Z-Binder Reinforcement

▶ Mechanics of 3D Fibre Reinforced Polymer Composites

Zerna, Wolfgang

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Wolfgang Zerna (*October 11th, 1916 in Berlin, German Empire; †November 14th, 2005 in Celle, Germany) was a civil engineer with focus on mechanics and construction.

Wolfgang Zerna

Early Years and Education

Wolfgang Zerna, after secondary school in Berlin, studied civil engineering at the Technische Hochschule Berlin-Charlottenburg. Even if he was called to military service, he managed to complete his studies with a diploma in 1940. After 4 years of military service and 2 years as prisoner of war in the USA, where together with other German scientists he organized a camp university, Zerna returned home in 1947. The same year, he became an assistant to Alf Pflüger at the chair of statics at the Technische Hochschule Hannover and finished his PhD thesis (On the membrane theory of general shells of revolution – Zur Membrantheorie der allgemeinen Rotationsschalen, Zerna 1949). In1948, he habilitated (On the basic equations of the theory of elasticity – Allgemeine Grundgleichungen der Elastizitätstheorie, Zerna 1950). In 1949 he followed an invitation of Albert E. Green as visiting scientist to the University of Durham. This was the starting point of a long-term cooperation resulting in the monograph on *Theoretical Elasticity* (Green and Zerna 1954), which for a long time became a standard monograph for tensor calculus and theory of elasticity.

Professional Career

Back from the UK, Zerna started working in the German building industry in 1950, first at

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Polensky and Zöllner in Cologne and then at Ph. Holzmann in Frankfurt, where he was responsible for the entire prestressed concrete construction. During these postwar reconstruction years, he developed the Holzmann prestressing method, was involved in many reinforced concrete and prestressed concrete constructions, and influenced bridge and engineering structures of both companies.

At the same time, since 1950, he was appointed as a professor (extraordinaire) in Hannover. Since 1957 he was a full professor for solid construction. Here he laid the basis for the introduction of electronic computing, as computer science was called at that time into the civil engineering sector and combined it with model static measuring techniques. His chair was the first engineering institute in Germany to receive an “electronic calculator” (Zuse Z 22R).

From 1963 he was a member of the founding committee of the Ruhr University Bochum. He was appointed as full professor in this new university in 1967 (up to his retirement in 1983). Zerna was thus the first civil engineer of the then Faculty of Mechanical Engineering and Structural Engineering in Bochum. He organized this engineering faculty like in the UK combining theoretical, numerical, and experimental chairs.

Pioneering work for large natural draft cooling towers and nuclear power plants began in 1964. Through practical engineering activities as a consulting engineer, test engineer, and scientific expert, he gave the impetus for the founding of two still very successful engineering bureaus in Bochum, which remain connected to him in gratitude even after his death.

Scientific Achievements and Honors

During the first term of collaboration with A.E. Green, he developed a tensor-based theory of thin shells. Together with his collaborator Hermann Flessner (who later became professor at the University of Hamburg), he introduced and developed the use of computers in civil engineering (static problems and measurements) in Germany. From 1983 at his chair in Bochum was established the first Unix computer at the Ruhr University.

Since 1963 Wolfgang Zerna was a member of the Braunschweigische Wissenschaftliche Gesellschaft. He was a founding member of the International Association for Shell and Spatial Structures (IASS) and since 1988 honorary member of this association. Wolfgang Zerna was awarded doctor honoris causa at the Universities of Stuttgart and Essen.

He was a member of the Verein Deutscher Ingenieure (Association of German Engineers) and the chairman of the civil engineering department. In 2014 the Verein Deutscher Ingenieure established in honor of the achievements of Wolfgang Zerna the Wolfgang Zerna honorary medal for engineers with outstanding contributions in the field of civil engineering.

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Zhilin, Pavel Andreevich

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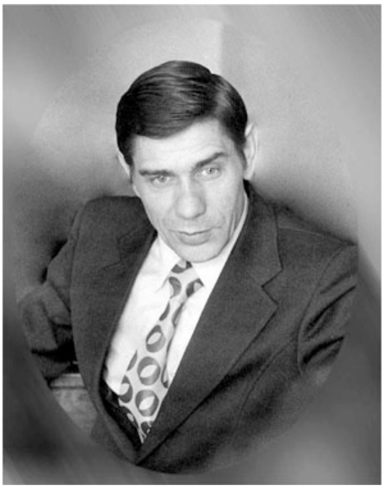
Pavel Andreevich Zhilin (*February 8, 1942, in Velikiy Ustyug, Vologda region, Soviet Union; †December 4, 2005, in St. Petersburg, Russian Federation) was a scientist in the field of rational



Pavel Andreevich Zhilin
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mechanics, shell theory, and tensor calculus. His scientific interests covered various branches of mechanics and theoretical physics. In his research he strived to pave a way based on rational mechanics to areas which are traditionally not associated with mechanics, such as physics of microstructures and electrodynamics.

Family and Education

Pavel Andreevich Zhilin was born in Velikiy Ustyug (Vologda region, Soviet Union), where his family found themselves during the World War II. He spent his childhood in Volkhov and Podporozhie, where his father, Andrey Pavlovich Zhilin, has been employed as a chief engineer at the coordinated hydroelectric system of Svir river. Zoya Alexeevna Zhilina, the mother of P.A. Zhilin, was bringing up the sons and kept the house.

In 1956 the family moved to Leningrad. In 1959 P.A. Zhilin left the secondary school and entered the Leningrad Polytechnic Institute. Yet at school Pavel Zhilin met his future wife, Nina Alexandrovna, who was his faithful friend and helpmate all his life long. While studying at the institute, P.A. Zhilin became keen on table tennis and was a captain of the student and later institute team for many years. P.A. Zhilin got a qualification of the candidate master of sports (the highest qualification in this sport discipline at that time).

Professional Carrier

In the period of 1959–1965, P.A. Zhilin studied at Leningrad Polytechnic Institute in the Department of *Mechanics and Control Processes* at the Faculty of Physics and Mechanics. Later on his daughter, Olga Zhilina, graduated from the same department (in addition, she got her Candidate of Physical and Mathematical Sciences degree for the thesis *On the influence of the elasticity of bodies, which are in contact, on the development of dynamical effects under wear conditions*). After graduation, P.A. Zhilin got the qualification of engineer-physicist (specialization *Dynamics and Strength of Machines*), and from 1965 to 1967, he worked as an engineer at the *Water Turbine Strength Department* in the Central Boiler Turbine Institute in Leningrad.

In 1967 he accepted a position of assistant professor at the Department of *Mechanics and Control Processes*; later he worked there as a senior researcher, as an associate professor, and as a full professor. The founder of the Department was the corresponding member of the Academy of Sciences of the USSR Prof. Anatoliy Isaakovich Lurie, worldwide recognized scientist in Mechanics and Control Processes. P.A. Zhilin became the closest disciple of A.I. Lurie and spent many hours working together with him. Scientific ideology of P.A. Zhilin was developing to a great extent under the influence of A.I. Lurie. He got his Candidate degree (CSc) in Physical and Mathematical Sciences in 1968 (the topic of his thesis was *The theory of ribbed shells*) and his Doctor degree (DSc) in Physical and Mathematical Sciences in 1984 (the topic of his DSc-thesis was *The theory of simple shells and its applications*). In 1989 he was appointed as a professor of rational mechanics at the Department of *Mechanics and Control Processes*.

In 1974–1975 P.A. Zhilin worked as a visiting researcher at the University of Technology of Denmark. While working at the Department of

Mechanics and Control Processes, P.A. Zhilin delivered lectures on analytical mechanics, theory of oscillations, theory of shells, tensor analysis, and continuum mechanics. In 1988 he was invited in the Yarmouk University (Jordan) to present a course on continuum mechanics at the Faculty of Physics. From 2001 he was several times a visiting scientist at the Martin-Luther-Universität Halle-Wittenberg (Chair of Engineering Mechanics). At the same time, P.A. Zhilin actively carried out scientific work in the field of the theory of plates and shells, nonlinear theory of rods, theory of elasticity, and continuum mechanics. He gained three certificates of invention in the area of vibroinsulation and hydroacoustics; he was awarded with the Inventor of the USSR insignia.

Since 1989 P.A. Zhilin was the head of the Department of *Theoretical Mechanics* at the Leningrad Polytechnic Institute (now Peter the Great State Polytechnic University). In the period of his direction, five of his colleagues defended their DSc-theses; for the four of them, P.A. Zhilin was a scientific advisor. While working in the Department of *Theoretical Mechanics*, P.A. Zhilin read original courses on tensor algebra, rational mechanics, and the rod theory. During this period P.A. Zhilin worked hard in the field of investigating and developing foundations of mechanics. His investigations on spinor motions in mechanics and physics, phase transitions and phenomena of inelasticity, binary media (suspensions), electrodynamics from the positions of the rational mechanics, and logical foundations of mechanics relate to this period.

Since 1994 P.A. Zhilin was the head of *Dynamics of Mechanical Systems* laboratory at the Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences. From 1999 he was a member of the Scientific Committee of the Annual International Summer School-Conference *Advanced Problems in Mechanics*, held by the Institute for Problems in Mechanical Engineering.

Pavel Andreevich Zhilin was a member of the Russian National Committee for Theoretical and Applied Mechanics, a member of the Society of Applied Mathematics and Mechanics (GAMM), and a member of Guidance Board Presidium for Applied Mechanics (Ministry of Higher Education of Russian Federation).

Academic Pupils

P.A. Zhilin prepared many PhD students. Sixteen PhD theses (Candidate of Science) and six Professorial theses (Doctor of Science) were defended under his supervision and advisory. Among his pupils are Anton M. Krivtsov (corresponding member of the Russian Academy of Sciences, professor and head of the Department of *Theoretical Mechanics*, Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russian Federation), Vitaly R. Skvortsov (professor of State Marine Technical University of St. Petersburg), Elena Grekova (senior researcher of the Institute of the Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg), Sergei Gavrilov (leading researcher of the Institute of the Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg), Alexander D. Sergeev (leading researcher of the Institute of the Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg), Dmitri P. Goloskokov (professor of Admiral Makarov State University of Maritime and Inland Shipping), and both authors of this entry.

Scientific Achievements

P.A. Zhilin obtained original and fundamental results in continuum mechanics, rational mechanics, shell theory, and tensor calculus. On these topics he published several monographs (Zhilin 2001, 2003, 2006, 2007, 2012, 2015). A collection of his papers was published in English (Indeitsev et al. 2006). Special problems of the direct approach in the shell theory were published in Zhilin (1976), on the theory of invariants; in Altenbach et al. (2006), on binary media; in Zhilin (2003), on rigid body mechanics; in Zhilin (1996), on rigid body oscillators; in Zhilin

(2000); and in Ivanova et al. (2007), on general problems of rational mechanics. Together with A.I. Lurie, he edited the Russian edition of Truesdell's *A First Course in Rational Continuum Mechanics* (see ▶ "Truesdell, Clifford Ambrose III"). A detailed description of his scientific achievements was published in Altenbach et al. (2009). In honor a special issue of the ZAMM (Zeitschrift für angewandte Mathematik und Mechanik) was prepared by his friends and colleagues in 2007 (Altenbach et al. 2007).

Cross-References

- ▶ Beams, Plates, and Shells
- ▶ Elastic Shells, Linear Shear-Deformable Theory
- ▶ Elastic Shells, Material Symmetry Group
- ▶ Elastic Shells, Resultant Nonlinear Theory
- ▶ Lurie, Anatoli Isakowitsch
- ▶ Thin Elastic Shells, Lagrangian Geometrically Nonlinear Theory
- ▶ Thin Elastic Shells, Linear Theory
- ▶ Truesdell's and Zhilin's Approaches: Derivation of Constitutive Equations
- ▶ Zhilin's Method and Its Modifications

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Zhilins Approach

▶ Zhilin's Method and Its Modifications

Zhilin's Method and Its Modifications

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Synonyms

Chemical potential; Entropy introduction; Zhilin's approach

Definition

The main idea of the method consists in transformation of the energy balance equation to a special form called the reduced equation of energy balance. This form is obtained by separation of the stress tensors into elastic and dissipative components and introduction of quantities characterizing the physical processes associated with neglected degrees of freedom. As a result the energy balance equation is divided into two or more parts: one of them is the reduced equation of energy balance and the rest carrying the meaning of heat conduction equation and equation of structural transformations.

Introduction

To describe inelastic processes associated with phase transitions and structural transformations, plastic flow, dynamics of bulk solids, dynamics of granular media, fragmentation and defragmentation of materials, particle diffusion, chemical reactions, etc., it is important to introduce additional state variables such as temperature, entropy, chemical potential, and particle distribution density. In fact, the introduction of these quantities in continuum mechanics should be considered as an attempt to take into account the microstructural processes at the macro level by means of some integral characteristics.

Usually the concepts of temperature, entropy, internal energy, and chemical potential are supposed to be well-known. However, in fact, there are no satisfactory definitions for them in continuum mechanics. Partly the problem is that it is impossible to prove that the temperature as it is introduced in thermodynamics or in statistical physics coincides with the temperature definition as it is used in continuum mechanics. A situation with the definition of unmeasurable variables such as the entropy, internal energy, or chemical potential is even more complicated. Such quantities are characteristics of a mathematical model, and they are necessary for obtaining some relations connecting measurable quantities. Consequently, the preference of this or that definition is determined by specific features of problems under consideration.

There are different ways of entropy introduction (see, e.g., Boltzmann 1874; Clausius 1960, reprint; Maugin 1999; Nowacki 1975), and it is difficult to say unambiguously which of them is more preferable. In fact, entropy is introduced as an attempt to take into account a dependence of the internal energy on the velocities of the ignored degrees of freedom. Another thermodynamical quantity – chemical potential – is introduced to describe a change of density of particles. Usually in thermodynamics the chemical potential is defined as the derivative of the internal energy with respect to the number of particles, (see Gibbs 1875; Prigogine 1955). However there exist other definitions of the chemical potential. For example, Baierlein (2001) proposed to introduce the chemical potential by describing its properties instead of explaining the chemical potential by relating it to an energy change. These ideas have a further development in Job and Herrmann (2006).

Zhilin suggested a new concept of the entropy and chemical potential introduction as a conjugate variables to the temperature and number of particles correspondingly, for example Zhilin (2003, 2006, 2012). As a result, the definitions of the chemical potential and entropy are given by means of pure mechanical arguments, which are based on using a special form of the energy balance equation.

Balance Equations for a Continuum with Microstructure

Consider an arbitrary volume V (control volume) at a fixed position $\mathbf{r}$ in space. The local form of the mass conservation law can be written as:

$$\frac{\delta\rho}{\delta t} + \rho\,\nabla\cdot\mathbf{v} = 0. \tag{1}$$

Here $\delta/\delta t$ is the material derivative (Ivanova et al. 2016), $\rho(\mathbf{r},t)$ is a mass density, $\mathbf{v}(\mathbf{r},t)$ is the velocity field, and ∇ denotes the nabla operator.

In addition to the mass density, a particle density $n(\mathbf{r}, t)$ is introduced as an independent variable. Consideration of this quantity independently of mass density allows to take into account microstructural changes in media. Such differentiation is important, for example, when the material tends to fragmentation, as in this case the mass is preserved but the number of particles changes. In other words considering the particle density as an independent characteristic corresponds to introducing an additional degree of freedom which accounts for structural changes. As a result an additional balance equation for the new variable has to be formulated. This equation can be written by analogy to Eq. (1) with a source term. Thus, the particle balance equation takes the form (see Altenbach et al. 2003; Zhilin 2012; Vilchevskaya et al. 2014)

$$\frac{\delta n}{\delta t} + n\nabla\cdot\mathbf{v} = \chi. \tag{2}$$

Here χ is the rate of particle production per unit volume.

From the combination of Eqs. (1) and (2), it follows that

$$\frac{\delta z}{\delta t} = -\frac{\chi(\mathbf{r}, t)}{n(\mathbf{r}, t)}, \quad z \equiv \ln\left(\frac{\rho(\mathbf{r}, t)n_0(\mathbf{r})}{\rho_0(\mathbf{r})n(\mathbf{r}, t)}\right), \tag{3}$$

where $n_0(\mathbf{r})$ and $\rho_0(\mathbf{r})$ are reference distributions of densities of particles and mass.

The local form of Euler's first dynamical law is

$$\rho\,\frac{\delta\mathbf{v}}{\delta t} = \nabla\cdot\mathbf{T} + \rho\mathbf{f}, \tag{4}$$

where $\mathbf{f}$ is an external specific body force and $\mathbf{T}$ is the symmetric Cauchy stress tensor. The symmetry of the stress tensor is related to the balance of angular momentum in the general nonpolar case (i.e., in the case where there are no assigned traction couples or body couples and no couple stresses).

The first law of thermodynamics (the energy balance equation) states that there is a function of state U (called internal energy) satisfying the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}(K + U) = N^{\mathrm{e}} + Q, \tag{5}$$

where K is the kinetic energy of the substance in the control volume, N^{e} is the power of external forces, and Q is the energy supply from external sources per unit time.

The kinetic energy is assumed to be a quadratic form of velocities. As usual it is an additive function of the mass and thus can be written in terms of the mass density of the kinetic energy κ

$$K = \int\limits_V \rho\kappa\,\mathrm{d}V. \tag{6}$$

The definition of internal energy is less formal than that of the kinetic energy. As a matter of fact, the internal energy is the energy of motion with respect to degrees of freedom that are ignored in the model under consideration. Indeed, the momentum balance equation and the angular momentum balance equation are obtained by choosing the kinetic energy as a quadratic form of velocity. Other degrees of freedom that are ignored in the kinetic energy are taken into account by means of the internal energy. As a rule the sense of the internal energy depends on the mathematical model used for description of the system. For example, in classical equilibrium thermodynamics, the internal energy of the ideal gas is an additive function of the number of particles and proportional to the temperature (Müller and Ruggeri 1998; Prigogine 1955). In statistical thermodynamics, the internal energy is determined by the elastic interactions of the particles, for example, Laurendeau (2005). The difference between the approaches cannot give the cause for doubts about their correctness. The fact is that the internal energy is a quantity that cannot be measured, and so there are no physical experiments that let us know what the internal energy of the system under consideration is.

Usually in many continuum mechanics applications, the internal energy is an additive function of the mass (see Truesdell and Toupin 1960; Truesdell 1965; Müller and Müller 2009). To take into account the structure changes in the media caused by a change of the number of particles in the medium, it is supposed in Zhilin (2003) that the internal energy is an additive function of the number of particles,

$$U = \int_V nu\,\mathrm{d}V, \tag{7}$$

where u is the internal energy per one particle.

The power of external forces can be represented in the following form:

$$N^e = \int_V \rho \cdot \mathbf{v}\,\mathrm{d}V + \int_\Sigma \mathbf{T}_n \cdot \mathbf{v}\,\mathrm{d}\Sigma, \tag{8}$$

where $\mathbf{T}_n$ is a stress vector acting upon an elementary surface of the volume boundary Σ, $\mathbf{T}_n = \mathbf{n} \cdot \mathbf{T}$, and $\mathbf{n}$ is normal to this surface.

The energy supply per unit time is determined due to the entering (to the leaving) of new particles into (out of) the control volume and by the heat supply per unit time t which is the sum of the heat supply per unit time directly in the volume and through the volume boundary,

$$Q = \int_V nq\,\mathrm{d}V - \int_\Sigma \mathbf{n} \cdot \mathbf{h}\,\mathrm{d}\Sigma - \int_\Sigma \mathbf{n} \cdot \mathbf{v}(\rho\kappa + nu)\mathrm{d}\Sigma, \tag{9}$$

where q is the energy supply per unit time into the particles of the medium and $\mathbf{h}$ is the heat flow.

Taking into account Gauss' theorem and balance laws (1), (2), and (4), the local form of energy balance equation is obtained:

$$n\frac{\delta u}{\delta t} = nu\frac{\delta z}{\delta t} + \mathbf{T} \cdot\cdot (\nabla \mathbf{v})^s - \nabla \cdot \mathbf{h} + nq. \tag{10}$$

Here $(\nabla \mathbf{v})^s = 1/2(\nabla \mathbf{v} + \mathbf{v}\nabla)$ is a symmetric part of the spatial gradient of the velocity, and the double dot product is defined by $(\mathbf{ab}) \cdot\cdot (\mathbf{cd}) = (\mathbf{b} \cdot \mathbf{c})(\mathbf{a} \cdot \mathbf{d})$.

However this form of the energy balance equation does not allow to see on which arguments the internal energy depends. The basic idea of Zhilin's method is to transform the energy balance equation (10) into a special form called reduced equation of energy balance (see Zhilin 2012; Altenbach et al. 2003). During this transformation, the stresses are represented as a sum of elastic and dissipative components; the temperature, entropy, and chemical potential are introduced, and the energy balance equation is divided into two or more parts: one of them is the reduced energy balance equation which shows clearly on which variables the internal energy depends, and the rest have a sense of heat conduction equation, diffusion equation, equation of structural transformations, etc.

Transformation of the Energy Balance Equation

The right-hand side of Eq. (10) contains the power of forces and moments. A part of this power leads to the change of the internal energy. The remaining part of the power is partly conserved within the body as heat and is partly emanated into the external medium. In order to separate these parts, the following decomposition is introduced:

$$\begin{aligned} \mathbf{T} &= -(p_e + p_f)\mathbf{I} + \boldsymbol{\tau}_e + \boldsymbol{\tau}_f, \\ \operatorname{tr} \boldsymbol{\tau}_e &= \operatorname{tr} \boldsymbol{\tau}_f = 0, \end{aligned} \tag{11}$$

where $\mathbf{I}$ is the identity tensor. The quantities with the index "e" are independent of rates. These quantities always affect the internal energy. The quantities with the index "f" account for an internal friction. These quantities may have an influence on the internal energy but only by means of additional parameters like entropy or chemical potential. Because of (11) it follows that:

$$\begin{aligned} n\frac{\delta u}{\delta t} = nu\frac{\delta z}{\delta t} &- p_e \nabla \cdot \mathbf{v} + \boldsymbol{\tau}_e \cdot\cdot (\nabla \mathbf{v})^s \\ &+ nq - \nabla \cdot \mathbf{h} - p_f \nabla \cdot \mathbf{v} + \boldsymbol{\tau}_f \cdot\cdot (\nabla \mathbf{v})^s. \end{aligned} \tag{12}$$

The part of the power of forces that does not depend on rates can be represented as:

$$\boldsymbol{\tau}_e \cdot\cdot (\nabla \mathbf{v})^s = -\frac{1}{2} I_3^{2/3}(\mathbf{g})\big(\mathbf{g}^{-1} \cdot \boldsymbol{\tau}_e \cdot \mathbf{g}^{-T}\big) \cdot\cdot \frac{\delta \mathbf{G}}{\delta t}. \tag{13}$$

Here the deformation measures are determined by

$$\mathbf{g} = \mathbf{I} - \nabla\mathbf{u}, \qquad I_3(\mathbf{g}) = \det\mathbf{g},$$
$$\mathbf{G} = I_3^{-2/3}(\mathbf{g})\,\mathbf{g}^T\cdot\mathbf{g}, \tag{14}$$

where **u** is a displacement field and **G** describes the shape deformation. From the mass balance, it follows that

$$\nabla\cdot\mathbf{v} = \frac{\rho}{\rho_0}\frac{\delta\varsigma}{\delta t}, \qquad \varsigma = \frac{\rho_0}{\rho} \tag{15}$$

and as a result the energy balance equation takes the form:

$$\begin{aligned} n\frac{\delta u}{\delta t} &= nu\frac{\delta z}{\delta t} - \frac{p_e}{\varsigma}\frac{\delta\varsigma}{\delta t} \\ &\quad -\frac{1}{2}I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\frac{\delta\mathbf{G}}{\delta t} \\ &\quad +\underline{nq - \nabla\cdot\mathbf{h} - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f^T\cdot\cdot(\nabla\mathbf{v})^s}. \end{aligned} \tag{16}$$

A transformation of the underlined terms is not as formal as the above ones. In order to state the full form of the reduced equation of the energy balance, the concepts of temperature, entropy, and chemical potential have to be introduced. Zhilin's idea is to introduce them in such a way that the material derivative of the internal energy in terms of independent variables (natural variables) has, as coefficients, the other thermodynamic variables. Zhilin's method tolerates various modifications of the definitions of entropy and chemical potential as well as other unmeasurable state variables.

Different Ways of Entropy and Chemical Potential Introduction

In classical thermodynamics, the internal energy of a system is expressed in terms of pairs of conjugate variables. Equation (16) partly has the desired structure. Further terms may be expressed as time derivatives of set functions by adding further assumptions. The underlined terms comprise of the nonmechanical energy supply and dissipative stress power and may lead to change of the temperature and/or particle density which conjugate variables are entropy and chemical potential, respectively. When the particle density changes due to diffusion, then in many thermodynamic approaches, a chemical potential is considered as a force that pushes changes in the particle number, and its introduction is required. Alternatively, the change of the particle density can be due to internal structure transformations like cracks or voids appearing or particle consolidation in granular and powderlike materials. In these cases the necessity of a chemical potential introduction is not obvious, since other thermodynamical variables can have the exact meaning of the chemical potential. Below different variants, with and without an explicit chemical potential introduction, are considered.

Variant 1

Let the temperature $\theta(\mathbf{r}, t)$ and entropy $\eta(\mathbf{r}, t)$ be introduced by the following equation:

$$n\theta\frac{\delta\eta}{\delta t} = nq - \nabla\cdot\mathbf{h} - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s. \tag{17}$$

Similar equation can be found in Truesdell and Toupin (1960), and Altenbach et al. (2003). The definition (17) brings about a few remarks. First, the temperature θ is considered to be a characteristic of the medium that is measured by a thermometer, and the entropy η related to one particle is introduced as a quantity conjugate with the temperature. Second, it is assumed that the entropy, as well as the internal energy, is an additive function of the number of particles. Note that this definition of entropy is different from the definition used, for example, in classical thermodynamics, where an inequality is introduced and the equality holds for a reversible process only (e.g., Truesdell 1984; Wilmanski 2008; Müller 2007; Gurtin et al. 2010). Equation (17) is the heat conduction equation, i.e., an equation describing a non-equilibrium process.

Owing to Eq. (17) Eq. (16) reads:

$$n\,\frac{\delta u}{\delta t}=nu\,\frac{\delta z}{\delta t}-p_e\frac{\rho}{\rho_0}\frac{\delta\varsigma}{\delta t}-\frac{1}{2}\,I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\,\frac{\delta\mathbf{G}}{\delta t}+n\theta\,\frac{\delta\eta}{\delta t}\tag{18}$$

It is seen that the internal energy is a function of the following arguments:

$$u=u(z,\varsigma,\eta,\mathbf{G})\tag{19}$$

Note that in Zhilin's method, the set of the natural variables is determined by the reduced equation of the energy balance in contrast to the usual approach where the assignment of these parameters is made a priori. In fact, within Zhilin's approach, the definition (17) is also a definition of the internal energy.

From (19) and (18), it follows that

$$u=\frac{\partial u}{\partial z}.\tag{20}$$

In thermodynamics the derivative of the internal energy with respect to the number of particles is usually called chemical potential (see, e.g., Gibbs 1875; Prigogine 1955). Thus, introduction of the temperature and entropy by (17) means that the internal energy can play the role of the chemical potential.

At the same time, in that case, the variable z can be excluded from the arguments of the internal energy. Indeed from (20) it follows that

$$u=u_*(\varsigma,\eta,\mathbf{G})\,\frac{\rho_0}{n_0}\exp z\quad\Rightarrow\quad u=\frac{\rho}{n}u_*,\tag{21}$$

where u_* is a mass density of the internal energy. It should be noted that the last equation is valid only if there are no massless particles in the system.

Insertion of (20) into (18) gives

$$\rho\,\frac{\delta u_*}{\delta t}=-\frac{p_e}{\varsigma}\frac{\delta\varsigma}{\delta t}+n\theta\,\frac{\delta\eta}{\delta t}-\frac{1}{2}\,I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\,\frac{\delta\mathbf{G}}{\delta t}.\tag{22}$$

Thus, in the case of the temperature and entropy definition by means of (17), ς, θ, and $\mathbf{G}$ are the natural variables of the internal energy, and other thermodynamic properties of the system can be found by taking partial derivatives of the internal energy with respect to its natural variables. From the reduced energy balance equation, one can derive the following equations of state:

$$p_e=-\frac{\partial(\rho_0u_*)}{\partial\varsigma},\qquad\theta=\frac{1}{\varsigma n}\frac{\partial(\rho_0u_*)}{\partial\eta},$$

$$\boldsymbol{\tau}_e=\frac{2}{3\varsigma}\left(\mathbf{G}\cdot\cdot\,\frac{\partial(\rho_0u_*)}{\partial\mathbf{G}}\right)\mathbf{I}-\frac{2I_3^{-2/3}(\mathbf{g})}{\varsigma}\,\mathbf{g}\cdot\frac{\partial(\rho_0\partial u_*)}{\partial\mathbf{G}}\cdot\mathbf{g}^T.\tag{23}$$

Note that the function ρ_0u_* is independent of z. It means that only the constitutive equation for the temperature depends on the particle density. The heat conduction equation (17) depends on n only by means of $n\theta\,\frac{\delta\eta}{\delta t}$, and the chemical potential does not appear in any equation.

Considering the function ρ_0u_* implies that the internal energy is an additive function of mass. In this case it is natural to assume that the entropy is also additive by mass. Thus instead of (17), the temperature and entropy η_* can be introduced by means of

$$\rho\theta\,\frac{\delta\eta_*}{\delta t}=\rho q_*-\nabla\cdot\mathbf{h}-p_f\nabla\cdot\mathbf{v}+\boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s.\tag{24}$$

Then the reduced equation of the energy balance has the form

$$\rho\,\frac{\delta u_*}{\delta t} = -\frac{p_e}{\varsigma}\frac{\delta\varsigma}{\delta t} + \rho\theta\,\frac{\delta\eta_*}{\delta t} - \frac{1}{2}\,I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\,\frac{\delta\mathbf{G}}{\delta t}. \tag{25}$$

All relations (23) are still valid except the one for the temperature, which now has the form

$$\theta = \frac{\partial u_*}{\partial\eta_*}. \tag{26}$$

It is seen that the heat conduction equation (24) and the state equations (23) do not depend on the particle density. Thus the influence of the mechanical and thermal processes on the change of the particle distribution can be taken into account only by means of the source term in the particle balance equation (2). So the stress-strain state and the temperature conditions can affect the changes of particle distribution density since the source term in the particle balance equation can depend on all these factors. Hence, this method of temperature and entropy introduction can be used to describe the structure transformations and phase transitions which occur without the release or absorption of heat and are not accompanied by significant changes in the mechanical and thermodynamical characteristics but only leads to changes in other physical characteristics such as, for example, electrical or magnetic properties.

Variant 2

An alternative form of the reduced energy balance equation makes use of the particle balance equation. Insertion of (3) into (16) gives

$$n\,\frac{\delta u}{\delta t} = -p_e\frac{\rho}{\rho_0}\frac{\delta\varsigma}{\delta t} - \frac{1}{2}\,I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\,\frac{\delta\mathbf{G}}{\delta t} - \chi u - \nabla\cdot\mathbf{h} + nq - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s \tag{27}$$

and as a result the source term in the particle balance equation χ appears in the energy balance equation.

Now let us define the temperature and entropy by the equation

$$n\theta\,\frac{\delta\eta}{\delta t} = -\chi u - \nabla\cdot\mathbf{h} + nq - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s. \tag{28}$$

This equation differs from (17) only due to the term χu standing for the rate of energy supply caused by the structural transformation of the medium. Then the reduced energy balance equation takes the form

$$n\,\frac{\delta u}{\delta t} = -\frac{p_e}{\varsigma}\frac{\delta\varsigma}{\delta t} - \frac{1}{2}\,I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\,\frac{\delta\mathbf{G}}{\delta t} + n\theta\,\frac{\delta\eta}{\delta t}. \tag{29}$$

Thus the internal energy is a function of the following independent arguments:

$$u = u(\varsigma, \eta, \mathbf{G}) \tag{30}$$

and the state equations are

$$p_e = -n\varsigma\frac{\partial u}{\partial\varsigma}, \qquad \theta = \frac{\partial u}{\partial\eta},$$

$$\boldsymbol{\tau}_e = \frac{2n}{3}\left(\mathbf{G}\cdot\cdot\,\frac{\partial(\rho_0 u_*)}{\partial\mathbf{G}}\right) \mathbf{I} - 2nI_3^{-2/3}(\mathbf{g})\,\mathbf{g}\cdot\frac{\partial(\rho_0\partial u_*)}{\partial\mathbf{G}}\cdot\mathbf{g}^T. \tag{31}$$

Note that now the internal energy does not play the role of a chemical potential as it was in Variant 1. At the same time, the heat conduction equation (28) has a term connected with particle distribution changes, and this term depends on the internal energy. Thus, this method of temperature and entropy introduction can be used to describe structure transformations and phase transitions accompanied by the release or absorption of heat. Note that the first and second variants of derivation of the constitutive equations and the heat conduction equation are correct both in the case when the mass density and the particle distribution density are independent quantities and in the case when they are linearly related (i.e., when the source term in the particle balance equation is equal to zero).

Variant 3

Now consider an explicit way of the chemical potential introduction. Then, instead of Eq. (17), a more general equation containing an additional term that accounts for structural transformation can be used.

$$n\theta\,\frac{\delta\eta}{\delta t}+\psi\,\frac{\delta n}{\delta t}=nq-\nabla\cdot\mathbf{h}-p_f\nabla\cdot\mathbf{v}+\boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s. \tag{32}$$

Analogously to the temperature and entropy, n and ψ appear in Eq. (32) as the conjugate variables. Equation (32) is the combined equation of structural transitions (e.g., fragmentation) and heat conduction.

Substitution of Eq. (32) into Eq. (16) leads after some transformation to the following form of the reduced energy balance equation:

$$\begin{aligned}\frac{\delta(n\,u)}{\delta t}&=\frac{p_e+n\,u}{\rho}\frac{\delta\rho}{\delta t}\\&\quad-\frac{1}{2}I_3^{2/3}(\mathbf{g})\left(\mathbf{g}^{-1}\cdot\boldsymbol{\tau}_e\cdot\mathbf{g}^{-T}\right)\cdot\cdot\frac{\delta\mathbf{G}}{\delta t}\\&\quad+n\theta\,\frac{\delta\eta}{\delta t}+\psi\,\frac{\delta n}{\delta t}.\end{aligned} \tag{33}$$

It should be particularly emphasized that such a form of the reduced energy balance equation is valid only if the mass density and the density of particle distribution are independent variables.

From Eq. (33) there follow the state equations

$$\begin{aligned}&p_e=\rho^2\frac{\partial}{\partial\rho}\left(\frac{n\,u}{\rho}\right),\quad \theta=\frac{1}{n}\frac{\partial(n\,u)}{\partial\eta},\\&\psi=\frac{\partial(n\,u)}{\partial n},\ \boldsymbol{\tau}_e=\frac{2}{3}\left(\mathbf{G}\cdot\cdot\frac{\partial(\rho_0u_*)}{\partial\mathbf{G}}\right)\\&\qquad\mathbf{I}-2I_3^{-2/3}(\mathbf{g})\,\mathbf{g}\cdot\frac{\partial(\rho_0\partial u_*)}{\partial\mathbf{G}}\cdot\mathbf{g}^T.\end{aligned} \tag{34}$$

From Eq. $(34)_3$ it is seen that ψ is the chemical potential. Similar expressions to $(34)_3$ are given in the classical textbooks (see Kondepudi and Prigogine 1998; Müller 2007; Müller and Müller 2009; Prigogine 1955).

Note that Eq. (32) characterizes only overall influence of the entropy and chemical potential on the internal energy. To clarify their roles in the considered processes, it is necessary to split Eq. (32) into two equations: the heat conduction equation and the equation of structural transitions.

$$\begin{aligned}&n\theta\,\frac{\delta\eta}{\delta t}+\tilde{Q}=nq_1-\nabla\cdot\mathbf{h}_1-p_1\nabla\cdot\mathbf{v}\\&\qquad+\boldsymbol{\tau}_f^T\cdot\cdot(\nabla\mathbf{v})^s,\\&\psi\,\frac{\delta n}{\delta t}-\tilde{Q}=nq_2-\nabla\cdot\mathbf{h}_2-p_2\nabla\cdot\mathbf{v}\end{aligned} \tag{35}$$

where the following decompositions are used:

$$\mathbf{h}=\mathbf{h}_1+\mathbf{h}_2,\quad q=q_1+q_2,\quad p_f=p_1+p_2 \tag{36}$$

The equivalence of Eqs. (32) and (35) is determined by the presence of the undefined quantity $\tilde{Q}$ characterizing the rate of energy exchange in the processes of the heat conductivity and the structural transitions.

The definition (35) given above brings about a necessity to formulate constitutive equations for all new quantities: $\mathbf{h}_i$, q_i, p_i $(i=1,2)$, and $\tilde{Q}$. The following circumstances have to be taken into account. First, suppose that the expression for the internal energy u and the source term χ are given. Then n and ψ can be determined from the particle balance equation and equation of state $(34)_3$. It means that the term $\psi\,\frac{\delta n}{\delta t}$ in the equation of structural transformations is known. Therefore the constitutive equations for $\mathbf{h}_2$, q_2, p_2, and $\tilde{Q}$ cannot be independent. Second, arbitrarily given constitutive equations for $\mathbf{h}_2$, q_2, p_2, and $\tilde{Q}$ together with the equation of structural transitions and corresponding equation of state, determine the quantities n and ψ. Then the source term χ can be found from the particle balance equation. Finally a third variant exists. The constitutive equations for $\mathbf{h}_2$, q_2, p_2, $\tilde{Q}$, and χ can be arbitrarily chosen, but then there is no freedom in internal energy choosing.

Note that instead of (32), the equation

$$n\theta \frac{\delta \eta}{\delta t} + n\bar{\psi}\frac{\delta z}{\delta t} = nq - \nabla\cdot\mathbf{h} - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s \qquad (37)$$

or

$$n\theta \frac{\delta \eta}{\delta t} + n\frac{\delta \tilde{\psi}}{\delta t} = nq - \nabla\cdot\mathbf{h} - p_f\nabla\cdot\mathbf{v} + \boldsymbol{\tau}_f\cdot\cdot(\nabla\mathbf{v})^s. \qquad (38)$$

can be considered. The quantities ψ, $\bar{\psi}$, and $\tilde{\psi}$ have, in general, a different physical sense. However, $\bar{\psi}$ or $\tilde{\psi}$, being the variable conjugate to the number of particles (or to z closely allied to n), can be treated as a chemical potential. The introduction of the chemical potential by means of (32) can be found in Altenbach et al. (2003) and Zhilin (2012) and by means of (37) in Zhilin (2012) and Vilchevskaya et al. (2014). Note that in contrast to the definition (28), the introduction of the chemical potential by means of (37) is valid also in a case where mass density and the density of particle distribution are dependent variables.

Closing Remarks

Sometimes the mass density and particle density can be considered as independent variables without the chemical potential introduction. In some cases the role of the chemical potential can be played by the internal energy or the source term in the particle balance equation. Of course there is no reason to say that there is no necessity for the chemical potential introduction in general. The preference of this or that approach is determined by specific features of the problems under consideration. For example, if experimental data allow to formulate the constitutive equation for the quantity $\tilde{Q}$ characterizing the rate of energy exchange in the processes of the heat conductivity and the structural transitions, then the third variant of unmeasurable parameters introduction looks more preferable. In the opposite case, an approach based on smaller amounts of constitutive equations should be chosen. The first and the second approaches require only the source term χ specification and do not impose any constraints on the internal energy definition. Thus they are easier in this sense, but of course a number of problems stays beyond the consideration.

Also it is important to emphasize the fact that the equations of structural transitions and heat conduction (24), (28), and (32) define not only the entropy and chemical potential but also the internal energy. Thus all these quantities should be introduced simultaneously.

Finally, note that the different forms of the reduced energy balance equation (18), (22), (29), or (33) used in Zhilin's method allow to obtain the equations of state for the temperature, chemical potential, and the elastic component of the stress tensor in the both cases of an elastic and inelastic medium. In order to obtain a closed system of equations, additional constitutive equations relating the remaining thermodynamical variables have to be formulated. Some examples of constitutive relations for the inelastic part of the stress tensor can be found in Ivanova and Vilchevskaya (2013) or https://meteor.springer.com/chapter/contribute.jsf?id=108542. Also a relation between the heat flux and the temperature gradient in a form of linear Fourier law (see Fourier 1822) or Maxwell-Cattaneo law (e.g., Cattaneo 1958; Vernotte 1958) has to be considered.

Cross-References

- ▶ Truesdell's and Zhilin's Approaches: Derivation of Constitutive Equations
- ▶ Zhilin, Pavel Andreevich

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Zienkiewicz, Olgierd Cecil

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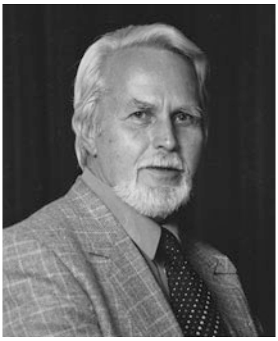
Olgierd (Olek) Cecil Zienkiewicz (*May 18, 1921, in Caterham, Surrey, England, UK; †January 2, 2009, in Swansea, Wales, UK) was a mathematician and civil engineer.



Olgierd Cecil Zienkiewicz

Education

Olgierd C. Zienkiewicz had a Polish father and an English mother. His early life was greatly influenced by the turbulence of European history over the first half of the twentieth century. In 1917 Zienkiewicz's father, Casimir, held the post of consul in Birmingham for the Russian Kerensky government. Toward the end of World War I, the Bolsheviks finally overthrew the revolutionaries who formed the government under Alexander Kerensky. Therefore, in 1922, the Zienkiewicz family returned to Poland, first to Warsaw and then to Łódź, before moving in 1926 to Katowice, where his father held the post of district judge up to the outbreak of World War II.

The young Zienkiewicz completed his high school studies in June 1939, planned to compete for admission to the Warsaw Polytechnic in September, and moved to Warsaw in preparation for the examination. The outbreak of World War II on 1 September 1939 stopped his plans. Warsaw was soon under siege. Many exciting days were to follow, including wandering through the countryside for 10 days, a return to Warsaw, and, eventually, a reuniting of the family in Katowice. The family finally managed to obtain a visa to Italy and then subsequently travelled onto France, where Casimir Zienkiewicz worked for the Polish government in exile. With the fall of France in June 1940, the family was able to escape to England.

With a special scholarship for Poles, he opted for a course in civil engineering at Imperial College. Placed first in his studies at the end of the first year, he got two scholarships for the remaining time at Imperial College. He graduated in 1943, one of two recipients of a first-class honors degree. On graduation, Zienkiewicz started his research work with Alfred John Sutton Pippard and Richard Vynne (later Sir Richard) Southwell on a dam analysis project. Southwell and Pippard were two of the principal figures in the development of structural mechanics in the 1930s and 1940s. Southwell's relaxation method bridged the gap between the classical methods developed over the century before 1930 and the large-scale computational methods of today, which emerged in the late 1950s. He was awarded the PhD degree in 1945 at the Imperial College with his thesis title: "Classical Theories of Gravity Dam Design in the Light of Modern Analytical Methods."

Professional Career

After his PhD in 1945, Zienkiewicz moved directly into engineering practice. He approached the consultancy firm of W. Halcrow and was offered a post leading a survey party on a dam project in Scotland. In 1949, he accepted a lectureship in the Department of Engineering at Edinburgh. Toward the end of 1956, he accepted a position as associate professor of civil engineering at Northwestern University, Evanston, Illinois. In 1961, the chair of civil engineering became vacant at Swansea, and Zienkiewicz was successful in the ensuing competition. He relished the attraction of a professorship in the UK system at that time, in which it was possible to influence the work and professional development of many others. Moreover, it was a time of unprecedented expansion of higher education in the UK. He remained at Swansea until his retirement at the age of 67 years in 1988 and subsequently became professor emeritus of the University of Wales, as well as held the UNESCO Chair of Numerical Methods in Engineering at the University of Technology of Catalunya, Barcelona, for 15 years. Although formally retired, he remained active in finite element research.

Scientific Achievements and Honors

Over his lifetime, Zienkiewicz published close to 600 papers. In 1967, he published his famous monograph *The Finite Element Method in Structural Mechanics* (Zienkeiwiecz and Cheung 1967).

Zienkiewicz received 28 honorary degrees. He was elected to a number of the Royal Society, Royal Academy of Engineering, US National

Academy of Engineering (foreign member), Polish Academy of Science, Italian National Academy of Sciences, and Chinese Academy of Sciences. He has been the recipient of many honors, awards, and medals including the Commander of the Order of the British Empire, Royal Medal (Royal Society), Carl Friedrich Gauß Medal, Nathan Newmark Medal (American Society of Civil Engineers), Newton Gauß Medal (International Association for Computational Mechanics), Gold Medal (Institution for Mathematics and its Applications), Gold Medal (Institution of Structural Engineers), Timoshenko Medal (American Society of Mechanical Engineers), and Prince Philip Medal (Royal Academy of Engineering). The ISI Web of Knowledge, Thomson Scientific Company, has listed Zienkiewicz as an ISI Highly Cited Author in Engineering. He was instrumental in setting up the association of computational mechanics in engineering (ACME) for the United Kingdom in 1992 and was the honorary president for the association for the rest of his life.

Further Reading

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