Public Announcements for Epistemic Models and Hypertheories

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Abstract. Artemov has recently proposed a modernization of the semantics and proof theory of epistemic logic. We take up his approach and extend his framework with public announcements and the corresponding belief change operation. We establish a soundness and completeness result and show that our model update operation satisfies the AGM postulate of minimal change. Further, we also show that the standard approach cannot be directly employed to capture knowledge change by truthful announcements.

Keywords: Modal logic · Public announcements · Epistemic models · Hypertheories

1 Introduction

Artemov [3–6] suggests a modernization of the semantics and proof theory of epistemic logic. He proposes new foundations for epistemic logic with

1. a semantics that does not assume models to be common knowledge and
2. a matching framework of hypertheories for reasoning with partial information.

He introduces the class of epistemic models, which includes Kripke models, but can cover many more epistemic situations. The main difference is that in epistemic models, the Kripkean definition of satisfiability of a belief formula

$$u \Vdash \Box_i A \;\Leftrightarrow\; R_i(u) \Vdash A, \tag{1}$$

is replaced by a weaker condition

$$u \models \Box_i A \;\Rightarrow\; R_i(u) \models A,$$

where we write $R_i(u) \Vdash A$ for $\forall v(R_i(u, v) \Rightarrow v \Vdash A)$ and similarly for $R_i(u) \models A$. Hence the fully explanatory property of models is avoided, i.e., we do not have that if a sentence holds at all possible states, then it is believed.

This work was supported by the Swiss National Science Foundation grant 200021_165549.

Md. A. Khan and A. Manuel (Eds.): ICLA 2019, LNCS 11600, pp. 64–75, 2019.
https://doi.org/10.1007/978-3-662-58771-3_7

In this paper, we extend Artemov's epistemic models and hypertheories with public announcements. The idea behind public announcements in the Kripkean case is that a public announcement induces a model change: after the public announcement of a formula A, the model is restricted in a way that preserves only the relations between states where A holds [11,14,16,18]. In the case of epistemic models, however, only restricting the model does not yield new beliefs since (1) does not hold. To model public announcements properly, we also have to explicitly take care of what the new beliefs are. We will make use of public announcements that are total [7,16,17], i.e., new information can always be announced. Moreover, our explicit treatment of belief change is influenced by dynamic epistemic justification logics [2,8,9,12,13,15].

Our approach gives us more control over the belief dynamics that takes place when an announcement occurs. In particular, we can define the updated model such that the minimal change property of the AGM postulates is satisfied [1,10]. This is not possible in the traditional Kripkean setting.

We also show that there is no straightforward adaptation of our approach to the case of knowledge change. Namely, with a help of one example, it is showed that a restricted $\mathsf{S4}_n$-models are not well-defined.

The content of this paper is as follows. In Sect. 2 we present epistemic models and hypertheories formally. In Sect. 3 we propose the logic KPA_n of public announcements and prove soundness and completeness of the corresponding hypertheories w.r.t. epistemic models. In Sect. 4 we discuss a problem concerning truthful public announcements over $\mathsf{S4}_n$-epistemic models. We conclude the paper in Sect. 5.

2 Epistemic Models and Hypertheories

Recall the multi-agent modal logic K_n. Let $\mathsf{Prop} = \{p, q, r, \dots\}$ be a countable set of propositional letters. The language of the logic K_n consists Prop, the classical propositional connectives $\neg$ and $\wedge$, and modalities $\Box_i$, for $i = 1, \dots, n$. The set of formulas Fml is generated by the following grammar:

$$A := p \mid \neg A \mid (A \wedge A) \mid \Box_i A.$$

The other connectives are defined as usual. An axiomatization of K_n contains, besides the axioms of classical propositional logic and Modus Ponens, the following belief postulates:

Distributivity: $\Box_i(A \to B) \to (\Box_i A \to \Box_i B)$;
Necessitation rule: From A, infer $\Box_i A$.

The semantics of K_n is standard Kripke semantics. Namely, a model is a tuple $\mathcal{K} = (W, R_1, \dots, R_n, \Vdash)$, where

(K1) $W \neq \emptyset$;
(K2) R_i is a binary relation on W, for $i = 1, \dots, n$;
(K3) $\Vdash$: $\mathsf{Prop} \to 2^W$.

We call $(W, R_1, \ldots, R_n)$ a *frame.*

Truth in Kripke semantics is then defined inductively, starting from atomic propositions, with classical conditions for Boolean connectives and

$$u \Vdash \Box_i A \;:\Leftrightarrow\; R_i(u) \Vdash A. \tag{2}$$

In epistemic models, the situation is quite different since we use belief sets instead of (2) to model the agents' beliefs. Belief sets contain all theorems of a logic and they are closed under Modus Ponens. We use the following closure operation.

Definition 1 (Closure of a Set of Formulas). *Let* L *be a logic and* T *be a set of formulas.*

1. $cl^0_{\mathsf{L}}(T) = T \cup \{A \mid \mathsf{L} \vdash A\}$;
2. $cl^{j+1}_{\mathsf{L}}(T) = cl^j_{\mathsf{L}}(T) \cup \{A \mid B \in cl^j_{\mathsf{L}}(T) \text{ and } B \to A \in cl^j_{\mathsf{L}}(T), \text{ for some } B\}$;
3. $F \in cl_{\mathsf{L}}(T)$ *iff* $F \in cl^j_{\mathsf{L}}(T)$, *for some* j.

Definition 2 (Belief Set). *For a given logic* L*, an* L-belief set *is a set of formulas* T *with* $T = cl_{\mathsf{L}}(T)$.

Remark 1. The set $cl_{\mathsf{L}}(T)$ is a belief set for any set of formulas T.

Remark 2. Instead of belief sets, Artemov uses complete truth assignments in his definition of epistemic model. For the purpose of this paper, however, belief sets are better suited.

For a set $Z \subseteq X \times Y$ and an element $x \in X$, we set $(Z)_x := \{y \mid (x, y) \in Z\}$.

Definition 3 (Pre-epistemic Model). *Let* L *be a logic. A* pre-epistemic L-model *is a tuple* $\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu^1_{\mathcal{B}}, \ldots, \nu^n_{\mathcal{B}})$*, where:*

- $W \neq \emptyset$ *is a non-empty set of states;*
- $R_1, \ldots, R_n$ *are binary relations on* W*;*
- $\nu \subseteq W \times \mathsf{Prop}$*;*
- $\nu^i_{\mathcal{B}} \subseteq W \times \mathsf{For}$*, such that for every* $u \in W$*,* $(\nu^i_{\mathcal{B}})_u$ *is an* L*-belief set.*

Definition 4 (Satisfaction Relation). *Let* $\mathcal{E}$ *be a pre-epistemic* L*-model and* $u \in W$*. The satisfaction relation,* $\models$*, is defined as follows:*

- $\mathcal{E}, u \models p$ *iff* $(u, p) \in \nu$*;*
- $\mathcal{E}, u \models A \wedge B$ *iff* $\mathcal{E}, u \models A$ *and* $\mathcal{E}, u \models B$*;*
- $\mathcal{E}, u \models \neg A$ *iff* $\mathcal{E}, u \not\models A$*;*
- $\mathcal{E}, u \models \Box_i A$ *iff* $(u, A) \in \nu^i_{\mathcal{B}}$.

Definition 5 (Epistemic Model). *Let* L *be a logic. An* epistemic L-model *is a pre-epistemic* L*-model that satisfies*

$$\mathcal{E}, u \models \Box_i A \;\Rightarrow\; \mathcal{E}, R_i(u) \models A. \tag{3}$$

In contrast to Kripke models, the truth value of belief formulas in epistemic models is provided by belief sets and (3) is a set of constraints. Also note that in (3) we only have the implication ‘from left to right’, while we have an equivalence in (2). Hence the fully explanatory property, which states that

if a sentence is valid at all possible states, then it is believed,

does not hold for epistemic models.

The following theorem shows the relationship between Kripke and epistemic models: for any given epistemic K_n-model, there is a Kripke model that contains it.

Theorem 1 (Embedding Theorem). *For any epistemic* K_n*-model*

$$\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu_{\mathcal{B}}^1, \ldots, \nu_{\mathcal{B}}^n),$$

there exists a Kripke model

$$\mathcal{K} = (\widetilde{W}, \widetilde{R_1}, \ldots, \widetilde{R_n}, \Vdash),$$

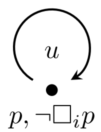
such that:

(a) $W \subseteq \widetilde{W}$*;*
(b) $R_i \subseteq \widetilde{R_i}$*;*
(c) for each $u \in W$ *and each formula* A*,*

$$\mathcal{E}, u \models A \qquad \text{iff} \qquad \mathcal{K}, u \Vdash A.$$

Theorem 1 tells us that epistemic models are contained in Kripke models, where the containing Kripke model is obtained by adding appropriate states to the epistemic model. The following example illustrates this fact.

Example 1. Consider W consisting of a single state u at which p is true but the agent does not believe that p. The appropriate epistemic model is:



Note that it is not a Kripke model (since p holds in every possible state but is not believed), but can be extended to one:

This example shows one important difference between epistemic and Kripke models: in epistemic models we do not have to add new states in order to represent situations where an agent does not believe a true fact, as it is the case in Kripke models.

Hypertheories provide the proof-theoretic framework that matches epistemic models.

Definition 6 (Hypertheory). *A* hypertheory *is a tuple*

$$\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T}),$$

where:

- $(W, R_1, \ldots, R_n)$ *is a frame;*
- $\mathcal{T}$ *assigns a set of formulas* T_u *to each* $u \in W$*.*

Note that T_u need not be maximal, i.e., we may have neither $p \in T_u$ nor $\neg p \in T_u$. This reflects the fact, mentioned in the introduction, that hypertheories represent a tool for dealing with partial information.

Definition 7. *An epistemic* L*-model* $\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu^1_{\mathcal{B}}, \ldots, \nu^n_{\mathcal{B}})$ *is a* model of $\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T})$ *if for each* $u \in W$*,*

$$u \models T_u \quad (\textit{i.e. } u \models A, \textit{ for each } A \in T_u).$$

A formula A logically L-follows *from a hypertheory* $\mathcal{H} = (W, R_1, \ldots, R_n, \mathcal{T})$ *at state* $u \in W$*, denoted by*

$$\mathcal{H}, u \models_{\mathsf{L}} A,$$

if $\mathcal{E}, u \models A$ *for each epistemic* L*-model* $\mathcal{E}$ *of* $\mathcal{H}$*.*

Hyperderivations provide the syntactic consequence relation for hypertheories.

Definition 8 (Hyperderivation). *Let* $\mathcal{H}$ *be a hypertheory. A formula A is* L-hyperderivable *at* $u \in W$ *(write* $\mathcal{H}, u \Vdash_{\mathsf{L}} A$*) if A can be obtained by the rules:*

(1) classical inference[1]*:*
  - *(a)* $u \Vdash_{\mathsf{L}} A$*, if* $A \in T_u$*;*
  - *(b)* $u \Vdash_{\mathsf{L}} A$*, if* $\mathsf{L} \vdash A$*;*
  - *(c)* $u \Vdash_{\mathsf{L}} A$*, if* $u \Vdash_{\mathsf{L}} B \to A$ *and* $u \Vdash_{\mathsf{L}} B$ *for some formula B;*

(2) transition: $u \Vdash_{\mathsf{L}} \Box_i A \;\Rightarrow\; R_i(u) \Vdash_{\mathsf{L}} A$*;*

(3) deduction: $u \cup A \Vdash_{\mathsf{L}} B \;\Rightarrow\; u \Vdash_{\mathsf{L}} A \to B$*, where for a hypertheory* $\mathcal{H}$*,* $\mathcal{H}^{v \cup A}$ *is defined as a hypertheory* $\mathcal{H}$ *where* T_v *is replaced by* $T_v \cup \{A\}$ *and* $u \cup A \Vdash_{\mathsf{L}} B$ *stands for* $\mathcal{H}^{u \cup A}, u \Vdash_{\mathsf{L}} B$*;*

(4) consistency: if uR_iv*, then* $v \Vdash_{\mathsf{L}} \bot \;\Rightarrow\; u \Vdash_{\mathsf{L}} \bot$*.*

[1] As usual, we write $u \quad _{\mathsf{L}} A$ instead of $\mathcal{H}$, $u \quad _{\mathsf{L}} A$ when $\mathcal{H}$ is clear from the context.

Note that the *transition* rule "goes only in one direction", which corresponds to (3).

Artemov established the following soundness and completeness result for K_n.

Theorem 2. *For a hypertheory* $\mathcal{H}$ *and any* Fml*-formula* A,

$$\mathcal{H}, u \Vdash_{\mathsf{K}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{K}_n} A.$$

3 Public Announcements

In this section we discuss how to model public announcements in epistemic models. For simplicity we follow an approach where the agents believe any formula that is announced—no matter whether the announcement is truthful or whether it is consistent with the current beliefs.

The logic KPA_n is an extension of the logic K_n with an announcement operator $[\cdot]$. The set of formulas $\mathsf{Fml}_{[\cdot]}$ is generated by the following grammar:

$$A := p \mid \neg A \mid (A \wedge A) \mid \Box_i A \mid [A]A.$$

Read $[A]$B as: "after the announcement of A, it holds that B". The logic KPA_n is given by the following axioms and rules:

Axiom schemes:

(B1) all instantiations of classical propositional tautologies
(B2) $\Box_i(A \to B) \to (\Box_i A \to \Box_i B)$
(B3) $[A]p \leftrightarrow p$
(B4) $[A]\neg B \leftrightarrow \neg[A]B$
(B5) $[A](B \wedge C) \leftrightarrow ([A]B \wedge [A]C)$
(B6) $[A]\Box_i B \leftrightarrow \Box_i(A \to B)$
(B7) $[A][B]C \leftrightarrow [A \wedge B]C$

Inference rules:

(IR1) Modus Ponens
(IR2) From A, infer $\Box_i A$.

Since atomic propositions represent facts, axiom $B3$ says that announcements of formulas do not change facts (only agents' beliefs). Axioms $B4$ and $B5$ state that negation and conjunction behave as expected, while axiom $B6$ explains what it means that after an announcements of a formula, an agent beliefs that B. Finally, axiom $B7$ says that announcing first a formula A and then B is the same as announcing $A \wedge B$.

We already have the definitions of KPA_n-belief sets and pre-epistemic KPA_n-models. To define the corresponding satisfaction relation, we add the following clause to Definition 4:

– $\mathcal{E}, u \models [A]B$ iff $\mathcal{E}|_A, u \models B$, where the restricted model $\mathcal{E}|_A = (W', R'_1, \ldots, R'_n, \nu', \nu'^1_{\mathcal{B}}, \ldots, \nu'^n_{\mathcal{B}})$ is given by:

$$\begin{aligned}
&W' = W,\\
&R'_i = R_i \cap ([\![A]\!]_{\mathcal{E}} \times [\![A]\!]_{\mathcal{E}}),\\
&\nu' = \nu,\\
&\nu'^i_{\mathcal{B}} = \{(w, B) \mid w \in W \text{ and } B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_w \cup \{A\})\},
\end{aligned}$$

with $[\![A]\!]_{\mathcal{E}} = \{w \in W \mid \mathcal{E}, w \models A\}$.

Epistemic KPA_n-models are now given by Definition 5. We show that the restriction $\mathcal{E}|_A$ of an epistemic KPA_n-model is well-defined.

Lemma 1. *For any formula A, if $\mathcal{E}$ is an epistemic KPA_n-model, then the restricted model $\mathcal{E}|_A$ is an epistemic KPA_n-model, too.*

Proof. Directly from the definition, we have that W' is non-empty, R'_i are binary relations on W', $\nu' \subseteq W' \times \mathsf{Prop}$, and for any $u \in W'$, $(\nu'^i_{\mathcal{B}})_u$ is a belief set and therefore $\mathcal{E}|_A$ is a pre-epistemic KPA_n-model. We need to prove that for any formula B, the constraint

$$\mathcal{E}|_A, u \models \Box_i B \;\Rightarrow\; \mathcal{E}|_A, R_i(u) \models B \tag{4}$$

holds as well. Suppose that $\mathcal{E}|_A, u \models \Box_i B$, i.e.,

$$B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\}).$$

By induction on the buildup of $cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\})$, we prove $\mathcal{E}|_A, R_i(u) \models B$.

(1) (i) If $B \in (\nu^i_{\mathcal{B}})_u$, from the assumption that $\mathcal{E}$ is an epistemic model we get that $\mathcal{E}|_A, R_i(u) \models B$.
(ii) If $B = A$ or $\mathsf{KPA}_n \vdash B$, the claim follows from the definition of an restricted model.

(2) There exists a formula C, such that both

$$C, C \to B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\}).$$

By induction hypothesis, $\mathcal{E}|_A, R_i(u) \models C$ and $\mathcal{E}|_A, R_i(u) \models C \to B$. Thus $\mathcal{E}|_A, R_i(u) \models B$. $\square$

Note that KPA_n is a conservative extension of the modal logic K_n with respect to announcement-free formulas. We have for each Fml-formula A,

$$\mathsf{KPA}_n \vdash A \quad \text{iff} \quad \mathsf{K}_n \vdash A.$$

This conservativity result can be transfered to logical consequence.

Lemma 2. *Let $\mathcal{H}$ be a hypertheory consisting of Fml-formulas and let u be a state of $\mathcal{H}$. We have for each Fml-formula A,*

$$\mathcal{H}, u \models_{\mathsf{KPA}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{K}_n} A.$$

We say that a formula A is KPA_n-*valid*, if for any epistemic KPA_n-model $\mathcal{E}$ and state u, we have that $\mathcal{E}, u \models A$. We have the following result.

Lemma 3. *Axioms B3–B7 are* KPA_n*-valid.*

Proof

(B3)

$$\mathcal{E}, u \models [A]p \quad \text{iff} \quad \mathcal{E}|_A, u \models p \quad \text{iff} \quad \mathcal{E}, u \models p.$$

(B4)

$$\begin{aligned} \mathcal{E}, u \models \neg[A]B \quad &\text{iff} \quad \mathcal{E}, u \not\models [A]B \\ &\text{iff} \quad \mathcal{E}|_A, u \not\models B \\ &\text{iff} \quad \mathcal{E}|_A, u \models \neg B \\ &\text{iff} \quad \mathcal{E}, u \models [A]\neg B. \end{aligned}$$

(B5)

$$\begin{aligned} \mathcal{E}, u \models [A](B \wedge C) \quad &\text{iff} \quad \mathcal{E}|_A, u \models B \wedge C \\ &\text{iff} \quad \mathcal{E}|_A, u \models B \text{ and } \mathcal{E}|_A, u \models C \\ &\text{iff} \quad \mathcal{E}, u \models [A]B \text{ and } \mathcal{E}, u \models [A]C \\ &\text{iff} \quad \mathcal{E}, u \models [A]B \wedge [A]C. \end{aligned}$$

(B6)

($\leftarrow$) $(u, A \to B) \in \nu^i_{\mathcal{B}}$ implies $(u, A \to B) \in \nu'^i_{\mathcal{B}}$, where

$$\nu'^i_{\mathcal{B}} = \{(w, B) \mid w \in W \text{ and } B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_w \cup \{A\})\}.$$

Obviously, $(u, A) \in \nu'^i_{\mathcal{B}}$ as well and hence $(u, B) \in \nu'^i_{\mathcal{B}}$, i.e., $\mathcal{E}, u \models [A]\Box_i B$.
($\rightarrow$)

$$\begin{aligned} \mathcal{E}, u \models [A]\Box_i B \quad &\text{iff} \quad \mathcal{E}|_A, u \models \Box_i B \\ &\text{iff} \quad B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\}). \end{aligned}$$

We prove that $\mathcal{E}, u \models \Box_i(A \to B)$ by induction on the construction of $cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\})$.

(1) (i) If $(u, B) \in \nu^i_{\mathcal{B}}$, since $(u, B \to (A \to B)) \in \nu^i_{\mathcal{B}}$ as well, we obtain that $(u, A \to B) \in \nu^i_{\mathcal{B}}$, i.e., $\mathcal{E}, u \models \Box_i(A \to B)$.
(ii) If $B = A$, since $\mathsf{KPA}_n \vdash A \to A$, we get $(u, A \to A) \in \nu^i_{\mathcal{B}}$.
(iii) If $\mathsf{KPA}_n \vdash B$, the claim follows from the same reasoning as in (i).

(2) Suppose that there exists a formula C, such that both

$$C, C \to B \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{A\}).$$

By induction hypothesis,

$$\mathcal{E}, u \models \Box_i(A \to C) \text{ and } \mathcal{E}, u \models \Box_i(A \to (C \to B)).$$

Since belief sets are closed under classical propositional reasoning, we finally obtain $\mathcal{E}, u \models \Box_i(A \to B)$.

(B7) Directly from the fact that $(\mathcal{E}|_A)|_B = \mathcal{E}|_{A\wedge B}$. □

From the axiomatization of the logic KPA_n, it is clear that we have the usual "rewriting" property for public announcements, which makes it possible to remove all announcements from an arbitrary formula (see, e.g., [18]). Namely, it can be proved that for any $\mathsf{Fml}_{[.]}$-formula A, there exists an Fml-formula A^{K} such that

$$\mathsf{KPA}_n \vdash A \leftrightarrow A^{\mathsf{K}}. \tag{5}$$

Hence we can do the usual completeness by reduction proof for KPA_n.

Theorem 3 (Soundness and Completeness Theorem). *Let $\mathcal{H}$ be a hypertheory containing Fml-formulas. For each $\mathsf{Fml}_{[.]}$-formula A, we have*

$$\mathcal{H}, u \Vdash_{\mathsf{KPA}_n} A \quad \textit{iff} \quad \mathcal{H}, u \models_{\mathsf{KPA}_n} A.$$

Proof. From Lemma 3 we know that axioms $B3$–$B7$ are sound, while from the definition of an epistemic KPA_n-model follows that the axioms $B1$–$B2$, both inference rules, as well as transition, deduction and consistency constraints from KPA_n-hyperderivations are sound as well, i.e., the direction from left to right is established.

Completeness is obtained from the following observation:

$$\begin{aligned} \mathcal{H}, u \models_{\mathsf{KPA}_n} A \quad & \text{implies} \quad \mathcal{H}, u \models_{\mathsf{KPA}_n} A^{\mathsf{K}} \quad \text{(by (5) and soundness)} \\ & \text{implies} \quad \mathcal{H}, u \models_{\mathsf{K}_n} A^{\mathsf{K}} \quad \text{(Lemma 2)} \\ & \text{implies} \quad \mathcal{H}, u \Vdash_{\mathsf{K}_n} A^{\mathsf{K}} \quad \text{(Theorem 2)} \\ & \text{implies} \quad \mathcal{H}, u \Vdash_{\mathsf{KPA}_n} A. \end{aligned}$$

□

Our belief change operation satisfies the AGM postulates [1,10] for belief expansion. First of all, it is obvious from our semantics that all announcements are *successful*, i.e., after any announcement of A, each agent beliefs A. Formally, we have that

$$[A]\Box_i A$$

is KPA_n-valid.

Further, we have *persistence of beliefs*, i.e, no announcement will change existing beliefs. The formula

$$\Box_i A \to [B]\Box_i A$$

is KPA_n-valid. Indeed, for an arbitrary epistemic KPA_n-model $\mathcal{E}$ and formula B:

$$\begin{aligned} \mathcal{E}, u \models \Box_i A \quad & \text{iff} \quad (u, A) \in \nu^i_{\mathcal{B}} \\ & \text{then} \quad A \in cl_{\mathsf{KPA}_n}((\nu^i_{\mathcal{B}})_u \cup \{B\}) \\ & \text{iff} \quad \mathcal{E}|_B, u \models \Box_i A \\ & \text{iff} \quad \mathcal{E}, u \models [B]\Box_i A. \end{aligned}$$

The belief sets of the restricted model are given as a least fixed point of a monotone operator. It is an immediate consequence of this definition that our model update satisfies the requirement of *minimal change*. We have the following lemma.

Lemma 4 (Minimal Change). *Let $\mathcal{E}$ be an epistemic KPA_n-model with a state w. Let $\mathcal{F}$ be any epistemic KPA_n-model with a state v such that*

1. $\mathcal{F}, v \models \Box_i A$
2. $\mathcal{E}, w \models \Box_i B$ *implies* $\mathcal{F}, v \models \Box_i B$ *for all formulas* B.

Then we find that for all formulas B,

$$\mathcal{E}|_A, w \models \Box_i B \textit{ implies } \mathcal{F}, v \models \Box_i B.$$

4 The Case of Knowledge Change in Epistemic Models

In this section we show that there is no straightforward adaptation of our approach to the case of knowledge change. Let us investigate public announcements over $\mathsf{S4}_n$. In order to model them, we consider so-called *truthful* announcements. For a given Kripke model $\mathcal{M} = (W, R_1, \ldots, R_n, \Vdash)$, we define satisfiability of announcement formulas by

$$\mathcal{M}, s \Vdash [A]B \quad \text{iff} \quad \mathcal{M}, s \Vdash A \ \text{ implies } \ \mathcal{M}|_A, s \Vdash B, \tag{6}$$

where $\mathcal{M}|_A = (W', R'_1, \ldots, R'_n, \Vdash')$ is the restriction of the model defined as

$$\begin{aligned}
&W' = [\![A]\!]_{\mathcal{M}},\\
&R'_i = R_i \cap ([\![A]\!]_{\mathcal{M}} \times [\![A]\!]_{\mathcal{M}}),\\
&\Vdash' = \Vdash \cap [\![A]\!]_{\mathcal{M}},
\end{aligned}$$

for $[\![A]\!]_{\mathcal{M}} = \{w \in W \mid w \Vdash A\}$.

Adapting this strategy for an epistemic $\mathsf{S4}_n$-model

$$\mathcal{E} = (W, R_1, \ldots, R_n, \nu, \nu^1_{\mathcal{B}}, \ldots, \nu^n_{\mathcal{B}})$$

yields the following definition of satisfiability:

$$\mathcal{E}, s \models [A]B \quad \text{iff} \quad \mathcal{E}, s \models A \ \text{ implies } \ \mathcal{E}|_A, s \models B,$$

where $\mathcal{E}|_A$ is given by

$$\begin{aligned}
&W' = [\![A]\!]_{\mathcal{E}},\\
&R'_i = R_i \cap ([\![A]\!]_{\mathcal{E}} \times [\![A]\!]_{\mathcal{E}}),\\
&\nu' = \nu,\\
&\nu'^i_{\mathcal{B}} = \{(w, B) \mid w \in [\![A]\!]_{\mathcal{E}} \text{ and } B \in cl_{\mathsf{S4}_n}((\nu^i_{\mathcal{B}})_w \cup \{A\})\},
\end{aligned}$$

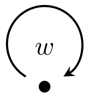
where cl_{S4_n} is given as in Definition 1 with the addition of

– if $A \in cl^{j}_{\mathsf{S4}_n}(T)$, then $\Box_i A \in cl^{j+1}_{\mathsf{S4}_n}(T)$,
– if $\Box_i A \in cl^{j}_{\mathsf{S4}_n}(T)$, then $A \in cl^{j+1}_{\mathsf{S4}_n}(T)$.

Unfortunately, restricted $\mathsf{S4}_n$-models are not well-defined. There exists an epistemic $\mathsf{S4}_n$-model $\mathcal{E}$ and a formula A such that the restriction $\mathcal{E}|_A$ is not an epistemic $\mathsf{S4}_n$-model. Consider $\mathcal{E} = (W, R, \nu, \nu_{\mathcal{B}})$ with

$$
\begin{aligned}
&W = \{w\},\\
&R = \{(w, w)\},\\
&\nu = \{(w, p)\},\\
&\nu_{\mathcal{B}} = \{(w, A) \mid A \in cl_{\mathsf{S4}_n}(\{\Box p \to \Box q\})\}.
\end{aligned}
$$

This is an epistemic $\mathsf{S4}_n$-model. We can depict it as follows:



$$p, \neg q, \neg\Box p, \neg\Box q, \Box(\Box p \to \Box q)$$

Since p holds at the state w, the restriction of $\mathcal{E}$ to the formula p yields

$$\mathcal{E}|_p = (W, R, \nu, \nu'_{\mathcal{B}}) \text{ with } \nu'_{\mathcal{B}} = \{(w, A) \mid A \in cl_{\mathsf{S4}_n}(\{\Box p \to \Box q, p\})\}.$$

Thus, by the closure conditions of cl_{S4_n}, we get $\Box p \in (\nu'_{\mathcal{B}})_w$, thus $\Box q \in (\nu'_{\mathcal{B}})_w$ and finally $q \in (\nu'_{\mathcal{B}})_w$. However, now we have the situation that $\mathcal{E}|_p, w \models \Box q$ but also $\mathcal{E}|_p, w \not\models q$. Since we also have $R(w, w)$, we find that condition (3) in the definition of an epistemic model is not satisfied.

5 Conclusion

We introduced public announcements for epistemic models and studied the corresponding belief dynamics. We showed that our model update operation satisfies the AGM postulate of minimal change. We also adapted hypertheories to support public announcements and established soundness and completeness.

In the case of knowledge change and truthful announcements, the situation gets more complicated. We presented an example showing that the standard approach cannot be used in a straightforward way to capture public announcements over epistemic $\mathsf{S4}_n$-models. This remains a topic for future research.

Moreover, it will be interesting to see how other belief change operations can be implemented in the framework of epistemic models. The fact that minimal change is satisfied for public announcements is a strong hint that using epistemic models is a promising approach to dealing with belief change.

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