Electoral Reform and Social Choice Theory: Piecemeal Engineering and Selective Memory

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Abstract. Most electoral reforms are dictated by recognized problems discovered in the existing procedures or - perhaps more often - by an attempt to consolidate power distributions. Very rarely, if ever, is the motivation derived from the social choice theory even though it deals with issues pertaining to what is possible and what is impossible to achieve by using given procedures in general. We discuss some reforms focusing particularly on a relatively recent one proposed by Eric Maskin and Amartya Sen. It differs from many of its predecessors in invoking social choice considerations in proposing a new system of electing representatives. At the same time it exemplifies the tradeoffs involved in abandoning existing systems and adopting new ones.

Keywords: Condorcet consistency · Plurality voting · Plurality with runoff · Black's method · Nanson's rule

1 Introduction

As basically all institutions, the voting procedures can be seen as problem-solving devices: their adoption starts with a recognized problem – be it one of including popular participation in a domain where decisions formerly were made by a single individual or one related to the working of an existing procedure – proceeding then to a mechanism that purportedly provides a solution to this problem. This piecemeal way of improving existing institutions may, however, lead to a paradox that carries the name of Marquis de Condorcet, the French social philosopher and science administrator of the 18th century. We shall give it a slightly unusual interpretation in the following Table 1. Consider three voting procedures—x, y and z – three performance criteria – I, II and III – and the following hypothetical configuration of procedures and criteria.

In Table 1 the procedures are ranked on each of the three performance criteria. Thus, e.g. on criterion II y performs best, z second best and x worst. Assuming that the criteria are of equal importance, it is reasonable to come up

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Table 1. Condorcet's paradox

Criterion I	Criterion II	Criterion III
x	y	z
y	z	x
z	x	y

with the overall conclusion that x is better than y since it is ranked higher than y on two criteria (I and III). Similarly, y can be deemed better than z since it is ranked higher than z on two criteria (I and II). One could then be led to conclude by transitivity that x is better than z as well. Yet, this is clearly not the case: z is ranked higher than x on two criteria (II and III). This is the crux of Condorcet's paradox: aggregating several rankings (i.e. complete and transitive preference relations) applying the majority principle in each pair of alternatives may lead to an endless cycle of alternatives where each replacement of an alternative can be justified by a majority rule, but at the same time no best alternative is to be found.

This is a version of the well-known money-pump. To wit, suppose the original procedure is x which is the best of the three in terms of criterion I. Assume now that an analyst or a proponent of y comes up with criterion II which is deemed more plausible than I. In terms of criterion II the ranking of the procedures is the one presented in the middle of Table 1. Now an advocate of z may take the floor and suggest criterion III where z is ranked first, x second and y third. This might be the historical sequence of events whereby criteria are considered one by one. Depending on the content of the criteria each step of the process from x to z via y may well be justified, but the overall result - i.e. when all criteria are taken into account and given a roughly equal weight – is a never-ending cycle of 'improvements'.

In the next sections we shall focus on a couple of examples of electoral reform. First, we discuss the proposal of Jean-Charles de Borda presented for the French Royal Academy in the 1770's. We then focus on a procedure designed to rectify an alleged weakness in Borda's proposal. Borda's proposal was primarily directed avoiding the main flaw of the first-past-the-post (plurality) voting. Other ways of avoiding this are discussed next. Finally, we deal with the recent proposal presented by Amartya Sen and Eric Maskin to replace the prevailing first-past-the-post system of the U.S. Congressional elections with a hybrid of two mutually incompatible procedures.

2 The First Attack on the Plurality System

The one-person-one-vote or plurality voting system is undoubtedly the most common voting procedure today. It is not only used extensively in political elections (e.g. in the U.K. parliamentary and in the U.S. elections of the members of the House of Representatives), but also in informal settings. In fact, it is the procedure people typically have in mind when saying 'let's take a vote' in informal settings. Its main virtue is simplicity, both for voters and for the persons determining the winner(s). Its outcomes also lend themselves for a straightforward interpretation: the winner is the alternative that has been voted upon by more voters than any of its contestants. But it is associated with an important shortcoming made visible by Borda in his example reproduced in Table 2 [4,11,16].

Table 2. Borda's paradox

1 voter	7 voters	7 voters	6 voters
A	A	B	C
B	C	C	B
C	B	A	A

Assuming, as Borda does, that each voter votes according to his/her preferences, alternative A wins with 8 votes against 7 (for B) and 6 (for C). And yet, A is the last-ranked alternative of a clear majority of voters, an absolute (Condorcet) loser one could say in modern terminology. As such it would be defeated in pairwise majority comparisons by all other alternatives. It seems that Borda's main motivation was to prevent such an outcome from happening by devising a system that would exclude the eventual Condorcet losers from being elected. Thus, he introduced a procedure: the method of marks or the Borda count in modern terminology. Given a profile of individual preference rankings over, say k, alternatives, the Borda count determines the Borda scores of alternatives as follows: each voter ranking an alternative first increases this alternative's score by $k-1$ points, each voter ranking an alternative second increases the score by $k-2$ points, etc. and each voter ranking an alternatives last increases its score by zero points. The sum of points given by all voters determines its Borda score. The collective ranking of alternatives is the same as the order of their Borda scores. Hence, it is natural to call the first ranked alternative the Borda winner. Borda showed that his method can also be implemented through exhaustive pairwise comparisons so that the score of each alternative is the sum of the votes it gets in every pairwise comparison with other alternatives.

That the Borda count really guarantees the exclusion of the eventual Condorcet loser was formally shown by P. C. F. Daunou in the beginning of the

Table 3. The strong Condorcet winner and the Borda winner can be distinct

4 voters	3 voters
A	B
B	C
C	A

19th century [11, 263–267].[1] Table 2 gives rise to another observation that came to play a central role in the emerging debate between Borda and Condorcet, viz. the possibility that the procedure investigated always elects a Condorcet winner, that is, an alternative that – on the basis of the information presented in the preference profile – defeats by a majority of votes all others in pairwise comparisons. This is often deemed an important desideratum in the theory of voting (see e.g. [5]; for more sceptical views, see [17,20]). In Table 2 there is a Condorcet winner[2] and it is simultaneously the Borda winner. This, however, is not always the case. Indeed, one could even envision a profile with a strong Condorcet winner which, nonetheless, is not the Borda winner. Table 3 provides an illustration.

Here A is the strong Condorcet winner, but B gets the largest Borda score.

3 The Search for Condorcet Consistency

For those convinced of the desirability of Condorcet consistency – i.e. the property guaranteeing the choice of Condorcet winner when one exists – the possibility of not electing a Condorcet winner is not acceptable. One of the early writers feeling uncomfortable at the possibility of not electing a Condorcet winner was

[1] It is relatively straightforward to see how this conclusion is derived. To wit, suppose that there is a Condorcet loser, say x, in a profile consisting of n voters and k alternatives. This means that in each pairwise comparison, the maximum number of votes for x is strictly less than $n/2$. Hence x's Borda score is less than $(k-1)\times(n/2)$. If all alternatives had the same or smaller Borda score than x (which would make x the Borda winner), the total number of Borda scores would be no larger than: $k \times (k-1) \times n/2$. Now this upper bound is strictly less than the sum total of Borda scores in any profile, viz. $n \times (k^2 - k)/2 = k \times (k-1) \times n/2$. (The number of pairwise comparisons involving different alternatives is $k^2 - k$ with the sum of entry (i, j) and entry (j, i) being equal to the number of voters, n, for all alternatives i and j.) Therefore, in any profile there must be at least one alternative with a strictly larger Borda score than that of the Condorcet loser. Hence, the latter cannot be elected by the Borda count.

[2] A strong Condorcet winner is an alternative ranked first by more than half of the electorate. Obviously, all procedures that elect a Condorcet winner also elect a strong Condorcet winner. The converse is not true, that is, there are procedures (e.g. plurality voting) that elect the strong Condorcet winner, but not necessarily a Condorcet winner.

E. J. Nanson. He set out to design a system that would be as 'Bordaesque' as possible while at the same time satisfying Condorcet consistency [13]. The resulting procedure – known as Nanson's rule – is based on a sequence of eliminations based on the Borda scores of alternatives so that in each counting round those alternatives with the average or smaller Borda score are discarded whereupon the Borda scores are computed in the remaining set of alternatives. Eventually the winner is found after one or more rounds of eliminations.

The main purpose of Nanson's rule is to preserve the eventual Condorcet winner, while determining the winner by using in full the positional information given by the voters just like in the Borda count. The survival of the Condorcet winner in the elimination process is guaranteed by a relationship that exists between the Condorcet and Borda winner in any profile: the Condorcet winner always has a Borda score that is strictly larger than the average.[3] Hence, the Condorcet winner, whenever it exists, coincides with the Nanson winner.

A few decades after the invention of Nanson's rule, another Borda elimination procedure was proposed by Baldwin [1]. Somewhat paradoxically Baldwin – fully cognizant of Nanson's rule – presented his method as a simplification of Nanson's even though it in general requires more computing rounds than Nanson's rule. The basic difference between these two is that Baldwin's method eliminates at each counting round the alternative with the smallest Borda score. It is easy to see that, as Nanson's, also Baldwin's method is Condorcet-consistent.[4]

4 Meanwhile Other Attacks Were Launched

The possibility that an alternative elected is considered very bad (even the worst) by more than a half of the electorate has motivated other voting procedure proposals. By far the most successful of them in terms of the frequency of adoption is the plurality with runoff procedure. It is clearly aimed at securing that the winner is supported by at least half of the electorate. If this is not the case in the original profile, the runoff is organized between those two (sometimes three or more) alternatives gaining the most votes in the original profile. With only two alternatives competing on the second round and barring ties, one of these is bound to receive the support of more than half of the electorate.

This method guarantees that an alternative ranked last by a majority of voters cannot be elected. For if the winner is found in the original profile, the winner cannot be such an alternative that is ranked last by a majority of voters (since such an alternative cannot simultaneously be first-ranked by a majority). If, on the other hand, the winner is found on the second round, it cannot be the alternative that is originally last-ranked by a majority (since that majority ranks whichever of its competitors first in the second round).

[3] Nanson's argument to that effect is pretty similar to the one in footnote 1. It amounts to showing that the lower bound of the Borda score of a Condorcet winner is strictly larger than the average of the Borda scores.

[4] The differences and similarities of Nanson's and Baldwin's rules are discussed in [7,14,15].

Table 4. Non-monotonicity of plurality with runoff and alternative vote

8 voters	9 voters	7 voters
A	B	C
C	C	B
B	A	A

A couple of decades earlier than Nanson presented his rule, Carl Andrae of Denmark and Thomas Hare of England introduced the alternative vote method. In contradistinction to Nanson's rule, this method is based on plurality eliminations of alternatives one by one until one of them occupies a majority of first ranks. In three-alternative profiles the plurality runoff and alternative vote are equivalent, but with the presence of more than three alternatives they can end up with different choices. A common feature in both these procedures is that they never choose a candidate ranked last by a majority of voters. The reason for this in the case of plurality runoff was just stated. In the case of the alternative vote, the reason is the observation that in order to become the alternative vote winner, the alternative has to defeat by a majority at least one other, viz. the one it is confronted with in the last sub-profile that determines the winner.

5 Advantages Gained and Lost

So both procedures dealt with in the preceding section avoid Borda's paradox and can thus be considered improvements over the plurality voting. The same is, of course, true of the Borda count which arguably was specifically designed to address Borda's paradox. All three methods (Borda count, plurality runoff and alternative vote), however, correct one major flaw, viz. avoid the choice of an eventual Condorcet loser, but two of them (plurality with runoff and the alternative vote) are accompanied with another flaw that the original culprit, plurality voting, is not plagued with: non-monotonicity. In other words, while in plurality voting the additional support for a winner, *ceteris paribus*, never makes it a non-winner, such an addition may displace winners of the plurality with runoff and alternative vote. Table 4 provides an illustration.

The plurality with runoff procedure first eliminates C whereupon B wins the second counting round. The same choice is made by the alternative vote. Suppose now that the winner B had somewhat more support so that two of the voters with ACB ranking lift B (the winner) at the top of their ranking which then becomes BAC. Then the runoff contestants are B and C with C the winner. The same outcome ensues from the alternative vote procedure. Thus additional support may, indeed, be detrimental for the winning alternative when plurality with runoff or alternative vote are being applied.

In contrast to Nanson's rule, however, the plurality runoff and alternative vote procedures do not necessarily elect the Condorcet winner when one exists. The failure is easy enough to demonstrate. See Table 5 where the Condorcet

Table 5. Plurality with runoff and alternative vote may not elect the Condorcet winner

4 voters	3 voters	2 voters
A	B	C
C	C	A
B	A	B

Table 6. Nanson's rule is non-monotonic

5 voters	4 voters	3 voters	2 voters	2 voters	1 voter
C	B	A	B	A	A
A	D	B	A	C	C
D	C	D	C	B	D
B	A	C	D	D	B

winner C is ranked first by the smallest number of voters and is thus eliminated in the first counting round by both procedures. Hence it is not elected. In fact, with more than three alternatives profiles can be envisaged where the Condorcet winner is ranked first by no voters at all.[5]

As said, Nanson's rule is, by design, Condorcet consistent. So, it is an improvement over plurality with runoff and alternative vote systems in that regard. However, it shares an important drawback with them: it is non-monotonic. See the 17-voter 4-alternative profile of Table 6 where A wins once first D and then both B and C are eliminated. Let now A's support be increased, *ceteris paribus*, so that the 2 voters with BACD ranking lift the winner A on top of their ranking so that their ranking is ABCD. As a result both B and D are eliminated in the first computing round, whereupon C wins.

A property intuitively related to monotonicity is participation. It requires that for any group of identically-minded voters and any profile of preferences, abstaining does not lead to an outcome that is preferable to the one ensuing when the group is voting according to its preferences, *ceteris paribus*. In a seminal paper Moulin established that Condorcet consistency is incompatible with participation if there are more than three alternatives and the number of voters is large enough [12].[6] In other words, all procedures that necessarily elect a Condorcet winner when one exists may encounter profiles of four or more alternatives where a group of voters with identical opinions about alternatives is better off abstaining than voting according to its preferences. Violations of participation are often called no-show paradoxes [6,9].

[5] For example, let a three-person four-alternative profile be the following: 1 voter: ABCD, 1 voter: CBDA, 1 voter: DBAC. Here B is the Condorcet winner.

[6] Moulin's lower bound for the number of voters was 25. This bound has more recently been lowered to 12 by Brandt et al. [3]. A stronger variant of Moulin's result has subsequently been proven by Pérez [19].

Table 7. Plurality with runoff and alternative vote fail on participation

7 voters	8 voters	5 voters
A	B	C
B	C	A
C	A	B

Table 8. Nanson's rule may lead to a strong no-show paradox

5 voters	4 voters	3 voters	3 voters	1 voter	5 voters
C	B	A	A	B	C
D	D	C	C	D	A
B	A	D	B	A	D
A	C	B	D	C	B

As Nanson's rule is Condorcet consistent, it is, by Moulin's result, vulnerable to the no-show paradox, but this result as such gives no clues as to whether plurality with runoff or alternative vote are consistent with participation. The following example, however, illustrates the possibility of a no-show paradox in both of the last mentioned procedures in a profile with only three alternatives (Table 7).

Both plurality with runoff and alternative vote result in A after C has been eliminated. This is the worst outcome for the eight voters in the middle. Suppose that four of them had abstained. Then B would have been eliminated, whereupon C would have won. C being preferable to A, we have an instance of the no-show paradox demonstrating that the procedures are vulnerable to it.

A far more dramatic instance of the no-show paradox – called the strong no-show paradox [19] – may occur when Nanson's rule is used. Table 8 gives an illustration. In this 21-voter profile A is the Condorcet winner and is, by Nanson's argument, elected. Suppose now that the five voters with CADB ranking would have abstained. In the ensuing 16-voter profile, there is no Condorcet winner. Instead C emerges as the Nanson winner. Thus, by abstaining the five voters not only improve the outcome (from their point of view), but bring about the victory of their first-ranked alternative.[7]

In terms of the social choice desiderata considered here, the price to be paid for the avoidance of Borda's paradox by plurality with runoff, alternative vote and Nanson's rules seems high: they all suffer from vulnerability to the monotonicity failures both in fixed and variable electorates, while the plurality voting is immune to these anomalies. The same observation can be made about Condorcet consistency, albeit with the significant reservation that Nanson's rule is

[7] Table 8 also illustrates the vulnerability to the strong no-show paradox of several other Condorcet consistent procedures: Baldwin, Black, Copeland and Kemeny. For further discussion, see [8].

Condorcet consistent. These conclusions have some bearing on the evaluation of the relatively recent reform proposal concerning the election system of the U.S. House of Representatives.

6 The Reform Proposal of Maskin and Sen

The relatively recent reform proposal by Maskin and Sen is a hybrid voting system intended for the replacement of the current plurality voting system commonly used in the election of the members of the House of Representatives of the U.S. Congress [10].[8] It is pretty similar to the system proposed by Black nearly seventy years earlier [2]. Black's suggestion is simply a combination of Condorcet's and Borda's winning intuitions: given a profile, elect the Condorcet winner if one exists, otherwise elect the Borda winner. In similar vein, Maskin and Sen suggest that the Condorcet winner be elected if one exists, but otherwise the plurality with runoff winner is elected. It is difficult to see how this proposal would improve upon Black's hybrid method. After all, when a Condorcet winner exists in a profile, both Maskin and Sen's procedure (MS, for brevity) and Black's method elect it. The differences can occur only in those profiles where there is no Condorcet winner. In those, MS resorts to plurality with runoff, while Black's method applies the Borda count. The latter is monotonic both in fixed and variable electorates, while the former is non-monotonic in both kinds of electorates. As observed above, both MS and Black's method avoid electing the Condorcet loser.

The primary objective of Black's method and MS is the election of the Condorcet winner. In this sense both procedures are fundamentally majoritarian, i.e. the winner should primarily be determined by pairwise majority comparisons. In the absence of a Condorcet winner, one then resorts to different positional procedures. It is not clear why such a leap from one intuition of winning (binary) to another (positional) is called for. Such a combination of intuitions may lead to quite astonishing occurrences. To wit, the alternative that comes close to being the Condorcet winner may not do well in terms of the plurality with runoff. In fact, even the Condorcet winner may not have sizable support in terms of first ranks of voters. Indeed, as was seen above (footnote 5), it may well be that no voter ranks the Condorcet winner first. Hence if an alternative comes close to being the Condorcet winner (without quite being one), its showing in the plurality with runoff may be the worst.

Overall MS exemplifies the typical tradeoffs involved in rectifying flaws in prevailing voting systems: by correcting one weakness one often ends up with another which may or may not characterize the original system. MS aims at making sure that the Condorcet winners are elected and that the Condorcet losers are not. The original plurality system does not guarantee either of these objectives. Instead, it does well in terms of monotonicity-related desiderata. So by suggesting the replacement of the plurality voting with MS one is in a way revealing a preference for Condorcet criteria over monotonicity-related ones.

[8] For a related discussion on the MS procedure, see [18].

Suppose, however, that we could find a procedure that satisfies the Condorcet criteria and does reasonably well in terms of monotonicity. By Moulin's theorem we know the variable electorate variants of monotonicity cannot be satisfied. Yet, there are Condorcet consistent procedures that are monotonic in fixed electorates, e.g. Copeland's method and Kemeny's median, to name two. Intuitively it would make sense to resort to these to guarantee that a uniform standard – the success in pairwise majority comparisons – be applied in determining the winner. This, of course, doesn't do away with the fact that all Condorcet extensions suffer from some form of non-monotonicity in variable electorates [6], but maintains the same winner intuition both in profiles where a Condorcet winner exists and in those where it doesn't.

7 Concluding Remarks

Electoral reforms are often made in order to avoid real or imagined problems in the working of existing procedures. To motivate systemic modifications it is common to concentrate on just one problem at a time rather than to engage in overall evaluation of available procedures in terms of all plausible criteria of performance. The latter, holistic, approach typically reveals theoretical incompatibilities between desiderata. Not all nice properties are achievable under any given procedure in all conceivable profiles. Above we have focused on a few modifications in procedures suggested as responses to observed problems. The tradeoffs involved in these are mainly related to the Condorcet criteria and monotonicity. The problem with these and other similar criteria is their general nature: they deal with all conceivable circumstances. Yet, in practice some circumstances may be excluded or extremely unusual. A plausible way of approaching the electoral reforms would be to take into account any information one might have on such circumstances. For example, are the profiles typically encountered such that a Condorcet winner exists? Are we typically dealing with a small number of alternatives? The vulnerability of various procedures to various anomalies may depend on answers to these kinds of questions [8]. In any event, a holistic multiple criteria evaluations are likely to yield more lasting procedural choices than strategies focusing on one criterion at a time.

References

1. Baldwin, J.M.: The technique of the Nanson preferential majority system. Proc. Roy. Soc. Victoria **39**, 42–52 (1926)
2. Black, D.: On the rationale of group decision-making. J. Polit. Econ. **56**, 23–34 (1948)
3. Brandt, F., Geist, C., Peters, D.: Optimal bounds for the no show paradox via SAT solving. Math. Soc. Sci. **90**, 18–27 (2017)
4. De Grazia, A.: Mathematical derivation of an election system. Isis **44**, 42–51 (1953)
5. Felsenthal, D.S., Machover, M.: After two centuries should Condorcet's voting procedure be implemented? Behav. Sci. **37**, 250–273 (1992)

6. Felsenthal, D.S., Nurmi, H.: Monotonicity Failures Afflicting Procedures for Electing a Single Candidate. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-51061-3
7. Felsenthal, D.S., Nurmi, H.: Monotonicity violations by Borda's elimination and Nanson's rules: a comparison. Group Decis. Negot. **27**(2018), 637–664 (2018)
8. Felsenthal, D.S., Nurmi, H.: Voting Procedures Under a Restricted Domain. An Examination of the (In)Vulnerability of 20 Voting Procedures to Five Main Paradoxes. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-12627-8
9. Fishburn, P.C., Brams, S.J.: Paradoxes of preferential voting. Math. Mag. **56**, 207–214 (1983)
10. Maskin, E., Sen, A.K.: How majority rule might have stopped Donald Trump. The New York Times, 28 April 2016 (2016). https://www.nytimes.com/2016/05/01/opinion/sunday/how-majority-rule-might-have-stopped-donald-trump.html
11. McLean, I., Urken, A.B.: Classics of Social Choice. University of Michigan Press, Ann Arbor (1995)
12. Moulin, H.: Condorcet's principle implies the no-show paradox. J. Econ. Theory **45**, 53–64 (1988)
13. Nanson, E.J.: Methods of elections. Trans. Proc. Roy. Soc. Victoria **19**, 197–240 (1883). McLean, I., Urken, A.B. (eds.) Classics of Social Choice, Chapter 14, pp. 321–359. University of Michigan Press (1995)
14. Niou, E.M.S.: A note on Nanson's rule. Public Choice **54**, 191–193 (1987)
15. Nurmi, H.: On Nanson's method. In: Borg, O., Apunen, O., Hakovirta, H., Paastela, J. (eds.) Democracy in the Modern World. Essays for Tatu Vanhanen, series A, vol. 260, pp. 199–210. Acta Universitatis Tamperensis, Tampere (1989)
16. Nurmi, H.: Voting Paradoxes and How to Deal with Them. Springer, Heidelberg (1999). https://doi.org/10.1007/978-3-662-03782-9
17. Nurmi, H.: Reflections on two old Condorcet extensions. Trans. Comput. Collective Intell. **XXXI**, 9–21 (2018)
18. Nurmi, H.: Voting theory: cui bono? Politeia **91**, 106–121 (2018)
19. Pérez, J.: The strong no show paradoxes are a common flaw in Condorcet voting correspondences. Soc. Choice Welfare **18**, 601–616 (2001)
20. Saari, D.G.: Capturing 'The Will of the People'. Ethics **113**, 333–349 (2003)