An Approach to the Dynamics and Kinematical Control of Motion Systems Consisting of a Chain of Bodies

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Abstract. Theoretical investigations concerning the motion of a straight chain of mass points interconnected with kinematical constraints are considered. The ground contact can be described by dry (discontinuous) friction. The controls are assumed in the form of periodic functions with zero average, shifted in phase to each other. There arises a spreading wave along the chain of mass points. In the case of small friction we derive a condition for the locomotion of the center of mass by means of an average method. Motion of the system can be generated both in case of isotropic and non-isotropic friction using specified controls, moreover the movement in the latter case in direction of the larger friction force. The obtained theoretical results give hints for the development of mobile robots applying the described principles of the motion.

1 Introduction

The rectilinear motion on a rough plane of bodies (mass points) connected by elastic elements is considered in a series papers. The system is moved by forces that changed harmonically and acting between the bodies (Miller, 1988). The force of normal pressure is not changed and the asymmetry of the friction force, required for a motion in a given direction, is provided by the dependence of the friction coefficient on the sign of the velocity of the bodies which make up the system (Blekhman, 2000). This effect can be achieved if the contact surfaces of the bodies are equipped with a special form of scales (needle-shaped plate with a required orientation of scales).

In the book, Zimmermann, Zeidis and Behn (2009), the dynamics of a system of two bodies moving along a rough plane joined by an elastic element are considered. The motion is excited by a harmonic force acting between the bodies. In the article, Zimmermann et al. (2004), a magnetizable polymer was employed as an elastic element and the motion was excited by a magnetic field. In the case of small friction, the analytical expression for the average velocity of steady motion of the whole system was found and it is shown, that the motion with this velocity is stable. A similar investigation for a system of two bodies joined by a spring with a nonlinear (cubic) characteristic was shown in the article, Zimmermann et al. (2007). Algebraic equations were obtained for average velocities of the steady motion. It was shown that there exist up to three different motion modes, one or two of them are stable. Steigenberger (1999) presented a numerical solution of the motion equations of a chain of bodies joined by viscoelastic elements for the case, when each body can move only in one direction. In the article Bolotnik et al. (2006), the rectilinear motion of a vibration-driven system on a horizontal rough plane consisting of a carrying body, which interacts with the plane directly, and of internal masses that perform harmonic oscillations relative to the carrying body, is considered. The vertical and horizontal oscillations of the internal masses have the same frequency, but they are shifted in phase. It is shown, that by controlling the phase shift of the horizontal and vertical oscillations, it is possible to change the velocity of the steady motion of the carrying body, and it is not necessary to use scales in order to provide friction asymmetry.

Zimmermann et al. (2009) considered the motion of two mass points connected by a linear spring, when the coefficient of friction does not depend on the direction of motion. Due to the change of the normal force in dependence on time asymmetry of friction is present. The change of normal force is realized by the rotation of two unbalanced rotors with various angular velocities. Chernousko (2005) investigated the rectilinear motion of a body with a movable internal mass moving along a straight line parallel to the line of the body motion on a rough plane. A periodic control mode was constructed for the relative motion of the internal mass for which the main body moves with a periodically changing velocity passing the same distance in a given direction. It is supposed that, at the beginning and the end of each period, the velocity of the main body is zero. The internal mass can move within fixed limits. The control modes relative to the velocity and acceleration of the internal mass were considered. The optimal parameters of both modes which lead to a maximum of the average velocity of motion of the main body for a period were found.

In the present paper[1] we consider the motion of a straight chain of three equal mass points interconnected with kinematical constraints. The ground contact can be described by dry (discontinuous) friction. The controls are assumed in the form of periodic functions with zero average, shifted on a

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phase one concerning each other. Thus, there is a travelling wave along the chain of mass points. It is shown that, using special control algorithms motion is also possible by isotropic friction and by constant normal force. In the case of non-isotropic friction the motion is possible even in the direction of the greater friction.

2 Equations of Motion

We consider the motion of a system of three mass points with the coordinates x_i $(i = 1\,,2\,,3)$ and with the masses m, connected by kinematical constraints along an axis Ox (see Figure 1). The motion of the system is excited by the kinematical constraints setting the distances $l_1(t)$ and $l_2(t)$ between mass points.

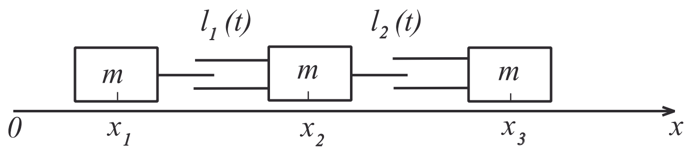

Figure 1. The schematic of the system of three mass points.

$$l_1(t) = l_0 + a_1(t), \quad l_2(t) = l_0 + a_2(t), \quad a_1(0) = a_2(0) = 0\,. \tag{1}$$

Let us consider the functions $l_1(t)$ and $l_2(t)$ (hence as well $a_1(t)$ and $a_2(t)$) that are periodic in time t with period T. The kind of functions $a_1(t)$ and $a_2(t)$ will be discussed below.

There is the force of friction $F(V_i)$ acting on each mass point from the surface, directed against motion and depending on the velocity $V_i = \dot{x}_i$ $(i = 1\,,2\,,3)$.

The velocity of the center of mass of system can be represented as $V = \frac{1}{3}(\dot{x}_1 + \dot{x}_2 + \dot{x}_3)$. The equation of the motion of the center of mass is:

$$3\,m\,\dot{V} = F(\dot{x}_1) + F(\dot{x}_2) + F(\dot{x}_3)\,. \tag{2}$$

Using $x_2(t) - x_1(t) = l_1(t)\,, \quad x_3(t) - x_2(t) = l_2(t)$ and substituting expressions (1) in (2) the equation of the motion (2) takes the form

$$3\,m\,\dot{V} = F\left(V - \frac{2}{3}\,\dot{a}_1 - \frac{1}{3}\,\dot{a}_2\right) + F\left(V + \frac{1}{3}\,\dot{a}_1 - \frac{1}{3}\,\dot{a}_2\right) + F\left(V + \frac{1}{3}\,\dot{a}_1 + \frac{2}{3}\,\dot{a}_2\right). \tag{3}$$

We assume, that in the initial moment $t = 0$ the velocity of the center of mass $V(0) = 0$.

Let us introduce dimensionless variables in according to the following formulas (the asterisk * is a symbol of dimensionless variables):

$$\begin{aligned} x_i^* &= \frac{x_i}{L}\ (i=1,2,3)\,, \quad V^* = V\,\frac{T}{L}\,, \quad t^* = \frac{t}{T}\,, \\ a_i^* &= \frac{a_i}{L}\,, \quad l_i^* = \frac{l_i}{L}\ (i=1,2)\,, \quad F^*(V^*) = \frac{F(V)}{F_s} = \frac{F(V^*\cdot L/T)}{F_s}\,. \end{aligned} \tag{4}$$

Here L is a characteristic linear dimension (for example the greatest value $a_1(t)$ or $a_2(t)$ in period T), F_s is a characteristic value of the friction force.

Hereafter, we use dimensionless variables. Introducing the dimensionless variables in equation (4) and denoting $u_1(t) = \dot{a}_1(t)$, $u_2(t) = \dot{a}_2(t)$, we rewrite equation (3) in dimensionless variables (the old symbols are hold)

$$\frac{dV}{dt} = \frac{\varepsilon}{3}\left[F\left(V-\frac{2}{3}\,u_1-\frac{1}{3}\,u_2\right)+F\left(V+\frac{1}{3}\,u_1-\frac{1}{3}\,u_2\right)+F\left(V+\frac{1}{3}\,u_1+\frac{2}{3}\,u_2\right)\right], \tag{5}$$

where $\varepsilon = F_s T^2/mL$.

Let us notice, that since $a_1(t)$ and $a_2(t)$ are periodic functions with period T, therefore $u_1(t) = \dot{a}_1(t)$ and also $u_2(t) = \dot{a}_2(t)$ are periodic functions with period T and have zero average.

Further we assume everywhere, that $\varepsilon \ll 1$. The smallness of the parameter ε shows that the value of the friction force F_s is small compared to the amplitude of the "driving" force mL/T^2. The equation (5) has a so called "standard" form (Bogolyubov and Mitropolslki, 1961). Averaging the right side of the equation (5) relative to the variable t in the period 1, we obtain

$$\frac{dV}{dt} = \frac{\varepsilon}{3}\,G(V)\,, \tag{6}$$

$$G(V) = \int_0^1 \left[F\left(V-\frac{2}{3}u_1-\frac{1}{3}u_2\right)+F\left(V+\frac{1}{3}u_1-\frac{1}{3}u_2\right)+F\left(V+\frac{1}{3}u_1+\frac{2}{3}u_2\right)\right]dt\,.$$

Now it is necessary to define the functions $u_1(t)$, $u_2(t)$ and the law of friction.

3 Smooth Control

Let us consider the functions $a_1(t)$ and $a_2(t)$, composed from the parabolas. These functions have continuous derivatives $u_1(t)$ and $u_2(t)$, shown in Figure 2 accordingly marked as a solid and as a dotted line and have the form:

$$u_1(t) = \begin{cases} 0, \ 0 \le t \le \dfrac{1}{3}, \\ 2\,(3\,t-1), \ \dfrac{1}{3} < t \le \dfrac{1}{2}, \\ -2\,(3\,t-2), \ \dfrac{1}{2} < t \le \dfrac{5}{6}, \\ 6\,(t-1), \ \dfrac{5}{6} < t \le 1\,. \end{cases} \quad u_2(t) = \begin{cases} 6\,t, \ 0 \le t \le \dfrac{1}{6}, \\ -2\,(3\,t-1), \ \dfrac{1}{6} < t \le \dfrac{1}{2}, \\ 2\,(3\,t-2), \ \dfrac{1}{2} < t \le \dfrac{2}{3}, \\ 0, \ \dfrac{2}{3} < t \le 1. \end{cases} \tag{7}$$

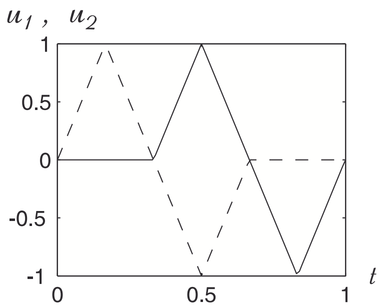

Figure 2. The function $u_1(t)$ (sol. line), $u_2(t)$ (dott. line).

They are equal to zero on an interval of length 1/3 and are shifted on time for this magnitude one relatively to another.

4 Dry (Discontinuous) Friction

We assume that Coulomb dry friction acts on the mass points. The dimension force of dry friction $F(V)$ satisfies the Coulomb law

$$F(V) = \begin{cases} F_- = k_- N\,, & \text{if } V < 0\,, \\ -F_0\,, & \text{if } V = 0\,, \\ -F_+ = -k_+ N\,, & \text{if } V > 0\,. \end{cases}$$

Here N is the force of normal pressure (in this case $N = mg$, where g is the free fall acceleration), k_- and k_+ are the coefficients of dry friction at the motion in a negative and in a positive direction respectively, $F_- \le F_+$ ($k_- \le k_+$), the value $F_0 \in [-F_-\,, F_+]$.

The expression for dimensionless friction force takes the form ($i = 1, 2, 3$):

$$F^* (\dot{x}_i^*) = \begin{cases} 1, & \dot{x}_i^* < 0, \\ -\mu_0, & \dot{x}_i^* = 0, \\ -\mu, & \dot{x}_i^* > 0. \end{cases} \tag{8}$$

Here, the value F_s in formulas (4) is $F_s = F_-$ (F_- is the magnitude of the friction force at the motion in a negative direction), $\mu = F_+/F_- = k_+/k_- \geq 0$, $\mu_0 \in [-1, \mu]$.

Let us prove this, assuming that $\dot{x}_i(t)$ is a piecewise continuous function of time. This assumption is quite sufficient for simulating feasible motion. The second condition (8) is connected with sticking ("stick-slip" motions). This effect is characteristic for systems with dry friction. Let us notice, that for the given control the velocity of each mass point could not be equal to zero on a finite time interval. Hence, the "stick-slip" effect is absent.

After substituting expressions (7) and (8) into equation (6), we receive

$$\frac{dV}{dt} = \frac{\varepsilon}{3} \begin{cases} 3, & V \leq -2/3, \\ 2 - \mu - 3V\, 1 + \mu/2, & -2/3 < V \leq 0, \\ 2 - \mu - 6V(1 + \mu), & 0 < V \leq 1/3, \\ -3\mu, & V > 1/3. \end{cases} \tag{9}$$

We consider the solution of equation (9) with the initial condition $V(0) = 0$. If $\mu = 2$ (friction in the positive direction is double the friction in the negative direction), the system remains in rest.

If $\mu < 2$ the chain of mass points moves to the right with the velocity

$$V = \frac{2 - \mu}{6(1 + \mu)} \left[1 - e^{-2\varepsilon(1+\mu)t}\right]$$

and tends to stationary value $V_s = \dfrac{2 - \mu}{6(1 + \mu)}$. In the case of isotropic friction, we find $V_s = 1/12$.

Thus, under this control algorithms, motion is possible in the case of isotropic friction as well as in the direction of the greater friction in the case of non-isotropic friction. If $\mu > 2$ the chain moves to the left with the velocity

$$V = \frac{2(2 - \mu)}{3(1 + \mu)} \left[1 - e^{-\frac{\varepsilon}{2}(1+\mu)t}\right]$$

and tends to stationary value $V_s = \dfrac{2(2 - \mu)}{3(1 + \mu)}$.

In Figure 3 the results of the numerical integration of the exact and averaged equations in the case of symmetric friction ($\mu = 1$) and for parameter $\varepsilon = 0.3$ are shown.

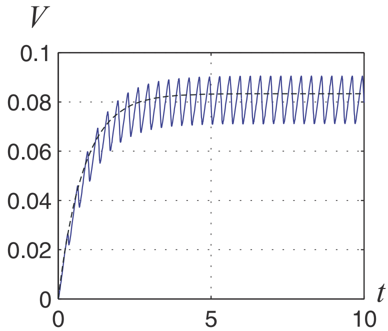

Figure 3. Solutions of the exact and the averaged equations for dry friction.

5 Conclusions

It is shown that, using periodical control algorithms, the motion of a chain of mass points is possible in the case of isotropic friction and in the case of non-isotropic friction in the direction of the greater friction. The locomotion is impossible without a shift of the phases in the control law.

In the case of small friction we derived a condition for the locomotion of the center of the mass with the help of an average method. In the case of smooth control we received explicit expressions for the average velocity of the motion of the center of mass. A prototype of this system was created (see Figure 4). Experiments with this system coincide with the qualitative predictions of the theory.

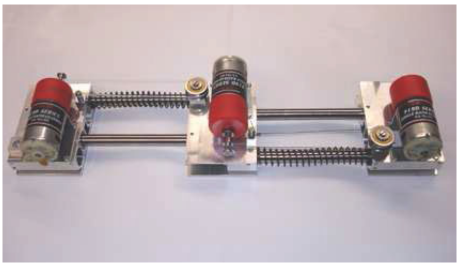

Figure 4. A first prototype of the motion system, Keil (2009).

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