An Introduction to Leonardo's Lattices

Among the architectural and mathematical treatises that flourished during the Renaissance period, Leonardo's codices deserve special attention. They are not didactic treatises, arranged in several books that must be read from the first page to the last, but information about the scientific research in the Renaissance flows from their pages, full of sketches and notes as from an endless font. The reader always bumps into something new or unexpected when going through the drawings, whichever codex or whatever page he is exploring.

The sketches that can be seen on folio 899 of the *Codex Atlanticus* illustrate the design of a roof system assembled from simple elements, and describe the building process of this system based on the weaving of wooden logs that will generate a vaulted roof covering a wide space without intermediate supports (fig. 1). These drawings are quite unique in the pages of Leonardo and no repetition has been noticed in other codices, but they are not unique in the vast amount of architectural literature of Middle Ages and Renaissance.

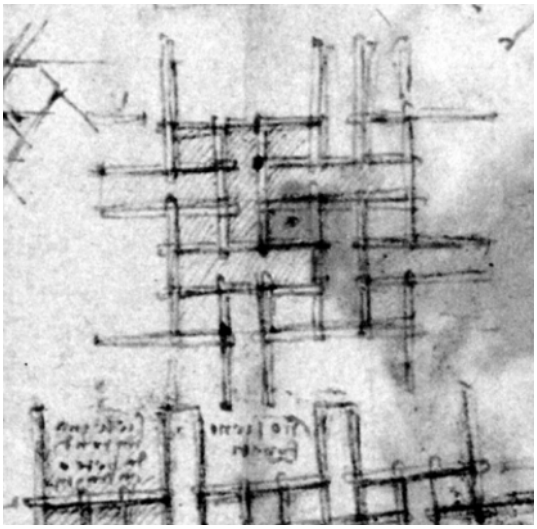

Fig. 1. Detail from Leonardo's *Codex Atlanticus*, folio 899

One of the drawings of Villard de Honnecourt's portfolio illustrates a building trick that can be considered to be an anticipation of Leonardo's research. The sketch by Villard shows a wooden floor built from beams that are all shorter than the dimensions of the room itself. The caption says: "in such a way you can work in a tower or a house with pieces of wood that are too short" (fig. 2). The beams are tied together according to a geometric pattern that makes it possible to cover of a span wider than the length of the beams themselves. We have no information whether Villard invented this trick together with some fellow builders, or if he inherited it from previous "know how", but the drawing is intended to transmit this technique to posterity and in fact it appears again, in drawing and in written description, in some treatises of the Renaissance.

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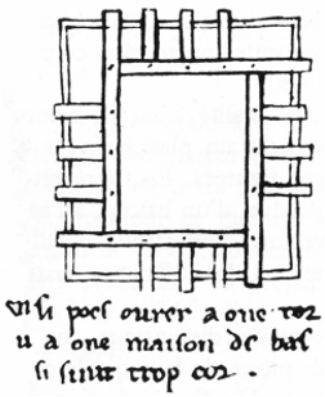


Fig. 2. How to build a floor with beams that are too short. From Villard de Honnecourt's portfolio (1225-1250)

In Book I of the *Seven Books on Architecture* by Sebastiano Serlio we can see a drawing that shows an application of the same technique: the system has been repeated and extended to build an even wider floor (fig. 3). Book I: *De Geometria* was first published in Paris in 1545, twenty-six years after Leonardo's death. Serlio's drawing looks like a combination of ancient problems and new solutions, but no explanation is given as far as design and science are concerned. Serlio only claims to present some possible solutions to inconveniences that often arise in the course of the architect's professional carreer. It is interesting to notice, however, that this practical trick of the trade appears in the more theoretical of the *Seven Books*, which deals with geometry.

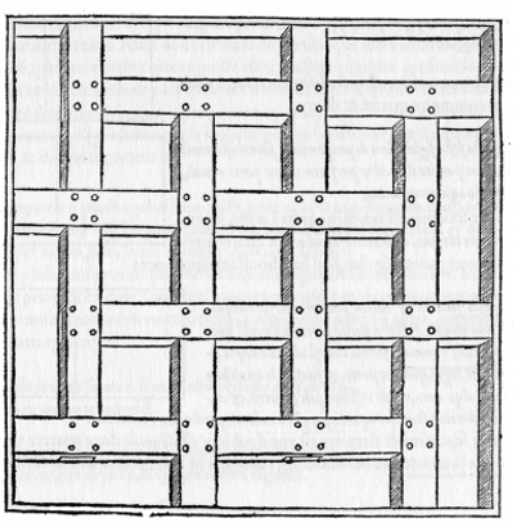


Fig. 3. How to build a floor of fifteen feet with beams that are one *braccio* short. From the *Seven Books on Architecture* by Sebastiano Serlio, Bk I: *De Geometria* (1545)

In the captions related to the drawings of folio 899 of the *Codex Atlanticus*, Leonardo does not claim to have discovered the construction technique that he illustrates. But while Villard and Serlio only deal with one single geometric pattern based on squares, Leonardo's investigates four different geometric patterns, not all of them based on orthogonality. In addition, since his beams are not abutted and nailed together they do not produce an horizontal surface, but rather a vaulted one. Because his notes are so specific in describing quantities and areas together with the details of the building process, it seems, however, quite certain that he actually took part in – or even directed – some building experiment of this kind.

The papers that are presented in this issue of the *Nexus Network Journal* are the result of a workshop that took place in Vinci, birthplace of Leonardo, in June 2003, sponsored by the Leonardo Museum and Library of Vinci and Kim Williams Books, dedicated to the study of "Leonardo's lattices".

The workshop was composed of two parts: first, theoretical studies, and then the experimental phase. For the first part of the project, the seminar, a team of experts in various fields and from various nationalities discussed the use of geometry in the architecture of Leonardo as found in his sketchbooks. Presentations were made by Biagio di Carlo, Sylvie Duvernoy, Christopher Glass, Vesna Petresin, Mark Reynolds, Rinus Roelofs and João Pedro Xavier. The papers in this present issue all grew out of the research for and exchange of ideas during the workshop.

The second part of the project involved the actual construction of domes based upon Leonardo's system. During this second phase, which acted as the verification process of the theoretical research, our group decided to build four vaults following the instructions and drawings of Leonardo. It was important for us to take the discussion from the realm of theory into the realm of practice, since the construction of the dome allowed the theory to be tested. The construction was directed by Dutch artist Rinus Roelofs, who has worked with Leonardo's system of bar grids since 1989.

Leonardo himself gives some starting instructions:

> *Sien legnami tondi, d'abete o castagni. Non sieno forati*
>
> (Let them be round logs, of fir or chessnut. Let them not be drilled with holes).

Our beams where four meters long, and in order to arrange them very regularly we first made four notches in each, to mark the precise position where they had to intersect with each other. The notches also helped prevent the beams from sliding, since they were not fixed to one another by either nails or ropes, but were only "woven". For our first experiment we chose the orthogonal pattern based on the composition of square and rectangular shapes (fig. 4). This pattern may adequately cover a square space if regularly expanded from the center in both directions, or it can be developed along a single direction to cover a rectangular space, or even form the structure for a kind of bridge.

Fig. 4. First dome completed

Unlike masonry vaults or cupolas, which are usually built from the exterior towards the interior, that is, starting from the supporting walls and proceeding towards the center of the space, this kind of woven wooden structure starts from the center and expands outwards, the vaulted form rising in proportion to the width or diameter of the covered space and the thickness of the individual elements: the thicker the beams are, the higher the dome will rise.

The second vault that we built was the one based on a geometrical pattern in which hexagons and equilateral triangles alternate. We later tried another kind of orthogonal pattern composed only of squares of two different sizes (figs. 5 and 6).

Fig. 5. Second dome completed

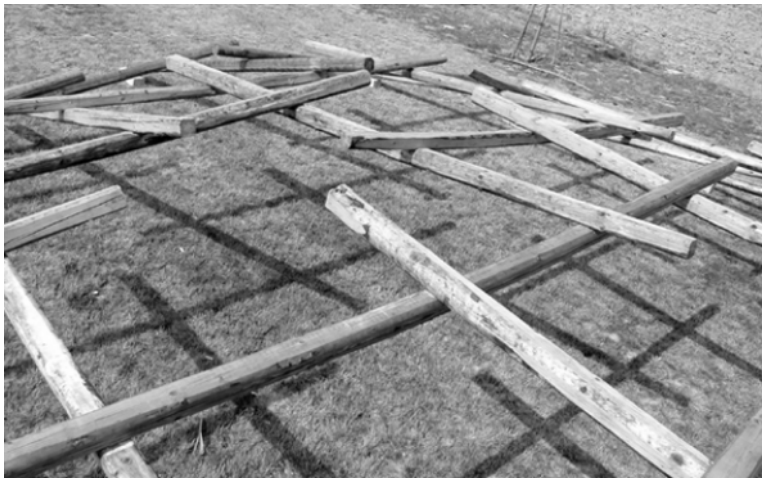


Fig. 6. Third dome completed

The rapidity of the building – and unbuilding – of the vaults allowed us to construct four of them in two days. The beams can be lifted and assembled by three or four men at most, no machinery being necessary, but the more the vault expands, the heavier it becomes and so the more difficult it is to lift and insert more beams at the edges to continue to enlarge the structure. The ultimate limit to the width of the vault is proportional to the strength and maximum resistance of the beams that touch the ground. Leonardo suggests doubling them in order to increase the stability of the structure:

> *ma con certezza si romperà li più deboli, li quali son li più carichi, e quelli che son piè carichi son quelli che toccan terra, che sostengano il tutto, li quali fien raddoppiati, che ne tocca a sostenere tre quarti di cantile*
>
> (but certainly the weakest ones will break, those which carry the greatest loads, and those carrying the greatest loads are those that touch the ground, which can be doubled, because it falls to them to carry three-quarters of the load).

Indeed, at one point one of our beams broke (one that was close to the edge but not actually resting on the ground). Part of the dome collapsed as a consequence, but not all of it, and we were able to repair the damage by removing the broken beam and inserting a new one, without having to dismantle it further.

Leonardo indicates that a vault covering a space of 45 *braccia* will have a height of 5 *braccia* (one Florentine *braccio* equals about 58.37 cm). The curvature of the vault is not very steep. The ratio of height to length is 1 to 9. The measurements we took of our own constructions confirmed this ratio. Some patterns produced a slightly lower curve than others, but overall the ratio of height to width varied between 1:8 and 1:10 (figs. 7 and 8).

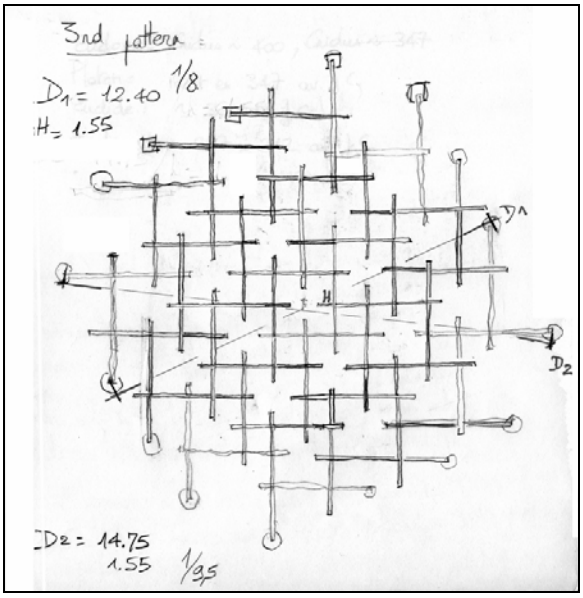


Fig. 7. Measurements of the third dome actually built: the space that has been covered can roughly been approximated to a square. The ratio between width and height of the dome varies from 1/8 to 1/9.5 according to the peripheral supports that are considered

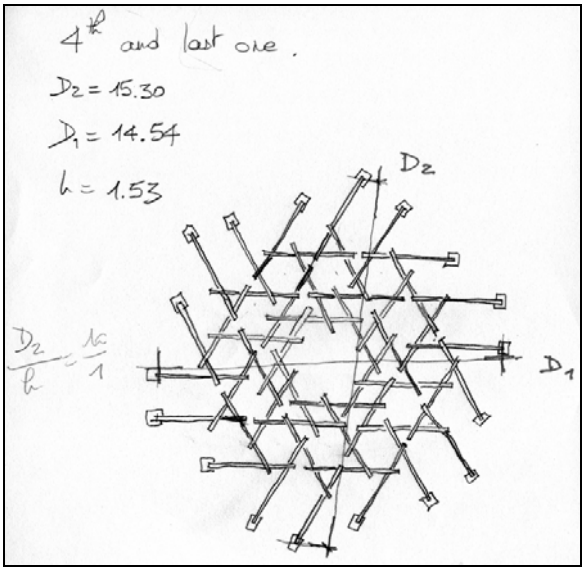


Fig. 8. Last but not least: geometric complexity increases in the pattern of the fourth built dome. Hexagons alternate with triangles and rhombuses. The dome is interrupted by a large central hexagonal oculus. The ratio between width and height – measured at the edge of the oculus – is close to 1/10

The fact that Leonardo speaks of beams that "touch the ground" shows that the experiment in which he participated was similar to ours. "His" vault too was erected on the ground and not on a peripheral masonry structure. But he also speaks of lifting the whole roof to raise it on some supports. For this operation machinery of ropes and levers appears to be required:

Debbon s'alzare tutti a un tratto colle lieve

(The whole should be lifted all at once with levers).

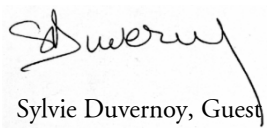
To complete the structure, this roof may be covered with fabric. Leonardo in his notes calculates the number of cloths necessary for a vault that covers a space of 45 *braccia*.

The whole building thus appears to be done with standard materials: small, identical timber beams, pieces of fabric *li quali ordinariamente si fan 30 braccia per ciascuno* (each of which is usually 30 *braccia*). The economy, ease and rapidity of this technology suggested to Carlo Pedretti, the major analyst of Leonardo's writings, that this kind of building was intended as some kind of temporary or emergency shelter. While this hypothesis is logical, nothing in Leonardo's own words of confirm the hypothesis, as he indicates nothing of the purpose and the function of such a structure.

We must consider this experiment as part of the general Renaissance research on relationships between mathematics and architecture. So far, those relationships have been investigated mainly with regards to the interaction between geometry and design, where geometrical shapes and patterns, together with their numerical proportions, guarantee the aesthetic result of the final architectural object. Here, in this experiment, geometry and mathematics are related to building technology, and Leonardo's concern obviously focuses on the role of geometrical shapes in structural stability.

Dome building is only one of the things that can be done with the Leonardo grids. The next step, by varying the basic building element, leads to building spheres, cylinders, and columns. Famous architects and engineers, such as Guastavino, Fuller and Snelson, would later study related structures.

We would like to take this opportunity to thank those who supported the 2003 Leonardo seminar and construction project: The Biblioteca and Museo Leonardiano of the city of Vinci, especially director Romano Nanni and librarian Monica Taddei, Laurent-Paul Robert for coordination of photography, and student helpers Lorenzo Matteoli and Alessio Mattu for help with the construction. And, of course, thank you very much to all those who participated for making the seminar and construction not only a learning experience, but fun as well.



Sylvie Duvernoy, Guest Editor