The Complexity of Synthesizing nop-Equipped Boolean Petri Nets from g-Bounded Inputs

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Abstract. Boolean Petri nets equipped with nop allow places and transitions to be independent by being related by nop. We characterize for any fixed $g \in \mathbb{N}$ the computational complexity of synthesizing nop-equipped Boolean Petri nets from labeled directed graphs whose states have at most g incoming and at most g outgoing arcs.

1 Introduction

Boolean Petri nets are a basic model for the description of distributed and concurrent systems. These nets allow at most one token on each place p in every reachable marking. Therefore, p is considered a Boolean condition that is *true* if p is marked and *false* otherwise. A place p and a transition t of a Boolean Petri net N are related by one of the following Boolean *interactions*: *no operation* (nop), *input* (inp), *output* (out), *unconditionally set to true* (set), *unconditionally reset to false* (res), *inverting* (swap), *test if true* (used), and *test if false* (free). The relation between p and t determines which conditions p must satisfy to allow t's firing and which impact has the firing of t on p: The interaction inp (out) defines that p must be *true* (*false*) first and *false* (*true*) after t has fired. If p and t are related by free (used) then t's firing proves that p is *false* (*true*). The interaction nop says that p and t are independent, that is, neither need p to fulfill any condition nor has the firing of t any impact on p. If p and t are related by res (set) then p can be both *false* or *true* but after t's firing it is *false* (*true*). Also, the interaction swap does not require that p satisfies any particular condition to enable t. Here, the firing of t inverts p's Boolean value.

Boolean Petri nets are classified by the interactions of I that they use to relate places and transitions. More exactly, a subset $\tau \subseteq I$ is called a *type of net* and a net N is of type τ (a τ-net) if it applies at most the interactions of τ. So far, research has explicitly discussed seven Boolean Petri net types: *Elementary net systems* {nop, inp, out} [9], *Contextual nets* {nop, inp, out, used, free} [6], *event/condition nets* {nop, inp, out, used} [2], *inhibitor nets* {nop, inp, out, free} [8], *set nets* {nop, inp, set, used} [5], *trace nets* {nop, inp, out, set, res, used, free} [3], and *flip flop nets* {nop, inp, out, swap} [10].

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However, since we have eight interactions to choose from, there are actually a total of 256 different types.

This paper addresses the computational complexity of the τ-*synthesis* problem. It consists in deciding whether a given directed labeled graph A, also called *transition system*, is isomorphic to the reachability graph of a τ-net N and in constructing N if it exists. It has been shown that τ-*synthesis* is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}\}$ [1], even if the inputs are strongly restricted [14,17]. On the contrary, in [10], it has been shown that it becomes polynomial if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{swap}\}$. These opposing results motivate the question which interactions of I make the synthesis problem hard and which make it tractable. In our previous work of [13,15,16], we answer this question partly and reveal the computational complexity of 120 of the 128 types that allow nop.

In this paper, we investigate for fixed $g \in \mathbb{N}$ the computational complexity of τ-synthesis restricted to g-bounded inputs, that is, every state of A has at most g incoming and at most g outgoing arcs. On the one hand, inputs of practical applications tend to have a low bound g such as benchmarks of digital hardware design [4]. On the other hand, considering restricted inputs hopefully gives us a better understanding of the problem's hardness. Thus, g-bounded inputs are interesting from both the practical and the theoretical point of view. In this paper, we completely characterize the complexity of τ-synthesis restricted to g-bounded inputs for all types that allow places and transitions to be independent, that is, which contain nop. Figure 1 summarizes our findings: For the types of §1 and §2, we showed hardness of synthesis without restriction in [15]. In this paper, we strengthen these results to 2- and 3-bounded inputs, respectively, and show that these bounds are tight. The hardness result of the types of §3 originates from [16]. This paper shows that a bound less than 2 makes synthesis tractable. Hardness for the types of §4 to §8 has been shown for 2-bounded inputs in [16]. In this paper, we strengthen this results to 1-bounded inputs. The hardness part for the types of §9 origin from [13]. In this paper, we argue that the bound 2 is tight. Finally, while the results of §10 are new, the ones of §11 have been found in [15].

For all considered types τ, the corresponding hardness results are based on a reduction of the so-called *cubic monotone one-in-three 3SAT* problem [7]. All reductions follow a common approach that represents clauses by directed labeled paths. Thus, this paper also contributes a very general way to prove NP-completeness of synthesis of Boolean types of nets.

2 Preliminaries

Transition Systems. A *transition system* (TS) $A = (S, E, \delta)$ is a directed labeled graph with states S, events E and partial *transition function* $\delta : S \times E \longrightarrow S$, where $\delta(s, e) = s'$ is interpreted as $s \xrightarrow{e} s'$. For $s \xrightarrow{e} s'$ we say s is a source and s' is a sink of e, respectively. An event e *occurs* at a state s, denoted by $s \xrightarrow{e}$, if $\delta(s, e)$ is defined. An *initialized* TS $A = (S, E, \delta, s_0)$ is a TS with a distinct state $s_0 \in S$ where every state $s \in S$ is *reachable* from s_0 by a directed labeled path. TSs in this paper are *deterministic* by design as their state

§	Type of net τ	g	Complexity	#
1	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{used}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$	≥ 2 < 2	NP-complete polynomial	4
2	$\{\mathsf{nop}, \mathsf{set}, \mathsf{res}\} \cup \omega$ and $\emptyset \neq \omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 3 < 3	NP-complete polynomial	3
3	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}\}$, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\} \cup \omega$ and $\omega \subseteq \{\mathsf{out}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{set}\} \cup \omega$ and $\omega \subseteq \{\mathsf{inp}, \mathsf{used}, \mathsf{free}\}$	≥ 2 < 2	NP-complete polynomial	16
4	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{set}\} \cup \omega$ or $\{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{res}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 1	NP-complete	8
5	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{used}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{used}, \mathsf{free}\}$	≥ 1	NP-complete	4
6	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{swap}\} \cup \omega$ or $\{\mathsf{nop}, \mathsf{out}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 1	NP-complete	8
7	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{out}, \mathsf{res}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{inp}, \mathsf{set}, \mathsf{used}, \mathsf{free}\}$	≥ 1	NP-complete	28
8	$\{\mathsf{nop}, \mathsf{inp}, \mathsf{out}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 1	NP-complete	4
9	$\{\mathsf{nop}, \mathsf{set}, \mathsf{swap}\} \cup \omega$, $\{\mathsf{nop}, \mathsf{res}, \mathsf{swap}\} \cup \omega$, $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ and $\emptyset \neq \omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 2 < 2	NP-complete polynomial	9
10	$\{\mathsf{nop}, \mathsf{inp}\}$, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{used}\}$, $\{\mathsf{nop}, \mathsf{out}\}$, $\{\mathsf{nop}, \mathsf{out}, \mathsf{free}\}$ $\{\mathsf{nop}, \mathsf{set}, \mathsf{swap}\}$, $\{\mathsf{nop}, \mathsf{res}, \mathsf{swap}\}$, $\{\mathsf{nop}, \mathsf{set}, \mathsf{res}\}$, $\{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{swap}\}$	≥ 0	polynomial	8
11	$\{\mathsf{nop}, \mathsf{res}\} \cup \omega$ and $\omega \subseteq \{\mathsf{inp}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{set}\} \cup \omega$ and $\omega \subseteq \{\mathsf{out}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{inp}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$, $\{\mathsf{nop}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$	≥ 0	polynomial	36

Fig. 1. The computational complexity of Boolean net synthesis from g-bounded TS for all types that contain nop.

transition behavior is given by a (partial) function. Let $g \in \mathbb{N}$. An initialized TS A is called g-bounded if for all $s \in S(A)$ the number of incoming and outgoing arcs at s is restricted by g: $|\{e \in E(A) \mid \xrightarrow{e} s\}| \leq g$ and $|\{e \in E(A) \mid s \xrightarrow{e}\}| \leq g$.

Boolean Types of Nets [2]**.** The following notion of Boolean types of nets serves as vehicle to capture many Boolean Petri nets in a uniform way. A *Boolean type of net* $\tau = (\{0,1\}, E_\tau, \delta_\tau)$ is a TS such that E_τ is a subset of the Boolean interactions: $E_\tau \subseteq I = \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{set}, \mathsf{res}, \mathsf{swap}, \mathsf{used}, \mathsf{free}\}$. The interactions $i \in I$ are binary partial functions $i : \{0,1\} \rightarrow \{0,1\}$ as defined in Fig. 2. For all $x \in \{0,1\}$ and all $i \in E_\tau$ the transition function of τ is defined by $\delta_\tau(x, i) = i(x)$. Notice that I contains all binary partial functions $\{0,1\} \rightarrow \{0,1\}$ except for the entirely undefined function $\perp$. Even if a type τ includes $\perp$, this event can never occur, so it would be useless. Thus, I is complete for deterministic Boolean types of nets, and that means there are a total of 256 of them. By definition, a Boolean type τ is completely determined by its event set E_τ. Hence, in the following we identify τ with E_τ, cf. Fig. 3. Moreover, for readability, we group interactions by $\mathsf{enter} = \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, $\mathsf{exit} = \{\mathsf{inp}, \mathsf{res}, \mathsf{swap}\}$, $\mathsf{keep}^+ = \{\mathsf{nop}, \mathsf{set}, \mathsf{used}\}$, and $\mathsf{keep}^- = \{\mathsf{nop}, \mathsf{res}, \mathsf{free}\}$.

x	nop(x)	inp(x)	out(x)	set(x)	res(x)	swap(x)	used(x)	free(x)
0	0		1	1	0	1		0
1	1	0		1	0	0	1	

Fig. 2. All interactions in I. An empty cell means that the column's function is undefined on the respective x. The entirely undefined function is missing in I.

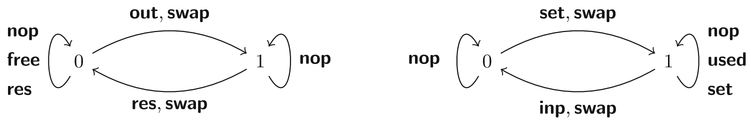


Fig. 3. Left: $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{swap}, \mathsf{free}\}$. Right: $\tilde{\tau} = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{swap}, \mathsf{used}\}$. τ and $\tilde{\tau}$ are isomorphic. The isomorphism $\phi : \tau \to \tilde{\tau}$ is given by $\phi(s) = 1 - s$ for $s \in \{0, 1\}$, $\phi(i) = i$ for $i \in \{\mathsf{nop}, \mathsf{swap}\}$, $\phi(\mathsf{out}) = \mathsf{inp}$, $\phi(\mathsf{res}) = \mathsf{set}$ and $\phi(\mathsf{free}) = \mathsf{used}$.

τ-Nets. Let $\tau \subseteq I$. A Boolean Petri net $N = (P, T, H_0, f)$ of type τ, (τ-net) is given by finite and disjoint sets P of places and T of transitions, an initial marking $H_0 : P \longrightarrow \{0, 1\}$, and a (total) flow function $f : P \times T \to \tau$. A τ-net realizes a certain behavior by firing sequences of transitions: A transition $t \in T$ can fire in a marking $M : P \longrightarrow \{0, 1\}$ if $\delta_\tau(M(p), f(p, t))$ is defined for all $p \in P$. By firing, t produces the next marking $M' : P \longrightarrow \{0, 1\}$ where $M'(p) = \delta_\tau(M(p), f(p, t))$ for all $p \in P$. This is denoted by $M \overset{t}{\longrightarrow} M'$. Given a τ-net $N = (P, T, H_0, f)$, its behavior is captured by a transition system A_N, called the reachability graph of N. The state set of A_N consists of all markings that, starting from initial state H_0, can be reached by firing a sequence of transitions. For every reachable marking M and transition $t \in T$ with $M \overset{t}{\longrightarrow} M'$ the state transition function δ of A is defined as $\delta(M, t) = M'$.

τ-Regions. Let $\tau \subseteq I$. If an input A of τ-synthesis allows a positive decision then we want to construct a corresponding τ-net N purely from A. Since A and A_N are isomorphic, N's transitions correspond to A's events. However, the notion of a place is unknown for TSs. So-called regions mimic places of nets: A τ-region of a given $A = (S, E, \delta, s_0)$ is a pair (sup, sig) of *support* $sup : S \to S_\tau = \{0, 1\}$ and *signature* $sig : E \to E_\tau = \tau$ where every transition $s \overset{e}{\longrightarrow} s'$ of A leads to a transition $sup(s) \overset{sig(e)}{\longrightarrow} sup(s')$ of τ. While a region divides S into the two sets $sup^{-1}(b) = \{s \in S \mid sup(s) = b\}$ for $b \in \{0, 1\}$, the events are cumulated by $sig^{-1}(i) = \{e \in E \mid sig(e) = i\}$ for all available interactions $i \in \tau$. We also use $sig^{-1}(\tau') = \{e \in E \mid sig(e) \in \tau'\}$ for $\tau' \subseteq \tau$. A region (sup, sig) models a place p and the corresponding part of the flow function f. In particular, $sig(e)$ models $f(p, e)$ and $sup(s)$ models $M(p)$ in the marking $M \in RS(N)$ corresponding to $s \in S(A)$. Every set $\mathcal{R}$ of τ-regions of A defines the *synthesized τ-net* $N_A^{\mathcal{R}} = (\mathcal{R}, E, f, H_0)$ with flow function $f((sup, sig), e) = sig(e)$ and initial marking $H_0((sup, sig)) = sup(s_0)$ for all $(sup, sig) \in \mathcal{R}, e \in E$. It is well known that $A_{N_A^{\mathcal{R}}}$ and A are isomorphic if and only if $\mathcal{R}$'s regions solve certain separation atoms [2], to be introduced next. A pair (s, s') of distinct states of A defines a *state separation atom* (SSP atom). A τ-region $R = (sup, sig)$ *solves* (s, s') if $sup(s) \neq sup(s')$. The meaning of R is to ensure that $N_A^{\mathcal{R}}$ contains at least one place R such that $M(R) \neq M'(R)$ for the markings M and M' corresponding to s and s', respectively. If there is a τ-region that solves (s, s') then s and s' are called *τ-solvable*. If every SSP atom of A is τ-solvable then A has the *τ-state separation property* (τ-SSP). A pair (e, s) of event $e \in E$ and state $s \in S$ where e does not occur at s, that is $\neg s \xrightarrow{e}$, defines an *event state separation atom* (ESSP atom). A τ-region $R = (sup, sig)$ *solves* (e, s) if $sig(e)$ is not defined on $sup(s)$ in τ, that is, $\neg\delta_\tau(sup(s), sig(e))$. The meaning of R is to ensure that there is at least one place R in $N_A^{\mathcal{R}}$ such that $\neg M \xrightarrow{e}$ for the marking M corresponding to s. If there is a τ-region that solves (e, s) then e and s are called *τ-solvable*. If every ESSP atom of A is τ-solvable then A has the *τ-event state separation property* (τ-ESSP). A set $\mathcal{R}$ of τ-regions of A is called *τ-admissible* if for every of A's (E)SSP atoms there is a τ-region R in $\mathcal{R}$ that solves it. The following lemma, borrowed from [2, p.163], summarizes the already implied connection between the existence of τ-admissible sets of A and (the solvability of) τ-synthesis:

Lemma 1 ([2]). *A TS A is isomorphic to the reachability graph of a τ-net N if and only if there is a τ-admissible set $\mathcal{R}$ of A such that $N = N_A^{\mathcal{R}}$.*

We say a τ-net N *τ-solves* A if A_N and A are isomorphic. By Lemma 1, deciding if A is τ-solvable reduces to deciding whether it has the τ-(E)SSP. Moreover, it is easy to see that if τ and $\tilde{\tau}$ are isomorphic then deciding the τ-(E)SSP reduces to deciding the $\tilde{\tau}$-(E)SSP:

Lemma 2 (Without proof). *If τ and $\tilde{\tau}$ are isomorphic types of nets then a TS A has the τ-(E)SSP if and only if A has the $\tilde{\tau}$-(E)SSP.*

In particular, we benefit from the isomorphisms that map nop to nop, swap to swap, inp to out, set to res, used to free, and vice versa.

$s_0 \xrightarrow{a} s_1 \xleftrightarrow{a} s_2$ TS A_1.

$s_0 \xrightarrow{a} s_1 \xrightarrow{a} s_2$ TS A_2.

$s_0 \xleftrightarrow{a} s_1 \xleftrightarrow{a} s_2$ TS A_3.

$s_0 \xrightarrow{a} s_1 \xrightarrow{a} s_2 \xrightarrow{a} s_3$ TS A_4.

Fig. 4. Let $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{swap}, \mathsf{free}\}$. The TSs $A_1, \dots, A_4$ give examples for the presence and absence of the τ-(E)SSP: TS A_1 has the τ-ESSP as a occurs at every state. It has also the τ-SSP: The region $R = (sup, sig)$ where $sup(s_0) = sup(s_2) = 1$, $sup(s_1) = 0$ and $sig(a) = \mathsf{swap}$ separates the pairs s_0, s_1 and s_2, s_1. Moreover, the region $R' = (sup', sig')$ where $sup'(s_0) = 0$ and $sup'(s_1) = sup'(s_2) = 1$ and $sig'(a) = \mathsf{set}$ separates s_0 and s_1. Notice that R and R' can be translated into $\tilde{\tau}$-regions, where $\tilde{\tau} = \{\mathsf{nop}, \mathsf{res}, \mathsf{swap}, \mathsf{used}\}$, via the isomorphism of Fig. 3. For example, if $s \in S(A_1)$ and $e \in E(A_1)$ and $sup''(s) = \phi(sup(s))$ and $sig''(e) = \phi(sig(e))$ then the resulting $\tilde{\tau}$-region $R'' = (sup'', sig'')$ separates s_0, s_1 and s_2, s_1. Thus, A_1 has also $\tilde{\tau}$-(E)SSP. TS A_2 has the τ-SSP but not the τ-ESSP as event a is not inhibitable at the state s_2. TS A_3 has the τ-ESSP (a occurs at every state) but not the τ-SSP as s_1 and s_2 are not separable. TS A_4 has neither the τ-ESSP nor the τ-SSP.

3 Hardness Results

In this section, for several types of nets $\tau \subseteq I$ and fixed $g \in \mathbb{N}$, we show that τ-synthesis is NP-complete even if the input TS A is g-bounded, cf. Fig. 1. Since τ-synthesis is known to be in NP for all $\tau \subseteq I$ [16], we restrict ourselves to the hardness part. All proofs are based on a reduction of the problem *cubic monotone one-in-three* 3-*SAT* which has been shown to be NP-complete in [7]. The input for this problem is a Boolean expression $\varphi = \{\zeta_0, \dots, \zeta_{m-1}\}$ of m negation-free three-clauses $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ such that every variable $X \in V(\varphi) = \bigcup_{i=0}^{m-1} \zeta_i$ occurs in exactly three clauses. Notice that the latter implies $|V(\varphi)| = m$. Moreover, we assume without loss of generality that if $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ then $i_0 < i_1 < i_2$. The question to answer is whether there is a subset $M \subseteq V(\varphi)$ with $|M \cap \zeta_i| = 1$ for all $i \in \{0, \dots, m-1\}$. For all considered types of nets τ and corresponding bounds g, we reduce a given instance φ to a g-bounded TS A^τ_φ such that the following two conditions are true: Firstly, the TS A^τ_φ has an ESSP atom α which is τ-solvable if and only if there is a one-in-three model M of φ. Secondly, if the ESSP atom α is τ-solvable then all ESSP and SSP atoms of A^τ_φ are also τ-solvable. A reduction that satisfies these conditions proves the NP-hardness of τ-synthesis as follows: If φ has a one-three-model then the conditions ensure that the TS A^τ_φ has the τ-(E)SSP and thus is τ-solvable. Conversely, if A^τ_φ is τ-solvable then, by definition, it has the τ-ESSP. In particular, there is a τ-region that solves α which, by the first condition, implies that φ has a one-in-three model. Consequently, A^τ_φ is τ-solvable if and only if φ has a one-in-three model. Due to space restrictions, we omit for all considered types the proof that A^τ_φ satisfies the second condition, that is, that the solvability of α implies the (E)SSP. However, the corresponding proofs can be found in the technical report [11].

A key idea, applied by all reductions in one way or another, is the representation of every clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$, $i \in \{0, \dots, m-1\}$, by a directed labeled path of A^τ_φ on which the variables of ζ_i occur as events:

$$s_{i,0} \dots s_{i,j} \xrightarrow{X_{i_0}} s_{i,j+1} \dots s_{i,j'} \xrightarrow{X_{i_1}} s_{i,j'+1} \dots s_{i,j''} \xrightarrow{X_{i_2}} s_{i,j''+1} \dots s_{i,n}$$

The reductions ensure that if a τ-region (sup, sig) solves the atom α then $sup(s_{i,0}) \neq sup(s_{i,n})$. This makes the image of this path under (sup, sig) a directed path from 0 to 1 or from 1 to 0 in τ. Thus, there has to be an event e on the path that causes the state change from $sup(s_{i,0})$ to $sup(s_{i,n})$ by $sig(e)$. We exploit this property and ensure that this state change is unambiguously done by (the signature of) exactly one variable event per clause. As a result, the corresponding variable events define a searched model of φ via their signature. The proof of the following theorem gives a first example of this approach, and Fig. 5 shows a full example reduction.

Theorem 1. *For any fixed $g \geq 2$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{free}\}$, $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{used}, \mathsf{free}\}$, $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{used}\}$ and $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$.*

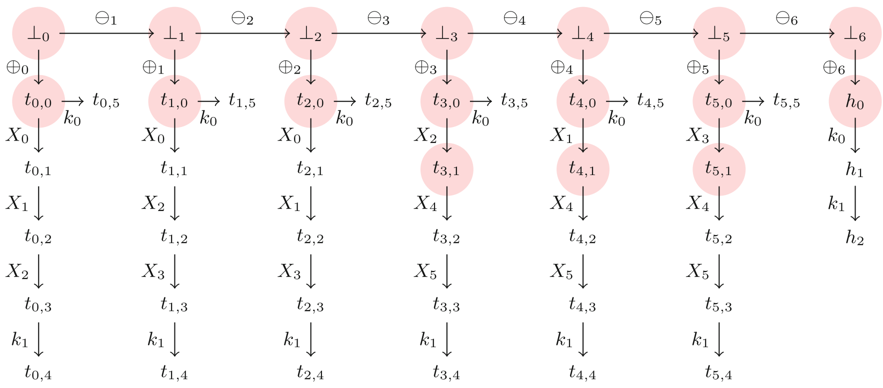


Fig. 5. The TS A_{φ}^{τ} for $\varphi = \{\zeta_0, \dots, \zeta_5\}$ with clauses $\zeta_0 = \{X_0, X_1, X_2\}$, $\zeta_1 = \{X_0, X_2, X_3\}$, $\zeta_2 = \{X_0, X_1, X_3\}$, $\zeta_3 = \{X_2, X_4, X_5\}$, $\zeta_4 = \{X_1, X_4, X_5\}$, $\zeta_5 = \{X_3, X_4, X_5\}$. The red colored area sketc.hes the τ-region (sup, sig) that solves (k_1, h_0) and corresponds to the one-in-three model $M = \{X_0, X_4\}$. (Color figure online)

Proof. We argue for $\tau \in \{\{\mathsf{nop}, \mathsf{inp}, \mathsf{free}\}, \{\mathsf{nop}, \mathsf{inp}, \mathsf{used}, \mathsf{free}\}\}$, which by Lemma 2 proves the claim for the other types, too.
Firstly, the TS A_{φ}^{τ} has the following gadget H (left hand side) that provides the events k_0, k_1 and the atom $\alpha = (k_1, h_0)$. Secondly, it has for every clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ the following gadget T_i (right hand side) that applies k_0, k_1 and $\zeta_i's$ variables as events.

$$h_0 \xrightarrow{k_0} h_1 \xrightarrow{k_1} h_2 \qquad\qquad \begin{array}{l} t_{i,0} \xrightarrow{X_{i_0}} t_{i,1} \xrightarrow{X_{i_1}} t_{i,2} \xrightarrow{X_{i_2}} t_{i,3} \xrightarrow{k_1} t_{i,4} \\ \downarrow k_0 \\ t_{i,5} \end{array}$$

Finally, A_{φ}^{τ} uses the states $\bot_0, \dots, \bot_m$ and events $\ominus_1, \cdots \ominus_m$ and $\oplus_0, \dots, \oplus_m$ to join the gadgets $T_0, \dots, T_{m-1}$ and H by $\bot_i \xrightarrow{\ominus_{i+1}} \bot_{i+1}$ and $\bot_i \xrightarrow{\oplus_i} t_{i,0}$, for all $i \in \{0, \dots, m-1\}$, and $\bot_m \xrightarrow{\oplus_m} h_0$, cf. Fig. 5.

The gadget H ensures that if (sup, sig) is a region that solves α then $sup(h_0) = 1$ and $sig(k_1) = \mathsf{free}$ which implies $sup(h_1) = 0$ and $sig(k_0) = \mathsf{inp}$. This is because $sig(k_1) \in \{\mathsf{inp}, \mathsf{used}\}$ and $sup(h_0) = 0$ implies $sig(k_0) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, which is impossible. Consequently, $s \xrightarrow{k_0}$ and $s' \xrightarrow{k_1}$ imply $sup(s) = 1$ and $sup(s') = 0$, respectively. The TS A_{φ}^{τ} uses these properties to ensure via $T_0, \dots, T_{m-1}$ that the region (sup, sig) implies a one-in-three model of φ.

More exactly, if $i \in \{0, \dots, m-1\}$ then for T_i we have by $t_{i,0} \xrightarrow{k_0}$ and $t_{i,3} \xrightarrow{k_1}$ that $sup(t_{i,0}) = 1$ and $sup(t_{i,3}) = 0$. Thus, there is an event X_{i_j}, where $j \in \{0, 1, 2\}$, such that $sig(X_{i_j}) = \mathsf{inp}$. Clearly, if $sig(X_{i_j}) = \mathsf{inp}$ then $sig(X_{i_\ell}) \neq \mathsf{inp}$ for all $j < \ell \in \{0, 1, 2\}$ as X_{i_ℓ}'s sources have a 0-support. Consequently, there is *exactly one* variable event $X \in \zeta_i$ such that $sig(X) = \mathsf{inp}$. Since i was arbitrary, this is simultaneously true for all clauses $\zeta_0, \dots, \zeta_{m-1}$. Thus, the set $M = \{X \in V(\varphi) \mid sig(X) = \mathsf{inp}\}$ is a one-in-three model of φ.

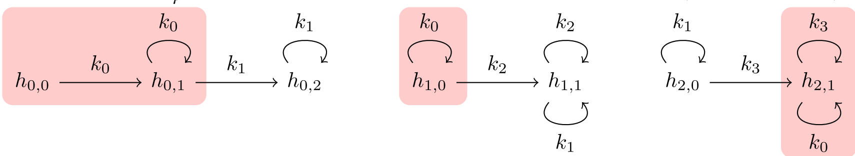
Conversely, if φ is one-in-three satisfiable then there is a τ-region (sup, sig) of A^τ_φ that solves α. In particular, if M is a one-in-three model of φ then we first define $sup(\bot_0) = 1$. Secondly, for all $e \in E(A^\tau_\varphi)$ we define $sig(e) = \mathsf{free}$ if $e = k_1$, $sig(e) = \mathsf{inp}$ if $e \in \{k_0\} \cup M$ and else $sig(e) = \mathsf{nop}$. Since A^τ_φ is reachable, by inductively defining $sup(s_{i+1}) = \delta_\tau(sup(s_i), sig(e_i))$ for all paths $\bot_0 \xrightarrow{e_1} s_1 \dots s_{n-1} \xrightarrow{e_n} s_n$, this defines a fitting region (sup, sig), cf. Fig. 5.

This proves that α is τ-solvable if and only if φ is one-in-three satisfiable.

In the remainder of this section, we present the remaining hardness results in accordance to Fig. 1 and the corresponding reductions that prove them.

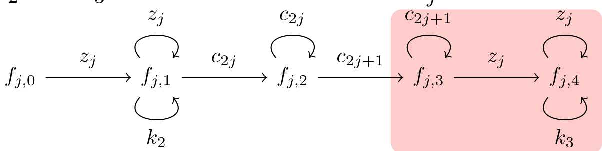
Theorem 2. *For any fixed $g \geq 3$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{res}\} \cup \omega$ and $\emptyset \neq \omega \subseteq \{\mathsf{used}, \mathsf{free}\}$.*

Proof. The TS A^τ_φ has the following gadgets H_0, H_1 and H_2 (in this order):



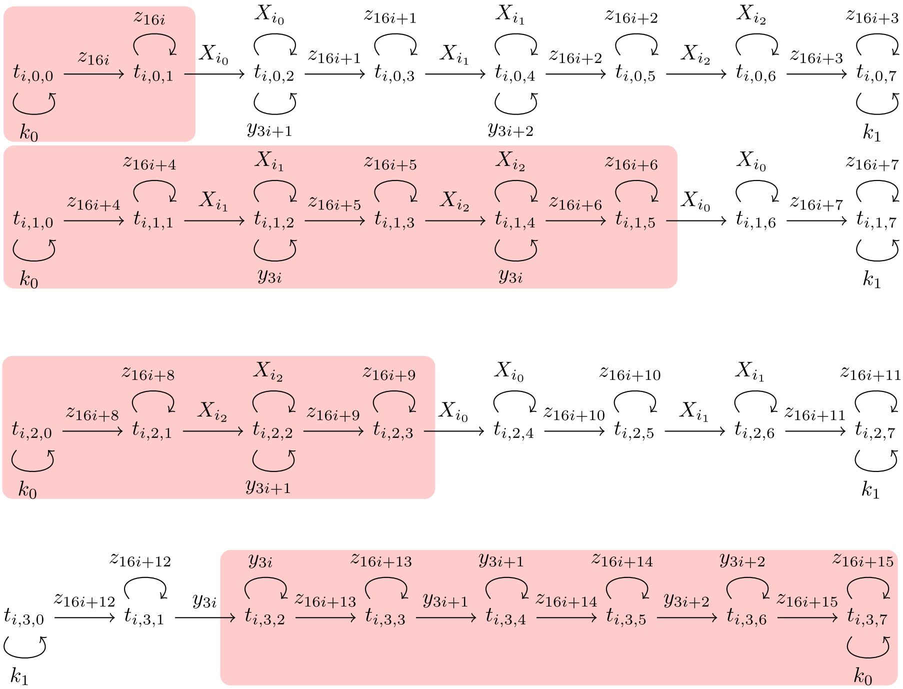
The gadget H_0 provides $\alpha = (k_0, h_{0,2})$. By symmetry, A^τ_I is $\{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{used}\}$-solvable if and only if it is $\{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{free}\}$- or $\{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{free}, \mathsf{used}\}$-solvable. Thus, in the following we assume $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{used}\}$, $sig(k_0) = \mathsf{used}$ and $sup(h_{0,2}) = 0$ if (sup, sig) τ-solves α. As a result, by $sig(k_0) = \mathsf{used}$, implying $sup(h_{0,1}) = 1$, and $sup(h_{0,2}) = 0$ we have $sig(k_1) = \mathsf{res}$. Especially, if $\xrightarrow{k_0} s$ then $sup(s) = 1$ and if $\xrightarrow{k_1} s$ then $sup(s) = 0$. Thus, $sup(h_{1,0}) = sup(h_{2,1}) = 1$ and $sup(h_{1,1}) = sup(h_{2,0}) = 0$ which implies $sig(k_2) = \mathsf{res}$ and $sig(k_3) = \mathsf{set}$.

The construction uses k_2 and k_3 to produce some neutral events. More exactly, the TS A^τ_φ implements for all $j \in \{0, \dots, 16m-1\}$ the following gadget F_j that uses k_2 and k_3 to ensure that the events z_j are neutral:



By $sig(k_2) = \mathsf{res}$ and $sig(k_3) = \mathsf{set}$ we have $sup(f_{j,1}) = 0$ and $sup(f_{j,4}) = 1$. This implies $\xrightarrow{sig(z_j)} 0$ and $\xrightarrow{sig(z_j)} 1$ and thus $sig(z_j) = \mathsf{nop}$.

Finally, for every $i \in \{0, \dots, m-1\}$ and clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$, the TS A^τ_φ has the following four gadgets $T_{i,0}, T_{i,1} T_{i,2}$ and $T_{i,3}$ (in this order):



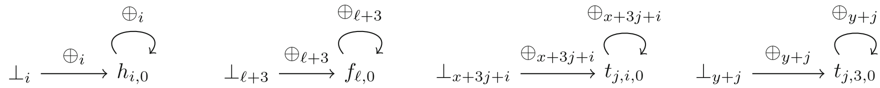
$T_{i,0}, \dots, T_{i,4}$ ensure that there is exactly one $X \in \zeta_i$ with $sig(X) = \textsf{res}$: By $sig(k_0) = \textsf{used}$ and $sig(k_1) = \textsf{res}$ we get $sup(t_{i,0,0}) = sup(t_{i,1,0}) = sup(t_{i,2,0}) = sup(t_{i,3,7}) = 1$ and $sup(t_{i,0,7}) = sup(t_{i,1,7}) = sup(t_{i,2,7}) = sup(t_{i,3,0}) = 0$. Since $z_{16i}, \dots, z_{16i+11}$ are neutral, this implies $sup(t_{i,0,6}) = sup(t_{i,1,6}) = sup(t_{i,2,6}) = 0$ and that there is a variable event with a res-signature. Moreover, by $sup(t_{i,3,0}) = 0$ and $sup(t_{i,3,7}) = 1$ and the neutrality of $z_{16i+12}, \dots, z_{16i+15}$ there is an event of $y_{3i}, y_{3i+1}, y_{3i+2}$ with a set-signature. We argue that there is exactly one variable event with a res-signature: By $sup(t_{i,0,6}) = sup(t_{i,1,6}) = sup(t_{i,2,6}) = 0$, we have $sig(X) \notin \{\textsf{set}, \textsf{used}\}$ for all $X \in \{X_{i_0}, X_{i_1}, X_{i_2}\}$. Hence, if $sig(X_{i_0}) = \textsf{res}$ then $sup(t_{i,0,2}) = \dots = sup(t_{i,0,6}) = 0$ which implies $sig(y_{3i+1}) \neq \textsf{set}$ and $sig(y_{3i+2}) \neq \textsf{set}$ and thus $sig(y_{3i}) = \textsf{set}$. By $sig(y_{3i}) = \textsf{set}$ we have $sup(t_{i,1,2}) = sup(t_{i,1,4}) = 1$ which implies $sig(X_{i_1}) \neq \textsf{res}$ and $sig(X_{i_2}) \neq \textsf{res}$.

If $sig(X_{i_1}) = \textsf{res}$, then $sup(t_{i,0,4}) = sup(t_{i,1,2}) = 0$ which implies $sig(y_{3i}) \neq \textsf{set}$ and $sig(y_{3i+2}) \neq \textsf{set}$ and thus $sig(y_{3i+1}) = \textsf{set}$. By $sig(y_{3i+1}) = \textsf{set}$ we have $sup(t_{i,0,2}) = sup(t_{i,2,2}) = 1$ which implies $sig(X_{i_0}) \neq \textsf{res}$ and $sig(X_{i_2}) \neq \textsf{res}$.

Since $sig(X_{i_0}) = \textsf{res}$ or $sig(X_{i_1}) = \textsf{res}$ implies $sig(X_{i_2}) \neq \textsf{res}$, we conclude that $sig(X_{i_2}) = \textsf{res}$ implies $sig(X_{i_0}) \neq \textsf{res}$ and $sig(X_{i_1}) \neq \textsf{res}$. Thus, there is exactly one variable of the i-th clause with a signature res. Hence, the set $M = \{X \in V(\varphi) \mid sig(X) = \textsf{res}\}$ is a one-in-three model of φ.

To finally build A^τ_φ, we use the states $\bot = \{\bot_0, \dots, \bot_{20m+2}\}$ and the events $\oplus = \{\oplus_0, \dots, \oplus_{20m+2}\}$ and $\ominus = \{\ominus_1, \dots, \ominus_{20m+2}\}$. The states of $\bot$ are connected by $\bot_j \xrightarrow{\ominus_{j+1}} \bot_{j+1}$ and $\bot_{j+1} \xrightarrow{\ominus_{j+1}} \bot_{j+1}$ for $j \in \{0, \dots, 20m+1\}$. Let $x = 16m + 3$ and $y = 19m + 3$. For all $i \in \{0, 1, 2\}$, for all $\ell \in \{0, \dots, 16m-1\}$ and for all $j \in \{0, \dots, m\}$ we add the following edges that connect the gadgets

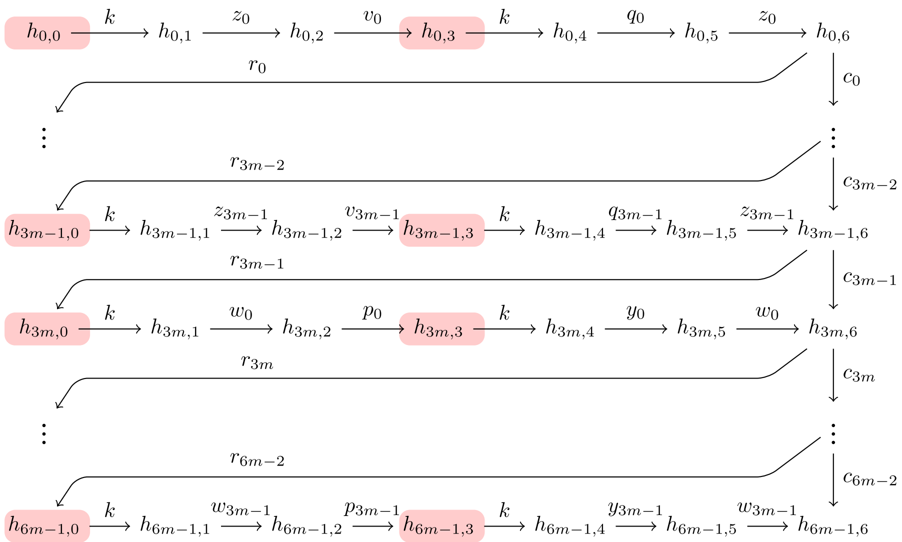
H_0, H_1, H_2 and $F_0, \dots, F_{16m-1}$ and $T_{0,0}, T_{0,1}, T_{0,2}, \dots, T_{m-1,0}, T_{m-1,1} T_{m-1,2}$ and



If M is a one-in-three model of φ then α is τ-solvable by a τ-region (sup, sig): If $s \in \{h_{0,0}, h_{1,0}, h_{2,1}\}$ or $\{f_{j,0} \mid j \in \{0, \dots, 16m-1\}\}$ then $sup(s) = 1$. The support values of the states of $T_{i,0}, \dots, T_{i,3}$, where $i \in \{0, \dots, m-1\}$, are defined in accordance to which event of $X_{i_0}, X_{i_1}, X_{i_2}$ belongs to M. The red colored area above sketches $X_{i_0} \in M$. Moreover, we define $sup(s) = 0$ for all $s \in \perp$. Let $e \in E(A^\tau_\varphi) \setminus \oplus$. We define $sig(e) = \textsf{used}$ if $e = k_0$ and $sig(e) = \textsf{res}$ if $e \in \{k_1\} \cup M$. For all $i \in \{0, \dots, m-1\}$ and clauses $\{X_{i_0}, X_{i_1}, X_{i_2}\}$ and all $j \in \{0, 1, 2\}$ we set $sig(e) = \textsf{set}$ if $e = y_{3i+j}$ and $X_{i_j} \in M$. Otherwise, we define $sig(e) = \textsf{nop}$. For all events $e \in \oplus$ and edges $s \xrightarrow{e} s'$ of A we define $sig(e) = \textsf{set}$ if $sup(s') = 1$ and, otherwise, $sig(e) = \textsf{nop}$. The resulting τ-region (sup, sig) of A^τ_φ solves α. □

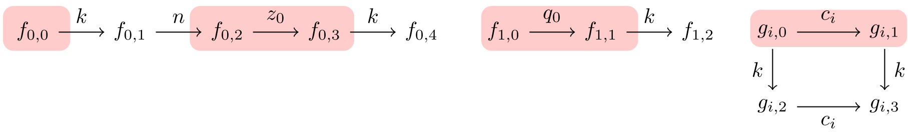
Theorem 3. *For any fixed $g \geq 2$, deciding if a g-bounded TS A is τ-solvable is NP-complete if (1) $\tau = \{\textsf{nop}, \textsf{inp}, \textsf{set}\}$ or $\tau = \{\textsf{nop}, \textsf{inp}, \textsf{set}, \textsf{used}\}$ or $\tau = \{\textsf{nop}, \textsf{inp}, \textsf{res}, \textsf{set}\} \cup \omega$ and $\omega \subseteq \{\textsf{out}, \textsf{used}, \textsf{free}\}$ or if (2) $\tau = \{\textsf{nop}, \textsf{out}, \textsf{res}\}$ or $\tau = \{\textsf{nop}, \textsf{out}, \textsf{res}, \textsf{free}\}$ or $\tau = \{\textsf{nop}, \textsf{out}, \textsf{res}, \textsf{set}\} \cup \omega$ and $\omega \subseteq \{\textsf{inp}, \textsf{used}, \textsf{free}\}$.*

Proof. We present a reduction for the types of (1). By Lemma 2, this proves the claim also for the types of (2). The TS A^τ_φ has the following gadget H:

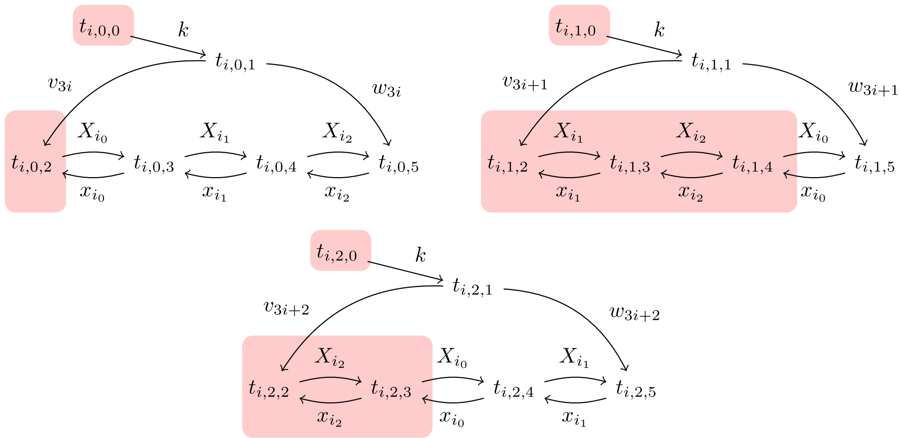


The intention of the gadget H is to provide the atom $\alpha = (k, h_{0,6})$ and the events of $Z = \{z_0, \dots, z_{3m-1}\}$, $V = \{v_0, \dots, v_{3m-1}\}$ and $W = \{w_0, \dots, w_{3m-1}\}$.

Moreover, the TS A^τ_φ has the following two gadgets F_0 and F_1 and for all $i \in \{0, \dots, 6m-2\}$ the following gadget G_i (in this order):



Finally, the TS A^τ_φ has for every clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$, $i \in \{0, \dots, m-1\}$, the following gadgets $T_{i,0}, T_{i,1}$ and $T_{i,2}$ (in this order):



In the following, we argue that H, F_0, F_1 and $G_0, \dots, G_{m-2}$ collaborate like this: If (sup, sig) is a τ-region solving α then either $sig(k) = \mathsf{inp}$, $V \subseteq sig^{-1}(\mathsf{enter})$ and $W \subseteq sig^{-1}(\mathsf{keep}^-)$ or $sig(k) = \mathsf{out}$ and $V \subseteq sig^{-1}(\mathsf{exit})$ and $W \subseteq sig^{-1}(\mathsf{keep}^+)$. Moreover, we prove that this implies by the functionality of $T_{0,0}, \dots, T_{m-1,2}$ that $M = \{X \in V(\varphi) \mid sig(X) \neq \mathsf{nop}\}$ is a one-in-three model of φ.

Let (sup, sig) be a τ-region that solves α. Since the interactions $\mathsf{res}, \mathsf{set}$ and nop are defined on both 0 and 1, this implies $sig(k) \in \{\mathsf{inp}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$. If $sig(k) = \mathsf{used}$ then $sup(s) = sup(s') = 1$ for every transition $s \xrightarrow{k} s'$. Hence, we have $sup(f_{0,3}) = sup(f_{1,1}) = sup(h_{0,4}) = 1$. By definition of $\mathsf{inp}, \mathsf{res}$ we have that $\xrightarrow{e} s$ and $sig(e) \in \{\mathsf{inp}, \mathsf{res}\}$ implies $sup(s) = 0$. Consequently, by $\xrightarrow{z_0} f_{0,3}$ and $\xrightarrow{q_0} f_{1,1}$ we get $sig(z_0), sig(q_0) \in \mathsf{keep}^+$ and thus $sup(h_{0,4}) = sup(h_{0,5}) = sup(h_{0,6}) = 1$ which contradicts $\neg sup(h_{0,6}) \xrightarrow{sig(k)}$. Hence, $sig(k) \neq \mathsf{used}$. Similarly, $sig(k) = \mathsf{free}$ implies $sup(h_{0,6}) = 0$, which is a contradiction. Thus, we have that $sig(k) = \mathsf{inp}$ and $sup(h_{0,6}) = 0$ or $sig(k) = \mathsf{out}$ and $sup(h_{0,6}) = 1$.

As a next step, we show that $sig(k) = \mathsf{inp}$ and $sup(h_{0,6}) = 0$ together imply $sig(v_0) \in \mathsf{enter}$ and $sig(z_0) \in \mathsf{keep}^-$. By $sig(k) = \mathsf{inp}$ and $\xrightarrow{k} h_{0,1}$ and $h_{0,3} \xrightarrow{k}$ we get $sup(h_{0,1}) = 0$ and $sup(h_{0,3}) = 1$. Moreover, by $\xrightarrow{z_0} h_{0,6}$ and $sup(h_{0,6}) = 0$ we obtain $sig(z_0) \in \mathsf{keep}^-$, which by $sup(h_{0,1}) = 0$ implies $sup(h_{0,2}) = 0$.

Finally, $sup(h_{0,2}) = 0$ and $sup(h_{0,3}) = 1$ imply $sig(v_0) \in \mathsf{enter}$. Notice that this reasoning purely bases on $sig(k) = \mathsf{inp}$ and $sup(h_{0,6}) = 0$. Moreover, A^τ_φ uses for every $j \in \{0, \dots, 6m-2\}$ the TS G_j to ensure $sup(h_{0,6}) = sup(h_{1,6}) = \dots = sup(h_{6m-1,6})$. This transfers $z_0 \in \mathsf{keep}^-$ and $v_0 \in \mathsf{enter}$ to $V \subseteq \mathsf{enter}$ and $W \subseteq \mathsf{keep}^-$. In particular, by $sig(k) = \mathsf{inp}$ we have $sup(g_{i,0}) = sup(g_{i,1}) = 1$ and $sup(g_{i,2}) = sup(g_{i,3}) = 0$, that is, $sig(c_i) = \mathsf{nop}$. Hence, if $sig(k) = \mathsf{inp}$ and $sup(h_{0,6}) = 0$ then $sup(h_{i,6}) = 0$ for all $i \in \{0, \dots, 6m-1\}$. Perfectly similar to the discussion for z_0 and v_0 we obtain that $V \subseteq sig^{-1}(\mathsf{enter})$ and $W \subseteq sig^{-1}(\mathsf{keep}^-)$, respectively. Similarly, $sig(k) = \mathsf{out}$ and $sup(h_{0,6}) = 1$ imply $V \subseteq sig^{-1}(\mathsf{exit})$ and $W \subseteq sig^{-1}(\mathsf{keep}^+)$.

We now argue that $T_{i,0}, \dots, T_{m-1,2}$ ensure that $M = \{X \in V(\varphi) \mid sig(X) \neq \mathsf{nop}\}$ is a one-in-three model of φ. Let $i \in \{0, \dots, m-1\}$ and $sig(k) = \mathsf{inp}$ and $sup(h_{0,6}) = 0$ implying $V \subseteq sig^{-1}(\mathsf{enter})$ and $W \subseteq sig^{-1}(\mathsf{keep}^-)$. By $sig(k) = \mathsf{inp}$ and $V \subseteq sig^{-1}(\mathsf{enter})$ and $W \subseteq sig^{-1}(\mathsf{keep}^-)$ we have that $sup(t_{i,0,2}) = sup(t_{i,1,2}) = sup(t_{i,2,2}) = 1$ and $sup(t_{i,0,5}) = sup(t_{i,1,5}) = sup(t_{i,2,5}) = 0$. As a result, every event $e \in \{X_{i_0}, X_{i_1}, X_{i_2}\}$ has a 0-sink, which implies $sig(e) \in \{\mathsf{nop}, \mathsf{inp}, \mathsf{res}\}$, and every event $e \in \{x_{i_0}, x_{i_1}, x_{i_2}\}$ has a 1-sink, which implies $sig(e) \in \{\mathsf{nop}, \mathsf{out}, \mathsf{set}\}$. By $sup(t_{i,0,2}) = 1$ and $sup(t_{i,0,5}) = 0$ there is a $X \in \{X_{i_0}, X_{i_1}, X_{i_2}\}$ such that $sig(X) \in \{\mathsf{inp}, \mathsf{res}\}$. We argue that $sig(Y) = \mathsf{nop}$ for $Y \in \{X_{i_0}, X_{i_1}, X_{i_2}\} \setminus \{X\}$. If $sig(X_{i_0}) \in \{\mathsf{inp}, \mathsf{res}\}$ then $sup(t_{i,0,3}) = 0$ which implies $sig(x_{i_0}) \in \{\mathsf{out}, \mathsf{set}\}$ and, therefore, $sup(t_{i,1,4}) = 1$. Since $sig(X_{i_1}), sig(X_{i_2}) \notin \{\mathsf{out}, \mathsf{set}\}$ and $sig(x_{i_1}), sig(x_{i_2}) \notin \{\mathsf{inp}, \mathsf{res}\}$, it holds $sup(t_{i,0,3}) = sup(t_{i,0,4}) = 0$ and $sup(t_{i,1,3}) = sup(t_{i,1,4}) = 1$, respectively. Thus, for all $e \in \{X_{i_1}, X_{i_2}\}$, there are edges $\xrightarrow{e} s$ and $\xrightarrow{e} s'$ such that $sup(s) = 0$ and $sup(s') = 1$. This implies $sig(e) = \mathsf{nop}$. Similarly, if $sig(X_{i_1}) \in \{\mathsf{inp}, \mathsf{res}\}$, then $sig(X_{i_0}) = sig(X_{i_2}) = \mathsf{nop}$, and if $sig(X_{i_2}) \in \{\mathsf{inp}, \mathsf{res}\}$, then $sig(X_{i_0}) = sig(X_{i_1}) = \mathsf{nop}$. Hence, every clause ζ_i has exactly one variable event with a signature different from nop. This makes $M = \{X \in V(\varphi) \mid sig(X) \neq \mathsf{nop}\}$ a one-in-three model of φ. Similarly, if $sig(k) = \mathsf{out}$ and $sup(h_{0,6}) = 1$, then M is also a one-in-three model of φ.

To join the gadgets and finally build A^τ_φ, we use the states $\bot = \{\bot_0, \dots, \bot_{9m+1}\}$ and the events $\oplus = \{\oplus_0, \dots, \oplus_{9m+1}\}$ and $\ominus = \{\ominus_1, \dots, \ominus_{9m+1}\}$. The states of $\bot$ are connected by $\bot_j \xrightarrow{\ominus_{j+1}} \bot_{j+1}$ for $j \in \{0, \dots, 9m+1\}$. Let $x = 6m+2$. For all $i \in \{0, \dots, 6m-2\}$, for all $j \in \{0, \dots, m-1\}$ and for all $\ell \in \{0, 1, 2\}$ we add the following edges that connect the gadgets $H_0, F_0, F_1, G_0, \dots, G_{6m-2}$ and $T_{0,0}, T_{0,1}, T_{0,2}$ up to $T_{m-1,0}, T_{m-1,1}, T_{m-1,2}$ to A^τ_φ:

$$\bot_0 \xrightarrow{\oplus_0} h_{0,0} \quad \bot_1 \xrightarrow{\oplus_1} f_{0,0} \quad \bot_2 \xrightarrow{\oplus_2} f_{1,0} \quad \bot_{i+3} \xrightarrow{\oplus_{i+3}} g_{i+3,0} \quad \bot_{x+j+3\ell} \xrightarrow{\oplus_{x+j+3\ell}} t_{j,\ell,0}$$

If M is a one-in-three model of φ then there is a τ-region (sup, sig) of A^τ_φ that solves α. The red colored area of the figures introducing the gadgets indicates already a positive support of some states. In particular, if $s \in \{h_{j,0}, h_{j,3} \mid j \in \{0, \dots, 6m-1\}\}$ or $s \in \{f_{0,0}, f_{0,2}, f_{0,3}, f_{1,0}, f_{1,1}\}$ $s \in \{g_{j,0}, g_{j,1} \mid j \in \{0, \dots, 6m-2\}\}$ then $sup(s) = 1$. The support values of the states of $T_{i,0}, \dots, T_{i,2}$, where

$i \in \{0, \ldots, m-1\}$, are defined in accordance to which of the events $X_{i_0}, X_{i_1}, X_{i_2}$ belongs to M. The red colored area above sketches the situation where $X_{i_0} \in M$. Moreover, for all $s \in \bot$, we define $sup(s) = 0$. Let $e \in E(A^\tau_\varphi) \setminus \oplus$. We define $sig(e) = \mathsf{inp}$ if $e \in \{k\} \cup M$. For all $i \in \{0, \ldots, m-1\}$ and clauses $\{X_{i_0}, X_{i_1}, X_{i_2}\}$ and all $j \in \{0, 1, 2\}$ we set $sig(e) = \mathsf{set}$ if $e = n$ or $e \in \{v_j, p_j \mid j \in \{0, \ldots, 3m-1\}\}$ or $e = x_{i_j}$ and $X_{i_j} \in M$. Otherwise, we define $sig(e) = \mathsf{nop}$. Finally, for all events $e \in \oplus$ and edges $s \xrightarrow{e} s'$ of A we define $sig(e) = \mathsf{set}$ if $sup(s') = 1$ and, otherwise, $sig(e) = \mathsf{nop}$.

Joining of 1-Bounded Gadgets. In the following, we consider types τ where τ-synthesis from 1-bounded inputs is NP-complete. All gadgets $A_0, \ldots, A_n$ of the reductions are directed paths $A_i = s^i_0 \xrightarrow{e_1} \ldots, \xrightarrow{e_n} s^i_n$ on pairwise distinct states $s^i_0, \ldots, s^i_n$. For all types, the joining is the concatenation

$$A^\tau_\varphi = \quad A_0 \xrightarrow{\ominus_1} \bot_1 \xrightarrow{\oplus_1} A_1 \xrightarrow{\ominus_2} \bot_2 \xrightarrow{\oplus_2} \quad \cdots \quad \xrightarrow{\ominus_n} \bot_n \xrightarrow{\oplus_n} A_n$$

with fresh states $\bot_1, \ldots, \bot_n$ and events $\ominus_1, \cdots \ominus_n, \oplus_1, \cdots \oplus_n$.

Theorem 4. *For any fixed $g \geq 1$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{set}\} \cup \omega$ or $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{res}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$.*

Proof. Our construction proves the claim for $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{out}\} \cup \omega$ with $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$. By Lemma 2, this proves the claim also for the other types.

The TS A^τ_φ has the following gadgets H_0, H_1, H_2 and H_3 (in this order):

$$h_{0,0} \xrightarrow{k_0} h_{0,1} \xrightarrow{z_0} h_{0,2} \xrightarrow{o} h_{0,3} \xrightarrow{k_1} h_{0,4} \xrightarrow{z_1} h_{0,5} \xrightarrow{z_0} h_{0,6} \xrightarrow{o} h_{0,7} \xrightarrow{k_0} h_{0,8}$$

$$h_{1,0} \xrightarrow{z_0} h_{1,1} \xrightarrow{k_0} h_{1,2} \qquad h_{2,0} \xrightarrow{z_1} h_{2,1} \xrightarrow{k_0} h_{2,2} \qquad h_{3,0} \xrightarrow{k_0} h_{3,1} \xrightarrow{k_1} h_{3,2}$$

If $\mathsf{used} \in \tau$ then A^τ_φ has the following gadget H_4:

$$h_{4,0} \xrightarrow{k_1} h_{4,1} \xrightarrow{z_0} h_{4,2} \xrightarrow{k_1} h_{4,3}$$

For all $i \in \{0, \ldots, m-1\}$, the TS A^τ_φ has for the clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ and the variable $X_i \in V(\varphi)$ the following gadgets T_i and B_i, respectively:

$$t_{i,0} \xrightarrow{k_1} t_{i,1} \xrightarrow{X_{i_0}} t_{i,2} \xrightarrow{X_{i_1}} t_{i,3} \xrightarrow{X_{i_2}} t_{i,4} \xrightarrow{k_0} t_{i,5} \qquad b_{i,0} \xrightarrow{X_i} b_{i,1} \xrightarrow{k_0} b_{i,2}$$

The gadget H_0 provides the atom $\alpha = (k_0, h_{0,6})$. Moreover, the gadgets $H_0, \ldots, H_4$ ensure that if (sup, sig) is a τ-region solving α then $sig(k_0) = \mathsf{out}$ and $sig(k_1) \in \{\mathsf{out}, \mathsf{set}\}$. In particular, H_4 prevents the solvability of α by used. As a result, such a region implies $sup(t_{i,1}) = 1$, $sup(t_{i,4}) = 0$ and $sup(b_{i,1}) = 0$ for all $i \in \{0, \ldots, m-1\}$. On the one hand, by $sup(b_{i,1}) = 0$ for all $i \in \{0, \ldots, m-1\}$ we have $sig(X) \notin \{\mathsf{out}, \mathsf{set}\}$ for all $X \in V(\varphi)$. On the other hand, by $sup(t_{i,1}) = 1$ and $sup(t_{i,4}) = 0$ there is an event $X \in \{X_{i_0}, X_{i_1}, X_{i_2}\}$

such that $sig(X) = \mathsf{inp}$. Since no variable event has an incoming signature we obtain immediately $sig(Y) \neq \mathsf{inp}$ for $Y \in \{X_{i_0}, X_{i_1}, X_{i_2}\} \setminus \{X\}$. Thus, $M = \{X \in V(\varphi) \mid sig(X) = \mathsf{inp}\}$ is a one-in-three model of φ.

We argue that $H_0, \dots, H_4$ behave as announced. Let (sup, sig) be a region that solves $(k_0, h_{0,6})$. If $sig(k_0) = \mathsf{inp}$ then $sup(h_{0,6}) = 0$ and $sig(h_{0,7}) = 1$, implying $sig(o) \in \{\mathsf{out}, \mathsf{set}\}$ and $sup(h_{0,3}) = 1$. Thus, there is an event $e \in \{k_1, z_0, z_1\}$ with $sig(e) = \mathsf{inp}$. By $sig(k_0) = \mathsf{inp}$ we have $sup(h_{1,1}) = sup(h_{2,1}) = 1$ and $sup(h_{3,1}) = 0$ implying $sig(e) \neq \mathsf{inp}$ for all $e \in \{k_1, z_0, z_1\}$, a contradiction.

If $sig(k_0) = \mathsf{free}$ then $sup(h_{0,6}) = 1$ and $sup(h_{0,1}) = sup(h_{0,7}) = sup(h_{1,1}) = 0$ which implies $sig(o) = \mathsf{inp}$ and $sup(h_{0,2}) = 1$. By $sup(h_{0,1}) = 0$ and $sup(h_{0,2}) = 1$ we have $sig(z_0) \in \{\mathsf{out}, \mathsf{set}\}$ which by $sup(h_{1,1}) = 0$ is a contradiction.

If $sig(k_0) = \mathsf{used}$ then $sup(h_{0,6}) = 0$ and $sup(h_{0,1}) = sup(h_{0,7}) = sup(h_{1,1}) = sup(h_{2,1}) = 1$. This implies $sig(o) \in \{\mathsf{out}, \mathsf{set}\}$ and $sup(h_{0,3}) = 1$. Thus, by $sup(h_{0,6}) = 0$ there is an event $e \in \{k_1, z_0, z_1\}$ with $sig(e) = \mathsf{inp}$. By $sup(h_{1,1}) = sup(h_{2,1}) = 1$, we have $e \notin \{z_0, z_1\}$. If $sig(k_1) = \mathsf{inp}$ then $sup(h_{4,1}) = 0$ and $sup(h_{4,2}) = 1$, implying $sig(z_0) \in \{\mathsf{out}, \mathsf{set}\}$ and $sup(h_{0,6}) = 1$. This is a contradiction. Altogether, this proves $sig(k_0) \notin \{\mathsf{inp}, \mathsf{used}, \mathsf{free}\}$.

Consequently, we obtain $sig(k_0) = \mathsf{out}$ and $sup(h_{0,6}) = 1$ which implies $sig(o) = \mathsf{inp}$ and $sup(h_{0,3}) = 0$. By $sup(h_{0,6}) = 1$, this implies that there is an event $e \in \{k_1, z_0, z_1\}$ with $sig(e) \in \{\mathsf{out}, \mathsf{set}\}$. Again by $sig(k_0) = \mathsf{out}$, we have $sup(h_{1,1}) = sup(h_{2,1}) = 0$, which implies $e = k_1$. The signatures $sig(k_0) = \mathsf{out}$ and $sig(k_1) \in \{\mathsf{out}, \mathsf{set}\}$ and the construction of $T_0, \dots, T_{m-1}$ and $B_0, \dots, B_{m-1}$ ensure that $M = \{X \in V(\varphi) \mid sig(X) = \mathsf{inp}\}$ is a one-in-three model of φ: By $sig(k_0) = \mathsf{out}$ and $sig(k_1) \in \{\mathsf{out}, \mathsf{set}\}$ we have $sup(t_{i,1}) = 1$ and $sup(t_{i,4}) = sup(b_{i,1}) = 0$ for all $i \in \{0, \dots, m-1\}$. By $sup(t_{i,1}) = 1$ and $sup(t_{i,4}) = 0$, there is an event $X \in \zeta_i$ such that $sig(X) = \mathsf{inp}$. Moreover, by $sup(b_{i,1}) = 0$, we get $sig(X_i) \notin \mathsf{enter}$ for all $i \in \{0, \dots, m-1\}$. Thus, X is unambiguous and thus M is a searched model.

Conversely, if M is a one-in-three model of φ then there is a τ-region (sup, sig) that solves α. The red colored area above sketches states with a positive support. Which states of T_i, besides of $t_{i,0}, t_{i,1}$ and $t_{i,5}$, get a positive support depends for all $i \in \{0, \dots, m-1\}$ on which of $X_{i_0}, X_{i_1}, X_{i_2}$ belongs to M. The red colored area above sketches the case $X_{i_0} \in M$. Moreover, we define $sup(s) = 1$ if $s = b_{i,0}$ and $X_i \in M$ or if $s \in \bot$. The signature is defined as follows: $sig(k_1) = \mathsf{set}$; for all $e \in E(A^\tau_\varphi) \setminus \{k_1\}$ and all $s \xrightarrow{e} s' \in A^\tau_\varphi$, if $sup(s') > sup(s)$, then $sig(e) = \mathsf{out}$; if $sup(s) > sup(s')$, then $sig(e) = \mathsf{inp}$; else $sig(e) = \mathsf{nop}$. □

Theorem 5. *For any $g \geq 1$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{free}\}$ or $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}, \mathsf{free}\}$ or $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{used}\}$ or $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{used}, \mathsf{free}\}$.*

Proof. Our reduction proves the claim for $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{free}\}$ and $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}, \mathsf{free}\}$ and thus by Lemma 2, for the other types, too.

The TS A^τ_φ has the following gadgets H_0 and H_1 providing the atom $(k_0, h_{0,3})$:

$$h_{0,0} \xrightarrow{k_0} h_{0,1} \xrightarrow{k_1} h_{0,2} \xrightarrow{z_0} h_{0,3} \xrightarrow{k_1} h_{0,4} \xrightarrow{z_1} h_{0,5} \xrightarrow{k_0} h_{0,6}$$

$$h_{1,0} \xrightarrow{k_0} h_{1,1} \xrightarrow{z_0} h_{1,2} \xrightarrow{k_0} h_{1,3}$$

For all $i \in \{0, \dots, m-1\}$, the A^τ_φ for the clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ and the variable $X_i \in V(\varphi)$ the gadgets T_i and B_i as previously defined for Theorem 4. The gadgets H_0 and H_1 ensure that a τ-region (sup, sig) solving $(k_0, h_{0,3})$ satisfies $sig(k_0) = \mathsf{free}$ and $sig(k_1) = \mathsf{set}$. This implies $sup(t_{i,1}) = 1$ and $sup(t_{i,4}) = sup(b_{i,2}) = 0$ for all $i \in \{0, \dots, m-1\}$. By $sup(t_{i,1}) = 1$ and $sup(t_{i,4}) = 0$, there is an event $X \in \zeta_i$ such that $sig(X) = \mathsf{inp}$ and, by $sup(b_{i,2}) = 0$ for all $i \in \{0, \dots, m-1\}$, we have $sig(X) \neq \mathsf{set}$ for all $X \in V(\varphi)$. Thus, the event $X \in \zeta_i$ is unique and $M = \{X \in V(\varphi) \mid sig(X) = \mathsf{inp}\}$ is a one-in-three model.

We briefly argue that H_0 and H_1 perform as announced: Let (sup, sig) be a τ-region that solves α. If $sig(k_0) = \mathsf{inp}$ then $sup(h_{1,1}) = 0$ and $sup(h_{1,2}) = 1$ which implies $sig(z_0) = \mathsf{set}$ and thus $sup(h_{0,3}) = 1$, a contradiction. Hence, $sig(k_0) \neq \mathsf{inp}$. If $sig(k_0) = \mathsf{used}$ then $sup(h_{0,1}) = sup(h_{1,2}) = 1$ and $sup(h_{0,3}) = 0$. Consequently, $sig(z_0) = \mathsf{inp}$ or $sig(k_1) = \mathsf{inp}$ but this contradicts $sup(h_{1,2}) = 1$ and $sup(h_{0,3}) = 0$. Thus, $sig(k_0) \neq \mathsf{used}$. Thus, we have $sig(k_0) = \mathsf{free}$ and $sup(h_{0,3}) = 1$, which implies that one of k_1, z_0 has a set-signature. By $sig(k_0) = \mathsf{free}$, we get $sup(h_{1,3}) = 0$ and thus $sig(k_1) = \mathsf{set}$.

If M is a one-in-three model of φ then we can define an α solving region similar to the one of Theorem 4, where we replace $sig(k_0) = \mathsf{inp}$ by $sig(k_0) = \mathsf{free}$.

Theorem 6. *For any fixed $g \geq 1$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{swap}\} \cup \omega$ or $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{used}, \mathsf{free}\}$.*

Proof. The TS A^τ_φ has the following gadgets H_0, H_1, H_2 and H_3:

$$h_{0,0} \xrightarrow{k} h_{0,1} \xrightarrow{y_0} h_{0,2} \xrightarrow{v} h_{0,3} \xrightarrow{k} h_{0,4} \qquad h_{1,0} \xrightarrow{k} h_{1,1} \xrightarrow{y_1} h_{1,2} \xrightarrow{y_0} h_{1,3} \xrightarrow{k} h_{1,4}$$

$$h_{2,0} \xrightarrow{k} h_{2,1} \xrightarrow{y_0} h_{2,2} \xrightarrow{y_1} h_{2,3} \xrightarrow{y_0} h_{2,4} \xrightarrow{k} h_{2,5} \qquad h_{3,0} \xrightarrow{y_1} h_{3,1} \xrightarrow{y_0} h_{3,2} \xrightarrow{v} h_{3,3} \xrightarrow{k} h_{3,4}$$

The gadgets $H_0, \dots, H_3$ provide the atom $\alpha = (k, h_{0,2})$ and ensure that a τ-region (sup, sig) solving α satisfies $sig(k) = \mathsf{inp}$ and $sup(h_{0,2}) = 0$. The TS A^τ_φ has the following gadgets F_0, F_1 and for all $j \in \{0, \dots, 10\}$ the gadget G_j:

$$f_{0,0} \xrightarrow{k} f_{0,1} \xrightarrow{z_0} f_{0,2} \xrightarrow{v} f_{0,3} \xrightarrow{k} f_{0,4} \qquad f_{1,0} \xrightarrow{k} f_{1,1} \xrightarrow{z_1} f_{1,2} \xrightarrow{v} f_{1,3} \xrightarrow{k} f_{1,4}$$

$$g_{j,0} \xrightarrow{k} g_{j,1} \xrightarrow{z_0} g_{j,2} \xrightarrow{u_j} g_{j,3} \xrightarrow{z_1} g_{j,4} \xrightarrow{k} g_{j,5}$$

For all $j \in \{0, \dots, 10\}$, the gadgets F_0, F_1, G_j ensure $sig(u_j) = \mathsf{swap}$ for any τ-region (sup, sig) solving α.

For all $i \in \{0, \dots, m-1\}$, the TS A^τ_φ has for the clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ some gadgets $T_{i,0}, \dots, T_{i,6}$ and B_i. The purpose of these gadgets is to make the one-and-three satisfiability of φ and the solvability of α the same. In particular, the TS $T_{i,0}$ is defined by:

$$t_{i,0,0} \xrightarrow{k} t_{i,0,1} \xrightarrow{u_0} t_{i,0,2} \xrightarrow{X_{i_0}} t_{i,0,3} \xrightarrow{u_1} t_{i,0,4} \xrightarrow{X_{i_1}} t_{i,0,5} \xrightarrow{u_2} t_{i,0,6} \xrightarrow{X_{i_2}} t_{i,0,7} \xrightarrow{u_3} t_{i,0,8} \xrightarrow{k} t_{i,0,9}$$

The gadgets $T_{i,1}, T_{i,2}$ and $T_{i,3}$ are defined (in this order) as follows:

$$t_{i,1,0} \xrightarrow{k} t_{i,1,1} \xrightarrow{u_4} t_{i,1,2} \xrightarrow{u_5} t_{i,1,3} \xrightarrow{X_{i_0}} t_{i,1,4} \xrightarrow{w_{3i}} t_{i,1,5} \xrightarrow{X_{i_1}} t_{i,1,6} \xrightarrow{u_6} t_{i,1,7} \xrightarrow{k} t_{i,1,8}$$

$$t_{i,2,0} \xrightarrow{k} t_{i,2,1} \xrightarrow{u_4} t_{i,2,2} \xrightarrow{u_5} t_{i,2,3} \xrightarrow{X_{i_2}} t_{i,2,4} \xrightarrow{w_{3i+1}} t_{i,2,5} \xrightarrow{X_{i_0}} t_{i,2,6} \xrightarrow{u_6} t_{i,2,7} \xrightarrow{k} t_{i,2,8}$$

$$t_{i,3,0} \xrightarrow{k} t_{i,3,1} \xrightarrow{u_4} t_{i,3,2} \xrightarrow{u_5} t_{i,3,3} \xrightarrow{X_{i_1}} t_{i,3,4} \xrightarrow{w_{3i+2}} t_{i,3,5} \xrightarrow{X_{i_2}} t_{i,3,6} \xrightarrow{u_6} t_{i,3,7} \xrightarrow{k} t_{i,3,8}$$

Moreover, the gadgets $T_{i,4}, T_{i,5}$ and $T_{i,6}$ are defined like this:

$$t_{i,4,0} \xrightarrow{k} t_{i,4,1} \xrightarrow{u_7} t_{i,4,2} \xrightarrow{w_{3i}} t_{i,4,3} \xrightarrow{u_8} t_{i,4,4} \xrightarrow{k} t_{i,4,5}$$

$$t_{i,5,0} \xrightarrow{k} t_{i,5,1} \xrightarrow{u_7} t_{i,5,2} \xrightarrow{w_{3i+1}} t_{i,5,3} \xrightarrow{u_8} t_{i,5,4} \xrightarrow{k} t_{i,5,5}$$

$$t_{i,6,0} \xrightarrow{k} t_{i,6,1} \xrightarrow{u_7} t_{i,6,2} \xrightarrow{w_{3i+2}} t_{i,6,3} \xrightarrow{u_8} t_{i,6,4} \xrightarrow{k} t_{i,6,5}$$

Finally, the gadget B_i is defined as follows:

$$b_{i,0} \xrightarrow{X_i} b_{i,1} \xrightarrow{u_9} b_{i,2} \xrightarrow{u_{10}} b_{i,3} \xrightarrow{k} b_{i,4}$$

Let (sup, sig) be a τ-region solving α. We first argue that the gadgets $H_0, \dots, H_3$ and F_0, F_1 and $G_0, \dots, G_{10}$ ensure that a τ-region (sup, sig) solving α satisfies $sig(k) = \mathsf{inp}$, $sup(h_{0,2}) = 0$ and $sig(u_0) = \dots = sig(u_{10}) = \mathsf{swap}$.

If $sig(k) = \mathsf{free}$ and $sup(h_{0,2}) = 1$ then $s \xrightarrow{k} s'$ implies $sup(s) = sup(s') = 0$. Especially, by $sup(h_{0,1}) = 0$ and $sup(h_{0,2}) = 1$ we have $sig(y_0) = \mathsf{swap}$. Moreover, by $sup(h_{2,1}) = sup(h_{2,4}) = 0$ and $sig(y_0) = \mathsf{swap}$ we have that $sup(h_{2,2}) = sup(h_{2,3}) = 1$. This implies $sig(y_1) \in \{\mathsf{nop}, \mathsf{used}\}$. By $sup(h_{1,1}) = 0$ and $h_{1,1} \xrightarrow{y_1}$ this implies $sig(y_1) = \mathsf{nop}$ and thus $sup(h_{1,2}) = 0$. Furthermore, by $sup(h_{1,2}) = sup(h_{1,3}) = 0$ and $h_{1,2} \xrightarrow{y_0} h_{1,3}$ this implies $sig(y_0) \neq \mathsf{swap}$, a contradiction. Thus, we have $sig(k) \neq \mathsf{free}$.

If $sig(k) = \mathsf{used}$ and $sup(h_{0,2}) = 0$ then $s \xrightarrow{k} s'$ implies $sup(s) = sup(s') = 1$. Thus, we get $sup(h_{0,1}) = sup(h_{0,3}) = sup(h_{1,3}) = 1$ which with $sup(h_{0,2}) = 0$ implies $sig(y_0) = sig(v) = \mathsf{swap}$. Moreover, $sup(h_{1,3}) = 1$ and $sig(y_0) = \mathsf{swap}$ imply $sup(h_{1,2}) = 0$. By $sup(h_{1,1}) = 1$, this implies $sig(y_1) \in \{\mathsf{inp}, \mathsf{res}\}$. Finally, $sup(h_{3,3}) = 1$ and $sig(v) = sig(y_0) = \mathsf{swap}$ imply $sup(h_{3,1}) = 1$. This contradicts

$sig(y_1) \in \{\textsf{inp}, \textsf{res}\}$. Thus, $sig(k) \neq \textsf{used}$. Altogether, this shows that $sig(k) = \textsf{inp}$ and $sup(h_{0,2}) = 0$, which implies $sig(v) = \textsf{swap}$.

By $sig(k) = \textsf{inp}$ we have $sup(f_{0,1}) = sup(f_{1,1}) = sup(g_{j,1}) = 0$ and $sup(f_{0,3}) = sup(f_{1,3}) = sup(g_{j,4}) = 1$. By $sig(v) = \textsf{swap}$, this implies $sup(f_{0,2}) = sup(f_{1,2}) = 0$ and thus $sig(z_0), sig(z_1) \in \{\textsf{nop}, \textsf{res}, \textsf{free}\}$. Moreover, $sup(g_{j,1}) = 0$, $sup(g_{j,4}) = 1$ and $sig(z_0), sig(z_1) \in \{\textsf{nop}, \textsf{res}, \textsf{free}\}$ imply $sup(g_{j,2}) = 0$ and $sup(g_{j,3}) = 1$ and thus $sig(u_j) = \textsf{swap}$.

Let $i \in \{0, \dots, m-1\}$. We now show that $T_{i,0}, \dots, T_{i,6}$ and B_i collaborate as announced. By $sig(k) = \textsf{inp}$ and $sig(u_9) = sig(u_{10}) = \textsf{swap}$, we have $sup(b_{i,1}) = 1$ for all $i \in \{0, \dots, m-1\}$. Since $\xrightarrow{X_i} b_{i,1}$ for all $i \in \{0, \dots, m-1\}$, the gadget B_i ensures for all $X \in V(\varphi)$ that $s \xrightarrow{X} s'$ and $sup(s) \neq sup(s')$ imply $sig(X) = \textsf{swap}$.

The

gadget $T_{i,0}$ works like this: By $sig(k) = \textsf{inp}$ we get that $sup(t_{i,0,1}) = 0$ and $sup(t_{i,0,8}) = 1$. Consequently, the image $sup(t_{i,0,1}) \xrightarrow{sig(X_{i_0})} \dots \xrightarrow{sig(u_3)} sup(t_{i,0,8})$ of the path $t_{i,0,1} \xrightarrow{X_{i_0}} \dots \xrightarrow{u_3} t_{i,0,8}$ performs an odd number of state changes between 0 to 1 in τ. Since $sig(u_0) = \dots = sig(u_3) = \textsf{swap}$, the events $u_0, \dots, u_3$ perform an even number of state changes. Thus, either all of $X_{i_0}, X_{i_1}, X_{i_2}$ are mapped to $\textsf{swap}$ or exactly one of them. The construction of $T_{i,1}, \dots, T_{i,6}$ guarantees that there is exactly one variable event mapped to $\textsf{swap}$.

In particular, the gadgets $T_{i,4}, T_{i,5}$ and $T_{i,6}$ ensure that if $e \in \{w_{3i}, w_{3i+1}, w_{3i+2}\}$ then $sig(e) \notin \{\textsf{nop}, \textsf{used}\}$. We argue for w_{3i}: By $sig(k) = \textsf{inp}$ we get $sup(t_{i,4,1}) = 0$ and $sup(t_{i,4,4}) = 1$ which, by $sig(u_7) = sig(u_8) = \textsf{swap}$, implies $sup(t_{i,4,2}) = 1$ and $sup(t_{i,4,3}) = 0$. Clearly, this implies $sig(w_{3i}) \notin \{\textsf{nop}, \textsf{used}\}$. Similarly, we obtain that $sig(w_{3i+1}) \notin \{\textsf{nop}, \textsf{used}\}$ and $sig(w_{3i+2}) \notin \{\textsf{nop}, \textsf{used}\}$.

Finally, the gadgets $T_{i,1}, T_{i,2}$ and $T_{i,3}$ ensure that no two variable events of the same clause can have a swap signature: By $sig(k) = \textsf{inp}$ we get that $sup(t_{i,1,1}) = 0$ and $sup(t_{i,1,7}) = 1$ which with $sig(u_4) = sig(u_5) = sig(u_6) = \textsf{swap}$ implies $sup(t_{i,1,3}) = 0$ and $sup(t_{i,1,6}) = 0$. Thus, if $sig(X_{i_0}) = sig(X_{i_1}) = \textsf{swap}$ then $sup(t_{i,1,4}) = sup(t_{i,1,5}) = 1$ which implies $sig(w_{3i}) \in \{\textsf{nop}, \textsf{used}\}$, a contradiction. Similarly, one uses $T_{i,2}$ and $T_{i,3}$ to show that neither X_{i_0} and X_{i_2} nor X_{i_1} and X_{i_2} can simultaneously be mapped to $\textsf{swap}$. As i was arbitrary, there is exactly one variable per clause that is mapped to $\textsf{swap}$. Thus, $M = \{X \in V(\varphi) \mid sig(X) = \textsf{swap}\}$ is a one-in-three model of φ.

Conversely, a one-in-three model M of φ allows a τ-region (sup, sig) that solves α: The red colored area above indicates which states of $H_0, \dots, H_3$, F_0, F_1, $G_0, \dots, G_{10}$ and $T_{0,4}, T_{0,5}, T_{0,6}, \dots, T_{m-1,4}, T_{m-1,5}, T_{m-1,6}$ have positive support. Moreover, we define $sup(s) = 1$ for all $s \in \perp$. Which states of $T_{i,0}, \dots, T_{i,3}$, where $i \in \{0, \dots, m-1\}$, besides of k's sources get a positive support depends on which of $X_{i_0}, X_{i_1}, X_{i_2}$ belongs to M. The red colored area sketches the situation for $X_{i_0} \in M$. It is easy to see that there is for all $e \in E(A^\tau_\varphi)$ a fitting sig-value making (sup, sig) a (solving) τ-region where $sig(k) = \textsf{inp}$ and $sup(h_{0,2}) = 0$. $\square$

Theorem 7. *For any fixed $g \geq 1$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{out}, \mathsf{res}, \mathsf{used}, \mathsf{free}\}$ or if $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{swap}\} \cup \omega$ and $\omega \subseteq \{\mathsf{inp}, \mathsf{set}, \mathsf{used}, \mathsf{free}\}$.*

Proof. We present the reduction for the types built by $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{swap}\} \cup \omega$ where $\omega \subseteq \{\mathsf{out}, \mathsf{res}, \mathsf{used}, \mathsf{free}\}$. Again, the other types are covered by Lemma 2.

The TS A^τ_φ has the following gadgets H_0, H_1, H_2 and H_3:

$$h_{0,0} \xrightarrow{k} h_{0,1} \xrightarrow{v_0} h_{0,2} \qquad h_{1,0} \xrightarrow{v_0} h_{1,1} \xrightarrow{k} h_{1,2}$$

$$h_{2,0} \xrightarrow{k} h_{2,1} \xrightarrow{v_0} h_{2,2} \xrightarrow{v_1} h_{2,3} \xrightarrow{k} h_{2,4} \qquad h_{3,0} \xrightarrow{k} h_{3,1} \xrightarrow{v_1} h_{3,2} \xrightarrow{v_0} h_{3,3}$$

If $\tau \cap \{\mathsf{used}, \mathsf{free}\} \neq \emptyset$ then A^τ_φ has also the following gadgets $H_4, \dots, H_{12}$:

$$h_{4,0} \xrightarrow{k} h_{4,1} \xrightarrow{x} h_{4,2} \xrightarrow{v_0} h_{4,3} \xrightarrow{k} h_{4,4} \qquad h_{5,0} \xrightarrow{k} h_{5,1} \xrightarrow{v_0} h_{5,2} \xrightarrow{x} h_{5,3} \xrightarrow{k} h_{5,4}$$

$$h_{6,0} \xrightarrow{k} h_{6,1} \xrightarrow{x} h_{6,2} \xrightarrow{y_0} h_{6,3} \xrightarrow{k} h_{6,4} \qquad h_{7,0} \xrightarrow{k} h_{7,1} \xrightarrow{y_0} h_{7,2} \xrightarrow{x} h_{7,3} \xrightarrow{k} h_{7,4}$$

$$h_{8,0} \xrightarrow{k} h_{8,1} \xrightarrow{x} h_{8,2} \xrightarrow{y_1} h_{8,3} \xrightarrow{k} h_{8,4} \qquad h_{9,0} \xrightarrow{k} h_{9,1} \xrightarrow{y_1} h_{9,2} \xrightarrow{x} h_{9,3} \xrightarrow{k} h_{9,4}$$

$$h_{10,0} \xrightarrow{k} h_{10,1} \xrightarrow{x} h_{10,2} \xrightarrow{y_2} h_{10,3} \xrightarrow{k} h_{10,4} \qquad h_{11,0} \xrightarrow{k} h_{11,1} \xrightarrow{y_2} h_{11,2} \xrightarrow{x} h_{11,3} \xrightarrow{k} h_{11,4}$$

$$h_{12,0} \xrightarrow{k} h_{12,1} \xrightarrow{y_0} h_{12,2} \xrightarrow{y_1} h_{12,3} \xrightarrow{y_2} h_{12,4} \xrightarrow{k} h_{12,5}$$

The gadgets $H_0, \dots, H_3$ ($H_4, \dots, H_{12}$, if added) provide $\alpha = (k, h_{3,3})$. They ensure that if (sup, sig) τ-solves α, then $sig(k) \in \{\mathsf{inp}, \mathsf{out}\}$. The TS A^τ_φ adds the following gadgets F_0, F_1, F_2 and, for all $i \in \{0, \dots, 13\}$, the gadgets G_i, N_i:

$$f_{0,0} \xrightarrow{k} f_{0,1} \xrightarrow{z_0} f_{0,2} \xrightarrow{v_0} f_{0,3} \xrightarrow{k} f_{0,4} \qquad f_{1,0} \xrightarrow{k} f_{1,1} \xrightarrow{z_1} f_{1,2} \xrightarrow{v_0} f_{1,3} \xrightarrow{k} f_{1,4}$$

$$f_{2,0} \xrightarrow{k} f_{2,1} \xrightarrow{z_0} f_{2,2} \xrightarrow{z_1} f_{2,3} \xrightarrow{z_2} f_{2,4} \xrightarrow{k} f_{2,5}$$

$$g_{i,0} \xrightarrow{k} g_{i,1} \xrightarrow{z_0} g_{i,2} \xrightarrow{u_i} g_{i,3} \xrightarrow{z_1} g_{i,4} \xrightarrow{k} g_{i,5} \qquad n_{i,0} \xrightarrow{k} n_{i,1} \xrightarrow{z_2} n_{i,2} \xrightarrow{u_i} n_{i,3} \xrightarrow{v_0} n_{i,4} \xrightarrow{k} n_{i,5}$$

The gadgets F_0, F_1, F_2 and $G_0, N_0, \dots, G_{13}, N_{13}$ guarantee that if (sup, sig) solves α then $sig(u_i) = \mathsf{swap}$. Similarly to the reduction of Theorem 6, the TS A^τ_φ has for every $i \in \{0, \dots, m-1\}$ gadgets $T_{i,0}, \dots, T_{i,6}$ and B_i to make the one-in-three satisfiability of φ and the τ-solvability of α the same. These gadgets and the ones for Theorem 6 have basically the same intention. However, since the current types have different interactions, the peculiarity of these gadgets is slightly different. In particular, A^τ_φ has for each clause $\zeta_i = \{X_{i_0}, X_{i_1}, X_{i_2}\}$ the following gadget $T_{i,0}$:

$$t_{i,0,0} \xrightarrow{k} t_{i,0,1} \xrightarrow{u_0} t_{i,0,2} \xrightarrow{X_{i_0}} t_{i,0,3} \xrightarrow{u_1} t_{i,0,4} \xrightarrow{X_{i_1}} t_{i,0,5} \xrightarrow{u_2} t_{i,0,6} \xrightarrow{X_{i_2}} t_{i,0,7} \xrightarrow{u_3} t_{i,0,8} \xrightarrow{k} t_{i,0,9}$$

Moreover, the gadgets $T_{i,1}, T_{i,2}$ and $T_{i,3}$ are defined as follows:

$$t_{i,1,0} \xrightarrow{k} t_{i,1,1} \xrightarrow{u_4} t_{i,1,2} \xrightarrow{X_{i_0}} t_{i,1,3} \xrightarrow{w_{3i}} t_{i,1,4} \xrightarrow{X_{i_1}} t_{i,1,5} \xrightarrow{u_5} t_{i,1,6} \xrightarrow{u_6} t_{i,1,7} \xrightarrow{k} t_{i,1,8}$$

$$t_{i,2,0} \xrightarrow{k} t_{i,2,1} \xrightarrow{u_4} t_{i,2,2} \xrightarrow{X_{i_2}} t_{i,2,3} \xrightarrow{w_{3i+1}} t_{i,2,4} \xrightarrow{X_{i_0}} t_{i,2,5} \xrightarrow{u_5} t_{i,2,6} \xrightarrow{u_6} t_{i,2,7} \xrightarrow{k} t_{i,2,8}$$

$$t_{i,3,0} \xrightarrow{k} t_{i,3,1} \xrightarrow{u_4} t_{i,3,2} \xrightarrow{X_{i_1}} t_{i,3,3} \xrightarrow{w_{3i+2}} t_{i,3,4} \xrightarrow{X_{i_2}} t_{i,3,5} \xrightarrow{u_5} t_{i,3,6} \xrightarrow{u_6} t_{i,3,7} \xrightarrow{k} t_{i,3,8}$$

Furthermore, the gadgets $T_{i,4}, T_{i,5}$ and $T_{i,6}$ are defined by

$$t_{i,4,0} \xrightarrow{k} t_{i,4,1} \xrightarrow{u_7} t_{i,4,2} \xrightarrow{u_8} t_{i,4,3} \xrightarrow{w_{3i}} t_{i,4,4} \xrightarrow{u_9} t_{i,4,5} \xrightarrow{u_{10}} t_{i,4,6} \xrightarrow{k} t_{i,4,7}$$

$$t_{i,5,0} \xrightarrow{k} t_{i,5,1} \xrightarrow{u_7} t_{i,5,2} \xrightarrow{u_8} t_{i,5,3} \xrightarrow{w_{3i+1}} t_{i,5,4} \xrightarrow{u_9} t_{i,5,5} \xrightarrow{u_{10}} t_{i,5,6} \xrightarrow{k} t_{i,5,7}$$

$$t_{i,6,0} \xrightarrow{k} t_{i,6,1} \xrightarrow{u_7} t_{i,6,2} \xrightarrow{u_8} t_{i,6,3} \xrightarrow{w_{3i+2}} t_{i,6,4} \xrightarrow{u_9} t_{i,6,5} \xrightarrow{u_{10}} t_{i,6,6} \xrightarrow{k} t_{i,6,7}$$

Finally, the TS A^τ_φ has for all $i \in \{0, \dots, m-1\}$ the following gadget B_i:

$$b_{i,0} \xrightarrow{X_i} b_{i,1} \xrightarrow{u_{11}} b_{i,2} \xrightarrow{k} b_{i,3}$$

We briefly argue for the announced functionality of the gadgets. Let (sup, sig) be a τ-region solving α. If $sig(k) = \mathsf{free}$ then $sup(h_{3,3}) = 1$ and $s \xrightarrow{k} s'$ implies $sup(s) = sup(s') = 0$. Since $sup(h_{3,1}) = 0$ and $sup(h_{3,3}) = 1$, there is an event $e \in \{v_0, v_1\}$ such that $sig(e) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$. If $sig(v_0) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, then, by $sup(h_{1,1}) = 0$, we get $sig(v_0) = \mathsf{swap}$. Moreover, if $sig(v_1) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, which implies $sig(h_{3,2}) = 1$, then, by $sup(h_{2,3}) = 0$, we get $sig(v_1) = \mathsf{swap}$. By $sig(v_1) = \mathsf{swap}$ and $sup(h_{2,3}) = 0$, we get $sup(h_{2,2}) = 1$. By $sup(h_{1,1})$, this implies $sig(v_0) = \mathsf{swap}$. Thus, in any case we get $sig(v_0) = \mathsf{swap}$. By $sig(v_0) = \mathsf{swap}$ and $sup(h_{4,3}) = sup(h_{5,1}) = 0$ we obtain $sup(h_{4,2}) = sup(h_{5,2}) = 1$ which implies $sig(x) = \mathsf{swap}$. Using this and $sup(s) = sup(s') = 0$ if $s \xrightarrow{k} s'$, we have that $sup(h_{j,2}) = 1$ for all $j \in \{6, \dots, 11\}$. This implies $sig(y_0) = sig(y_1) = sig(y_2) = \mathsf{swap}$. By $sup(h_{12,1}) = sup(h_{12,4}) = 0$, the image of $h_{12,1} \xrightarrow{y_0} \dots \xrightarrow{y_2} h_{12,4}$ is a path from 0 to 0 in τ. The number of state changes between 0 and 1 on such a path is even. This contradicts $sig(y_0) = sig(y_1) = sig(y_2) = \mathsf{swap}$. Thus, $sig(k) \neq \mathsf{free}$. The assumption that $sig(k) = \mathsf{used}$ and $sup(h_{3,3}) = 0$ yields a contradiction, too.

We conclude that $sig(k) = \mathsf{inp}$ and $sup(h_{3,3}) = 0$. This implies $sig(v_0) \notin \{\mathsf{out}, \mathsf{set}\}$ and if $s \xrightarrow{k} s' \in A^\tau_I$, then $sup(s) = 1$ and $sup(s') = 0$. Thus, by $sup(h_{2,1}) = 0$ and $sup(h_{2,3}) = 1$ there is an event $e \in \{v_0, v_1\}$ such that $sig(e) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$. If $e = v_0$ then $sig(v_0) = \mathsf{swap}$. Moreover, if

$e = v_1$ then $sup(h_{3,2}) = 1$ which with $sup(h_{3,3}) = 0$ and $sup(h_{1,1}) = 1$ implies $sig(v_0) = \mathsf{swap}$. Consequently, any case implies $sig(v_0) = \mathsf{swap}$. This results in $sig(u_j) = \mathsf{swap}$ for all $j \in \{0, \dots, 13\}$ as follows. By $sup(f_{0,3}) = sup(f_{1,3}) = 1$ and $sig(v) = \mathsf{swap}$ we obtain $sup(f_{0,2}) = sup(f_{1,2}) = 0$ which with $sup(f_{0,1}) = sup(f_{1,1}) = 0$ implies $sig(z_0), sig(z_1) \in \{\mathsf{nop}, \mathsf{res}, \mathsf{free}\}$. Moreover, by $sig(z_0), sig(z_1) \in \{\mathsf{nop}, \mathsf{res}, \mathsf{free}\}$ and $sup(f_{2,1}) = 0$ we get $sup(f_{2,3}) = 0$ which with $sup(f_{2,4}) = 1$ implies $sig(z_2) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$. By $sig(z_0) \in \{\mathsf{nop}, \mathsf{res}, \mathsf{free}\}$ and $sup(g_{i,1}) = 0$, we get $sup(g_{i,2}) = 0$. Furthermore, $sig(z_1) \in \{\mathsf{nop}, \mathsf{res}, \mathsf{free}\}$ and $sup(g_{i,4}) = 1$ yields $sig(z_1) = \mathsf{nop}$ and $sup(g_{i,3}) = 1$. This implies $sig(u_i) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$. Finally, by $sup(n_{i,1}) = 0$ and $sig(z_2) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, we get $sup(n_{i,1}) = 1$ and, by $sup(n_{i,4}) = 1$ and $sig(v_0) = \mathsf{swap}$, we have $sup(n_{i,3}) = 0$. Since $sig(u_i) \in \{\mathsf{out}, \mathsf{set}, \mathsf{swap}\}$, this yields $sig(u_i) = \mathsf{swap}$ for all $i \in \{0, \dots, 13\}$. The gadgets $T_{i,0}, \dots, T_{i,6}$, where $i \in \{0, \dots, m-1\}$, use $sig(k) = \mathsf{inp}$ and $sig(u_i) = \mathsf{swap}$ for all $i \in \{0, \dots, 13\}$ similarly to the ones of Theorem 6 to ensure that $M = \{X \in V(\varphi) \mid sig(X) = \mathsf{swap}\}$ is a one-in-three model of φ: By $sup(t_{i,4,6}) = sup(t_{i,5,6}) = sup(t_{i,6,6}) = 1$ and $sig(u_5) = sig(u_6) = \mathsf{swap}$ we have $sup(t_{i,4,4}) = sup(t_{i,5,4}) = sup(t_{i,6,4}) = 1$ for all $i \in \{0, \dots, m-1\}$. Thus, if $X \in V(\varphi)$, $s \xrightarrow{X} s'$ and $sup(s) \neq sup(s')$ then $sig(X) = \mathsf{swap}$. Using this, one argues in a manner quite similar to that of the proof of Theorem 6 that $T_{i,0}, \dots, T_{i,6}$ collaborate in such a way that there is exactly one variable event $X \in \{X_{i_0}, X_{i_1}, X_{i_2}\}$ such that $sig(X) = \mathsf{swap}$. Thus, M is a corresponding model. Moreover, if $sig(k) = \mathsf{out}$ and $sup(h_{3,3}) = 1$ then we obtain again that $sig(u_i) = \mathsf{swap}$ for all $i \in \{0, \dots, 13\}$ which also guarantees that M is a searched model.

Conversely, if M is a one-in-three model of φ then we can define analogously to Theorem 6 a τ-region solving α. ◻

Theorem 8 ([12]). *For any fixed $g \geq 1$, deciding if a g-bounded TS A is τ-solvable is NP-complete if $\tau \in \{\mathsf{nop}, \mathsf{inp}, \mathsf{out}\} \cup \{\mathsf{used}, \mathsf{free}\}$.*

Proof. The claim follows directly from our result of [12]. There we use 1-bounded cycle free gadgets to prove that synthesis of (pure) b-bounded Petri nets is NP-complete. The joining of [12] yields a 2-bounded TS. However, it is easy to see that the 1-bounded joining of this paper fits, too. The (pure) 1-bounded Petri net type is isomorphic to $\{\mathsf{nop}, \mathsf{inp}, \mathsf{out}, \mathsf{used}\}$ ($\{\mathsf{nop}, \mathsf{inp}, \mathsf{out}\}$). By symmetry, τ-solving ESSP atoms by used is equivalent to solving them by free. ◻

4 Polynomial Time Results

Theorem 9. *For any fixed $g < 2$, one can decide in polynomial time if a g-bounded TS A is τ-solvable if $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ or $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}\}$ or $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}\}$ or $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{free}\}$ or $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{res}\} \cup \omega$ with non-empty $\omega \subseteq \{\mathsf{inp}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$.*

Proof. If A is τ-solvable then no event e of A occurs twice in a row. Otherwise, the SSP atom (s', s'') of a sequence $s \xrightarrow{e} s' \xrightarrow{e} s''$ is not τ-solvable. Thus, in what follows, we assume that A has no event occurring twice in a row. Moreover, it is easy to see that a 1-bounded TS $A = s_0 \xrightarrow{e_1} \dots \xrightarrow{e_m} s_m$ is a simple directed path on pairwise distinct states $s_0, \dots, s_m$ or a directed cycle, that is, all states $s_0, \dots, s_m$ except s_0 and s_m are pairwise distinct. This proof proceeds as follows. First, we assume that $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ and that A is a directed cycle and argue that the τ-solvability of a given ESSP atom (k, s) or a SSP atom (s, s') of A is decidable in polynomial time. Secondly, we argue that the presented algorithmic approach is applicable to directed paths, too. Thirdly, we show that the procedure introduced for $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ can be extended to $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}\}$. By Lemma 2, this proves the claim for $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}\}$ and $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{free}\}$, too. After that we investigate the case where $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{res}\} \cup \omega$ with non-empty $\omega \subseteq \{\mathsf{inp}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$. We argue that it is sufficient to decide the $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$- and $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{used}\}$-solvability of A and that this is doable in polynomial time. The corresponding procedures again modify those introduced for $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$.

Let $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ and A be 1-bounded (cycle) TS with event $k \in E(A)$ that occurs m times. Since A is a cycle, we can assume that k occurs at A's initial state: $\iota \xrightarrow{k}$. Moreover, since k does not occur twice in a row, its occurrences partition A into m k-free subsequences $I_0, \dots, I_{m-1}$ such that $I_i = s_0^i \xrightarrow{y_1^i} s_1^i \dots s_{n_i-1}^i \xrightarrow{y_{n_i}^i} s_{n_i}^i$, $i \in \{0, \dots, m-1\}$, and $s_{n_{m-1}}^{m-1} = \iota$, cf. Fig. 6.

Obviously, defining $sup(\iota) = 1$, $sig(k) = \mathsf{inp}$ and $sig(e) = \mathsf{set}$ for all $e \in E(A) \setminus \{k\}$ inductively yields a region (sup, sig) solving the ESSP atoms (k, s) where $\xrightarrow{k} s$. Thus, it remains to consider the case $\neg(\xrightarrow{k} s)$. Since $\neg(\xrightarrow{k} s)$, there is an $i \in \{0, \dots, m-1\}$ such that s is a state of the i-th subsequence I_i. In particular, there is a $j \in \{1, \dots, n_i - 1\}$ such that $s = s_j^i$. The state s_j^i divides I_i into the sequences $I_i^0 = s_0^i \xrightarrow{y_1^i} \dots \xrightarrow{y_j^i} s_j^i$ and $I_i^1 = s_j^i \xrightarrow{y_{j+1}^i} \dots \xrightarrow{y_{n_i}^i} s_{n_i}^i$, cf. Fig. 6.

If (sup, sig) is a region that solves α then $sig(k) = \mathsf{inp}$ and $sup(s_j^i) = 0$ is true. This implies for all $\ell \in \{0, \dots, m-1\}$ that $sup(s_0^\ell) = 0$ and $sup(s_{n_\ell}^\ell) = 1$. Thus, it remains to define the signature of the events of $\bigcup_{\ell=0}^{m-1} E(I_\ell)$ such that $0 \xrightarrow{sig(y_1^\ell)} \dots \xrightarrow{sig(y_{n_\ell}^\ell)} 1$, for all $\ell \in \{0, \dots, m-1\} \setminus \{i\}$, and $0 \xrightarrow{sig(y_1^i)} \dots \xrightarrow{sig(y_j^i)} 0$ and $0 \xrightarrow{sig(y_{j+1}^i)} \dots \xrightarrow{sig(y_{n_i}^i)} 1$.

If there is for all $\ell \in \{0, \dots, m-1\} \setminus \{i\}$ an event $e_\ell \in E(I_\ell)$ such that $e_\ell \notin E(I_i^0)$ and if there is an event $e_i \in E(I_i^1)$ so that $e_i \notin E(I_i^0)$ then $sup(\iota) = 1$,

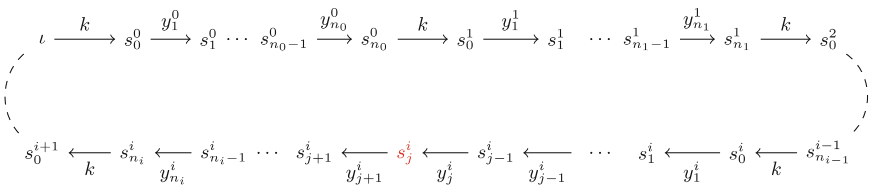


Fig. 6. A sketch of a cyclic 1-bounded input A with ESSP atom $\alpha = (k, s_j^i)$.

$sig(k) = \mathsf{inp}$, $sig(e_\ell) = \mathsf{set}$ for all $\ell \in \{0,\dots,m-1\}$, and $sig(e) = \mathsf{nop}$ for all $e \in E(A) \setminus \{k, e_0, \dots, e_\ell\}$ yields a τ-region (sup, sig) of A that solves α. Clearly, whether A satisfies this property is decidable in polynomial time.

Otherwise, there is a sequence $I \in \{I_0, \dots, I_{i-1}, I_i^1, I_{i+1}, \dots, I_{m-1}\}$ so that $E(I) \subseteq E(I_i^0)$. Thus, if (sup, sig) is a τ-region that solves α then there is a $\ell \in \{1,\dots,j-1\}$ such that $sig(y_\ell^i) = \mathsf{set}$. Consequently, there has to be a $\ell' \in \{\ell + 1,\dots,j\}$ such that $sig(y_{\ell'}^i) = \mathsf{inp}$ and, in particular, $sig(y_{\ell''}^i) = \mathsf{nop}$ for all $\ell'' \in \{\ell'+1,\dots,j\}$. Using this, one finds that (sup, sig) implies a region (sup', sig') that solves α and gets along with at most two inp-events. More exactly, defining $sup'(\iota) = 1$, $sig'(k) = sig'(y_{\ell'}^i) = \mathsf{inp}$, $sig'(e) = \mathsf{nop}$ for all $e \in \{y_{\ell'+1}^i, \dots, y_j^i\}$ and $sig'(e) = \mathsf{set}$ for all $e \in E(A) \setminus (\{k, y_{\ell'}^i, \dots, y_j^i\})$ yields a valid τ-region (sup', sig') that solves α. Since (sup, sig) was arbitrary, these deliberations show that in the second case the atom α is τ-solvable if and only if there is a corresponding region (sup', sig'). This yields the following polynomial procedure that decides whether α is τ-solvable: For ℓ from j to 2 test if (sup_ℓ, sig_ℓ) (inductively) defined by $sup_\ell(\iota) = 1$, $sig_\ell(y_\ell^i) = \mathsf{inp}$, $sig_\ell(y_{\ell'}^i) = \mathsf{nop}$ for all $\ell' \in \{\ell+1,\dots,j\}$ and $sig_\ell(e) = \mathsf{set}$ for all $e \in E(A) \setminus (\{k, y_\ell^i, \dots, y_j^i\})$ yields a τ-region of A. If the test succeeds for any iteration then return yes, otherwise return no.

We can modify this approach to test the τ-solvability of an SSP atom $\beta = (s, s')$ as follows. Since $A = \iota \xrightarrow{e_1} \dots \xrightarrow{e_m} \iota$ is a cycle we can assume without loss of generality that $s = \iota$ and $s' = s_i$ for some $i \in \{1,\dots,m-1\}$. The states ι and s_i partition A into two subsequences $I_0 = \iota \xrightarrow{e_1} \dots \xrightarrow{e_i} s_i$ and $I_1 = s_i \xrightarrow{e_{i+1}} \dots \xrightarrow{e_m} \iota$. If β is solvable by a region (sup', sig') such that $sup'(\iota) = 1$ and $sup'(s_i) = 0$ then there is an event $e \in I_0$ such that $sig(e) = \mathsf{inp}$. In particular, there is a region (sup, sig) as follows: $sup(\iota) = 1$, $sig(e_j) = \mathsf{inp}$ and $j \in \{1,\dots,i\}$, $sig(e_\ell) = \mathsf{nop}$ for all $\ell \in \{j+1,\dots,i\}$ and $sig(e) = \mathsf{set}$ for all $e \in E(A) \setminus \{e_j, \dots, e_i\}$. Similar to the approach for α, we can check if such a region exists in polynomial time. Moreover, the case where $sup(\iota) = 0$ and $sup(s_i) = 1$ works symmetrically.

So far we have shown that the τ-solvability of (E)SSP atoms of A are decidable in polynomial time if A is a cycle. If $A = \iota \xrightarrow{e_1} \dots \xrightarrow{e_m} s_m$ is a directed path then its *cycle extension* A_c has a fresh event $\oplus \notin E(A)$ and is defined by $A_c = \iota \xrightarrow{e_1} \dots \xrightarrow{e_m} s_m \xrightarrow{\oplus} \iota$. The event $\oplus$ is unique thus an (E)SSP atom of A is solvable by a τ-region of A if and only if it is solvable by a τ-region of A_c. Thus, we can decide the solvability of atoms of A via A_c. Altogether, this proves that the τ-solvability of (E)SSP atoms of 1-bounded inputs is decidable in polynomial time. Since we have at most $|S|^2 + |E| \cdot |S|$ atoms to solve, the decidability of the $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$-solvability for 1-bounded TS is polynomial.

Similar to the discussion for $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$, one argues that the following assertion is true: If $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}\}$ then there is a τ-region (sup', sig') with $sig'(k) = \mathsf{used}$ and $sup(s_j^i) = 0$ if and only if there is a τ-region (sup, sig) and an number $\ell \in \{1,\dots,j\}$ such that $sup(\iota) = 1$, $sig(k) = \mathsf{used}$, $sig(y_\ell^i) = \mathsf{inp}$, $sig(y_{\ell'}^i) = \mathsf{nop}$ for all $\ell' \in \{\ell+1,\dots,j\}$ and $sig(e) = \mathsf{set}$ for all $e \in E(A) \setminus \{k, y_\ell^i, \dots, y_j^i\}$. Clearly, the procedure introduced for $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ can be extended appropriately to a procedure that works for $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{used}\}$.

It remains to investigate the case where $\tau = \{\mathsf{nop}, \mathsf{res}, \mathsf{set}\} \cup \omega$ with non-empty $\omega \subseteq \{\mathsf{inp}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$. For a start, let's argue that deciding the τ-solvability is equivalent to deciding the $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$-solvability or the $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{used}\}$-solvability of A. This can be seen as follows: If (sup, sig) is a region that solves an ESSP atom $\alpha = (k, s)$ such that $sig(k) = \mathsf{inp}$ then there is a $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$-region (sup', sig') that solves (k, s), too. The region (sup', sig') originates from (sup, sig) by $sup' = sup$, $sig'(k) = \mathsf{inp}$ and for all $e \in E(A) \setminus \{k\}$ by $sig'(e) = \mathsf{nop}$ if $sig(e) \in \{\mathsf{nop}, \mathsf{used}, \mathsf{free}\}$, $sig'(e) = \mathsf{res}$ if $sig(e) \in \{\mathsf{inp}, \mathsf{res}\}$ and, finally, $sig'(e) = \mathsf{set}$ if $sig(e) \in \{\mathsf{out}, \mathsf{set}\}$. Similarly, one argues that α is τ-solvable such that $sig(k) = \mathsf{out}$ if and only if it is $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{set}\}$-solvable. Moreover, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$ and $\{\mathsf{nop}, \mathsf{out}, \mathsf{res}, \mathsf{set}\}$ are isomorphic thus τ-solvability with inp or out reduces to $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$-solvability. Similarly, the τ-solvability with used or free reduces to $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{used}\}$-solvability. It is easy to see that the procedure introduced for $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$ can be extended to the types $\{\mathsf{nop}, \mathsf{inp}, \mathsf{res}, \mathsf{set}\}$ and $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{used}\}$. The only difference is that we now search for an event y^i_ℓ such that $sig(y^i_\ell) = \mathsf{res}$ instead of $sig(y^i_\ell) = \mathsf{inp}$.

Finally, we observe that a SSP atom $\beta = (s, s')$ is τ-solvable if and only if it is $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}\}$-solvable. The states s and s' induce again a partition I_0 and I_1 of A and we can adapt the approach above to $\{\mathsf{nop}, \mathsf{res}, \mathsf{set}\}$. □

Theorem 10. *For any fixed $g \in \mathbb{N}$, deciding whether a g-bounded TS A is τ-solvable is polynomial if one of the following conditions is true:*

1. $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{free}\}$ *or* $\tau = \{\mathsf{nop}, \mathsf{inp}, \mathsf{used}, \mathsf{free}\}$ *or* $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{used}\}$ *or* $\tau = \{\mathsf{nop}, \mathsf{out}, \mathsf{used}, \mathsf{free}\}$ *and* $g < 2$.
2. $\tau = \{\mathsf{nop}, \mathsf{set}, \mathsf{res}\} \cup \omega$ *and* $\emptyset \neq \omega \subseteq \{\mathsf{used}, \mathsf{free}\}$ *and* $g < 3$.
3. $\tau = \tau' \cup \omega$ *and* $\tau' \in \{\{\mathsf{nop}, \mathsf{set}, \mathsf{swap}\}, \{\mathsf{nop}, \mathsf{res}, \mathsf{swap}\}, \{\mathsf{nop}, \mathsf{res}, \mathsf{set}, \mathsf{swap}\}\}$ *and* $\emptyset \neq \omega \subseteq \{\mathsf{used}, \mathsf{free}\}$ *and* $g < 2$.
4. $\tau \in \{\{\mathsf{nop}, \mathsf{inp}\}, \{\mathsf{nop}, \mathsf{inp}, \mathsf{used}\}, \{\mathsf{nop}, \mathsf{out}\}, \{\mathsf{nop}, \mathsf{out}, \mathsf{free}\}\}$ *or* $\tau \in \mathcal{T} = \{\{\mathsf{nop}, \mathsf{set}, \mathsf{swap}\}, \{\mathsf{nop}, \mathsf{res}, \mathsf{swap}\}, \{\mathsf{nop}, \mathsf{set}, \mathsf{res}\}, \{\mathsf{nop}, \mathsf{set}, \mathsf{res}, \mathsf{swap}\}\}$,

Proof. (1): It is easy to see that A is a loop, $A \cong s \xrightarrow{e} s$ or that A is cycle free, since there is an unsolvable SSP atom otherwise. Moreover, if an event e occurs twice consecutively, $s \xrightarrow{e} s' \xrightarrow{e} s''$, then (s, s') is not τ-solvable. Thus, for every $e \in E(A)$ there is a $s \in S(A)$ such that (e, s) has to be solved by $sig(e) = \mathsf{inp}$ ($sig(e) = \mathsf{out}$) and $sup(s) = 0$ ($sup(s) = 1$). If e occurs twice on the directed path A then such a region does not exist. On the other hand, A is τ-solvable if every event occurs exactly once. Consequently, A is τ-solvable if and only if it is 1-bounded and every event occurs exactly once.

(2): Since ESSP atoms of a τ-solvable input A are only solvable by used and free, we have that if $s \xrightarrow{e} s' \in A$ then $s' \xrightarrow{e} s'' \in A$. If $s = s'' \neq s'$ or if s, s', s'' are pairwise distinct then (s, s') is not τ-solvable. This implies $s' \xrightarrow{e} s'$. As a result, τ-solvable inputs have the shape

$$A = \iota \xrightarrow{e_0} s_1 \circlearrowleft^{e_1} \quad \dots \quad s_{m-1} \xrightarrow{e_m} s_m \circlearrowleft^{e_m}$$

Thus, if the *loop erasement* A' of A originates from A by erasing all loops $s \xrightarrow{e} s$, that is, $A' = \iota \xrightarrow{e_1} \dots \xrightarrow{e_m} s_m$, then deciding the τ-solvability of A reduces to deciding if A' has the τ-SSP and if all ESSP atoms (e, s) with $\neg(\xrightarrow{e} s)$ of A' are τ-solvable. This is doable in polynomial time by the approach of Theorem 9.

(3): Since ESSP atoms of an input A are only solvable by used and free, if $s \xrightarrow{e} s'$ and $s \neq s'$ then $s' \xrightarrow{e}$. If $s \xrightarrow{e} s' \xrightarrow{e} s'' \xrightarrow{e} s''' \in A$ and s, s', s'', s''' are pairwise different, then the SSP atom (s', s''') is not τ-solvable. As a consequence, τ-solvable inputs can have at most 3 different states.

(4): Let $\tau \in \{\{\mathsf{nop}, \mathsf{inp}\}, \{\mathsf{nop}, \mathsf{inp}, \mathsf{used}\}\}$. If A is τ-solvable, then for all $e \in E(A)$ holds $\iota \xrightarrow{e}$. Otherwise, (e, ι) is not τ-solvable. Similarly, if $\tau \in \mathcal{T}$, then ESSP atoms are not τ-solvable thus, every event occurs at ι. A is g-bounded. This implies $|E(A)| \leq g$. Thus, A has at most $2 \cdot |\tau|^g$ τ-regions. Since g is fixed, τ-synthesis is polynomial by brut-force. By Lemma 2, the claim follows.

□

5 Conclusion

In this paper, we fully characterize the computational complexity of nop-equipped Boolean Petri nets from g-bounded TS for any fixed $g \in \mathbb{N}$. Our results show that if τ-synthesis is hard then it remains hard even for low bounds g. Moreover, they also show that when g becomes very small, sometimes it makes the difference between hardness and tractability, cf. Fig. 1 §1–§3 and §9, but sometimes it does not, cf. Fig. 1 §4–§7. In this sense, the parameter g helps to recognize interactions that contribute to the power of a type. By Theorem 3 and Theorem 9, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}\}$-synthesis is hard if $g \geq 2$ and tractable if $g < 2$, respectively. By Theorem 5, $\{\mathsf{nop}, \mathsf{inp}, \mathsf{set}, \mathsf{free}\}$-synthesis remains hard for all $g \geq 1$. Thus, if restricted to 1-bounded inputs then the test interaction free makes the difference between hardness and tractability of synthesis. Surprisingly enough, by Theorem 9, replacing free by used makes synthesis from 1-bounded TS tractable again. It remains future work, to characterize the computational complexity of synthesis for the remaining 128 types which do not contain nop. Moreover, since τ-synthesis generally remains hard even for (small) fixed g, the bound of a TS is ruled out for FPT-algorithms. Future work might be concerned with parameterizing τ-synthesis by the *dependence number* of the searched τ-net: If $N = (P, T, f, M_0)$ is a Boolean net, $p \in P$ and if the *dependence number* d_p of p is defined by $d_p = |\{t \in T \mid f(p,t) \neq \mathsf{nop}\}|$ then the *dependence number* d of N is defined by $d = \max\{d_p \mid p \in P\}$. At first glance, d appears to be a promising parameter for FPT-approaches because this parameterization puts the problem into the complexity class XP: Since a τ-region of $A = (S, E, \delta, \iota)$ is determined by $sup(\iota)$ and sig, for each (E)SSP atom α there are at most $2 \cdot |\tau|^d \cdot \sum_{i=0}^{d} \binom{|E|}{i}$ fitting τ-regions solving α. Thus, by $|\tau| \leq 8$, τ-synthesis parameterized by d is decidable in $\mathcal{O}(|E|^d \cdot |S| \cdot \max\{|S|, |E|\})$.

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