9. Percussion Musical Instruments

Andrew C. Morrison, Thomas D. Rossing

Percussion instruments are an important part of every musical culture. Although they are probably our oldest musical instruments (with the exception of the human voice), there has been less research on the acoustics of percussion instruments, as compared to wind or string instruments. Quite a number of scientists, however, continue to study these instruments.

Over the years we have written several review articles on the acoustics of percussion instruments [9.1, 2] as well as a book [9.3]. They are also the subject of chapters in most books on musical acoustics and on musical instruments [9.4–8].

9.1 Drums

Drums generally have membranes of animal skin or synthetic material stretched over some type of air enclosure. Nowadays synthetic materials, such as Mylar (polyethylene terephthalate), are more common, although some percussionists still prefer animal skin (leather). Some type of tensioning device is nearly always included. The speed of waves on the membrane (and thus the frequency of the various modes) depends upon the tension, the thickness, and the density of the membrane. Some drums (e.g., timpani, tabla, boobams) sound a definite pitch; others convey almost no sense of pitch at all. Some drums have a single membrane (drumhead), while others include two membranes coupled together by vibrations of the drum shell and the enclosed air. The first 12 modes of vibration of a circular membrane are shown in Fig. 9.1. Above each sketch are given the values of m (the number of nodal diameters) and n (the number of nodal circles), and below it the frequency of vibration for that mode divided by the frequency of the lowest (01) mode. Mathematically, the mode frequencies of an ideal membrane are proportional to those of the mn Bessel function.

R. Bader (Ed.), *Springer Handbook of Systematic Musicology*, https://doi.org/10.1007/978-3-662-55004-5_9

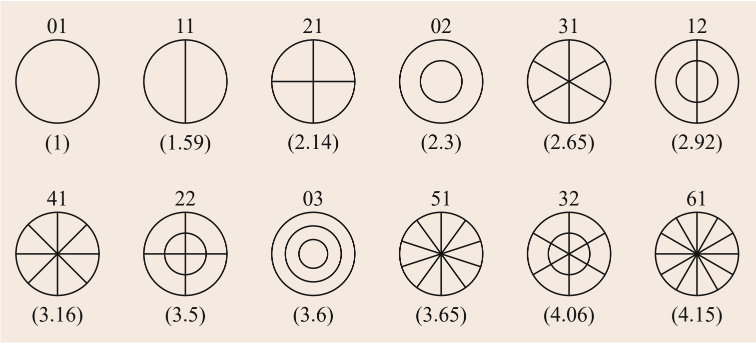

Fig. 9.1 Modes of vibration of a circular membrane

9.1.1 Timpani

The timpani or kettledrums are the most important drums in the orchestra, with one member of the percussion section usually devoting attention exclusively to them. Most modern timpani have a pedal-operated tensioning mechanism in addition to six or eight tensioning screws around the rim of the kettle. Although the modes of vibration of an ideal membrane are not harmonic, a carefully tuned kettledrum will sound a strong principal note plus two or more nearly harmonic overtones. *Rayleigh* [9.9] recognized the principal note as coming from the (11) mode and identified overtones about a perfect fifth ($f : f_1 = 3 : 2$), a major seventh (15 : 8), and an octave (2 : 1) above the principal tone. The inharmonic modes of an ideal membrane are shifted into a nearly harmonic series mainly by the effect of air loading [9.10]. Mode frequencies of a kettledrum, with and without the kettle, are given in Table 9.1.

Normal striking technique produces prominent partials with frequencies in the ratios 0.85 : 1 : 1.5 : 1.99 : 2.44 : 2.89. If we ignore the heavily damped fundamental, the others are nearly in the ratios 1 : 1.5 : 2 : 2.5, a harmonic series built on an octave below the principal note. Measurements on timpani of other sizes give similar results [9.11].

9.1.2 Snare Drums

The snare drum is a two-headed instrument about 33–38 cm in diameter and 13–20 cm deep. The shell is made from wood, metal, or Mylar. Strands of wire or gut are stretched across the lower (snare) head. When the upper (batter) head is struck, the snare head vibrates against the snares. The coupling between the snares and the snare head depends upon the mass and the tension of the snares. At a sufficiently large amplitude of the snare head, properly adjusted snares will leave the head at some point during the vibration cycle and then return to strike it, thus giving the snare drum its characteristic sound. The greater the tension on the snares, the larger the amplitude needed for this to take place [9.12]. Vibrational modes of a snare drum shell, with and without the drumheads, are shown in [9.12].

Table 9.1 Mode frequencies and ratios of kettledrum membranes with and without the kettle

Mode	Kettledrum		Drumhead alone		Ideal membrane
	f (Hz)	f/f_{11}	f (Hz)	f/f_{11}	f/f_{11}
01	127	0.85	82	0.53	0.63
11	150	1.00	155	1.00	1.00
21	227	1.51	229	1.48	1.34
02	252	1.68	241	1.55	1.44
31	298	1.99	297	1.92	1.66
12	314	2.09	323	2.08	1.83
41	366	2.44	366	2.36	1.98
22	401	2.67	402	2.59	2.20
03	418	2.79	407	2.63	2.26
51	434	2.89	431	2.78	2.29
32	448	2.99	479	3.09	2.55
61	462	3.08	484	3.12	2.61
13	478	3.19	497	3.21	2.66
42			515	3.32	2.89

9.1.3 Bass Drums

The bass drum is capable of radiating up to 20 W of peak acoustical power, probably the most of any instrument in the orchestra. A concert bass drum usually has a diameter of 80–100 cm, although smaller drums (50–75 cm) are popular in marching bands. Most bass drums have two heads, set at different tensions, although single-headed gong drums are used when a more defined pitch is desired. Mylar heads with a thickness of 0.25 mm are widely used, although calfskin heads are preferred by some drummers for large concert bass drums. Generally the batter or beating head is tuned to a greater tension than the carry or resonating head.

9.1.4 Tom-Toms

Tom-toms range from 20 to 45 cm in diameter, and they may have either one or two heads. Although often characterized as untuned drums, tom-toms do convey an identifiable pitch, especially the single-headed type. When a tom-tom is struck a hard blow, the deflection of the drumhead may be great enough to cause a significant change in the tension, which momentarily raises the frequencies of all modes of vibration and thus the apparent pitch. The fundamental frequency in a 33 cm tom-tom, for example, was found to rise about eight percent (slightly more than a semitone) during the first 0.2 s after the strike [9.13], resulting in a perceptible pitch glide. The pitch glide can be enhanced by loading the outer portion of the drumhead with a Mylar ring.

9.1.5 Indian Drums

Foremost among the drums of India are the tabla (north India) and mrdanga (south India). The overtones of both these drums are tuned harmonically by loading the drumhead with a paste of starch, gum, iron oxide, charcoal, or other materials. The tabla has a rather thick head made from three layers of animal skin (calf, sheep, goat, or buffalo skins are apparently used in different regions). The innermost and outermost layers are annular, and the layers are braided together at their outer edge and fastened to a leather hoop. Tension is applied to the head by means of a long leather thong that weaves back and forth between the top and bottom of the drum. The tabla is usually played together with a larger drum, called the banya or left-handed tabla. The head of the larger drum is also loaded, but slightly off center. The mrdanga is a two-headed drum that functions, in many respects, as a tabla and banya combined into one. The smaller head, like that of the tabla, is loaded with a patch of dried paste, while the larger head is normally loaded with a paste of wheat and water shortly before playing. A tabla and a mrdanga are shown in Fig. 9.2.

The acoustical properties of these drums have been studied by a succession of Indian scientists, including Nobel laureate C.V. Raman. *Raman* and his colleagues recognized that the first four overtones of the tabla are harmonics of the fundamental, and they identified these five harmonics as coming from nine normal modes [9.14]. For example, Fig. 9.3 shows how combinations of the (0,2) and (2,1) modes produce the third harmonic partial.

9.1.6 Japanese Drums

Drums have been used for centuries in Japanese temples. In Buddhist temples, it has been said that the sound of the drum is the voice of Buddha. In Shinto temples it is said that drums have a spirit (kumi) and that with a drum one can talk to the spirits of animals, water, and fire. Drums were often used to motivate warriors into battle and to entertain in town festivals and weddings [9.15]. The Japanese taiko (drum) has broken out of its traditional setting, and today's taiko bands have given new life to this old tradition. Japan's famous taiko band, the Kodo drummers, have performed in many countries of the world. Taiko bands exist in many Western countries. The o-daiko is a large drum consisting of two cowhide membranes stretched tightly across the ends of a wooden cylinder 50–100 cm in diameter and about 1 m in length. The drum, which hangs in a wooden frame, is struck with large felt-padded beaters. It is often used in religious functions at shrines, where its deep sound adds solemnity to the occasion. *Obata* and *Tesima* [9.15] found modes of vibration in the o-daiko to be somewhat similar to those in the bass drum. The tsudzumi

Fig. 9.2a,b The tabla **(a)** and mrdanga **(b)**

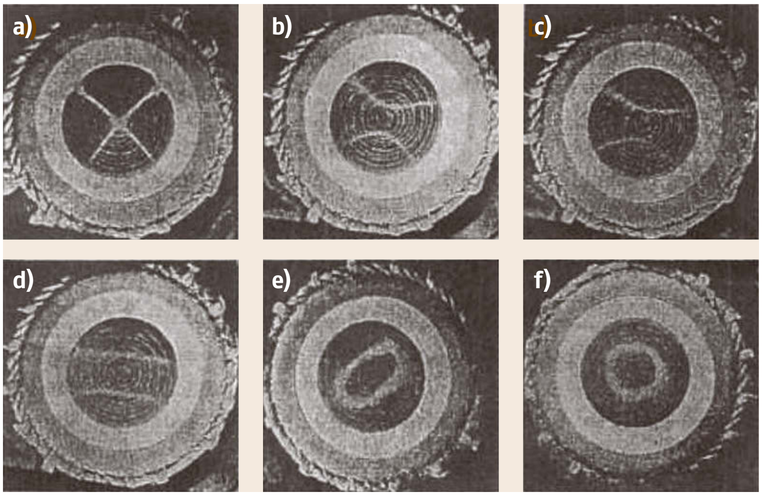

Fig. 9.3a–f Modes of vibration of the tabla (after [9.14]). **(a)** The (0,2) normal mode. **(b–e)** Combination of (0,2) and (2,1) normal modes. **(f)** The (2,1) normal mode

(or tsuzumi) is a braced drum whose body has cup-shaped ends and leather heads on both ends. A few sheets of paper wet with saliva cover an area of about 1 cm^2 at the center of the bottom head, which tunes the modes of this head into a nearly harmonic relationship [9.16].

9.2 Mallet Percussion Instruments

9.2.1 Vibrating Bars

Bars or rods can vibrate either longitudinally or transversely. The most important vibrations in percussion instruments are the transverse bending vibrations in which internal elastic forces supply the necessary restoring force. When a bar is bent, the outer part is stretched and the inner part is compressed. Somewhere in between is a neutral axis whose length remains unchanged, as shown in Fig. 9.4.

A filament located at a distance z below the neutral axis is compressed by an amount $z\,\mathrm{d}\phi$. The strain is $z\,\mathrm{d}\phi/\mathrm{d}x$, and the amount of force required to produce the strain is $E\,\mathrm{d}S z\,\mathrm{d}\phi/\mathrm{d}x$, where $\mathrm{d}S$ is the cross-sectional area of the filament and E is Young's modulus. This leads to a fourth-order differential equation whose solution can be found in [9.4, Sect. 2.15]. The solution leads to different modal frequencies, depending upon whether the ends of the bar or rod are free, clamped, or simply supported (hinged). The most commonly used bars in percussion instruments are bars that are free at both ends, whose relative frequencies are given by

$$f_n = \frac{\pi K}{8L^2}\sqrt{\frac{E}{\rho}}\left[3.011^2, 5^2, 7^2, ..., (2n+1)^2\right] .$$

The frequencies and nodal positions for the first four bending vibrational modes in a thin bar with free ends are given in Table 9.2.

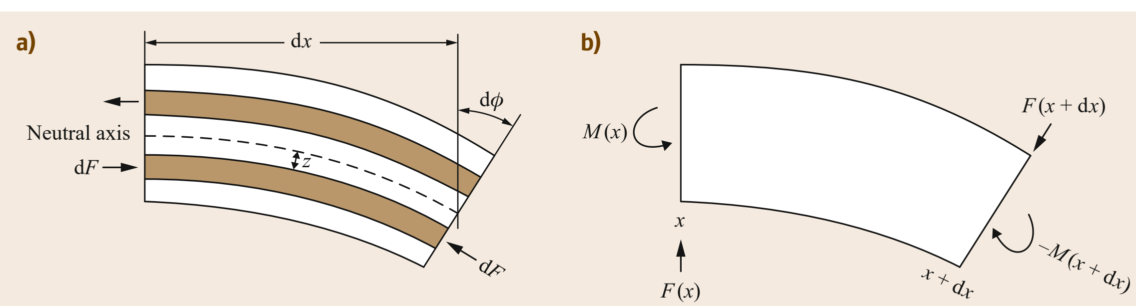

Fig. 9.4 **(a)** Bending strains in a bar. **(b)** Bending moments and shear forces in a bar

Table 9.2 Properties of transverse vibrations in a bar free at both ends

Frequency (Hz)	Wavelength (m)	Nodal positions (m from end of $1-m$ bar)
$f_1 = 3.5607 \frac{K}{L} \sqrt{\frac{E}{\rho}}$	$1.330\,L$	0.224, 0.776
$2.756\,f_1$	$0.800\,L$	0.132, 0.500, 0.868
$5.404\,f_1$	$0.572\,L$	0.094, 0.356, 0.644, 0.906
$8.933\,f_1$	$0.445\,L$	0.073, 0.277, 0.500, 0.723, 0.927

9.2.2 Marimbas

In most of the world, the term marimba denotes a deep-toned instrument with tuned bars and resonator tubes that evolved from the early Latin American instrument. The marimba typically includes three to four-and-a-half octaves of tuned bars of rosewood or synthetic material with a deep arch cut to tune the overtones. The first overtone, which is radiated by the second bending mode, is normally tuned to the fourth harmonic of the fundamental in the first two to three-and-a-half octaves, after which the interval decreases [9.17]. Details of bar shapes for harmonic tuning are given by *Bork* [9.18]. Below each marimba bar is a cylindrical resonator pipe tuned to the fundamental mode of the bar. A pipe with one closed end and one open end resonates when its acoustical length is one-fourth of a wavelength of the sound. The tubular resonators emphasize the fundamental and also increase the loudness, which is done at the expense of shortening the decay time of the sound. The statement is sometimes made that the resonator prolongs the sound but that is incorrect. That impression may be conveyed when it is played with other instruments in an ensemble since the sound decay curve begins higher and may cross the background sound at a slightly later time. Some companies now make large five-octave concert marimbas that cover the range C2 to C7. In such instruments, generally the second bending mode is accurately tuned to the fourth harmonic in the first three-and-a-half octaves. The third bending mode is tuned to the tenth harmonic in the first two octaves, after which the interval decreases. The fourth mode varies from the 20th harmonic in the lowest bars to about the sixth harmonic in the highest bars [9.19]. Relative frequencies of the first four bending modes in a Malletech marimba are shown in Fig. 9.5a, while those of several torsional modes in the same marimba are shown in Fig. 9.5b. The first torsional mode frequency ranges from about 1.9 times the nominal frequency (largest bars) to about 1.2 times the nominal frequency (smallest bars).

In normal playing, the bars are struck near their centers, where the torsional (twisting) modes have nodes, and thus they will not be excited to any great extent. On the other hand, if the bars are struck away from the center, deliberately or not, the torsional modes may contribute to the timbre. Applying finite element methods to marimba and xylophone bars showed that a small curvature in the bars has very little effect on the relative frequencies of the vibrational modes. Henrique and Antunes have used finite element methods both to optimize the shape of marimba and xylophone bars and to model the sound. They employ a physical modeling approach that addresses the spatial aspects of the problem and is suitable for both dispersive and nondispersive systems [9.20]. The sound field radiated by a simulated marimba bar has been calculated by assuming the vibrating bar to be equivalent to a linear array of oscillating spheres. This sound pressure excites a monodimensional lossy tube of finite length terminated by a radiation impedance at its open end, which represents the tubular resonator. The amount of frequency decrease as the resonator is moved closer to the bar is then calculated [9.21].

9.2.3 Xylophones

Xylophones also use bars of wood or synthetic material, but the arch is not cut as deep as that of a marimba. The first overtone is tuned to the third rather than the fourth harmonic of the fundamental. The closed-tube resonators placed below the bars reinforce the third harmonic as well as the fundamental, thus producing a brighter sound than the marimba. This is further enhanced by using hard mallets.

9.2.4 Vibes

Vibraphones or vibraharps, as they are called by different manufacturers, have aluminum bars deeply arched (as in marimbas) so that the first overtone has a frequency four times that of the fundamental. The aluminum bars in vibes have much longer decay times than the wood or synthetic bars of the marimba, and so vibes are equipped with pedal-operated dampers. The most distinctive feature of vibes, however, is the vibrato introduced by motor-driven discs at the top of the resonators, which alternately open and close the tubes. The vibrato produced by these rotating discs of pulsators produces a vibrato (hence the name). The speed of rotation of the discs may be adjusted to produce a slow

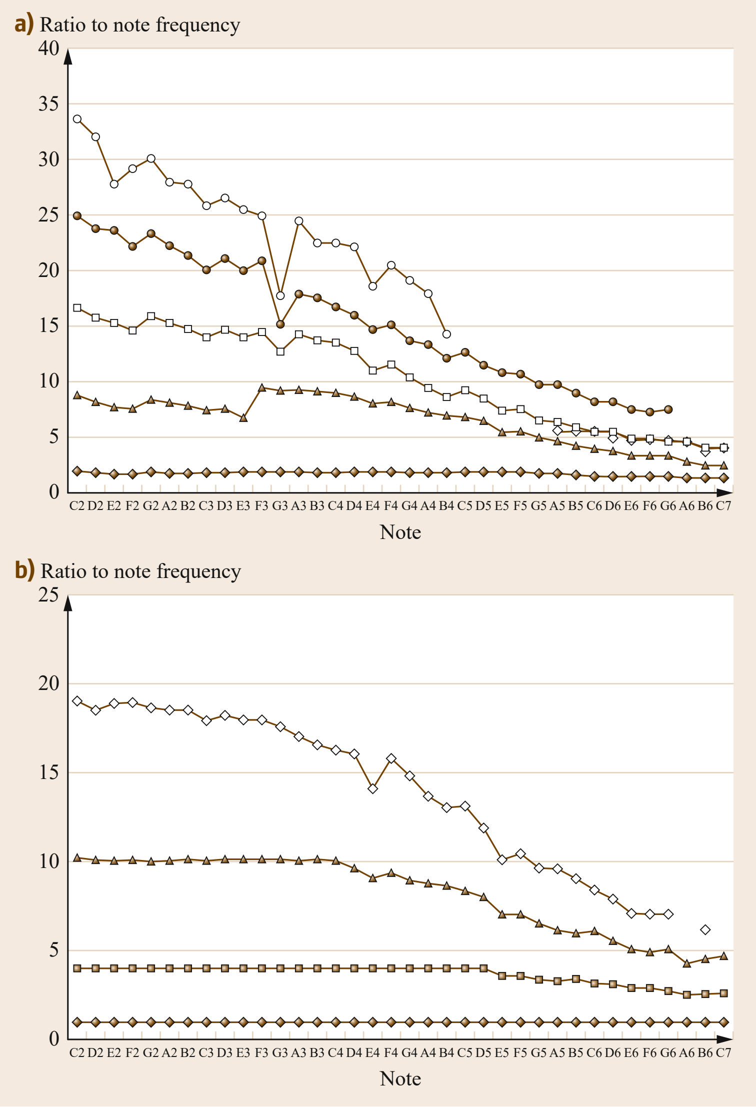

Fig. 9.5a,b Torsional modes **(a)** and bending modes **(b)** in a Malletech five-octave marimba

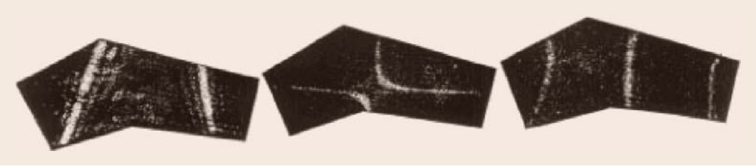

Fig. 9.6 Holographic interferogram showing vibrational modes of a jade chime stone

vibe or a fast vibe. Sometimes vibes are played without vibrato by switching off the motor. Vibes are generally played with soft mallets that produce a mellow tone.

9.2.5 Glockenspiel

The glockenspiel, or orchestra bells, uses rectangular steel bars 2.5–3.2 cm wide and 8–9 cm thick. Its range is customarily from G5 to C8, although it is scored two octaves lower than it sounds. The glockenspiel is usually played with brass or hard plastic mallets. The bell lyra is a portable version, popular in marching bands, that uses aluminum bars. Because the high overtones die out quickly, no effort is made to tune the overtones harmonically, as in the marimba, xylophone, and vibes.

Fig. 9.7 Set of 16 pyeon-gyoung (stone chimes) from the Chosun Dynasty in Korea

9.2.6 Chimes

Chimes or tubular bells are generally fabricated from lengths of brass tubing 32–38 mm in diameter. The upper end of each tube is partially or completely closed by a brass plug with a protruding rim. The rim forms a convenient and durable striking point. The modes of transverse vibration in a pipe are essentially those of a thin bar. One of the most interesting characteristics of chimes is that there is no mode of vibration with a frequency at the pitch of the strike tone one hears. Modes four, five and six, which are near the ratios 2 : 3 : 4 in a beam or tube, appear to determine the strike tone, which is heard one octave below the fourth mode [9.1].

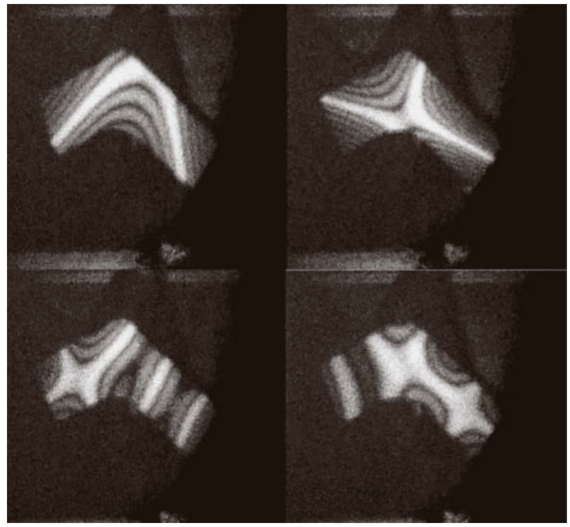

Fig. 9.8 Interferograms showing vibrational modes of a Korean pyeon-gyoung stone

9.2.7 Lithophones

Lithophones are stones that vibrate and produce sound. The ancient Chinese were fond of stone chimes, many of which have been found in ancient Chinese tombs. A typical stone chime was shaped to have arms of different lengths joined at an obtuse angle. The stones were generally struck on their longer arm with

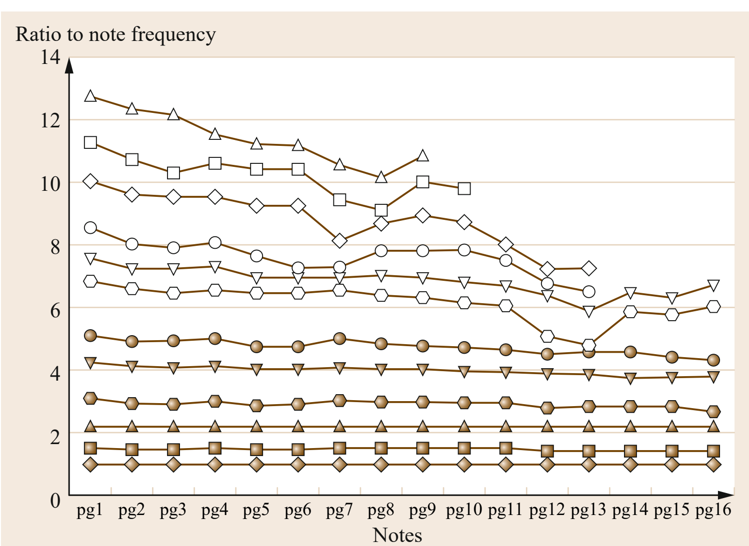

Fig. 9.9 Relative frequencies of the pyeon-gyoung stone

a wooden mallet. Sometimes the stones were richly ornamented. A lithophone of 32 stone chimes found in the tomb of the Marquis Yi (which also contained a magnificent set of 65 bells) was scaled in size, although the dimensions of the chimes do not appear to follow a strict scaling law [9.22]. In later times, the Chinese made stone chimes of jade. Holographic interferograms showing some of the modes of vibration of a small jade chime are shown in Fig. 9.6. Korean chime stones, called pyeon-gyoung, were originally brought from China to Korea in the 12th century.

A set of 16 stone chimes from the Chosun Dynasty is shown in Fig. 9.7. Unlike the Chinese stone chimes, these stones all have the same size but differ from each other only in thickness. The fundamental frequency is essentially proportional to the thickness, just as in a rectangular bar such as a marimba bar. The second mode in each stone is approximately 1.5 times the fundamental, while the third mode is about 2.3 times the nominal frequency. The fourth mode is about three times the nominal frequency up to the 12th stone, after which the ratio drops to about 2.7 [9.23]. Holographic interferograms of several modes of vibration in a pyeon-gyoung stone tuned to D♯6 are shown in Fig. 9.8. Relative frequencies of the modes in the pyeon-gyoung are shown in Fig. 9.9.

9.3 Cymbals, Gongs, and Plates

The vibrations of plates have fascinated scientists, as well as musicians, for many years. Nearly 200 years ago, *E.F.F. Chladni* published a book describing his well-known method of sprinkling sand on vibrating plates to made the nodal lines visible [9.24]. Chladni's lectures throughout Europe attracted any famous persons, including Napoleon. The nodal lines in the vibrational modes of a circular plate are not too different from those in a circular membrane, shown in Fig. 9.1. The modal frequencies are very different, however, because the stiffness of the plate contributes a substantial amount of elastic restoring force. In fact, a plate will vibrate without externally applied tension. The modes of a circular plate are often given the labels m and n, like those of a membrane, to designate the numbers of nodal diameters and nodal circles. Chladni observed that the frequencies of the various modes of a circular plate are nearly proportional to $(m+2n)^2$, a relationship that has been called Chladni's law [9.25].

9.3.1 Cymbals

Cymbals are very old instruments and have had both religious and military use in a number of cultures. The Turkish cymbals generally used in orchestras and bands are saucer-shaped with a small dome in the center, in contrast to Chinese cymbals, which have a turned-up edge. Orchestral cymbals are often designated as French, Viennese, and Germanic in order of increasing thickness. Jazz drummers use cymbals designated by such onomatopoeic terms as *crash*, *ride*, *swish*, *splash*, *ping*, and *pang*. Cymbals range from 20 to 75 cm in diameter. The strong aftersound that gives cymbal sound its characteristic shimmer is known to involve nonlinear processes [9.17]. There is considerable evidence that the vibrations exhibit chaotic behavior. A mathematical analysis of cymbal vibrations using nonlinear signal processing methods reveals that there are between three and seven active degrees of freedom, and that physical modeling will require a like number of equations [9.26]. One procedure is to calculate Lyapunov exponents from experimental time series, so that the complete spectrum of exponents can be obtained. The chaotic regime can be quantified in terms of the largest Lyapunov exponent [9.27].

9.3.2 Gongs

Gongs of many different sizes and shapes are popular in both Eastern and Western music. They are usually cast of bronze with a deep rim and a protruding dome. Tamtams are similar to gongs and are often confused with them. The main differences between the two are that tamtams do not have the dome of the gong, their rim is not as deep, and the metal is thinner. Tamtams generally sound a less definite pitch than do gongs. In fact, the sound of a tamtam may be described as somewhere between the sounds of a gong and a cymbal. The sound of a large tamtam develops slowly, changing from a sound of low pitch at strike to a collection of high-frequency vibrations, which are described as shimmer. These high-frequency modes fail to develop if the tamtam is not hit hard enough, indicating that the conversion of energy takes place through a nonlinear process [9.28].

9.3.3 Chinese Gongs

Among the many gongs in Chinese music are a pair of gongs used in Chinese opera orchestras, shown in Fig. 9.10. These gongs show a pronounced nonlinear behavior. The pitch of the larger gong glides downward

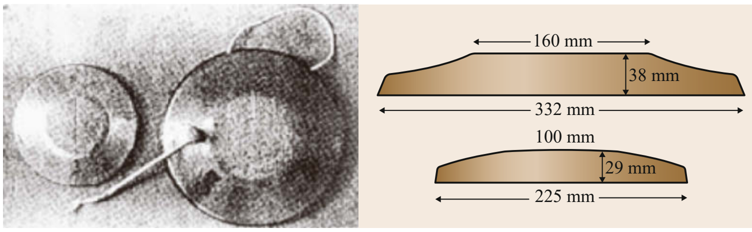

Fig. 9.10 Examples of gongs used in Chinese opera

as much as three semitones after striking, whereas that of the smaller gong glides upward by about two semitones [9.28]. Several vibrational modes of the larger gong are shown in Fig. 9.11.

In some of the modes, vibrations are confined pretty much to the flat inner portion of the gong, some to the sloping shoulders, and some involve considerable motion in both parts. When the gong is hit near the center, the central modes (178, 362, 504, 546 Hz) dominate the sound. When the gong is hit lightly on the shoulder, the lowest mode at 118 Hz is heard. The vibrations of a large tamtam were studied by *Chaigne* et al. [9.29]. They found that the nonlinear phenomena have the character of quadratic nonlinearity. Forced excitation at sufficiently large amplitude at a frequency close to one mode leads to a bifurcation with the appearance of lower frequencies corresponding to other modes. Varying the excitation frequency at constant force yielded subharmonics that were not observed at constant excitation frequency. This is quite similar to the nonlinear behavior of cymbals [9.17] [9.26].

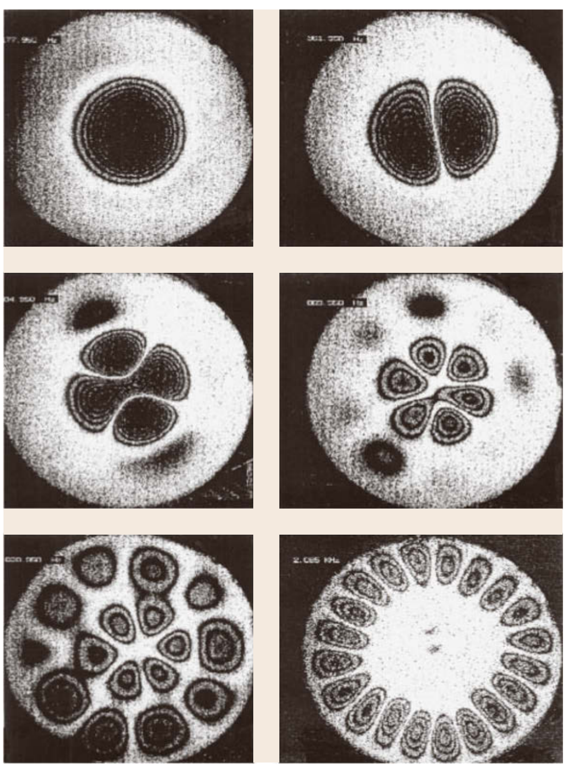

Fig. 9.11 Holographic interferograms of the modes of vibration of the larger gong shown in Fig. 9.10

9.3.4 The Caribbean Steelpan

The Caribbean steelpan is one of the most widely used acoustical instruments developed in the last 70 years. The instrument was developed on the islands of Trinidad and Tobago when local craftsman discovered methods of transforming surplus 50-gallon oil barrels into tuned drums. The Caribbean steelpan is an object of considerable acoustical study, both in its home country of Trinidad and Tobago and in the United States. Modern steel bands include a variety of instruments, such as tenor, double second, double tenor, guitar, cello, quadraphonic, and bass. Our earlier review paper [9.17] included holographic interferograms of several instruments showing how individual notes vibrate, how the entire instrument vibrates, and how the skirts of the instruments vibrate. Another piece of the puzzle, so to speak, is to understand how the vibrating components radiate sound. An effective aid to understanding sound radiation is to map the sound intensity field around the instrument. Since sound intensity is the product of sound pressure (a scalar quantity) and the acoustic fluid velocity (a vector), a two-microphone system is used. The acoustic fluid velocity can be readily calculated from the difference in sound pressure at the two accurately spaced microphones. Both the active intensity and the reactive intensity can be obtained at the desired points in the sound field. The active intensity represents the outward flow of energy, while the reactive intensity represents energy that is stored in the sound field near the instrument. While the active intensity is the most significant field in a concert hall, both active and reactive intensity fields have to be considered in recording a steelpan. Figure 9.12 shows a map of active and reactive sound intensity in a plane that bisects a double second steelpan when a single note ($F\sharp 3$) is excited at its fundamental frequency [9.30].

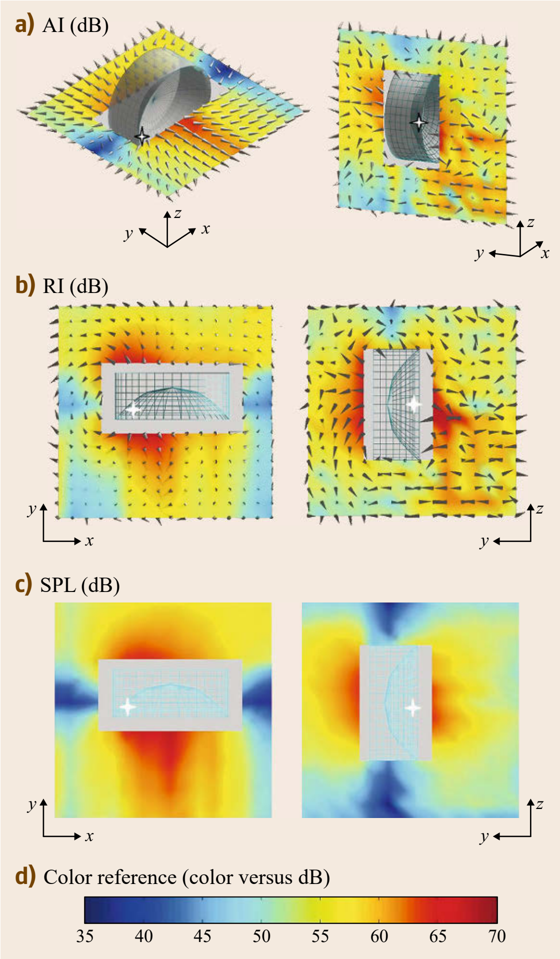

Fig. 9.12a–d Active intensity (AI) **(a)** and reactive intensity (RI) **(b)** of a Caribbean steelpan. **(c)** Sound pressure level (SPL), **(d)** color reference

9.3.5 The Hang

The Hang is a new steel percussion instrument, consisting of two spherical shells of steel, suitable for playing with the hands. Seven to nine notes are harmonically tuned around a central deep note, which is formed by the Helmholtz (cavity) resonance of the instrument body. The Hang shown in Fig. 9.13 has eight notes that can be tuned in any tonal systems between A3 and G5, including 30 tonal systems suggested by the tuners. The central note is usually tuned a fifth or fourth below the lowest note of the scale. Although it is a new instrument, many units have been shipped all over the world by PanArt, its creators. Holographic interferograms in Fig. 9.14 show the first five vibrational modes in the G3 note area of the Hang. The second and third modes are tuned to the second and third harmonics of the fundamental mode respectively [9.31].

Fig. 9.13 The Hang (image credit: Michael Paschko)

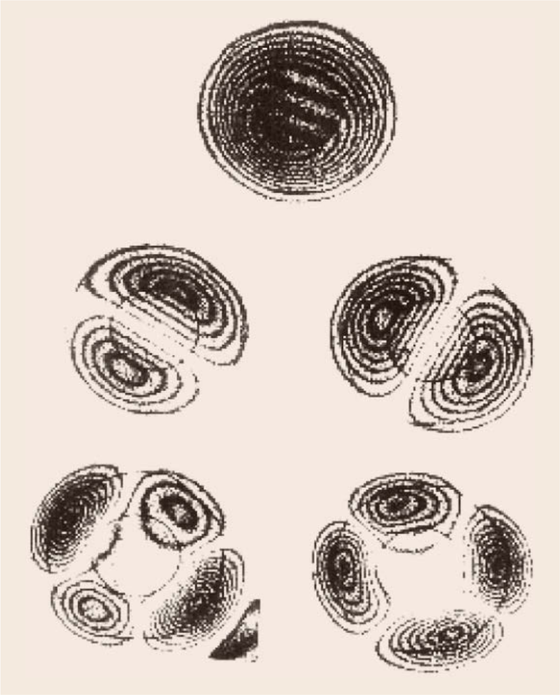

Fig. 9.14 The first five modes of vibration of the G3 note of the Hang

Figure 9.15 shows the active sound intensity in a plane 8 cm above the E4 note. The arrowheads show the direction of the sound intensity at each point in the plane, while the gray scale shows the sound pressure level. Note the sound level is greatest at the fundamental frequency, and the sound intensity is strongly upward from the note, while at the frequency of the second and third modes, considerable sound is radiated laterally.

Part A | 9.3

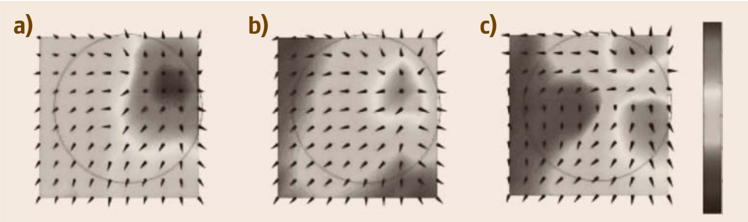

Fig. 9.15a–c Active intensity of the Hang when the E4 note is excited in **(a)** its fundamental mode **(b)** its second harmonic mode **(c)** its third mode

9.3.6 Bells

Bells have been a part of nearly every culture in history. Bells existed in the Near East before 1000 BCE, and a number of Chinese bells from the time of the Shang dynasty (1600–1100 BCE) can be found in museums around the world. In 1978 set of tuned bells from the fifth century BCE was discovered in the Chinese province of Hubei [9.32]. Bells developed as Western musical instruments in the seventeenth century when bell founders discovered how to tune their partials harmonically. The founders in the Low Countries, especially the Hemony brothers (François and Pieter) and Jacob van Eyck, took the lead in tuning bells, and many of their fine bells are found in carillons today. When struck by its clapper, a bell vibrates in a complex way. In principle, its vibrational motion can be described in terms of a linear combination of the normal modes of vibration whose initial amplitudes are determined by the distortion of the bell when struck. In practice, such a description becomes quite complex because of the large number of normal modes of diverse character that contribute to the motion. The first five modes of a church bell or carillon bell are shown in Fig. 9.16. Lines show the locations of the nodal lines. The numbers at the top denote the numbers of complete nodal meridians extending over the top of the bell and the number of nodal circles respectively. Note that there are two modes with $m = 3$ and $n = 1$, one with a circular node at the waist and one with a node near the sound bow. Thus we denote the one as (3,1♯) in Fig. 9.16. The ratio of each modal frequency to that of the prime is given at the bottom of each diagram.

When a large church bell or carillon bell is struck by its clapper, one first hears the sharp sound of metal on metal. This sound quickly gives way to a strike note that is dominated by the prominent partials of the bell. Most observers identify the metallic strike note as having a pitch at or near the frequency of the second partial. Finally, as the sound of the bell ebbs, the slowly decaying hum tone (an octave below the prime) lingers on. A new type of carillon bell, that has the dominating minor-third partial (Fig. 9.16) replaced by a partial tuned a major-third above the prime, has been developed at the Royal Eijsbouts bell foundry in The Netherlands [9.32]. The new bell design evolved partly from the use of a technique for structural optimization using finite element methods [9.33]. This technique allows a designer to make changes in the profile of an existing structure, and then to compute the resulting changes in the vibrational modes. Based on the results of the structural optimization procedure, *André Lehr* and his colleagues have designed two different bells, each having a major-third partial [9.34].

9.3.7 Handbells

Although handbells date back to at least several millennia BCE, handbells developed as Western musical instruments in the 18th century. One early use was to provide tower bell-ringers with a convenient means to practice change ringing. In more recent years, handbell choirs have become popular in schools and churches; some 40 000 choirs are reported in the United States alone. Handbells have modes of vibration somewhat similar to those of church bells or carillon bells. Hologram interferograms of a number of modes in a C5 handbell are shown in Fig. 9.17. Nodes show as bright lines, and the bullseyes locate the antinodes. In a well-tuned handbell, the (3,0) mode with three nodal meridians is tuned to a frequency three times that of the fundamental (2,0) mode.

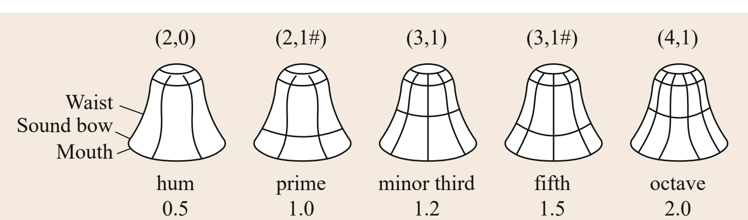

Fig. 9.16 The first five modes of a church or carillon bell

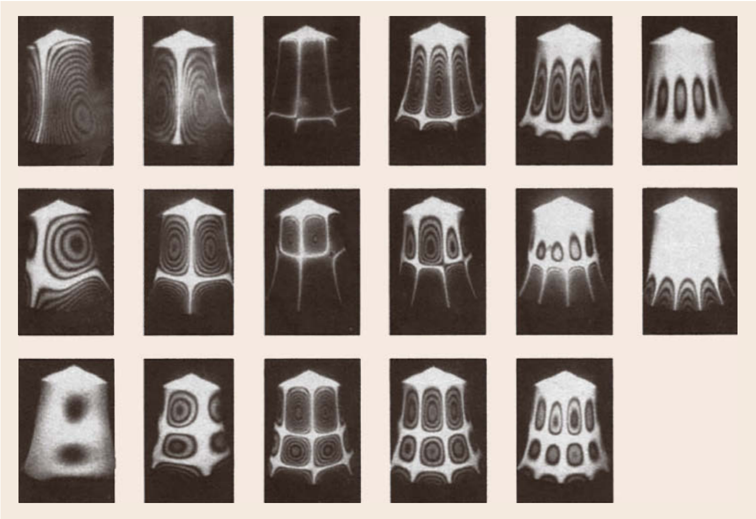

Fig. 9.17 Holographic interferograms of the vibrational modes of a C5 handbell

9.4 Methods for Studying the Acoustics of Percussion Instruments

Recent studies of the acoustics of percussion instruments have included:

1. Theoretical studies of modes of vibration
2. Experimental studies of modes of vibration
3. Sound radiation studies
4. Physical modeling
5. Studies of nonlinear behavior.

9.4.1 Finite Element and Boundary Element Methods

For all but the simplest vibrator shapes, it is difficult to calculate vibrational modes analytically. Fortunately, there are powerful numerical methods that can be carried out quite nicely by use of digital computers. These are generally described as finite element methods or boundary element methods.

9.4.2 Experimental Studies of Modes of Vibration

When a percussion instrument is excited by striking (or bowing or plucking), it vibrates in a rather complicated way. The motion can be conveniently described in terms of normal modes of vibration. A normal mode of vibration represents the motion of a linear system at a normal frequency (eigenfrequency). It should be possible to excite a normal mode of vibration at any point in a structure that is not a node and to observe motion at any other point that is not a node. It is a characteristic only of the structure itself, independent of the way it is excited or observed. In practice, however, it is difficult to avoid small distortions of the normal modes due to interaction with the exciter, the sensor, and especially the supports. Normal modes shapes are unique for a structure, whereas the deflection of a structure at a particular frequency, called its operating deflection shape (ODS), may result from the excitation of more than one normal mode [9.35].

Normal mode testing has traditionally been done using sinusoidal excitation, either mechanical or acoustical. Detection of motion may be accomplished by attaching small accelerometers, although optical and acoustical methods are less obtrusive. Modal testing with impact excitation, which became popular in the 1970s, offers a fast, convenient way to determine the normal modes of a structure. In this technique, an accelerometer is generally attached to one point on the structure, and a hammer with a load cell is used to impact the structure at carefully determined positions. Estimates of modal parameters are obtained by applying some type of curve-fitting program. Experimentally, all modal testing is done by measuring operating deflection shapes and then interpreting them in a specific manner to define mode shapes [9.35]. Strictly speaking, some type of curve-fitting program should be used to determine the normal modes from the observed ODSs, even when an instrument is excited at a single frequency. In practice, however, if the mode overlap is small, the single-frequency ODSs provide a pretty good approximation to the normal modes.

9.4.3 Scanning with a Microphone or an Accelerometer

Probably the simplest method for determining ODSs (and hence normal modes) is to excite the structure at single frequency with either a sinusoidal force or a sinusoidal sound field, and to scan the structure with an accelerometer or else to scan the near-field sound with a small microphone [9.36]. With practice, it is possible to determine mode shapes rather accurately by this method.

9.4.4 Holographic Interferometry

Holographic interferometry offers by far the best spatial resolution of operating deflection shapes (and hence of normal modes). Whereas experimental modal testing and various procedures for mechanical, acoustical, or optical scanning may look at the motion at hundreds (or even thousands) of points, optical holography looks at an almost unlimited number of points. Recording holograms on photographic plates or film (as in the holographic interferograms shown in Fig. 9.17) tends to be rather time consuming since each mode of vibration must be recorded and viewed separately. TV holography, on the other hand, is a fast, convenient way to record ODSs and to determine the normal modes. An optical system for TV holography is shown in Fig. 9.18.

A beam splitter (BS) divides the laser light to produce a reference beam and an object beam. The reference beam reaches the charge-coupled device (CCD) camera via an optical fiber, while the object beam is reflected by phase modulated (PM) mirror so that it illuminates the object to be studied. Reflected light from the object reaches the CCD camera, where it interferes with the reference beam to produce the holographic image. The speckle-averaging mechanism (SAM) alters the illumination angle in small steps in order to reduce laser speckle noise in the interferograms. Generally holographic interferograms show only variations in amplitude. It is possible, however, to recover phase information by modulating the phase of the reference beam by moving PM mirror at the driving frequency. This is a useful technique for observing motion of very small amplitude or resolving normal modes of vibration that are very close in frequency.

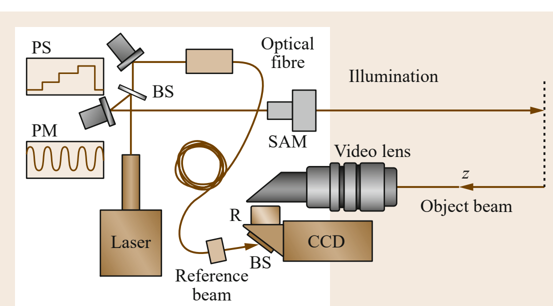

Fig. 9.18 Optical layout for a TV holography system

9.4.5 Experimental Modal Testing

Modal testing may be done with sinusoidal, random, pseudorandom, or impulsive excitation. In the case of sinusoidal excitation, the force may be applied at a single point or at several locations. The response may be measured mechanically (with accelerometers or velocity sensors), optically, or indirectly by observing the radiated sound field. In modal testing with impact excitation, an accelerometer is typically attached to a force transducer (load cell). Each force and acceleration waveform is Fourier transformed and a transfer function H_{ij} is calculated. Several different algorithms may be used to extract the mode shape and modal parameters from the measured transfer functions [9.35].

9.4.6 Radiated Sound Field

The best way to describe sound radiation from complex sources such as percussion instruments is by mapping the sound intensity field. Sound intensity is the rate at which sound energy flows outward from various points on the instrument. The sound intensity field represents the direction and the magnitude of the sound intensity at every point in the space around the source. A single microphone measures the sound pressure at a point, but not the direction of the sound energy flow. In order to determine the sound intensity it is necessary to compare the signals from two identical microphones spaced a small distance apart. The resulting pressure gradient can be used to determine sound intensity. When this is done at a large number of locations, a map of the sound intensity field results [9.30, 37, 38].

9.4.7 Physical Modeling

Synthesizing sounds by physical modeling has attracted a great deal of interest in recent years. The basic notion of physical modeling is to write equations that describe how particular sets of physical objects vibrate and then to solve those equations in order to synthesize the resulting sound. Percussion instruments have proven particularly difficult to model completely enough to be able to synthesize their sounds entirely based on a physical model. Physical modeling is complicated by their nonlinear behavior and by the strong role that transients play in their sound.

References

9.1 T.D. Rossing: Acoustics of percussion instruments part I, Phys. Teach. **14**, 546–556 (1976)
9.2 T.D. Rossing: Acoustics of percussion instruments part II, Phys. Teach. **15**, 278–288 (1977)
9.3 T.D. Rossing: *Science of Percussion Instruments* (World Scientific, Singapore 2000)
9.4 N.H. Fletcher, T.D. Rossing: *The Physics of Musical Instruments*, 2nd edn. (Springer, New York 1998)
9.5 T.D. Rossing, F.R. Moore, P.A. Wheeler: *The Science of Sound*, 3rd edn. (Addison Wesley, San Francisco 2002)
9.6 D.E. Hall: *Musical Acoustics* (Brooks/Cole, Pacific Grove 2002)
9.7 M. Campbell, C. Greated, A. Myers: *Musical Instruments* (Oxford Univ. Press, Oxford 2004)
9.8 J. Meyer: *Acoustics and the Performance of Music* (Springer, New York 2009), transl. by U. Hansen
9.9 Lord Rayleigh (J.W. Strutt): *The Theory of Sound*, Vol. 1, 2nd edn. (Macmillan, London 1894), reprinted by Dover, New York 1945
9.10 T.D. Rossing: The physics of kettledrums, Sci. Am. **247**(5), 172–178 (1982)
9.11 T.D. Rossing, G. Kvistad: Acoustics of timpani: preliminary studies, The Percussionist **13**, 90–98 (1976)
9.12 T.D. Rossing, I. Bork, H. Zhao, D. Fystrom: Acoustics of snare drums, J. Acoust. Soc. Am. (1992), https://doi.org/10.1121/1.404080
9.13 C.D. Rose: *A New Drumhead Design: An Analysis of the Nonlinear Behavior of a Compound Membrane*, M.S. Thesis (Northern Illinois University, DeKalb 1978)
9.14 C.V. Raman: The Indian musical drum, Proc. Indian Acad. Sci. A1 (1934), https://doi.org/10.1007/BF03035705
9.15 J. Obata, T. Tesima: Experimental studies on the sound and vibration of drum, J. Acoust. Soc. Am. **6**(4), 267–274 (1935)
9.16 S. Ando: Acoustical studies of Japanese traditional drums. In: *Jt. Meet. Acoust. Soc. Am/Acoust. Soc. Jn, Honolulu* (1996), Paper 4aMUb3
9.17 T.D. Rossing: Acoustics of percussion instruments: Recent progress, Acoust. Sci. Technol. **22**, 177–188 (2001)
9.18 I. Bork: Practical tuning of xylophone bars and resonators foobar, Appl. Acoust. **46**, 103–127 (1995)
9.19 J. Yoo, T.D. Rossing, B. Larkin: Vibrational modes of five-octave concert marimbas. In: *Proc. Stockholm Music Acoust. Conf. (SMAC03), Stockholm* (2003) pp. 355–357
9.20 L. Henrique, J. Antunes: Optimal design and physical modelling of mallet percussion instruments, Acta Acustica/Acustica **89**, 948–963 (2003)
9.21 V. Doutaut, A. Chaigne, G. Bedrane: Time-domain simulation of the sound pressure radiated by mallet percussion instruments. In: *Proc. ISMA, Dourdan* (1995) pp. 519–524
9.22 A. Lehr: Designing chimes and carillons in history, Acustica/Acta Acustica **83**, 320–336 (1997)
9.23 J. Yoo, T.D. Rossing: Vibrational modes of pyen-gyoung, Korean chime stone. In: *Proc. ISMA 2004, Nara* (2004) pp. 312–315
9.24 E.F.F. Chladni: *Entdeckungen über die Theorie des Klanges* (Breitkopf Härtel, Leipzig 1787), translated excerpts in R.B. Lindsay: *Acoustics: Historical and Philosophical Development* (Dowden Hutchinson Ross, Stroudsburg 1973) pp. 156–165
9.25 T.D. Rossing: Chladni's law for vibrating plates, Am. J. Phys. **50**, 271–274 (1982)
9.26 C. Touze, A. Chaigne, T. Rossing, S. Schedin: Analysis of cymbal vibrations and sound using nonlinear signal processing methods. In: *Proc. ISMA98* (1998)
9.27 C. Touzé, A. Chaigne: Lyapunov exponents from experimental time series: application to cymbal vibrations, Acta Acustica/Acustica **86**(3), 557–567 (2000)
9.28 T.D. Rossing, N.H. Fletcher: Acoustics of a tamtam, Bull. Australian Acoust. Soc. **10**(1), 21–26 (1982)
9.29 A. Chaigne, C. Touzé, O. Thomas: Nonlinear axisymmetric vibrations of gongs. In: *Proc. ISMA 2001, Perugia* (2001) pp. 147–152
9.30 B. Copeland, A. Morrison, T.D. Rossing: Sound radiation from Caribbean steelpans, J. Acoust Soc. Am. **117**, 375–383 (2005)
9.31 T.D. Rossing, U.J. Hansen, F. Rohner, S. Schärer: The HANG: A hand-played steel drum. In: *Proc. SMAC 93, Stockholm* (2003) pp. 351–354
9.32 T.D. Rossing: Acoustics of Eastern and Western bells, old and new, J. Acoust. Soc. Jpn. (E) **10**, 241–252 (1989)
9.33 B. Schoofs, F. van Asperen, P. Maas, A. Lehr: Computation of bell profiles using structural optimization, Music Percept **4**, 245–254 (1985)
9.34 A. Lehr: *The Designing of Swinging Bells and Carillon Bells in the Past and Present* (Athanasius Kircher Foundation, Asten 1987)
9.35 M.H. Richardson: Is it a mode shape, or an operating deflection shape?, Sound Vibr. **31**(1), 54–61 (1997)
9.36 T.D. Rossing, D.A. Russell: Laboratory observation of elastic waves in solids, Am. J. Phys. **58**, 1153–1162 (1990)
9.37 F.J. Fahy: *Sound Intensity*, 2nd edn. (E. F.N. Spon, London 1995)
9.38 A. Morrison: *Acoustical Studies of the Steelpan and HANG: Phase-Sensitive Holography and Sound Intensity Measurements*, Ph.D. Thesis (Northern Illinois University, DeKalb 2005)