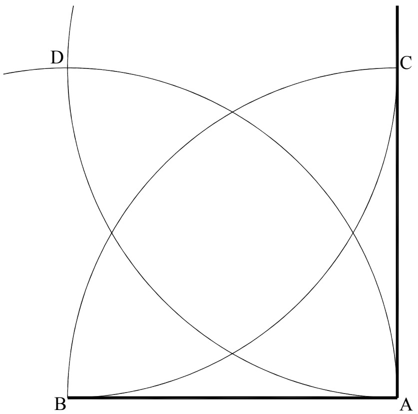


Fig. 2

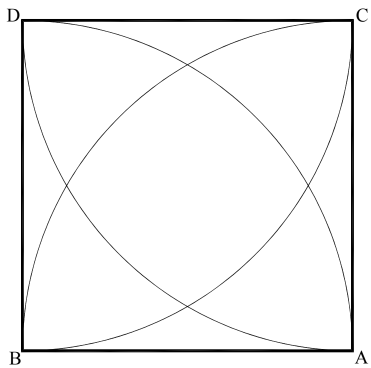


Fig. 3

- Draw the diagonal BC through the square (DBAC).

The side (DB) and the diagonal (BC) are in the ratio l : √2.

- Place the compass point at B. Draw an arc of radius BC that intersects the extension of line BA at point E (fig. 7).

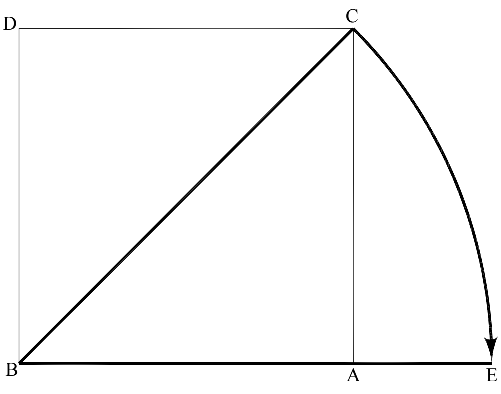
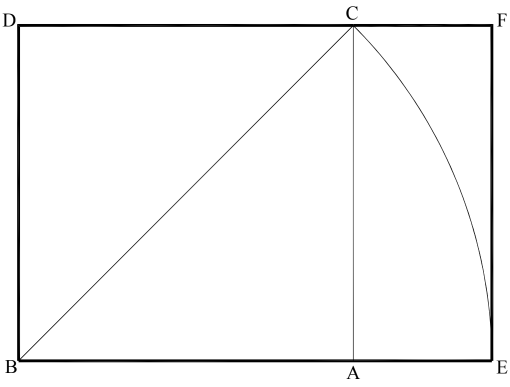


Fig. 7

- From point E, draw a line perpendicular to line EB that intersects the extension of line DC at point F.
- Connect points D, B, E, and F.

The result is a root-two rectangle (DBEF) with short and long sides of 1 and √2 (1 and 1.4142…) (fig. 8.)

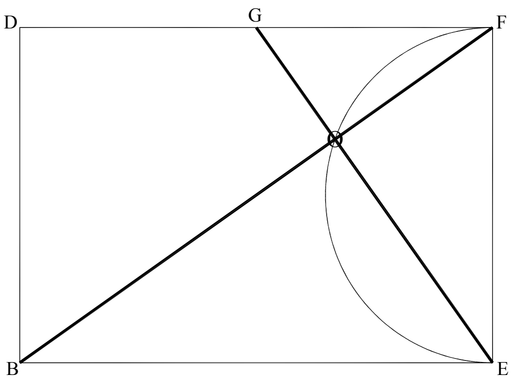


DB : BE :: 1 : √2

Fig. 8

- Locate the diagonal BF of the rectangle DBEF.
- Locate the line EF. Draw a semicircle that intersects the diagonal BF at point O, as shown.
- From point E, draw a line through point O that intersects line FD at point G.

The diagonal (BF) of the major rectangle (DBEF) and the diagonal (EG) intersect at right angles (fig. 9).



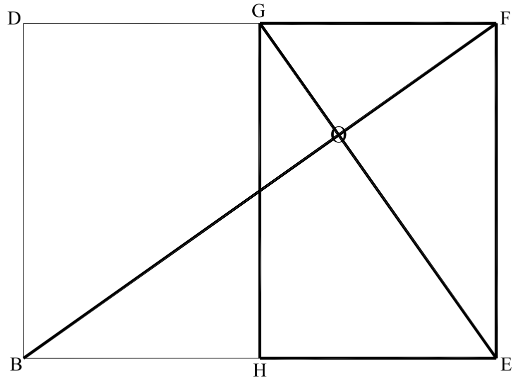
EG : BF :: 1 : √2

Fig. 9

- From point G, draw a line perpendicular to line FD that intersects line BE at point H.
- Connect points H, E, F, and G.

The result is a smaller root-two rectangle (HEFG) with short and long sides of 1/√2 and 1 (√2/2 : 1 or 0.7071… : 1). Rectangle HEFG is the reciprocal of the major rectangle DBEF.

The major 1 : √2 rectangle DBEF divides into two reciprocals (HEFG and BHGD) that are proportionally smaller in the ratio 1 : √2. The area of each reciprocal is one-half the area of the whole (fig. 10).



GF : FE :: FE : EB :: l : √2

Fig. 10

The diagonal (BF) of the major rectangle (DBEF) and the diagonal (EG) of the reciprocal (HEFG) locate endless divisions in continual proportion. A root-two rectangle of any size divides into two reciprocals in the ratio l : √2. If the process continues, the side lengths of successively larger rectangles form a perfect geometric progression (l ,√2, 2, 2√2…). The side lengths of successively smaller rectangles decrease in the ratio of 1 : 1/√2 toward a fixed point of origin known as the pole or eye (point O). (See figure 11.)[8]

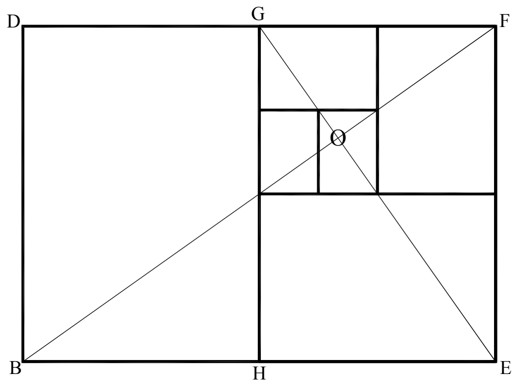


Fig. 11

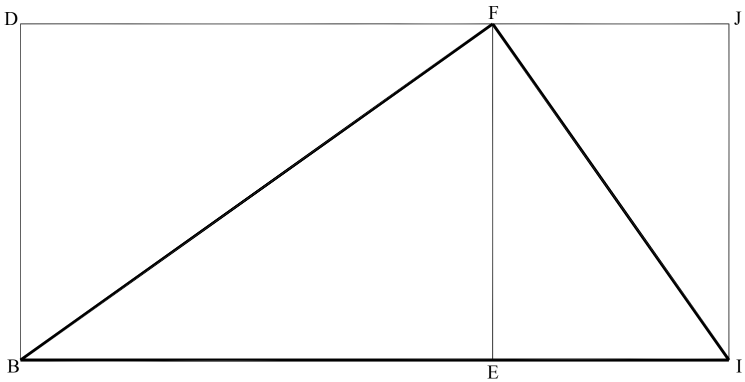
- Locate the root-two rectangle (DBEF) of sides 1 and $\sqrt{2}$, and its diagonal BF.
- From point F, draw an indefinite line perpendicular to the diagonal FB.
- Extend the line BE until it intersects the indefinite line at point I, as shown.
- Connect points B, F, and I.

The result is a right triangle (BFI).

- From point I, draw a line perpendicular to line IB that intersects the extension of line DF at point J.
- Connect points E, I, J, and F.

The rectangle EIJF is the reciprocal of the root-two rectangle DBEF.

If the long side (BF) of a right triangle (BFI) equals the diagonal of a major rectangle, the short side (IF) of the triangle equals the diagonal of the reciprocal (fig. 12).[9]



EI : IJ :: DB : BE :: 1 : $\sqrt{2}$
IF : FB :: 1 : $\sqrt{2}$

Fig. 12

Application of Areas

Definition:

Application of areas is Jay Hambidge's term for dividing rectangles into proportional components, where shapes applied on the short and long sides of a figure are equal in area.

Hambidge's technique for proportioning areas, which he traces to ancient Egyptian and Classical Greek design, produces harmonious compositions that are based on the proportions of the enclosing rectangle. An area of any rectangular shape may be "applied" on the short and long sides of a rectangle. When a square is applied on the short side, then "applied" on the long side, the new area is the same as the square, but the shape becomes rectangular. The reciprocal of the major rectangle is accomplished by locating the point where the diagonal of the major rectangle and the inside edge of the true square intersect [Hambidge 1960, 28–29; 1967, 35, 60–72].

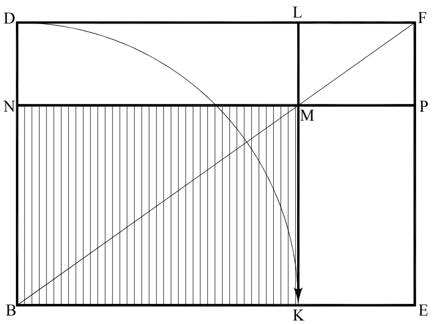
How to apply areas to a root-two rectangle

- Locate the root-two rectangle (DBEF) of sides 1 and √2, and its diagonal BF.
- From point B, draw a quarter-arc of radius BD that intersects line BE at point K.
- From point K, draw a line perpendicular to line BE that intersects line FD at point L.

The result is a square (DBKL) "applied" on the short side (DB) of the root-two rectangle (DBEF).

- Locate the diagonal BF of the root-two rectangle (DBEF).
- Locate point M where the diagonal (BF) intersects line KL.
- Draw a line through point M that is perpendicular to line KL and intersects line DB at point N and line EF at point P.
- Locate the rectangle NBKM.

The rectangle (NBKM) is the reciprocal of the root-two rectangle DBEF. Line BM is the diagonal of the reciprocal (NBKM) (fig. 13.)



NB : BK :: DB : BE :: 1 : √2
BM : BF :: 1 : √2

Fig. 13

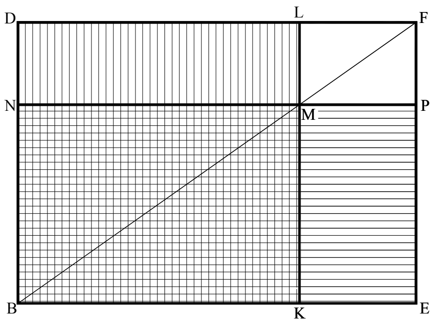


Fig. 14

- Locate the square (DBKL) and the rectangle BEPN.

The area of the rectangle (BEPN) and the area of the square (DBKL) are equal. In Hambidge's system, the "square" is applied on the short and long sides of the root-two rectangle (fig. 14).[10]

- Locate the reciprocal NBKM of the major root-two rectangle DBEF.
- Locate the rectangles DNML and KEPM.

Rectangle DNML is the complement of the reciprocal NBKM.[11]

The areas of rectangles DNML and KEPM are equal.

- Locate rectangles NBKM and LMPF.

The rectangles NBKM and LMPF share the same diagonal and are similar (fig. 15).[12]

Any quadrilateral figure can be applied to the long and short sides of a root rectangle in this fashion. In figure 16 the areas of rectangles DBQR and BEUT are equal and the areas of rectangles DTSR and QEUS are equal.

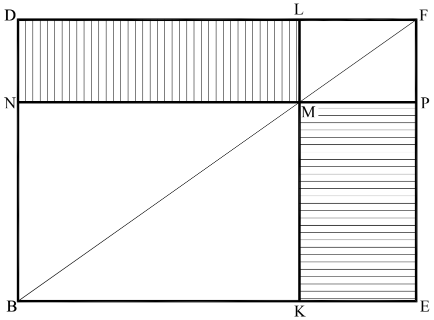


Fig. 15

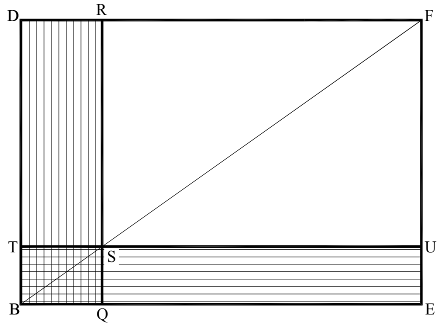


Fig. 16

Definitions:

Hambidge identifies three ways to apply one area to another. The applied area can be less than, equal to, or in excess of the other. If the applied area is less, it is **elliptic**; if equal, it is **parabolic**; and if in excess, it is **hyperbolic**.

If a square is applied to the short side of a rectangle, it "falls short" and is elliptic. If a square is applied to the short side of a root-two rectangle, the excess area contains a square and a root-two rectangle (fig. 17a). If squares are applied to both short sides of a root-two rectangle, they overlap and the total rectangle divides into three squares and three root-two rectangles (fig. 17b). If a square is applied to the long side of a rectangle, it "exceeds" the rectangle and is hyperbolic. If a square is applied to the long side of a root-two rectangle, the excess area contains two squares and a root-two rectangle (fig. 17c).[13]

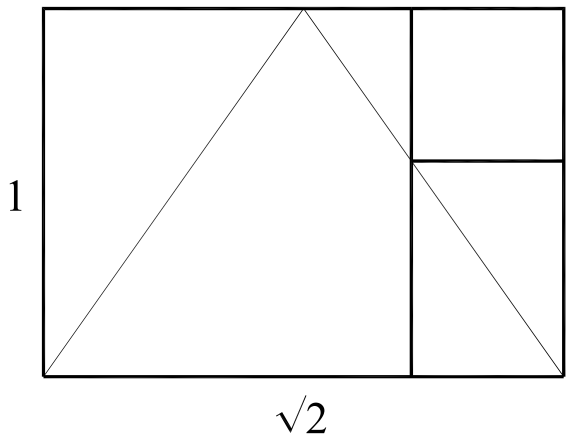


Fig. 17a

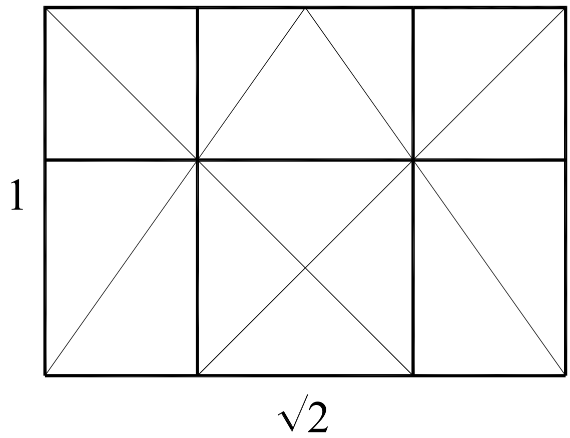


Fig. 17b

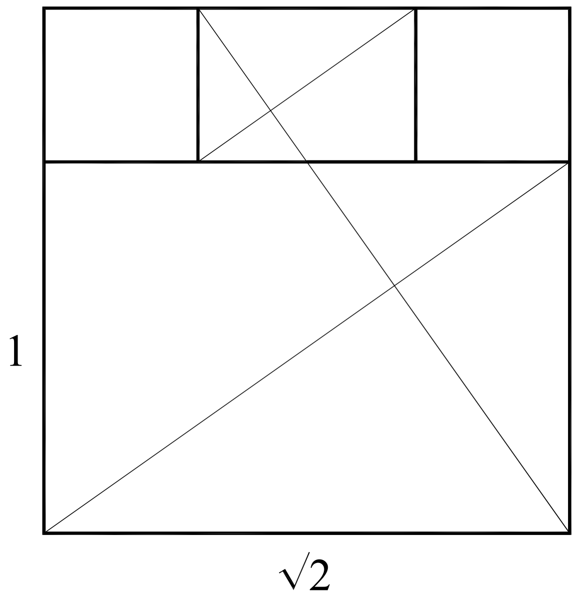


Fig. 17c

The Root-Two Rectangle and Variations on Area Themes

The application of areas permits the division of root rectangles into harmonious compositions that are based on the ratio of the overall figure. Let us consider the root-two rectangle and related figures.[14]

<u>How to divide a square into root-two proportional areas</u>

- Draw a square (DBAC) of side 1. (Repeat figures 2 and 3.)

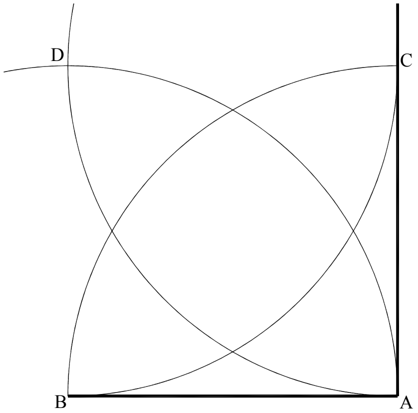


Fig. 2

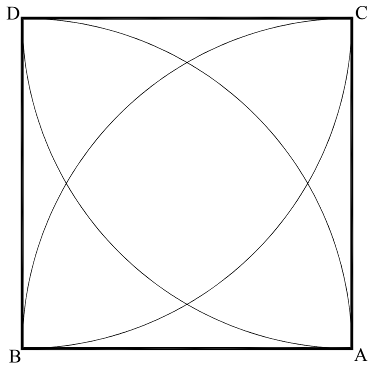


Fig. 3

- Draw the diagonals AD and CB through the square (DBAC).
- Locate point O, where the two diagonals intersect.
- Place the compass point at A. Draw an arc of radius AO that intersects line AC at point E.
- Place the compass point at B. Draw an arc of radius BO that intersects line BD at point F.
- Connect points E and F.

The result is a root-two rectangle (AEFB) of sides $1/\sqrt{2}$ and 1 (0.7071… and 1) (fig. 18.)

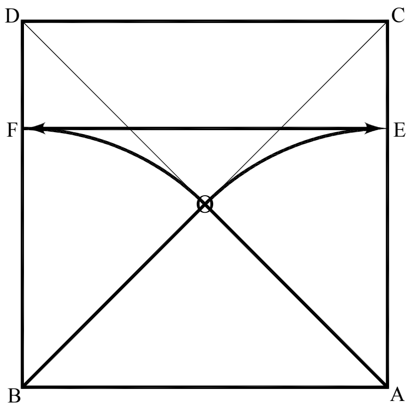


Fig. 18

- Place the compass point at A. Draw a quarter-arc of radius AO that intersects line AC at point E and line AB at point G.

- Place the compass point at B. Draw a quarter-arc of radius BO that intersects line BD at point F and line BA at point H.
- Place the compass point at D. Draw a quarter-arc of radius DO that intersects line DC at point I and line DB at point J.
- Place the compass point at C. Draw a quarter-arc of radius CO that intersects line CD at point K and line CA at point L.
- Connect points J and L. Connect points I and H. Connect points G and K.

The results are three root-two rectangles (CKGA, DJLC, and BHID). When root-two rectangles are drawn on all four sides of a square, the result is a composition that contains one center square, four smaller corner squares, and four root-two rectangles (fig. 19).[15]

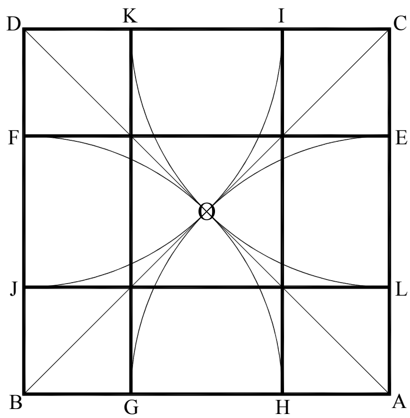


Fig. 19

<u>How to divide a root-two rectangle into segments that progress in the ratio 1 : √2</u>

- Draw a root-two rectangle (DBEF) of sides 1 and √2. (Repeat figures 2, 3, 7, and 8.)

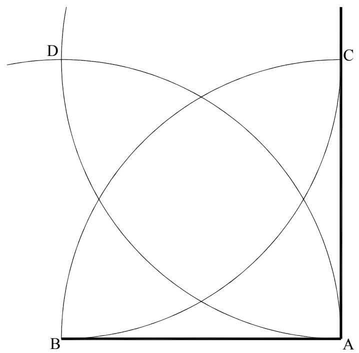


Fig. 2

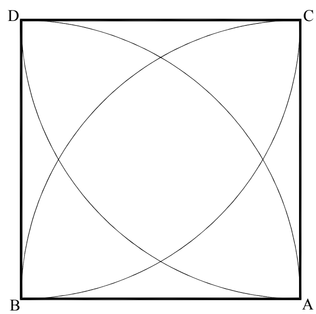


Fig. 3

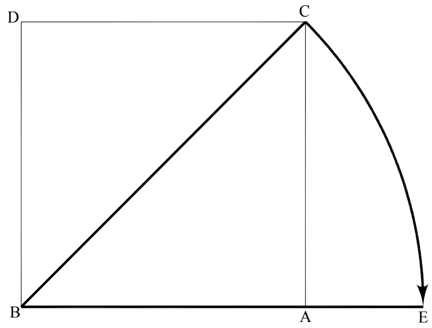


Fig. 7

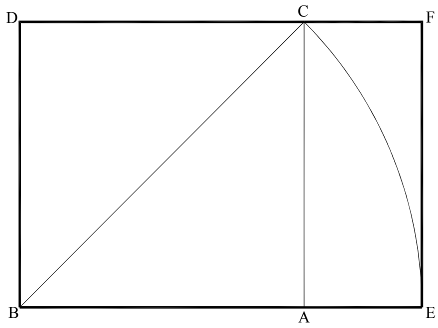


Fig. 8

- Draw the diagonal (BF) of the major rectangle (DBEF), the reciprocal HEFG, and its diagonal (EG). (Repeat figures 9 and 10.)

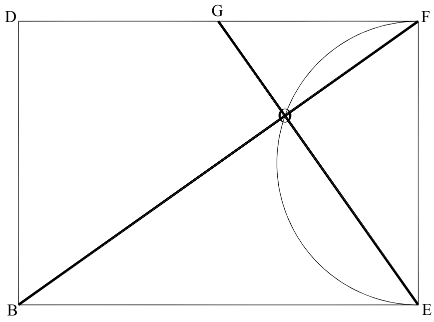


Fig. 9

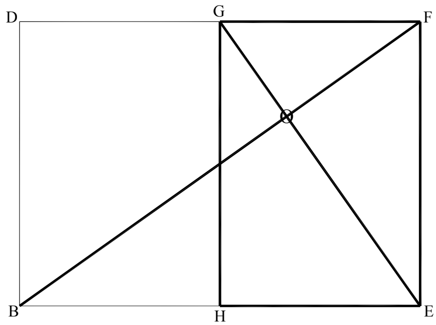


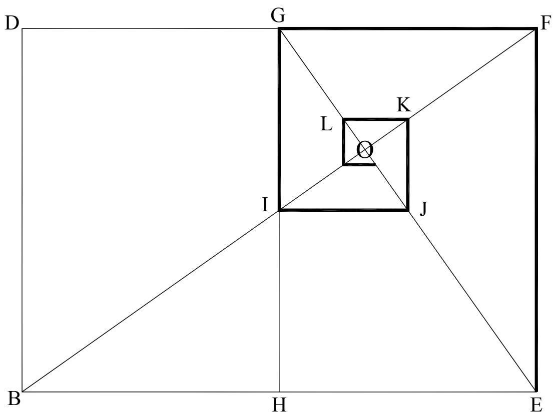
Fig. 10

The diagonal (BF) of the major rectangle (DBEF) and the long side (GH) of the reciprocal (HEFG) intersect at point I.

- Connect points G and I.
- From point I, draw a line perpendicular to line IG that intersects the diagonal (EG) of the reciprocal (GHEF) at point J.
- From point J, draw a line perpendicular to line JI that intersects the diagonal BF at point K.
- From point K, draw a line perpendicular to line KJ that intersects the diagonal EG at point L.

The process can continue infinitely.

The lines LK, KJ, JI, IG, GF, FE, and EB compose a rectilinear spiral that increases in root-two proportion and decreases in the ratio of $1 : 1/\sqrt{2}$ toward the pole or eye (point O) (fig. 20).



KJ : JI :: JI : IG :: IG : GF :: GF : FE :: l : √2

Fig. 20

- Locate the root-two rectangle (DBEF) of sides 1 and √2.
- Draw the diagonals (BF and DE) of the root-two rectangle DBEF, the diagonals (EG and HF) of the reciprocal HEFG, and the diagonals (HD and BG) of the reciprocal BHGD (fig. 21).

* * *

Use the diagonals to repeat the equiangular spiral three times, as shown (fig. 22).

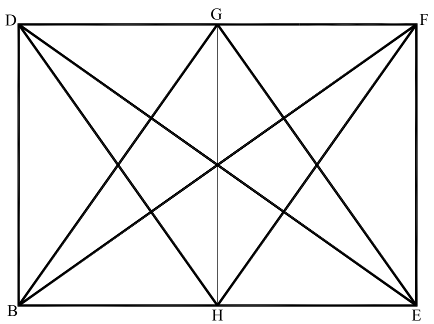


Fig. 21

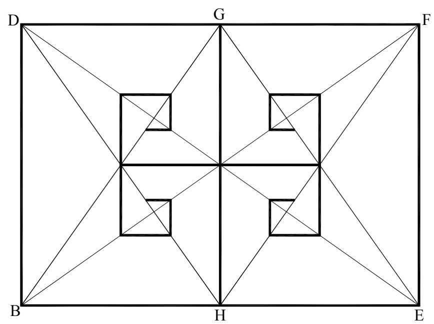


Fig. 22

<u>How to divide a root-two rectangle into smaller root-two rectangles of equal area</u>

- Locate the root-two rectangle (DBEF) of sides 1 and √2.
- Locate the diagonals (BF and DE) of the root-two rectangle DBEF.

- Locate the diagonals (HD and BG) of the reciprocal root-two rectangle BHGD and the diagonals (EG and HF) of the reciprocal root-two rectangle HEFG.
- Locate the midpoints (G, H, M, and N) of the root-two rectangle DBEF.
- Connect midpoints G and H, then midpoints M and N.

The result is a root-two rectangle divided into four smaller root-two rectangles of equal area (fig. 23).

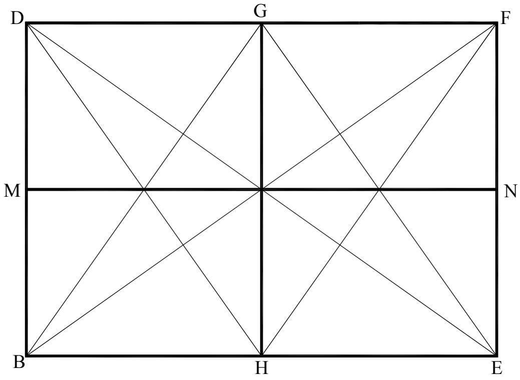


Fig. 23

- Locate points O, P, Q, and R, where the diagonals intersect, as shown.
- Through these points, extend two vertical and two horizontal lines to the sides of the original rectangle, as shown.

The result is a root-two rectangle divided into nine smaller root-two rectangles of equal area (fig. 24).

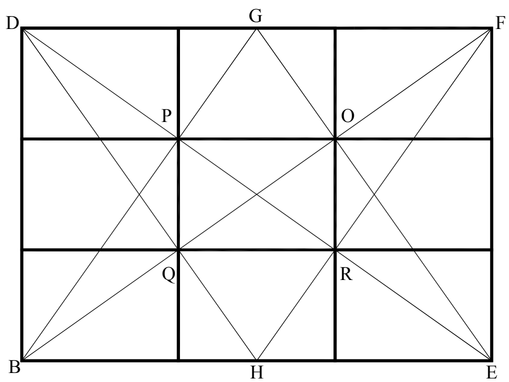


Fig. 24

- From midpoint M, draw the half diagonals ME and MF through the root-two rectangle, as shown.
- From midpoint N, draw the half diagonals ND and NB through the root-two rectangle, as shown (fig. 25).

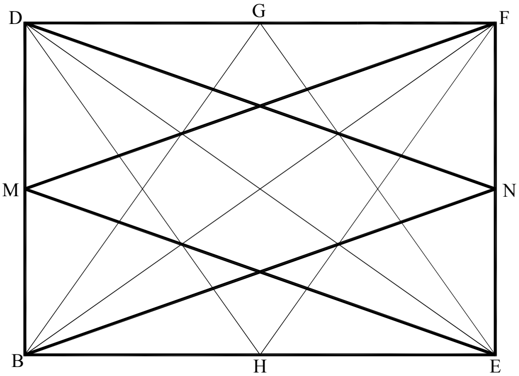


Fig. 25

- Locate points S, T, U, and V, where the diagonals and half diagonals intersect, as shown.
- Through these points, extend three vertical and three horizontal lines to the sides of the original rectangle, as shown.

The result is a root-two rectangle divided into sixteen smaller root-two rectangles of equal area (fig. 26).

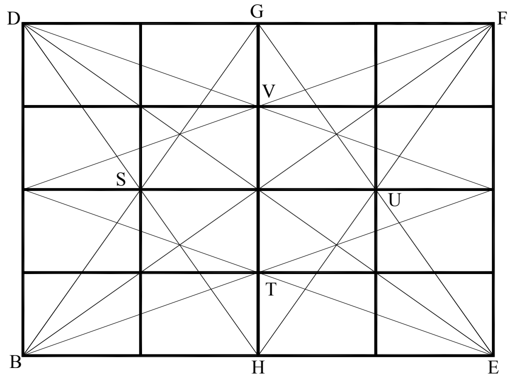


Fig. 26

- Extend four vertical and four horizontal lines to the sides of the original rectangle, through the points of intersection, as shown.

The result is a root-two rectangle divided into twenty-five smaller root-two rectangles of equal area (fig. 27).[16]

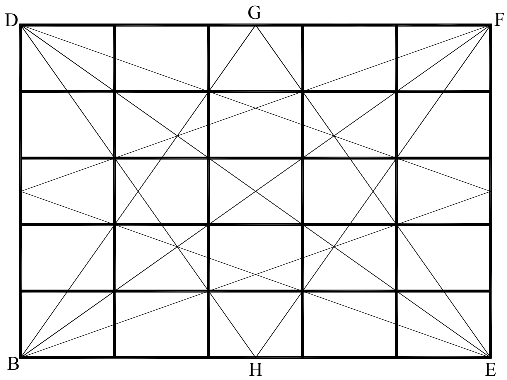
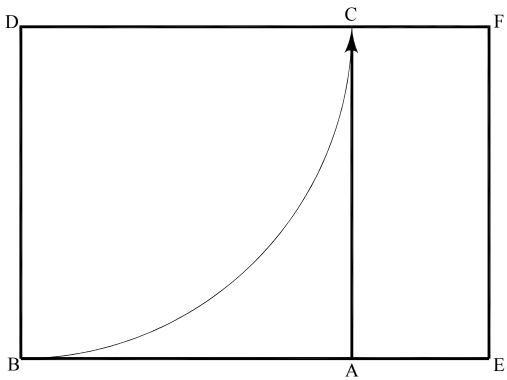


Fig. 27

<u>How to divide a root-two rectangle into patterns of proportional areas</u>

- Locate the root-two rectangle (DBEF) of sides 1 and √2.
- Apply a square (DBAC) to the left side (DB) of the root-two rectangle, as shown (fig. 28).



DB : BE :: 1 : √2

Fig. 28

- From the midpoint G of line FD, draw the half diagonals GB and GE through the root-two rectangle, as shown.
- Locate point W where the half diagonal GE intersects the right side (AC) of the square (DBAC), as shown.
- From point W, draw a line perpendicular to line AC that intersects line EF at point X.

When a square (DBAC) is "applied" on the short side of a root-two rectangle (DBEF), it falls short and is elliptic. The excess area (AEFC) is composed of a square (WXFC) and a root-two rectangle (AEXW) (fig. 29).

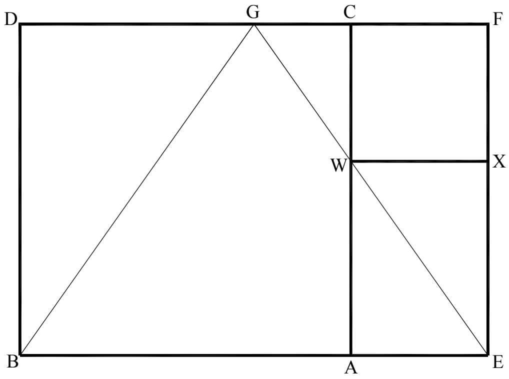


Fig. 29

- Apply a square (EFYZ) to the right side (EF) of the root-two rectangle, as shown (fig. 30).

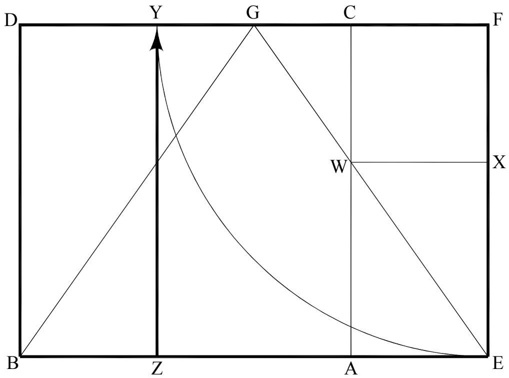


Fig. 30

- From point D, draw the diagonal DA of the square DBAC. From point F, draw the diagonal FZ of the square EFYZ, as shown.
- Locate point A′ where the diagonal DA and the half diagonal GB intersect, as shown.
- Extend the line XW through point A′, as shown, until it intersects the line DB at point B′.

When squares (DBAC and EFYZ) are "applied" on both short sides of a root-two rectangle (DBEF), they overlap. The various diagonals reveal a pattern of three squares and three root-two rectangles (fig. 31).

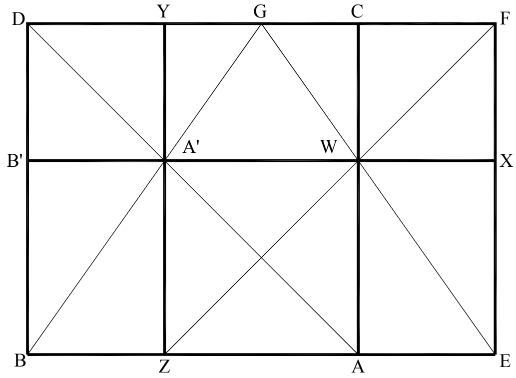


Fig. 31

- Locate the root-two rectangle (DBEF) of sides 1 and √2.
- Locate the squares DBAC and EFYZ applied to the root-two rectangle (DBEF), as shown.
- From point B, draw the diagonal BC of the square DBAC. From point E, draw the diagonal EY of the square EFYZ, as shown.
- Locate point C′ where the two diagonals intersect (fig. 32).

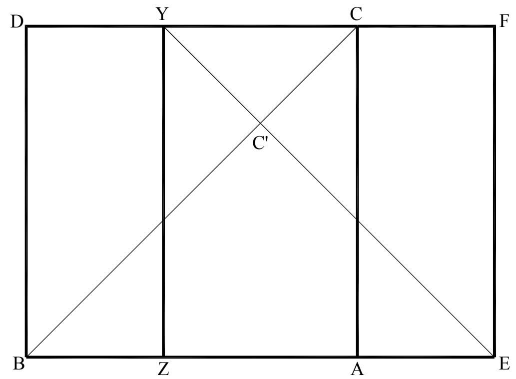


Fig. 32

- Draw a line through point C′ that is perpendicular to and intersects line FD at point G and line BE at point H.
- Draw a line through point C′ that is perpendicular to line GH and intersects line DB at point D′ and line EF at point E′ (fig. 33).

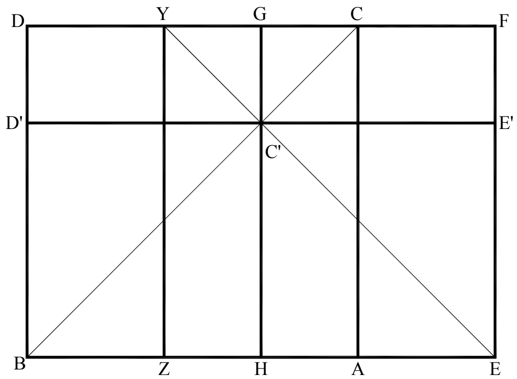


Fig. 33

- From point D, draw the diagonal DA of the square DBAC. From point F, draw the diagonal FZ of the square EFYZ, as shown.
- Locate point F′ where the two diagonals intersect.
- Draw a line through point F′ that is perpendicular to line GH and intersects line DB at point G′ and line EF at point H′.

The four diagonals reveal a pattern of six squares and six root-two rectangles (fig. 34).

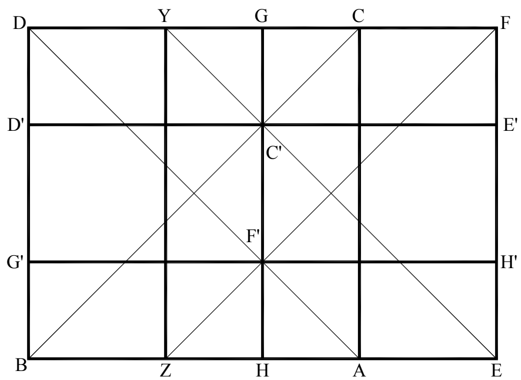


Fig. 34

- Locate the root-two rectangle (DBEF) of sides 1 and $\sqrt{2}$.
- Locate the square DBAC applied to the root-two rectangle (DBEF), as shown.
- From point B, draw the diagonal BF of the root-two rectangle DBEF.
- Locate point I′ where the diagonal BF intersects the right side (AC) of the square (DBAC), as shown.
- Draw a line through point I′ that is perpendicular to line AC and intersects line DB at point D′ and line EF at point E′.
- Locate the rectangle BEE′D′.

The area of the square DBAC and the area of the rectangle BEE′D′ are equal. The "square" is applied on the short and long sides of the root-two rectangle.

- Locate the rectangle D′BAI′.

Rectangle D′BAI′ is the reciprocal of the major root-two rectangle DBEF.

- Locate the rectangles DD′I′C and AEE′I′.

Rectangle DD′I′C is the complement of the reciprocal D′BAI′.

The areas of rectangles DD′I′C and AEE′I′ are equal.

- Locate the rectangles D′BAI′ and CI′E′F.

The rectangles D′BAI′ and CI′E′F share the same diagonal and are similar (fig. 35).

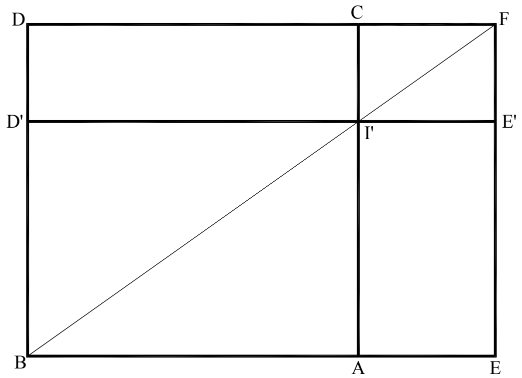


Fig. 35

- Locate the square EFYZ applied to the root-two rectangle (DBEF), as shown.
- Locate the diagonals (BF and DE) of the root-two rectangle DBEF.
- Locate point J′ where the diagonal DE intersects the left side (YZ) of the square (EFYZ), as shown.
- Locate point K′ where the diagonal BF intersects the left side (YZ) of the square (EFYZ), as shown.
- Locate point L′ where the diagonal DE intersects the right side (AC) of the square (DBAC), as shown.
- Draw a line through points K′ and L′ that intersects line DB at point G′ and line EF at point H′.

The diagonals BF and DE of the major root-two rectangle (DBEF) reveal a pattern of six squares and five root-two rectangles (fig. 36).

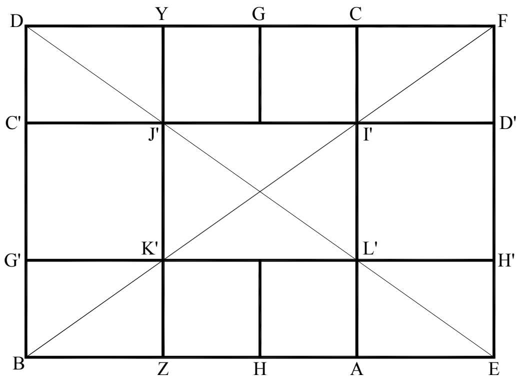


Fig. 36

Design applications

- Locate the root-two rectangle (DBEF) of sides 1 and √2.
- Construct a diagonal grid composed of
  - the diagonals of the major root-two rectangle (DBEF)
  - the diagonals of the two reciprocals (BHGD and HEFG)
  - the diagonals of the two squares (DBAC and EFYZ). (See figure 37.)

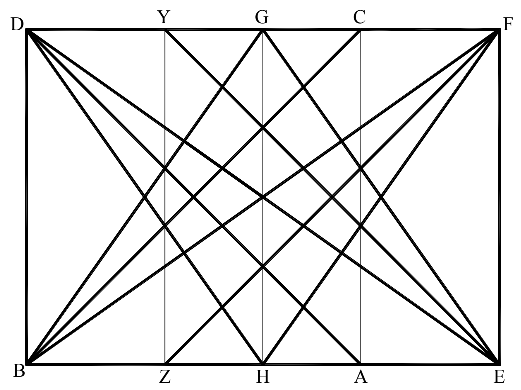


Fig. 37

Figures 38–42 illustrate patterns of proportional areas based on the ratio 1 : √2.

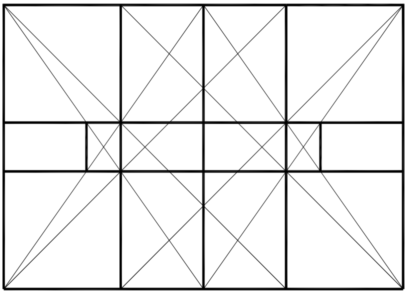


Fig. 38

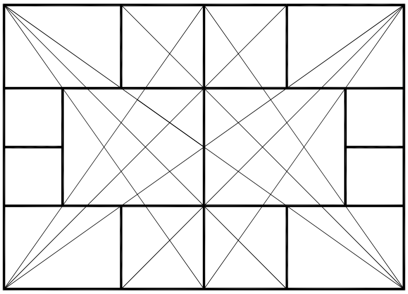


Fig. 39

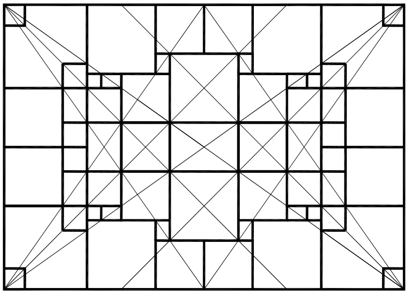


Fig. 40

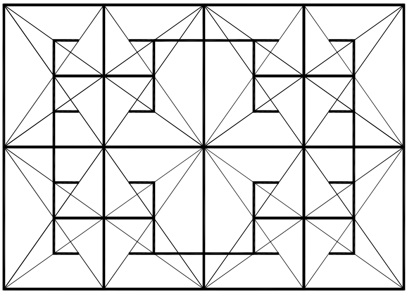


Fig. 41

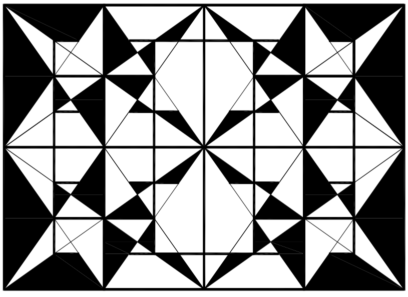


Fig. 42

Rectangles based on root-two proportions

Besides the square and the root-two rectangle, the designer may use a variety of quadrilateral figures when composing root-two patterns of dynamic symmetry.

The rectangle formed by a square plus a root-two rectangle is in the ratio 1:1 + √2 or 1:2.4142… (fig. 43).

The rectangle formed by the reciprocal of a root-two rectangle plus a square is in the ratio 1 : 1 + 1/√2 or 1: 1.7071… (fig. 44).

The rectangle formed by a square plus the reciprocal of a root-two rectangle on either side is in the ratio 1 : 1 + 2/√2 or 1 : 2.4141… (fig. 45).

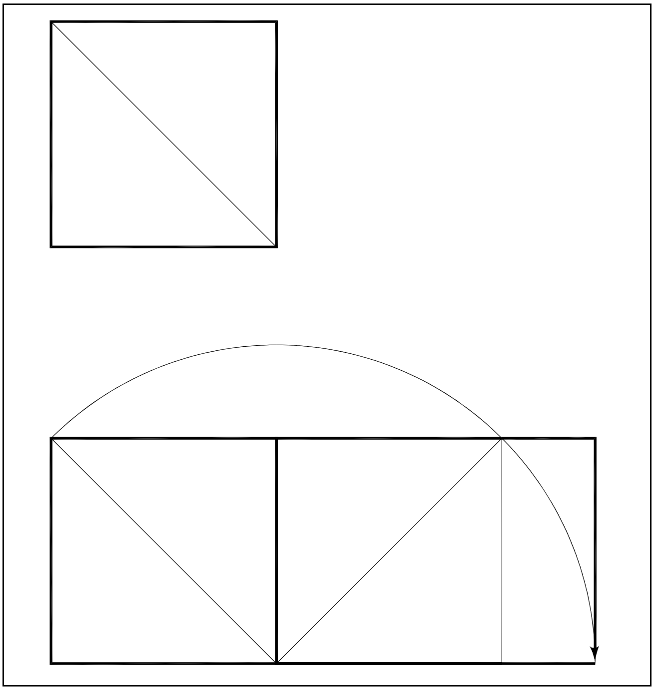


Fig. 43

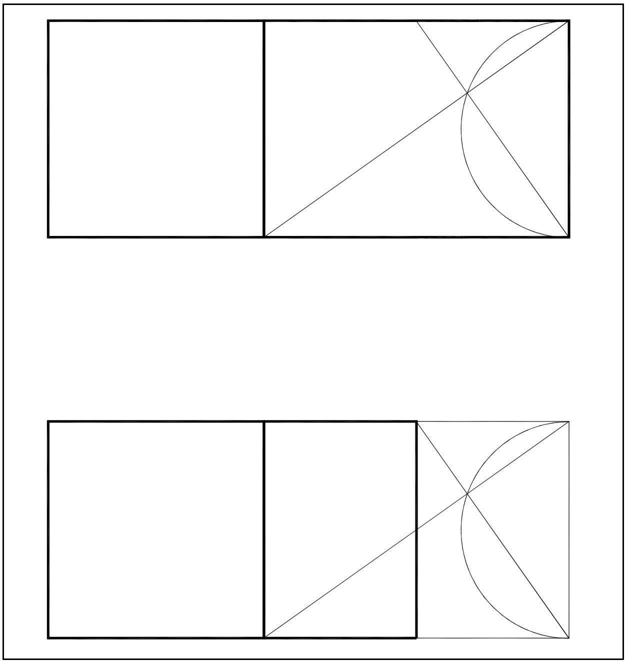


Fig. 44

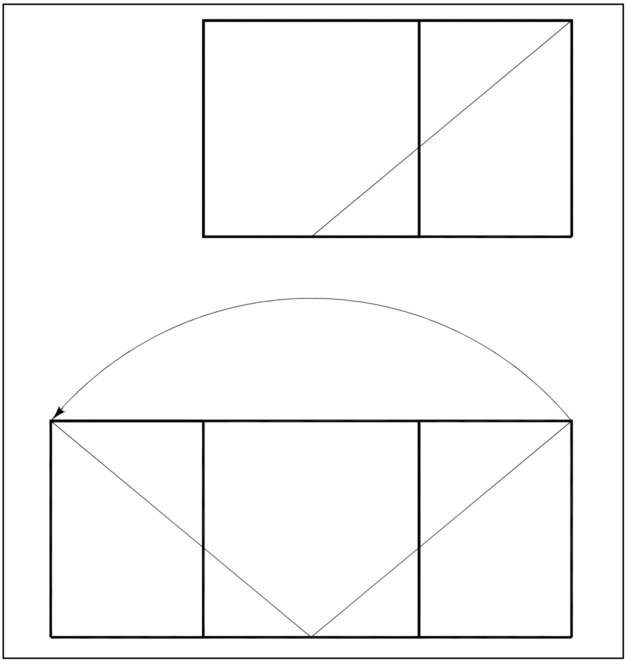


Fig. 45

Conclusion

Ratios inherent in geometric forms offer a rich vocabulary for achieving unity among a diversity of elements. But such systems cannot guarantee good design any more than rules of scale ensure good musical composition. They are mere theories that must be judged by how they serve. No ordering system has merit that does not inform the situation at hand. The capacity of proportional techniques to support and enhance specific cultural and functional requirements remains a matter of individual talent.

Notes

1. "Dynamic symmetry" appears in root rectangles based on square root proportions. The edge lengths of such rectangles are incommensurable and cannot divide into one another. But a square constructed on the long side of the rectangle can be expressed in whole numbers, relative to a square constructed on the shorter side [Hambidge 1960, 22–24; 1967, 17–18]. See [Fletcher 2007] for more on dynamic symmetry and its fundamental components.
2. Hambidge believes the artistic application of dynamic symmetry disappeared after the Classical Greek period of the sixth and seventh centuries BC, but its theoretical expression remained in Euclidean geometry. He speculates that the dynamic proportioning of rectangular forms evolved from the ancient Egyptian practice of "cording the temple" to lay out building plans, including the cord's division into twelve (3 + 4 + 5) equal units to form a right angle [Hambidge 1920, 7–12]. Edward B. Edwards argues that the artistic application of dynamic symmetry persisted well beyond classical Greece, in numerous styles of ornament [1967, viii–ix]. See also [Scholfield 1958, 116–119].
3. See [Fletcher 2007, 329–334] to draw the construction.
4. The construction is taken from [Edwards 1967, 10–11].
5. See [Fletcher 2007, 345–346] to draw the construction.
6. The diagonal is the straight line joining two nonadjacent vertices of a plane figure, or two vertices of a polyhedron that are not in the same face. The reciprocal of a major rectangle is a figure similar in shape but smaller in size, such that the short side of the major rectangle equals the long side of the reciprocal [Hambidge 1967, 30, 131]. See [Fletcher 2007, 336] and [Hambidge 1967, 33–37] for more on these elements.
7. A root-two rectangle is in the ratio 1 : 1.4142… . Its reciprocal ($\sqrt{2}/2 : 1$) is in the ratio 0.7071… : 1. A root-three rectangle is in the ratio 1 : 1.732… . Its reciprocal ($\sqrt{3}/3 : 1$) is in the ratio 0.5773… : 1. A root-four rectangle is in the ratio 1 : 2.0. Its reciprocal ($\sqrt{4}/4 : 1$) is in the ratio 0.5 : 1. A root-five rectangle is in the ratio 1 : 2.236… . Its reciprocal ($\sqrt{5}/5 : 1$) is in the ratio 0.4472… : 1.
8. For more on the pole or eye, see [Fletcher 2004, 105–106].
9. The construction applies Euclid's method of obtaining the mean proportional of two given lines [1956, II: 216 (bk. VI, prop. 13)].
10. For the geometrical proof, see [Hambidge 1967, 46].
11. In Hambidge's system, each rectangle has a reciprocal and each rectangle and reciprocal have complementary areas. The complementary area is the area that remains when a rectangle is produced within a unit square. If the rectangle exhibits properties of dynamic symmetry, its complement will also. See [Fletcher 2007, 347–354] and [Hambidge 1967, 128].
12. Rectangles are similar if their corresponding angles are equal and their corresponding sides are in proportion. Similar rectangles share common diagonals.
13. See [Hambidge 1920, 19–20; 1960, 35].
14. See [Hambidge 1967, 40–47].
15. See [Fletcher 2005b, 56–62] for similarities to the Sacred Cut.
16. See [Schneider 2006, 30-32, 34] for similar constructions.

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About the Geometer

Rachel Fletcher is a theatre designer and geometer living in Massachusetts, with degrees from Hofstra University, SUNY Albany and Humboldt State University. She is the creator/curator of two museum exhibits on geometry, "Infinite Measure" and "Design By Nature". She is the co-curator of the exhibit "Harmony by Design: The Golden Mean" and author of its exhibition catalog. In conjunction with these exhibits, which have traveled to Chicago, Washington, and New York, she teaches geometry and proportion to design practitioners. She is an adjunct professor at the New York School of Interior Design. Her essays have appeared in numerous books and journals, including *Design Spirit*, *Parabola*, and *The Power of Place*. She is the founding director of Housatonic River Walk in Great Barrington, Massachusetts, and is currently directing the creation of an African American Heritage Trail in the Upper Housatonic Valley of Connecticut and Massachusetts.