Chapter 9
Features and Characteristics of Chinese New Century Mathematics Textbooks

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Abstract Unified curriculum standards (or teaching and learning syllabus) provide uniform guidelines from all teaching and learning activities in different grades levels across China under a centralized education system. Without exception, the textbooks using in schools are also designed in accordance with the beliefs advocated by these official documents. This chapter reports the common features and characteristics (such as the emphasis on the knowledge learning and its applications) of several series school textbooks were and are used in the twenty-first century with the major beliefs presented in the standards or syllabus. It aims to show the essences kept in the New Century mathematics textbooks which reflects the major reforms happened on the textbooks development.

9.1 Background

The Chinese education system is centralized, because it uses unified curriculum standards (previously called the "teaching and learning syllabus") across the nation and uniform guidelines for all teaching and learning activities in different grade levels, and serves as a direct channel for major educational reforms (Li, Zhang, & Ma, 2014; Liu & Li, 2010; Tang, Peng, Cheng, Kuang, & Song, 2013).

In China, mathematics textbooks are developed according to the national curriculum standards, namely the *Full-time Compulsory Education Mathematics Curriculum Standard* (*Experimental Version*) (Ministry of Education, 2001) and the *Senior Secondary School Curriculum Standard* (*Experimental Version*) (Ministry of Education, 2003). The two standards reflect the two phases of Chinese fundamental education. A nine-year compulsory education system, which includes

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six-year primary education and three-year junior secondary education, is followed by a three-year senior secondary education.

Usually, different versions of textbooks have been developed under the same curriculum standard; this is referred to as "one standard, many versions." These versions have common characteristics and features, but also have their own unique characteristics and features.

Compared with the previous textbooks, the current versions have changed greatly in knowledge, content organization, and presentation. The nature of these changes will be addressed first in this chapter. As well, a set of typical textbooks, developed by the People's Education Press and utilized in junior secondary schools, will be introduced as an example to illustrate the characteristics and features of Chinese mathematics textbooks.

9.2 Common Features and Characteristics

The Curriculum Reform for the New Century has led to many new ideas for developing textbooks, as stated in the curriculum standards issued by the Ministry of Education. Under the unified guidance of the curriculum standards, there are common features and characteristics in these versions of textbooks, as described below.

9.2.1 Emphasis on the Relationship Between Knowledge Learning and Its Applications

New Century textbooks emphasize the relationship between mathematical knowledge learning in the classroom and its application in the real world. To introduce the new knowledge, practical problems occurring in students' lives serve as the teaching materials, to help students easily experience and find mathematical knowledge abstracted from life. This is not only shown in the single lessons but also in the whole chapters. The textbooks create chances for students to solve problems through model constructing. These factors can contribute to students developing a deep understanding of mathematical knowledge and its applications, and mathematical modeling as well as their abilities to apply mathematics.

For example, to introduce the concept of Function to senior secondary school students, all the textbook versions present a number of examples for students to try to solve before making a generalization for function definition. Version A, published by the People's Education Press, presents three examples: the relationship between the height and time of a shell launching, the relationship between time and the area of the ozone hole in the atmosphere, and the relationship between Engel's coefficient and time (Liu, 2007, pp. 15–16). Version B, also published by the

People's Education Press, presents five examples: the relationship between the distance and time of free fall; the relationship between people's curiosity and their ages; the relationship between the height and growth period of corn; the relationship between GDP and time; and the relationship between current and resistance under constant voltage (Gao, 2006, pp. 31–32). The version issued by the Beijing Normal University Publishing Group generalizes the concept of Function from the example of the relationship between mileage and time on the national highway (Yan & Wang, 2006, pp. 25–26). The version issued by the Jiangsu Education Publisher also presents three examples: the relationship between the population and year; the relationship between height and time of free fall; and the relationship between the temperature and the time in one day (Shan, 2008, p. 21). All of these examples are selected in view of their relationships with students' life experiences, or with real applications and science. In these contexts, it is indicated that mathematics has an intensive relationship with the development of society and social life. This makes students feel that mathematics learning is worthwhile.

Moreover, mathematical knowledge is shown alongside its applications in actual life, which is usually the major content of one chapter. Taking the chapter of *Liner Equation with One Unknown* as an example, the previous textbooks usually presented in the style of "the concept-solution—application," while the new textbooks have improved upon this style. The new style is to use equations to analyze and solve actual problems, with the actual problems being described throughout the chapter. An equation can be regarded as a kind of mathematical model. In one such example, the concept and solution of an equation are introduced and discussed to promote the process of using liner equations with one unknown to solve practical problems. The structure of this chapter (Lin, 2007, pp. 78–114) can be seen in Fig. 9.1.

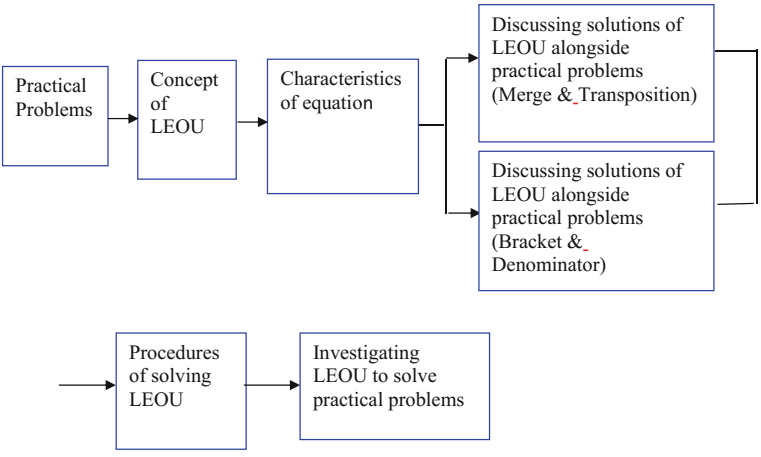

Fig. 9.1 Content structure of chapter Liner Equation with One Unknown (LEOU)

This approach provides much more opportunity and time for students to learn mathematics in practical contexts, and enhances their ability to construct mathematical models and to analyze and solve problems by using equations.

To understand this style better, the topic of *Linear Inequality in One Unknown*, issued by the Jiangsu Scientific Publisher, will be given as another example (Yang & Dong, 2006, pp. 6–20). The first section of this chapter is titled Inequality in Life. To introduce the concept of inequality, several practical problems are provided that are familiar to students in their lives. Students can generalize the concept of inequality from these examples with the guidance of the teacher. When the solutions for linear inequality in one unknown are discussed, a problem relating to a young tree's growth height is discussed, which provides an actual context. The next section, *Solve Problems via Linear Inequality in One Unknown*, focuses on solving problems. Several practical problems are provided to demonstrate the problem-solving process by means of establishing proper inequalities. Practical problems can be found from the beginning to the end of this chapter, which reflects the importance of mathematical knowledge in actual life.

9.2.2 *Emphasis on Knowledge Development, Heuristics and Investigation*

One of the beliefs underpinning the New Century education reform is that education should be student-centered. Thus, lots of mathematical tasks are designed to help students experience the knowledge acquirement and application through observing, experimenting, conjecturing, reasoning, discussing, and reflection activities.

When designing mathematical tasks, the New Century textbooks demonstrate knowledge in the sequence of "knowledge background—knowledge establishment —description of knowledge relationship." This process can attract students'

Fig. 9.2 Counting, ordering, and producing 1, 2, 3, …

Fig. 9.3 Describing “Null” “Vacancy,” and then 0

Fig. 9.4 Dividing, measuring, and then fraction $\frac{1}{2}, \frac{1}{3}, \ldots$

learning interest and help them to understand the mathematical knowledge and methods and form the good habits of mathematical thinking and application. For example, the textbook, published by the People’s Education Press, presents three figures (see the following Figs. 9.2, 9.3, and 9.4) before introducing negative numbers (Lin, 2007, p. 2).

The above three figures provide backgrounds for introducing positive integers, zero, and positive fraction origins. Based on these, students can generalize that the development and processing of numbers only occurred throughout history as a response to people’s needs. Negative number learning is also treated in a similar way. The whole process reflects number development and relationships among different numbers.

Furthermore, the New Century textbooks encourage active participation in mathematics activities, which enlivens the learning and provokes students’ interest in mathematics. Many investigation tasks have been designed for textbooks, for instance the topic of *Making an Eye Chart* in the Beijing Normal University textbooks (Ma, 2007, pp. 170–173), which uses the following activities:

- Measuring the height and width of "E" and investigating the relationships between Es of different sizes;
- Copying the Es corresponding to the respective vision levels of 0.1, 0.2, 0.3, 0.5, and 1.0, moving in horizontally and investigating their relationship with the knowledge of similarity;
- Making eye charts for 3 and 8 m vision testing distances, respectively, based on the Vision Table of 5 m testing distance;
- Investigating the conversion relationship among all the "Es" on eye charts that need different testing distances.

Students are familiar with eye charts in their daily lives, but they rarely notice the mathematics knowledge contained in the chart design. Given such a context, they can investigate the principle of designing the chart and find the ratio between the size of the E and the testing distance. By this mathematical activity, students calculate the height and width of the E based on their knowledge of similar triangles. Then, they can improve their ability to analyze practical problems by using mathematical knowledge and enhance the integration of theory with practice.

9.2.3 Improving the Content Presentation to Inspire Students' Interest in Mathematics

Regarding to this characteristics, two aspects are considered. One is on the presentation style which means in an appearance way. The other is on the content related to its nature. The two are explained in the following.

9.2.3.1 Improving the Content Presentation Style to Lead Students' Thinking and Investigating

The previous textbook versions usually gave mathematical definitions, concepts, and theorems directly, then explained them in detail, and finally introduced how to use them to solve problems. In the New Century textbooks, content is presented mostly as the sequence of **Problem Background—Problem Analysis—Abstracting Solution—Extending Application**. In this process, mathematical thinking can be shown through various activities, as illustrated below.

- Observing: Making conjectures through observing (i.e., introduction).
- Thinking: Drawing conclusions from thinking and enhancing the comprehension of conclusions through reflection (i.e., deduction and extension).
- Investigating: Finding conclusions through hands-on activities (often followed by the conclusions).

- Discussion: Finding conclusions through communication with other students, which is appropriate for deepening the recognition of content, the extension of the conclusion, and the relationship between different contents.
- Generalizing: Making generalizations about some mathematical features through observing, thinking, investigating, and discussion, which is appropriate for the generalizing of conclusions and the guidance of mathematical thinking.

For example, the knowledge of function monotonicity in Version A of the People's Education Press and the Beijing Normal University version is dealt with as follows. The reasons for investigating features of function are given first (Liu, 2007, pp. 27–29). Then, three function images are shown, and students are asked to find the variation laws of functions through observing (one of the laws is Monotonicity). The following requires the students to observe the images of $f(x) = x$ and $f(x) = x^2$ and articulate their monotonicity in natural language. One question "How to use analytical expression of $f(x) = x^2$ to describe the monotonicity of this function?" is posed to help students describe the function monotonicity in mathematical language.

The Beijing Normal University Publisher version (Yan & Wang, 2006, pp. 40–41) starts with observing expressions and images of $y = x + 1$ and $f(x) = x^2$ to find the laws of the function value y changing with the independent variable x and articulate it in natural language. Then, an activity named *Thinking and Communication* is set up to make students describe how general function changes based on the images of this function, and respond to the question how to describe the variation law of the function value in mathematical language.

It can be seen that the concept of monotonicity is not given directly and upfront in the new textbooks. Instead, they analyze some specific functional examples and pose inquiry questions to guide students to describe monotonicity, not only in natural language but also in mathematical language. This can help students to have a deep understanding of mathematical knowledge.

Another example is the conditions of congruent triangles; the textbook of the People's Education Press provides the following eight investigation problems (Lin, 2008, pp. 6–13):

- Investigation 1: If two triangles satisfy one or two of the conditions that three sides and three angles are correspondingly equal, must they be congruent?
- Investigation 2: If two triangles satisfy the condition that three sides are correspondingly equal, must they be congruent?
- Investigation 3: If two triangles satisfy the condition that two sides and their included angle are correspondingly equal, must they be congruent?
- Investigation 4: If two triangles satisfy the condition that two sides and one opposite angle are correspondingly equal, must they be congruent?
- Investigation 5: If two triangles satisfy the condition that two angles and their included sides are correspondingly equal, must they be congruent?
- Investigation 6: If two triangles satisfy the condition that two angles and one opposite side are correspondingly equal, must they be congruent?

- Investigation 7: If two triangles satisfy the condition that three angles are correspondingly equal, must they be congruent?
- Investigation 8: If two triangles satisfy the condition that the hypotenuse and one right-angle side are correspondingly equal, must they be congruent?

Through the above investigation activities, students are guided to explore the conditions of congruent triangles and master the methods, procedures, and presentation of the proof from conducting experiments to logical reasoning. As for the presentation, the new textbooks strengthen the investigation activities rather than listing conclusions directly, which reflects the focus on the students' learning process.

9.2.3.2 Enhancing Readability and Affinity

Compared with the previous ones, much attention has been paid to the readability of the New Century textbooks. Photos, figures, tables, words, and mathematical symbols are integrated to help students understand the mathematical knowledge and stimulate their interest in learning mathematics.

In terms of the external form, the textbook enlivens the presentation form by posing interesting questions, which guide students to ponder and comprehend the learning content. And there are not only black-and-white textbooks, but also colored and bicolored versions to be chosen. Along with improved readability, the inclusion of illustrations and pictures adds interest. For instance, in addition to the necessary illustrations of the mathematical content, all the new textbooks show a lot of pictures of actual items. Some of them, related to mathematical content, support the teaching, while some are purely decorative illustrations to make the layout lively and stimulate students' interest.

In terms of the content, the new textbooks present mathematical knowledge in ways that students like rather than in totally didactic ways. For example, a reading and thinking column, presented as a dialogue between numbers and the letter X, is presented in the chapter titled Addition and Subtraction of Integral Expressions in the People's Education Press's textbook (Lin, 2007, pp. 61–62), inspired by the form of a science sketch. Through the controversial dialogue, the meaning of using letters to represent numbers can be reflected. Students can realize the impact of progression from arithmetic to algebra in mathematics history. Textbook exercises are also presented in age-appropriate ways. The following exercise from the People's Education Press's textbook is an example (Lin, 2008, p. 81):

> On his way abroad, mathematician Loo-Keng Hua saw the passenger next to him reading an intellectual problem in a magazine. The problem was to find the cube root of 59819. Hua blurted out that it was 39. The people around him were all astonished and wanted to know how to get the answer so quickly.
>
> How do you think Hua calculated it so quickly and accurately? Try to solve the following questions:
>
> 1. Since $10^3 = 1000$ and $100^3 = 1000000$, can you know how many digits of $\sqrt[3]{59319}$?

2. If you know the single digit of 59819 is 9, can you know the single digit of $\sqrt[3]{59319}$?
3. If the last three digits 819 of 59819 were struck out of get 59, and $3^3 = 27$, $4^3 = 64$ can know the tens digit of $\sqrt[3]{59319}$?

The exercise is from the content of "cube root" and is known as an "extensional exploration" activity. It is also an estimation problem. A simple mathematics problem is integrated into a story, which makes the problem more interesting. This way of introducing a problem can attract more students' interest and help them to learn mathematics pleasantly instead of the old boring ways of doing exercises.

9.2.4 Providing Mathematics Context Knowledge to Embody Mathematical Cultural Value

It is one of the requirements that mathematical culture should be valued in the New Century textbooks. Most textbooks have provided a lot of reading materials about mathematicians' lives, mathematics development, and interaction between mathematics and the development of human society, which can help students to experience mathematicians' academic spirits and scientific attitudes, enable them to understand the history of the development of mathematics, and enable them to appreciate the scientific value, application value, and cultural value of mathematics. All of these can help students to broaden their horizons to understand the value of mathematics and to enhance students' cultural literacy.

In the section on *Number and Algebra*, junior secondary textbooks introduce the history of algebra and algebra language. The history of positive and negative numbers, the production and evolution of some important mathematical symbols, reading materials related to equations and their solutions, such as the *Nine Chapters in the Mathematical Art* and *Qin Jiushao's Method*, and the origin, development, and evolution of function concept, are all presented in these textbooks. In the section on *Space and Shape*, *Euclid's Geometry* is introduced to help students sense the value brought about by a deduced geometry system for mathematics development and human civilization. The introduction of several classical proofs of the *Pythagorean Theorem*, such as the Euclidean method and Zhao Shuang's method, and some important issues related to the *Pythagorean Theorem*, is provided to make students feel the flexible, graceful, and delicate features of mathematical proof and the rich cultural connotation of the *Pythagorean Theorem*. A brief introduction to the history of π is shown to help students to have an overview of methods, numerical value, formulae, and features involved in π and its value in modern life (e.g., computing the value of π accurately has been one of the best methods for evaluating the performance of a computer). Introducing cyclotomy in the context of Ancient Greece and China, with appropriate content, can give students a sense of approximation thinking and mathematical connotations in different cultures. For mathematical appreciation, the *Golden Section* and *Seven Bridges of Königsberg* can be introduced to students to encourage mathematical thinking and appreciate the aesthetic value of

mathematical propositions and methods. Included in the *Statistics and Probability* section are the origins of probability theory, coin toss experiments, Buffon needle problems, and geometric probability, which enable students to have a brief understanding of the random phenomenon and contribute to their further study.

Here are a few selected examples from junior secondary textbooks published by the People's Education Press. These examples are offered as elective content: denoting the tolerant error in machining by positive and negative numbers; Chinese as the first to use negative numbers; the dialogue between the number 1 and the letter X; the history of the equation; the origin of geometry; the measurement of length; representing a location by latitude and longitude; why to prove; modern and ancient representations and solutions to liner equations; why $\sqrt{2}$ is not a rational number; Yanghui (Pascal)'s triangle; proofs of the *Pythagorean Theorem*; the *Helen—Qin Jiushao formula*; the *Golden Section*; π; the origin of probability theory; probability and winning; wonderful fractal graphics; ancient trigonometric tables; the generation and application of view.

In addition to providing specialized reading materials on mathematical history and culture, mathematical culture is also incorporated into the introductions to chapters, marginalia, and project learning. For example, a poem is presented in the chapter on *Exponential and Logarithmic Functions* in the first senior textbook from Hunan Education Press. The essence of this chapter is embodied in the poem. Infusing mathematical knowledge with some literary flavor can help students to enjoy learning mathematics in a delightful atmosphere. One example of such a poem is (Zhang, 2005, p. 71):

> Moring mists hinder transportation, nuclear mushroom clouds cover the sky.
> Fossil years calculate clever, text searching faster than wind.
> Index and logarithm reflect each other, cube and square look symmetrical.
> Do not affect the interpretation of the world, three kinds of function make outstanding contributions.

In the objective world, there is a lot of monotonically increasing and decreasing phenomena in number. Exponential, logarithmic, and power functions are three basic function models that describe the increasing or decreasing process. The poem not only shows mathematical essence, but also connects mathematics and culture. This can help to improve students' cultural literacy.

9.2.5 *Stress the Integration of Information Technology and Mathematics Curriculum to Improve the Effectiveness of Mathematics Teaching and Learning*

IT is a powerful cognitive tool which should be utilized. This is another requirement of the New Century curriculum. Thus, New Century textbooks have used the

power of information technology to help students understand the nature of mathematics. In textbooks, learning how to use a scientific calculator is compulsory. Students can do complex calculations, check calculations, and attempt some exploration activities with the help of a calculator. In addition to the use of calculators, textbooks integrate information technology into content as appropriate, which makes calculators and computers new learning tools. By means of computer software, students can observe the process of formation and change of learning objects. The software, with conversion and measurement functions, can help students to summarize constant positions and quantity relationships in graphic motions. Textbooks integrated with information technology can help students to find the nature of mathematics. There are several ways to integrate information technology into the mathematics curriculum.

One way is to provide examples embedding information technology, such as setting up specialized activities that involve the application of information technology, exercises to be completed via a computer, and mathematical experiments to introduce information technology in detail or allow students to do mathematical experiments. Second, indicating information technology applications in appropriate places, such as, "This can also use a calculator or computer to …" "IT is recommended to use …" can prompt information technology use. The *Application of Information Technology*, in the junior middle school textbooks published by the People's Education Press, incorporates data calculations using spreadsheets, exploring the positional relationship between two straight lines, drawing to find a rule, drawing statistical charts with the help of a computer exploring the nature of axial symmetry, drawing function images through computers, exploring features of inverse functions, doing statistics with computers, exploring the features of rotation, exploring the features of quadratic functions, and exploring homothetic features.

With computers and calculators appearing in classrooms, students have opportunities to see the formation and change of learning objects. They are also given chances to observe, experiment, conjecture, and find mathematical facts. Computers and calculators have some advantages that other learning tools do not. Students can do some activities that were difficult to complete in the past. For example, in the senior high school textbook issued by the Jiangsu Education Press (Shan, 2008, pp. 85–87), the topic of "data fitting," which requires the use of information technology, is presented in the section on function application. In this topic, the method of drawing function images with EXCEL is introduced first. The values of independent variables are generated by the "arithmetic trend" function, and the corresponding values are obtained via the EXCEL function of relative referencing through dragging. Then, three problems are presented to be solved: one on population modeling, one on the modeling of car brakes, and one on the law of celestial movement. Using the given data tables, students are required first to draw a scatter plot in EXCEL, then select an appropriate fitting function, and finally forecast further populations, analyze a car's speed, and interpret Kepler's third law.

9.2.6 Summary

Overall, the new textbooks stress mathematics learning through application to students' lives, attention to their developing, heuristic, and inquiring abilities, and attracting their mathematics learning interest and appreciation of culture, with attention also given to the application of information technology.

9.3 A Specific Example: The Textbooks for Grades 7–9 by People's Education Press (Lin, 2012–2014)

To show more characteristics of Chinese mathematics textbooks, we take the latest version of the People's Education Press textbooks as an example. Before the year 2000, most of the textbooks were issued by this publisher. Even after 2000, more than 60% of the junior middle school textbooks and nearly 80% of the senior middle school ones were provided by this press. Due to its popularity, we selected this version as the example. These textbooks will be described from the perspectives of specific content, the structures of each lesson and each chapter and each volume, and the unique characteristics and features.

9.3.1 Specific Content

The textbooks follow the *Compulsory Education Mathematics Curriculum Standards* (*2011 Edition*). All the content required by the standards for Grade 6 to Grade 9 has been covered in these textbooks. The four areas *Number and Algebra*, *Graphics and Geometry*, *Statistics and Probability*, and *Comprehensive and Practical Activities* are presented in a mixed way. The following table shows the details of this content.

Grade 7 (Semester 1)

Chapter 1 Rational Numbers	**Chapter 2 Addition and Subtraction of Integral Expression**
1.1 Positive and Negative Numbers	2.1 Integral Expressions
1.2 Rational Numbers	2.2 Addition and Subtraction of Integral Expressions
1.3 Addition and Subtraction of Rational Numbers	
1.4 Multiplication and Division of Rational Numbers	
1.5 Power of Rational Numbers	

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Chapter 3 Linear Equations with One Unknown 3.1 From Arithmetic Formula to Equation 3.2 Solving a Linear Equation with One Unknown (1)—Transpose and Merge 3.3 Solving a Linear Equation with One Unknown (II)—Remove Parentheses and Denominator 3.4 Practical Problems and Linear Equations with One Unknown	**Preliminary of Geometric Figure** 4.1 Geometrical Figures 4.2 Line, Ray, and Segment 4.3 Angle 4.4 Topic Learning: Learn How to Make a Rectangular Packing Box

Grade 7 (Semester 2)

Chapter 5 Intersecting and Parallel Lines 5.1 Intersecting Lines 5.2 Parallel Lines and Determination 5.3 Properties of Parallel Lines 5.4 Translation	**Chapter 6 Real Numbers** 13.1 Square Roots 13.2 Cube Roots 13.3 Real Numbers
Chapter 7 Rectangular Coordinate System 7.1 Rectangular Coordinate System 7.2 Simple Application of Coordinate Method	**Chapter 8 System of Linear Equations in Two Unknowns** 8.1 System of Linear Equations in Two Unknowns 8.2 Elimination—Solve Systems of Linear Equations in Two Unknowns 8.3 Practical Problems and System of Linear Equations in Two Unknowns 8.4 Solutions to Systems of Linear Equations in Three Unknowns
Chapter 9 Inequality and System of Inequalities 9.1 Inequality 9.2 Linear Inequality with One Unknown 9.3 System of Linear Inequalities with One Unknown	**Chapter 10 Data Collection, Processing, and Presentation** 10.1 Statistical Survey 10.2 Histogram 10.3 Topic Learning: Water Saving from the Perspective of Data

Grade 8 (Semester 1)

Chapter 11 Triangle 11.1 Segments Related to Triangle 11.2 Angels Related to Triangle 11.3 Polygons and the Sums of their Interior Angles	**Chapter 12 Congruent Triangles** 12.1 Congruent Triangles 12.2 Determinations of Congruent Triangles 12.3 Features of Angular Bisectors
Chapter 13 Axisymmetric 13.1 Asymmetry 13.2 Draw Axisymmetric Figures 13.3 Isosceles Triangle 13.4 Topic Learning: The Shortest Path Problem	**Chapter 14 Multiplication of Integral Expressions and Factorization** 14.1 Multiplication of Integral Expressions 14.2 Multiplication Formula 14.3 Factorization

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Chapter 15 Fractional Expression 15.1 Fractional Expression 15.2 The Operation of Fractional Expressions 15.3 Fractional Equation	

Grade 8 (Semester 2)

Chapter 16 Quadratic Radical 16.1 Quadratic Radical 16.2 Multiplication and Division of Quadratic Radical 16.3 Addition and Subtraction of Quadratic Radical	**Chapter 17 Pythagoras Theorem** 17.1 Pythagoras Theorem 17.2 Inverse Theorem of Pythagoras Theorem
Chapter 18 Parallelogram 18.1 Parallelogram 18.2 Special Parallelograms	**Chapter 19 Linear Function** 19.1 Function 19.2 Linear Function 19.3 Topic Learning: Plan Selection
Chapter 20 Data Analysis 20.1 Central Tendency of Data 20.2 Volatility of Data 20.3 Topic Learning: Data of Physical Health Test	

Grade 9 (Semester 1)

Chapter 21 Quadratic Equations with One Unknown 21.1 Quadratic Equations with One Unknown 21.2 Solving Quadratic Equations with One Unknown 21.3 Practical Problem and Quadratic Equations with One Unknown	**Chapter 22 Quadratic Functions** 22.1 The Images and Features of Quadratic Functions 2.2 Studying Quadratic Equations with One Unknown from the Perspective of Function 22.3 Practical Problems and Quadratic Functions
Chapter 23 Rotation 23.1 Rotation of Graphics 23.2 Centrosymmetric 23.3 Topic Learning: Graphics Design	**Chapter 24 Circle** 24.1 Properties of Circle 24.2 Positional Relationship between Dot and Circle, and Line and Circle 24.3 Regular Polygon and Circle 24.4 Arc Length and Sectorial Area

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Chapter 25 Preliminary Probability 25.1 Random Events and Probability 25.2 Finding Probability by Enumeration Method 25.3 Finding Probability by Frequency Estimation	

Grade 9 (Semester 2)

Chapter 26 Inverse Proportional Function 26.1 Inverse Proportional Function 26.2 Practical Problems and Inverse Proportional Function	**Chapter 27 Similarity** 27.1 Similarity of Graphics 27.2 Similar Triangles 27.3 Homothetic
Chapter 28 Trigonometric Function of Acute Angle 28.1 Trigonometric Function of Acute Angle 28.2 Solving Right Angled Triangle and Its Application	**Chapter 29 Projection and View** 29.1 Projection 29.2 Three-View 29.3 Topic Learning: Learn How to Make a Stereo Model

9.3.2 Structures of Each Lesson, Each Chapter, and Each Volume

In each lesson, the structure is determined by the content, which is basically composed of the following parts (see Fig. 9.5). The background for the content of this lesson is provided first. Then, the problem is analyzed through mathematical activities, such as thinking, exploration, and generalization activities, to explain the knowledge underpinning the lesson. After this, a summary of previous content, illustrated examples, and exercises are prepared. At the end of this lesson, the summary is given, along with possible extensions for the lesson and other review or consolidation tasks. Selective content may also be added.

To provide a more detailed description of these textbooks, the structures of each chapter and each lesson are shown in Fig. 9.6.

This figure shows the sequence arrangement of each chapter. First, the name, a picture indicating the topic, and an introduction are presented at the beginning of each chapter. Then, sections on specific content are organized in a sequential way. Each section (e.g., one or two lessons) usually includes exercises for students to do. Sometimes, topics relevant to this chapter are recommended. At the end of each chapter, summary, review, or consolidated tasks are provided.

Each volume of these textbooks also has its own structures. The following introduce the characteristics of each volume of the textbooks.

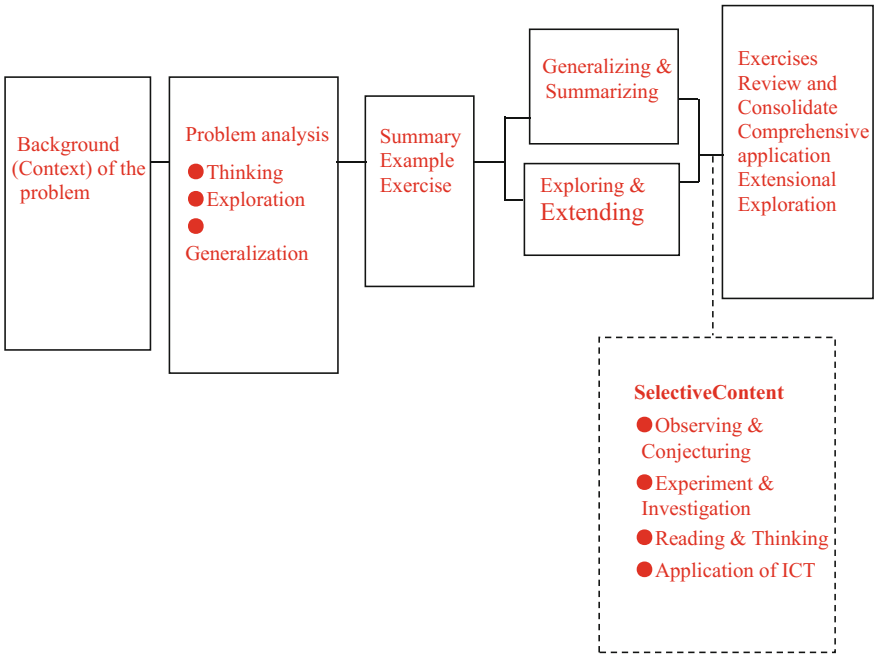

Fig. 9.5 Structure of each lesson

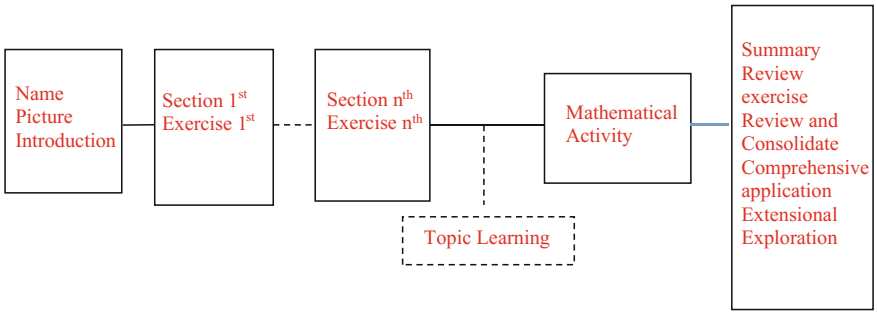

Fig. 9.6 Structure process of each chapter

- Introduction of each volume

The introduction is presented at the beginning of each volume of junior middle school textbooks. In addition to introducing the main content, the teaching columns and learning methods are also presented. The introductions are presented in a friendly way, which shows respect for students and views them as being at the center of the learning.

- The first page of each chapter

The name, theme map, and introduction to each chapter are shown on the first page. The style is usually lively, with illustrations and pictures, to introduce what will be learnt in the chapter. In addition to the name of the chapter, one or several physical pictures related to the chapter's topics are used as the theme map. The introduction is not only presented in a narrative way but also in a problem-based way. First, the real situation or mathematical context is provided, then the main content, learning methods, and mathematical thinking methods required in the chapter.

- Activities in each section

Each section presents the content in the sequence of **Problem Background—Problem Analysis—Abstracting Solution—Extending** Application. In the extension section, some kinds of thinking, investigating, or generalizing activities are included to motivate students to show their thinking processes and lead to generalizations. As well as these activities, Tips, and Clouds are provided to help students understand the content. Relevant content background is included in the Tips, and problems which can help students understand the topic are shown in the Clouds.

- Exercises

There are three kinds of exercises in the new textbooks: those used in classroom teaching, those appearing at the end of each section, and those for review. The first of these are used by students and the teacher during classroom teaching, to help students consolidate what they have learnt. The exercises at the end of each section are for use in classroom teaching or as homework. These exercises can not only help students to consolidate what they have learnt in the section, but also contribute to knowledge application. The exercises arranged at the end of a chapter are mainly for review purposes. These exercises are not simply grouped by difficulty levels but according to their teaching functions of consolidation, comprehensive application, and extended exploration. The first of these reviews addresses the knowledge learnt in a section or a chapter. Comprehensive application exercises call for the application of this knowledge to solve practical and mathematical problems. Extended exploration activities are optional and can be used to extend the knowledge addressed in the section or chapter.

- Selective content

Selective contents include expansion of the body of knowledge, the background of the body content and its applications, the history of mathematics, the introduction of mathematical thinking methods, and the application of information technology.

Based on the characteristics of the selective content, the materials are categorized under the headings of *Reading and Thinking*, *Observation and Conjecture*, *Experiment and Exploration*, and *Application of Information Technology*.

- Mathematical activity

There are some comprehensive, practical, and open-ended mathematical activities in each chapter, which require the comprehensive use of mathematical knowledge. These are optional for either teaching or reviewing the chapter.

- Summary of the chapter

Each chapter has a summary of the knowledge which can be used to help students to review what they have learned. There are two parts to a summary. One is the *Structure of Knowledge* in the chapter, which presents the main knowledge points, development context, and linkages. The other is *Review and Reflection*, which summarizes the main content and the thinking methods. Problems which can motivate students to think in depth are proposed as key or difficult knowledge points, which can help students have a better understanding of the core contents of the chapter and the relevant mathematical thinking methods.

9.3.3 Characteristics and Features

The mathematics knowledge in the textbooks has the characteristics of being universal, basic, and developmental. The textbooks follow the development of technology, consider the needs of a developing society, and focus on the students' long-term development. To ensure students develop fully in basic mathematics knowledge, basic skills, basic thinking, and basic activity experience, ample learning resources are provided in the textbooks. All of these purposes are requirements of the *Mathematics Curriculum Standard of Compulsory Education* (*2011 Edition*) (Ministry of Education, 2012). In addition to the five common characteristics listed in the above section, there are other characteristics:

Constructing the system in line with mathematical logic and students' psychological status, and presenting core concepts and thinking methods spirally

In the previous mathematics curriculum standards, mathematical content was classified into algebra and geometry. However, in the 2011 version, there are four areas: "*Number and Algebra*, *Graphics and Geometry*, *Statistics and Probability*, and *Comprehensive and Practical Activities* (Ministry of Education, 2012)". In the new textbooks, algebra and geometry textbooks are not presented separately. Instead, the four kinds of contents are arranged in a unified way.

The order of different content, coordination, and interconnections between different content, subjects related to mathematics, and students' cognitive understanding are all considered in constructing the textbook system. This system is in line with mathematical logic and students' psychological status in order to facilitate students' understanding of mathematics and mathematical ability.

The relationships between different content are emphasized in the textbooks, i.e., between *Number and Algebra*, *Graphs and Geometry*, and *Statistics and Probability*, showing the entirety of mathematics. The process of solving problems by drawing on knowledge from different content areas helps to give students a comprehensive view of mathematics. For example, in order to reflect the intrinsic link between *Numbers and Figures*, a *Rectangular Coordinate System* is arranged in advance (Chapter 6, 2nd Semester, Grade 7). Since the coordinate tool can fully reflect the number-shape combination thinking, teaching the knowledge of coordinate systems is beneficial for analyzing translation transformation, knowing the essential characteristics of symmetry transformation, handling some graphics issues, and deepening students' understanding of function and system of linear equations in two variables, etc.

Textbooks not only provide the content required by the curriculum standards, but also focus on students' age and cognitive characteristics. Core mathematical concepts and mathematical thinking methods are laid out step-by-step. Reasonable and effective learning approaches are designed and suggestions are provided for teachers. For example, *Function* is the core learning content for *Number and Algebra*, but it is also one of the most difficult concepts in junior secondary education. Considering the difficulty of this topic, the content is scattered and appears in a spiraling way. In accordance with the number sequence, *Equation* and *Function* are presented alternately. Thus, the sequence is: *Linear Equation (System)*, *Linear Function*, *Quadratic Equation*, and then *Quadratic Function*. There are three chapters on Function. *Linear Function* (including *Concept of Function*, *Proportional Function*, and *Linear Function*) is presented in the second volume of Grade 8. *Quadratic Function* is presented in the first volume of Grade 9 and *Inverse Proportional Function* in the second volume of Grade 9. This sequencing can help to overcome the difficulties caused by knowledge presented in a linear way, and to deepen the students' understanding of equations and functions in stages.

In another instance, developing the ability of logical thinking is the key issue in promoting students' mathematical literacy. In view of this, textbook activities are designed carefully to develop logical thinking processes. These activities are based on different levels: giving brief reasons, giving reasons, simple reasoning, and symbolic reasoning. For example, proof appears in Chapter 5 (*Intersecting Lines and Parallel Lines*) in the second semester of Grade 7, but only at the level of completing critical steps and giving reasons. The requirements are increased gradually in subsequent activities. Logical reasoning ability can be developed naturally across topic areas rather than be limited to the areas of *Graphics and Geometry*.

Emphasizing thinking and enhancing learning methods to help students have a deep understanding of core content

The value of thinking in core mathematics is emphasized in the textbooks, and students are guided to use relevant learning methods. Most mathematics knowledge is presented by posing problems. Through problem thinking and problem-solving, students can experience the process of generalizing mathematical concepts and abstracting mathematical principles. Hence, students can experience the mathematical processes used by "real" mathematicians.

For example, algebraic operations, such as addition and multiplication, are used in analyzing correlations between different quantities. In the operation, the universality of operation law can help us analyze the correlations of known and unknown quantities effectively. In solving actual problems, algebraic tools are used to express the quantity (or expression) of actual contexts, or reflect their relationships (equation and function) and changing processes (function). Through this, actual problems are solved using algebraic tools. In the *Number and Algebra* section, extension of number systems, expansion of expressions, enrichment of equations, and the introduction of variables and functions are presented successively. This is a progression from simple to complex, from concrete to abstract, from constant to continuous variables, which reflects the basic method of studying algebra-generalization methods. In the process, the function of rational number is given full attention in relation to real numbers, operations of integral expressions, fractional expressions, and quadratic radicals. Mathematical thinking and generalization are enhanced. Rules of calculation and laws of operation are compared. In the summary, the research method of extension from number to expression is elaborated.

In the area of *Graphics and Geometry*, the basic thinking, content, and methods for studying geometric problems are shown in the textbooks. Taking the topic of the parallelogram as an example, the research perspective from to specializing to generalizing is shown. That is, specialized parallelograms—rectangles, diamonds, and squares—can be obtained from the specialized sides and angles of parallelogram. Then, their properties can be investigated from the location and quantity relationships among their constituent elements (sides, angles, and diagonals). The methods of determination can be studied from the reciprocal relationship between the properties and determination of these figures.

In the case of *Statistics*, the core is data analysis and inference. In the textbooks, the focus is very much on finding rules through investigating data about actual problems. Students gather information through data collection, organization, description, and analysis. They learn to choose the appropriate method based on the problem backgrounds and experience the randomness of data. This can develop student's ideas about data analysis and statistical thinking.

The role of mathematics in other subjects also receives attention in the textbooks: arithmetic to algebra, experimental geometry to argument geometry, constant to variable, and certain mathematics to random mathematics. The role of basic knowledge and methods is emphasized in accomplishing the transition.

This establishes foundations for further learning and mathematics application. For example, the strategy of delimitating unknowns is used in solving the system of linear equations with two variables. The strategy of delimiting an unknown and decreasing the index of an unknown can help students catch the nature of thinking in solving equation problems.

Encourage basic mathematical thinking

As has been established earlier in this chapter, the development of basic mathematical thinking is the most important aim of the school mathematics curriculum. Mathematical thinking can also be used to assess students' mathematical literacy. It is the foundation of mathematics production and development, the basis of investigations, and the essence of the mathematics curriculum. The actual or theoretical background of each concept or theory is emphasized in its introduction. This can help students know the natural development of knowledge. Through problem-solving, students experience the process of generalizing concepts and abstracting mathematical principles. This can help them to understand the research methods of mathematics.

The role of the chapter introduction and summary

A good introduction plays an important role in stimulating students' learning interest, enhancing the teaching of mathematical thinking, and developing the ability to identify and pose questions. The role of the textbook chapter introduction is to pave the way for the exploration from practical problems to mathematical problems and to introduce the content in a natural way.

The summary of each chapter focuses on generalizations arising from the mathematical thinking methods. This can help students to review what they have learnt through the chapter and help the teachers to improve their teaching of mathematical thinking. There are two kinds of summary in each chapter. One is an overview of the knowledge, linkages, and interrelationships covered in the chapter and the other is for review of the types of mathematical thinking they have learnt.

9.4 Brief Summary

In summary, New Century textbooks not only keep good traditional aspects but also show new visions. Traditionally, Chinese textbooks emphasized presenting content in a systematic and rigorous way and stressed students' calculation skills, logical thinking, and imaginary ability. These are all evident in the New Century textbooks but, in addition, the New Century textbooks use content related to students' lives, highlight the knowledge, development, and application, and focus on students' ability to self-explore and collaborate with others in learning.

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