Chapter 9
Casimir Frictional Drag Force in Low-Dimensional Systems

Coulomb drag is a frictional coupling between electric currents flowing in spatially separated conducting layers. It is caused by interlayer electron–electron interactions. The frictional drag between quantum wells makes it possible to directly probe the inter-particle interaction. Inter-particle interactions form the cornerstone of many-body physics. Usually it is considered using time-dependent perturbation theory. In the lowest order of this theory, the friction force is determined by the Kubo formula, which gives the friction to linear order in relative drift velocity of the electrons in the different layers. In this section, we calculate the frictional drag force in low-dimensional systems at arbitrary relative sliding velocity using the theory of Casimir friction.

We study the frictional drag force in low-dimensional systems (2D electron and 2D liquid systems) mediated by a fluctuating electromagnetic field, which originates from the Brownian motion of the ions in the liquid. The analysis is focused on the (2D system–2D system), (2D system–semi-infinite liquid), and (2D system–infinite liquid) configurations. We show that for the 2D electron systems, the friction drag depends linearly on the relative velocity of the free carries in the different media, but for 2D liquid systems, the frictional drag depends nonlinear on the relative velocity. For 2D systems, the frictional drag force induced by liquid flow may be several orders of magnitude larger than the frictional drag induced by an electronic current.

9.1 Introduction

The presence of the fluctuating electromagnetic field leads to a coupling between bodies even when they are isolated from each other by a vacuum gap or a dielectric layer. In the non-retarded limit (short separation between the bodies), this fluctuating electromagnetic field is reduced to the fluctuating Coulomb field, which determines the electron–electron (e–e) and electron–hole (e–h) interaction, which plays a leading

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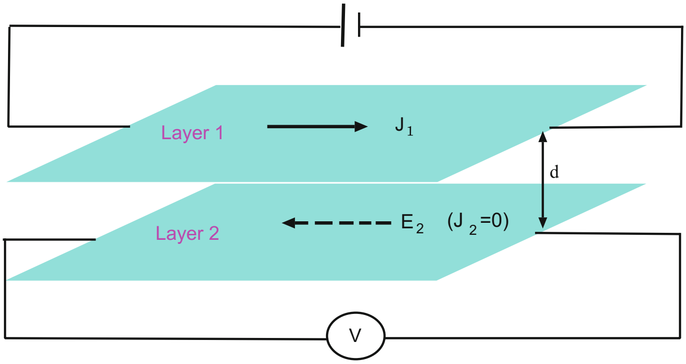

Fig. 9.1 Scheme of experiment for observation of the drag effect

role in a wide range of condensed matter phenomena such as the fractional quantum Hall effect, high-temperature superconductivity, Wigner crystallization, exciton condensates and the Mott transition. In addition, the e–e interaction is central to problems involving quantum coherence since it is a leading mechanism of electron dephasing.

Many-body effects are particularly important in low-dimensional systems. This leads to many intriguing phenomena, such as Luttinger liquid behavior in quantum wires, and the fractional quantum Hall effect and Wigner crystallization in 2D electron gases in a magnetic field. As technology improves and semiconductor devices shrink further in size, interaction effects become even more pronounced and it may become possible to probe these effects in novel experiments.

Despite its importance, the direct measurement of the e–e interaction through transport experiments is difficult. This is a consequence of the e–e interaction's momentum conserving nature. However, the e–e interaction can be tested using frictional drag. The frictional drag effect consists in driving an electric current in one metallic layer and registration of the effect of the frictional drag of the electrons in a second (parallel) metallic layer (Fig. 9.1). Such experiments were predicted by Pogrebinskii [255] and Price [256] and were performed for 2D quantum wells [112–114]. In these experiments, two quantum wells are separated by a dielectric layer thick enough to prevent electrons from tunneling across it while still allowing interlayer interaction between them. A current of density $J_2 = n_2 ev$ is driven through layer **2** (where n_2 is the carrier concentration per unit area in the second layer), see Fig. 9.1. Due to the proximity of the layers, the interlayer interactions will induce a current in layer **1** due to a friction stress $\sigma = \gamma v$ acting on the electrons in layer **1** from layer **2**. If layer **1** is an open circuit, an electric field E_1 will develop in the layer whose influence cancels the frictional stress σ between the layers. Thus the frictional stress $\sigma = \gamma v$ must equal the induced stress $n_1 e E_1$ so that

$$\gamma = n_1 e E_1 / v = n_2 n_1 e^2 E_1 / J_2 = n_1 n_2 e^2 \rho_{12}, \tag{9.1}$$

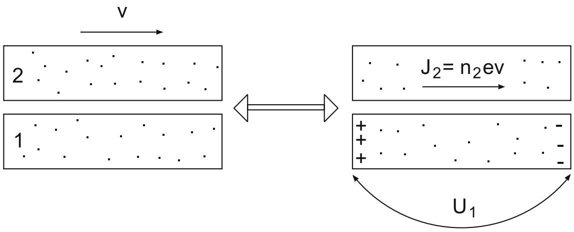

Fig. 9.2 Two ways of studying of Casimir friction. *Left* A metallic block sliding relative to the metallic substrate with the velocity v. An electronic frictional stress will act on the block (and on the substrate). *Right* The shear stress σ can be measured if instead of sliding the upper block, a voltage U_2 is applied to the block resulting in a drift motion of the conduction electrons (velocity v). The resulting frictional stress σ on the substrate electrons will generate a voltage difference U_1 (proportional to σ), as indicated in the figure. Both approaches are equivalent if in the upper block is possible to neglect scattering of the free carries by lattice defects

where the *transresistivity* $\rho_{12} = E_1/J_2$ is defined as the ratio of the induced electric field in the first layer to the driving current density in the second layer. The transresistivity is often interpreted in terms of a drag rate, which, in analogy with the Drude model, is defined by $\tau_D^{-1} = \rho_{12} n_2 e^2/m^* = \gamma/n_1 m^*$. These experiments spurred by a large body of theoretical work both on electron–hole systems and on electron–electron systems. Most of this work focused on interlayer Coulomb interaction, the most obvious coupling mechanism and the one considered in the original theoretical papers [255], though the contributions due to an exchange of phonons between the layers have also been considered [113]. The most widely used approach to study the drag effect is based on the Boltzmann equation [114, 257–259] and the Kubo formalism [260, 261]. In [114], a theory of the drag effect was developed based on the semi-classical theory of the fluctuating electromagnetic field. The retardation effects are automatically included in this approach.

The close connection of Casimir friction with frictional drag effect is illustrated in Fig. 9.2. At present, both these phenomena attract considerable attention in connection with the possibility of using them in the micro-and nano-electromechanical systems (MEMS and NEMS), and biological objects, in which local dynamic effects are intensively studied.

9.2 Fluctuating Electromagnetic Field

We consider two parallel 2D electron layers separated by a distance d. We introduce two reference systems K and K', with coordinate axes xyz and $x'y'z'$. The xy- and $x'y'$-planes coincide with layer **1**, with the x- and x'-axes pointing in the same direction, and the z- and z'-axes pointing toward layer **2**. The layers **1** and **2** are located at $z = 0$ and $z = d$, respectively. In the K system both layers are at rest. Assume now that in layer **2**, the conduction electrons move with the drift velocity v, corresponding to the current density $j_2 = n_2 ev$, while no current flows in layer **1**. The K' reference system moves with velocity v along to the x-axis relative to frame K. In the K' frame there is no current density in layer **2**, while the surrounding dielectric moves with velocity $-v\hat{\mathrm{x}}$. Following Lifshitz [42], to calculate the fluctuating field we shall use the general theory due to Rytov, which is described in his books [5–7] (see also Sect. 3.1). This method is based on the introduction of a ‘random’ field in the Maxwell equations (just as, for example, one introduces a ‘random’ force in the theory of Brownian motion of a particle). In the K-system for $z < d$, for a monochromatic field (time factor $\exp(-\mathrm{i}\omega t)$) in a dielectric, nonmagnetic medium, these equations are:

$$\nabla \times \mathbf{E} = i\frac{\omega}{c}\mathbf{B}, \tag{9.2}$$

$$\nabla \times \mathbf{H} = -i\frac{\omega}{c}\mathbf{D} + \frac{4\pi}{c}(\mathbf{j}_1 + \mathbf{j}_{1f})\delta(z), \tag{9.3}$$

where, following Rytov, we divided the total current density $\mathbf{j}_{1tot}$ in layer **1** into two parts, $\mathbf{j}_{1tot} = \mathbf{j}_1 + \mathbf{j}_{1f}$, the fluctuating current density $\mathbf{j}_{1f}$ associated with thermal and quantum fluctuations, and the current density $\mathbf{j}_1$ induced by the electric field $\mathbf{E}$:

$$j_{1\alpha}(\mathbf{r}) = \int \mathrm{d}^2 r'\, \sigma_{1\alpha\beta}(\mathbf{r} - \mathbf{r}')E_\beta(\mathbf{r}'), \tag{9.4}$$

where $\mathbf{r}$ is the 2D vector in the xy-plane, and $\sigma_{1\alpha\beta}(\mathbf{r}-\mathbf{r}')$ is the conductivity tensor in layer **1**. $\mathbf{D}$, $\mathbf{H}$ and $\mathbf{B}$ are the electric displacement field, the magnetic and the magnetic induction fields, respectively. For nonmagnetic medium $\mathbf{B} = \mathbf{H}$ and $\mathbf{D} = \varepsilon\mathbf{E}$, where ε is the dielectric constant of the surrounded media. According to the fluctuation-dissipation theorem [184], the correlation function of the fluctuating current density $\mathbf{j}_f$, determining the average value of the product of component of $\mathbf{j}_f$ at two different point in space, is given by (3.59). In this equation the dielectric tensor $\varepsilon_{\alpha\beta}$ can be expressed through the conductivity tensor $\sigma_{\alpha\beta}$ according to the relation $\varepsilon_{\alpha\beta} = 1 + 4\pi i\sigma_{\alpha\beta}/\omega$. As a result we get

$$\left\langle j_{f\alpha}(\mathbf{r},\omega) j^*_{f\beta}(\mathbf{r}',\omega')\right\rangle = \left\langle j_{f\alpha}(\mathbf{r},\omega) j^*_{f\beta}(\mathbf{r}',\omega)\right\rangle_\omega \delta(\omega - \omega'), \tag{9.5}$$

$$\left\langle j_{f\alpha}(\mathbf{r},\omega) j^*_{f\beta}(\mathbf{r}',\omega)\right\rangle_\omega = \frac{\hbar\omega}{\pi}\left(\frac{1}{2} + n(\omega)\right)\mathrm{Re}\,\sigma_{\alpha\beta}(\mathbf{r} - \mathbf{r}', \omega), \tag{9.6}$$

where $\mathrm{Re}\,\sigma_{\alpha\beta}(\mathbf{r}-\mathbf{r}')$ is the real part of the conductivity. We write the current density in the form of a Fourier integral

$$\mathbf{j}(\mathbf{r}) = \int \mathrm{d}^2 q\, \mathbf{j}(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{9.7}$$

where $\mathbf{q}$ is a 2D vector in the xy-plane. For the Fourier components $\mathbf{j}_f(\mathbf{q})$, the correlation function corresponding to the spatial correlation (9.6) is

$$\langle j_{f\alpha}(\mathbf{q},\omega) j^*_{f\beta}(\mathbf{q}',\omega)\rangle_\omega = \frac{\hbar\omega}{4\pi^3}\left(\frac{1}{2}+n(\omega)\right) \mathrm{Re}\, \sigma_{\alpha\beta}(\mathbf{q},\omega)\delta(\mathbf{q}-\mathbf{q}'), \tag{9.8}$$

where

$$\sigma_{\alpha\beta}(\mathbf{q},\omega) = \int \mathrm{d}^2 r\, \sigma_{\alpha\beta}(\mathbf{r},\omega) e^{-i\mathbf{q}\cdot\mathbf{r}}.$$

For the layers with assumed isotropy in the xy-plane, the conductivity tensor can be written in the form

$$\sigma_{\alpha\beta}(\mathbf{q},\omega) = \frac{q_\alpha q_\beta}{q^2}\sigma_l(\mathbf{q},\omega) + \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2}\right)\sigma_t(\mathbf{q},\omega), \tag{9.9}$$

where $\sigma_t(\mathbf{q},\omega)$ and $\sigma_l(\mathbf{q},\omega)$ are the transverse and longitudinal conductivity of the layer.

After decomposition of the components of the electromagnetic field into Fourier integrals, the general solution of the Maxwell equations for $z < d$ can be written in the form

$$\mathbf{E} = \begin{cases} \mathbf{v}e^{\mathrm{i}k_z z} + \mathbf{w}e^{-ik_z z}, & 0 < z < d, \\ \mathbf{u}_1 e^{-ik_z z}, & z < 0, \end{cases} \tag{9.10}$$

$$\mathbf{B} = -i\frac{c}{\omega} \begin{cases} \left([\mathbf{q}\times\mathbf{v}] + k_z[\hat{\mathrm{z}}\times\mathbf{v}]\right)e^{\mathrm{i}k_z z} + \left([\mathbf{q}\times\mathbf{w}] - k_z[\hat{\mathrm{z}}\times\mathbf{w}]\right)e^{-\mathrm{i}k_z z}, & 0 < z < d, \\ \left([\mathbf{q}\times\mathbf{u}_1] - k_z[\hat{\mathrm{z}}\times\mathbf{u}_1]\right)e^{-\mathrm{i}k_z z}, & z < 0, \end{cases} \tag{9.11}$$

where $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{u}_1$ satisfy the transversality conditions

$$\mathbf{v}\cdot\mathbf{q} + k_z v_z = 0, \quad \mathbf{w}\cdot\mathbf{q} - k_z w_z = 0, \quad \mathbf{u}_1\cdot\mathbf{q} - k_z u_{1z} = 0, \tag{9.12}$$

where

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 \varepsilon - q^2} \tag{9.13}$$

and $\hat{\mathrm{z}}$ is a unit vector along the z-axis. We now decompose the electromagnetic field into s- and p-polarized waves. For the p-polarized waves, the electric field $\mathbf{E}$ is in the plane determined by the vectors $\hat{\mathrm{q}} = \mathbf{q}/q$ and $\hat{\mathrm{z}}$, and perpendicular to this plane, along the vector $\mathbf{n} = \hat{\mathrm{z}}\times\hat{\mathrm{q}}$, for s-polarized waves. The boundary conditions at $z = 0$ for s- and p-polarized waves are given by

$$E_{\mathbf{n}}(z=+0) = E_{\mathbf{n}}(z=-0), \tag{9.14}$$

$$\frac{dE_{\mathbf{n}}}{dz}\Big|_{z=+0} - \frac{dE_{\mathbf{n}}}{dz}\Big|_{z=-0} = -\frac{4\pi i\omega}{c^2}\left(\sigma_{1t}(\mathbf{q},\omega)E_{\mathbf{n}} + j_{f1\mathbf{n}}\right), \tag{9.15}$$

$$E_{\mathbf{q}}(z=+0) = E_{\mathbf{q}}(z=-0), \tag{9.16}$$

$$\frac{dE_{\mathbf{q}}}{dz}|_{z=+0} - \frac{dE_{\mathbf{q}}}{dz}|_{z=-0} = -\frac{4\pi i k_z^2}{\varepsilon\omega}\left(\sigma_{1l}(\mathbf{q},\omega)E_{\mathbf{q}} + j_{f1\mathbf{q}}\right), \tag{9.17}$$

where $E_{\mathbf{q}} = \hat{\text{q}} \cdot \mathbf{E}$, $E_{\mathbf{n}} = \mathbf{n} \cdot \mathbf{E}$ and so on. From (9.15) and (9.17), we get the following equations:

$$v_{\mathbf{q}} + R_{1p}w_{\mathbf{q}} = -\frac{2\pi k_z j_{f1\mathbf{q}}}{\varepsilon\omega\varepsilon_{1p}}, \tag{9.18}$$

$$v_{\mathbf{n}} + R_{1s}w_{\mathbf{n}} = -\frac{2\pi\omega j_{f1\mathbf{n}}}{k_z c^2 \varepsilon_{1s}}, \tag{9.19}$$

where $v_{\mathbf{q}} = \hat{\text{q}} \cdot \mathbf{v}$ and so on, the reflection amplitudes for the layer

$$R_{1s(p)} = \frac{\varepsilon_{1s(p)} - 1}{\varepsilon_{1s(p)}},$$

and the dielectric functions of the layer

$$\varepsilon_{1s} = \frac{2\pi\omega\sigma_{1t}}{k_z c^2} + 1, \qquad \varepsilon_{1p} = \frac{2\pi k_z \sigma_{1l}}{\omega\varepsilon} + 1.$$

The Maxwell equations in the K'-system for $z > 0$ have the same form as (9.2) and (9.3) with $\mathbf{j} \to \mathbf{j}_2$ and $\mathbf{j}_f \to \mathbf{j}_{f2}$. To first order in v/c the relations between $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{B}$, $\mathbf{H}$ are [191]

$$\mathbf{D} = \varepsilon\mathbf{E} - (\varepsilon - 1)\frac{v}{c}\hat{\text{x}} \times \mathbf{B}, \tag{9.20}$$

$$\mathbf{H} = \mathbf{B} - (\varepsilon - 1)\frac{v}{c}\hat{\text{x}} \times \mathbf{E}. \tag{9.21}$$

Under a Lorentz transformation, we have to linear order in v/c: $\omega' = \omega - q_x v$ and $\mathbf{q}' = \mathbf{q} - \hat{\text{x}}\omega v/c^2$. Note also that k_z is invariant under the Lorentz transformation, i.e. $k_z = k_z'$. It can be shown that the last terms in (9.20) and (9.21) give rise only to a coupling between s- and p-polarized waves. However, as in Sect. 7.3 it can be shown that this coupling gives a correction $\sim (v/c)^2$ to the frictional drag force between the layers, so this term can be omitted. The solution of the Maxwell equations in the K' reference frame can be written as

$$\mathbf{E}' = \begin{cases} \mathbf{v}'e^{ik_z z} + \mathbf{w}'e^{-ik_z z}, & 0 < z < d \\ \mathbf{u}_2 e^{ik_z z}, & z > d \end{cases} \tag{9.22}$$

From the boundary conditions for the s- and p-polarized waves we get the equations

$$w'_{\mathbf{q}'} + R_{2p}(\mathbf{q}',\omega')e^{2ik_z d}v'_{\mathbf{q}'} = -\frac{2\pi k_z j_{f2\mathbf{q}'}(\mathbf{q}',\omega')e^{ik_z d}}{\varepsilon\omega'\varepsilon_{2p}(\mathbf{q}',\omega')}, \tag{9.23}$$

$$w'_{\mathbf{n}'} + R_{2s}(\mathbf{q}',\omega')e^{2ik_z d}v'_{\mathbf{n}'} = -\frac{2\pi\omega' j_{f2\mathbf{n}'}(\mathbf{q}',\omega')e^{ik_z d}}{pc^2\varepsilon_{2s}(\mathbf{q}',\omega')}. \tag{9.24}$$

The relations between the fields in the K and K' reference frames are determined by the Lorentz transformation. As shown in Sect. 7.3, such a Lorentz transformation gives the terms of the order v/c, which couples the s- and p-polarized waves but this results in a contribution to the frictional drag of the order $(v/c)^2$. Thus, we can take this transformation in zero order in v/c so that $v'_{\mathbf{q}'}(\omega') = v_{\mathbf{q}}(\omega)$, $v'_{\mathbf{n}'}(\omega') = (\omega'/\omega)\, v_{\mathbf{n}}(\omega)$ and similar equations for $\mathbf{w}$. After the transformation, the solution of the system of the equations (9.18), (9.19), (9.23) and (9.24) takes the form

$$v_{\mathbf{q}} = \frac{2\pi k_z}{\Delta_p \varepsilon}\left[\frac{j_{f2\mathbf{q}'}(\mathbf{q}',\omega')\mathrm{e}^{ik_z d}R_{1p}(q,\omega)}{\varepsilon_{2p}(\mathbf{q}',\omega')\omega'} - \frac{j_{f1\mathbf{q}}(\mathbf{q},\omega)}{\varepsilon_{1p}(\mathbf{q},\omega)\,\omega}\right], \tag{9.25}$$

$$w_{\mathbf{q}} = \frac{2\pi k_z}{\Delta_p \varepsilon}\left[\frac{j_{f1\mathbf{q}}(q,\omega)\,\mathrm{e}^{2ik_z d}R_{2p}(q',\omega')}{\varepsilon_{1p}(\mathbf{q},\omega)\,\omega} - \frac{j_{f2\mathbf{q}'}(q',\omega')\mathrm{e}^{ik_z d}}{\varepsilon_{2p}(\mathbf{q}',\omega')\omega'}\right], \tag{9.26}$$

$$v_{\mathbf{n}} = \frac{2\pi\omega}{\Delta_s k_z c^2}\left[\frac{j_{f2\mathbf{n}'}(q',\omega')\mathrm{e}^{ik_z d}R_{1s}(q,\omega)}{\varepsilon_{2s}(q',\omega')} - \frac{j_{f1\mathbf{n}}(q,\omega)}{\varepsilon_{1s}(q,\omega)}\right], \tag{9.27}$$

$$w_{\mathbf{n}} = \frac{2\pi\omega}{\Delta_s k_z c^2}\left[\frac{j_{f1\mathbf{n}}(\mathbf{q},\omega)\mathrm{e}^{2ik_z d}R_{2s}(q',\omega')}{\varepsilon_{1s}(\mathbf{q}',\omega)} - \frac{j_{f2\mathbf{n}'}(\mathbf{q}',\omega')\mathrm{e}^{ik_z d}}{\varepsilon_{2s}(\mathbf{q}',\omega')}\right], \tag{9.28}$$

$$v_z = -\frac{q v_{\mathbf{q}}}{k_z}, \qquad w_z = \frac{q w_{\mathbf{q}}}{k_z}, \tag{9.29}$$

where we have introduced the notation

$$\Delta_p = 1 - e^{2ik_z d}R_{2p}(\mathbf{q}',\omega')R_{1p}(\mathbf{q},\omega),$$
$$\Delta_s = 1 - e^{2ik_z d}R_{2s}(\mathbf{q}',\omega')R_{1s}(\mathbf{q},\omega).$$

9.3 Casimir Frictional Drag Force Between Two Quantum Wells

The frictional drag stress σ, which acts on the conduction electrons in layer **1** can be obtained from the xz-component of the Maxwell stress tensor σ_{ij}, evaluated at $z = \pm 0$

$$\sigma = \frac{1}{8\pi}\int_{-\infty}^{+\infty} \mathrm{d}\omega \left\{\left[\varepsilon\left\langle E_z E_x^*\right\rangle + \left\langle B_z B_x^*\right\rangle + c.c\right]_{z=+0} - [\ldots]_{z=-0}\right\}. \tag{9.30}$$

Here the $\langle \ldots \rangle$ denotes statistical averaging over the fluctuating current densities. The averaging is carrying out with the aid of (9.5). Note that the components of the fluctuating current density $\mathbf{j}_{f1}$ and $\mathbf{j}_{f2}$ refer to different layers, and are statistically independent, so that the average of their product is zero. Expanding the electric field and the magnetic induction in the Fourier series, we obtain

$$\sigma = \frac{1}{8\pi}\int \mathrm{d}\omega \mathrm{d}^2 q \Big\{ \big[\varepsilon \left\langle E_z(\mathbf{q},\omega) E_x^*(\mathbf{q},\omega)\right\rangle + \left\langle B_z(\mathbf{q},\omega) B_x^*(\mathbf{q},\omega)\right\rangle + \\ + c.c\big]_{z=+0} - [\ldots]_{z=-0} \Big\}. \tag{9.31}$$

For a given value of $\mathbf{q}$, it is convenient to express the component E_x and B_x in terms of the components along the vectors $\hat{\mathrm{q}}$ and $\mathbf{n}$

$$E_x = (q_x/q)E_{\mathbf{q}} - (q_y/q)E_{\mathbf{n}}, \tag{9.32}$$

$$B_x = (q_x/q)B_{\mathbf{q}} - (q_y/q)B_{\mathbf{n}}. \tag{9.33}$$

After substitution of expressions (9.32) and (9.33) into (9.31) and taking into account that the term that is proportional to q_y is equal to zero, we obtain

$$\sigma = \frac{1}{8\pi}\int \frac{\mathrm{d}\omega \mathrm{d}^2 q}{(2\pi)^2}\frac{q_x}{q} \Big\{ \big[\varepsilon \left\langle E_z(\mathbf{q},\omega) E_{\mathbf{q}}^*(\mathbf{q},\omega)\right\rangle + \left\langle B_z(\mathbf{q},\omega) B_{\mathbf{q}}^*(\mathbf{q},\omega)\right\rangle + \\ + c.c\big]_{z=+0} - [\ldots]_{z=-0} \Big\}, \tag{9.34}$$

where

$$E_z(z=+0) = (v_z + w_z) = (q/k_z)(w_{\mathbf{q}} - v_{\mathbf{q}}) = (q k_z^* / \mid k_z \mid^2)(w_{\mathbf{q}} - v_{\mathbf{q}}), \tag{9.35}$$

$$E_z(z=-0) = u_{1z} = (q/k_z)u_{\mathbf{q}} = (q/k_z)(w_{\mathbf{q}} + v_{\mathbf{q}}), \tag{9.36}$$

$$E_{\mathbf{q}}(z=+0) = E_{\mathbf{q}}(z=-0) = v_{\mathbf{q}} + w_{\mathbf{q}}, \tag{9.37}$$

$$B_z(z=+0) = (qc/\omega)(v_{\mathbf{n}} + w_{\mathbf{n}}) = B_z(z=-0) = (qc/\omega)u_{1\mathbf{n}}, \tag{9.38}$$

$$B_{\mathbf{q}}(z=+0) = (k_z c/\omega)(w_{\mathbf{n}} - v_{\mathbf{n}}), \tag{9.39}$$

$$B_{\mathbf{q}}(z=-0) = (k_z c/\omega)u_{1\mathbf{n}}, \tag{9.40}$$

After substituting these expressions into (9.34), we get

$$\sigma = \frac{1}{16\pi^3}\int_0^{+\infty}\mathrm{d}\omega\int \mathrm{d}^2 q q_x \Bigg(\frac{\varepsilon}{\mid k_z \mid^2}\Big[(k_z + k_z^*)\Big(\langle \mid w_{\mathbf{q}} \mid^2 \rangle - \langle \mid v_{\mathbf{q}} \mid^2 \rangle - \\ - \langle \mid v_{\mathbf{q}} + w_{\mathbf{q}} \mid^2 \rangle\Big) + (k_z - k_z^*)\langle \big(v_{\mathbf{q}} w_{\mathbf{q}}^* - v_{\mathbf{q}} w_{\mathbf{q}}^*\big)\rangle\Big] + \\ + \left(\frac{c}{\omega}\right)^2 \Big[(k_z + k_z^*)\Big(\langle \mid w_{\mathbf{n}} \mid^2 \rangle - \langle \mid v_{\mathbf{n}} \mid^2 \rangle - \langle \mid v_{\mathbf{n}} + w_{\mathbf{n}} \mid^2 \rangle\Big) - \\ - (k_z - k_z^*)\langle \big(v_{\mathbf{n}} w_{\mathbf{n}}^* - v_{\mathbf{n}} w_{\mathbf{n}}^*\big)\rangle\Big]\Bigg), \tag{9.41}$$

where we integrate only over positive values of ω, which gives an extra factor of two.

Substituting (9.25) and (9.29) into (9.41) and taking into account that $k_z = k_z^*$ for $q < \omega/c$ and $k_z = -k_z^*$ for $q > \omega/c$, we obtain

$$\sigma = \frac{\hbar}{8\pi^3}\int_0^\infty \mathrm{d}\omega \int_{q<(\frac{\omega}{c})\sqrt{\varepsilon}} \mathrm{d}^2 q q_x \times$$
$$\times\left[\frac{T_{1p}(\omega)T_{2p}(\omega-q_x v)\big(n(\omega-q_x v)-n(\omega)\big)}{\mid 1-\mathrm{e}^{2\mathrm{i}k_z d}R_{1p}(\omega)R_{2p}(\omega-q_x v)\mid^2}-\right.$$
$$\left.-\frac{T_{1p}(\omega)\big(\mid 1-R_{2p}(\omega-q_x v)\mid^2+\mid 1-\mathrm{e}^{2\mathrm{i}k_z d}R_{2p}(\omega-q_x v)\mid^2\big)\big(n(\omega)+1/2)\big)}{\mid 1-\mathrm{e}^{2\mathrm{i}k_z d}R_{1p}(\omega)R_{2p}(\omega-q_x v)\mid^2}\right]+$$
$$+\frac{\hbar}{2\pi^3}\int_0^\infty \mathrm{d}\omega \int_{q>(\frac{\omega}{c})\sqrt{\varepsilon}} \mathrm{d}^2 q q_x \mathrm{e}^{-2|k_z|d}\times$$
$$\times\frac{\mathrm{Im}R_{1p}(\omega)\mathrm{Im}R_{2p}(\omega-q_x v)}{\mid 1-\mathrm{e}^{-2|k_z|d}R_{1p}(\omega)R_{2p}(\omega-q_x v)\mid^2}\big(n(\omega-q_x v)-n(\omega)\big)+$$
$$+\left[p\to s\right], \tag{9.42}$$

$$T_{ip}(\omega) = 1-\mid R_{ip}\mid^2 - \mid 1-R_{ip}\mid^2 = \frac{4\pi \mathrm{Re}\sigma_{il}(\omega)k_z}{\omega\varepsilon|\varepsilon_{il}|^2},$$
$$T_{is}(\omega) = 1-\mid R_{is}\mid^2 - \mid 1-R_{is}\mid^2 = \frac{4\pi \mathrm{Re}\sigma_{it}(\omega)\omega}{k_z c^2|\epsilon_{it}|^2}.$$

The first integral in (9.42) is the contribution to the frictional drag force from propagating electromagnetic waves. The second term in (9.42) is derived from the evanescent field.

When the separation between quantum wells $d \ll \lambda_T$, the contribution to friction from propagating waves can be neglected. In this case, the first integral in (9.42) can be neglected, and the second integral is reduced to (7.30), where $R_i(\omega)$ is the reflection amplitude for layer i. In the random phase approximation, the equations for reflection amplitude are given in Appendix M. For $d < v_F\hbar/k_B T$, the reflection amplitude for the p-polarized electromagnetic waves is given by [13, 117]

$$R_p = 1 + \frac{i\hbar\varepsilon\omega}{2k_F e^2}, \tag{9.43}$$

where ε is the dielectric constant for surrounded dielectric, $k_F = \sqrt{2\pi n_s}$ is the Fermi wavevector and n_s is the electron concentration 2D electron layer. After substituting (9.43) in (10.13) we obtain the contribution to the drag resistivity due to the p-polarized waves

$$\rho_D = \frac{\gamma}{(ne)^2} = \frac{h}{e^2}\frac{\pi\zeta(3)}{32}\left(\frac{k_B T}{\epsilon_F}\right)^2 \frac{1}{(k_F d)^2}\frac{1}{(k_{TF} d)^2}, \tag{9.44}$$

where $q_{TF} = 2a_0^{-1}/\varepsilon$ is the single-layer Tomas-Fermi screening wavevector, $a_0 = \hbar^2/m^* e^2$, and ϵ_F is the Fermi energy. Equation (9.44) is a factor of two larger than the result obtained by Gramila et al. using an approach based on the Boltzmann equation

[113], and approximately a factor of two smaller that the result obtained by Persson and Zhang using a simple model of the van der Waals friction [117].

Figure 9.3 shows the friction coefficient γ as a function of distance d between two quantum wells at $T = 3\,\mathrm{K}$, and with $n_s = 1.5 \times 10^{15}\,\mathrm{m}^{-2}$, $m^* = 0.067\,\mathrm{m}_e$, $v_F = 1.6 \times 10^7\,\mathrm{cm\,s}^{-1}$, and, for the electron mean free path, $l = v_F\tau = 1.21 \times 10^5$ Å. We have also assumed $\varepsilon = 10$, which corresponds to the condition of the experiment [112, 113]. In this case, the s-wave contribution is negligibly small in comparison with the p-wave contribution. For $d = 175$ Å, we find $\gamma = 3.3 \times 10^{-9}$ $\mathrm{kg{\cdot}s^{-1}m^{-2}}$, which corresponds to a drag rate $\tau_D^{-1} = 3.3 \times 10^7\ \mathrm{s}^{-1}$, which is close to the experimental value $(\tau_D^{-1})_{exp} = 1.5 \times 10^7\,\mathrm{s}^{-1}$ [112, 113].

Figure 9.4 shows the friction coefficient for 2D-quantum wells with high electron density $n_s = 10^{19}\,\mathrm{m}^{-2}$, $T = 273\,\mathrm{K}$, $\tau = 4 \times 10^{-14}\,\mathrm{s}$, and $\varepsilon = 1$, where the result for other ε can be obtained using the scaling $\tau_{Dp} \sim \varepsilon^2$ and τ_{Ds} is independent of ε. In Figs. 9.3 and 9.4, the p- and s-wave contributions are shown separately. The

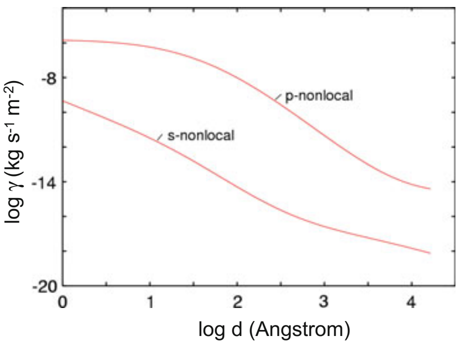

Fig. 9.3 The frictional drag coefficient for two quantum wells at $T = 3$ K as a function of separation d. The s- and p-wave contributions are shown separately. The calculations were performed with surface electron density $n_s = 1.5 \times 10^{15}\,\mathrm{m}^{-2}$, damping constant $\eta = 1.3 \times 10^{10}\,\mathrm{s}^{-1}$, effective electron mass $m^* = 0.067\,m_e$, and dielectric constant $\varepsilon = 10$, which corresponds to the condition of the experiment [112, 113]. (The base of the logarithm is 10.)

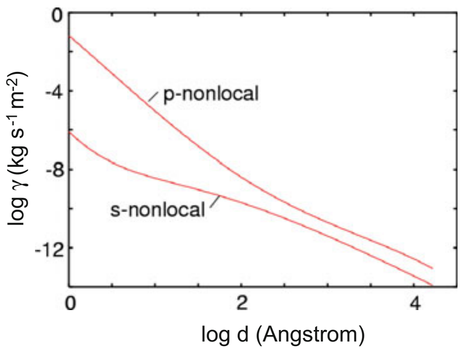

Fig. 9.4 The same as Fig. 9.3 but now for at $T = 273$ K, $n_s = 1.05 \times 10^{19}\,\mathrm{m}^{-2}$, $\eta = 2.5 \times 10^{13}\,\mathrm{s}^{-1}$, $m^* = m_e$, and $\varepsilon = 1$. (The base of the logarithm is 10)

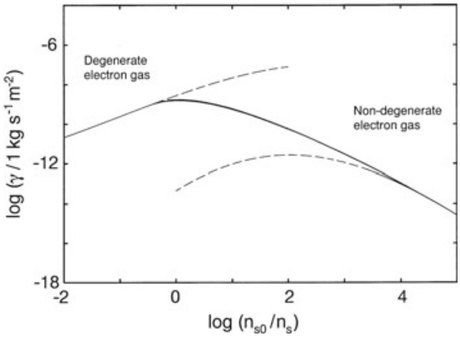

Fig. 9.5 The frictional drag coefficient for two quantum wells at $T = 3$ K as a function of electron concentration n_s. The *full curve* was obtained by interpolation between the curves (*dashed lines*) obtained within the non-local optic dielectric approach, with the dielectric functions corresponding to a degenerate electron gas ($n_s > n_F \sim 10^{14}\,\mathrm{m}^{-2}$), and to the non-degenerate electron gas ($n_s < n_F$). The electron density parameter $n_{s0} = 1.5 \times 10^{15}\,\mathrm{m}^{-2}$, damping constant $\eta = 1.3 \times 10^{10}$ s^{-1}, effective electron mass $m^* = 0.067\,m_e$, separation $d = 175\,\text{Å}$ and the dielectric constant $\varepsilon = 10$. (The base of the logarithm is 10)

calculations show that p-waves give a larger contribution for friction both for low-density and high-density 2D quantum wells.

Figure 9.5 shows the dependence of the friction coefficient on the electron density for the same parameters as in Fig. 9.3. In this case, the boundary between degenerate and non-degenerate electron density is determined by the Fermi density $n_F = 3k_BTm^*/2\pi\hbar^2 = 1.09 \times 10^{14}\,\mathrm{m}^{-2}$. From the calculations, we find the maximum of the frictional drag force for the electron density $n_{max} \approx 1 \times 10^{15}\,\mathrm{m}^{-2}$; this means that the experiment [112, 113] was performed the near optimum conditions.

The friction force per unit charge in the layer is determined by $E = \sigma_{\|}/n_s e$, where n_s is the 2D-electron concentration in the layer. For $v \ll v_F$, where v_F is the Fermi velocity, the friction force depends linearly on the velocity v. For $d = 175\,\text{Å}$ at $T = 3\,\mathrm{K}$, and with $n_s = 1.5 \times 10^{15}\,\mathrm{m}^{-2}$, the electron effective mass $m^* = 0.067\,\mathrm{m}_e$, $v_F = 1.6 \times 10^7\,\mathrm{cm/s}$, the electron mean free path $l = v_F\tau = 1.21 \times 10^5\,\text{Å}$, and $\varepsilon = 10$ (which corresponds to the condition of the experiment [113]) we get $E = 6.5 \times 10^{-6} v\,\mathrm{V/m}$, where the velocity v is in m/s. For a current 200 nA in a 2D layer with the width $w = 20\,\mu\mathrm{m}$, the drift of electrons (drift velocity $v = 60\,\mathrm{m/s}$) creates a frictional drag force per unit charge in the adjacent quantum well $E = 4 \times 10^{-4}\,\mathrm{V/m}$. Note that for the electron systems, the frictional drag force decreases when the electron concentration increases. For a example, for a 2D quantum wells with high electron density ($n_s = 10^{19}\,\mathrm{m}^{-2}$, $T = 273\,\mathrm{K}$, $\tau = 4 \times 10^{-14}\,\mathrm{s}$, $\varepsilon = 10$, $m^* = m_e$) at $d = 175\,\text{Å}$ we get $E = 1.2 \times 10^{-9} v\,\mathrm{V/m}$.

9.4 Casimir Frictional Drag Induced by Liquid Flow in Low-Dimensional Systems

In [262, 263], it was observed that the flow of a liquid over bundles of single-walled carbon nanotubes (SWNT) induces a voltage in the sample along the direction of the flow. Although several mechanisms were proposed to explain this effect [262–266, 268], only one of these mechanisms is related to a fluctuating electromagnetic field created by chaotic Brownian motion of ions in liquid flow. The free carriers in a low-dimensional system will experience frictional drag force due to this electromagnetic field in direction of liquid flow. The intriguing idea of using frictional drag as a non-contact means to detect motion in surrounding liquid was considered in [269].

9.4.1 Casimir Frictional Drag Between Two 2D Systems Induced by Liquid Flow

Consider a fluid with the ions in a narrow channel with thickness d_c. For $d \gg q_D^{-1} \gg d_c$, where $q_D = \sqrt{4\pi N_0 Q^2/\varepsilon_c k_B T}$ is the Debye screening wave number (N_0 is the concentration of ions, and ε_c is the dielectric constant of the liquid in the channel, and Q is the ion charge), the channel can be considered to be 2D. The Fourier transform of the diffusion equation for the ions (of type a) in the channel can be written in the form

$$\frac{i\omega}{D_a}\sigma_q^a = q^2\left(\sigma_q^a + \frac{N_a Q^2 d_c}{k_B T}\varphi_q\right), \tag{9.45}$$

where σ_q^a and ϕ_q are the Fourier components of the surface charge density and the electric potential, respectively, and D_a is the diffusion coefficient of the ions in the liquid in the channel. From (9.45), we get

$$\sigma_q^a = -\frac{N_a Q^2 d_c}{k_B T} q^2 \frac{\varphi_q}{q^2 - i\omega/D_a}. \tag{9.46}$$

The surface current density resulting from the diffusion and drift of the ions of type a, is determined by the formula

$$\begin{aligned} j_{iq}^a &= -iqD\left(\sigma_q^a + N_a Q^2 d_c k_B T \varphi_q\right) = \\ &= -i\omega \frac{N_a Q^2 d_c}{k_B T}\frac{1}{q^2 - i\omega/D_a}E_q, \end{aligned} \tag{9.47}$$

where $E_q = -iq\varphi_q$ is the Fourier component of the electric field. Furthermore, there is a surface current density connected with the polarization of the liquid, which is determined by the formula

$$j_{pq} = -i\omega p_q = -i\omega d_c \frac{\varepsilon - 1}{4\pi} E_q, \tag{9.48}$$

where p_q and ε_c are the surface polarization and dielectric permeability of liquid in the channel, respectively. Thus the total current density $j_q = \sigma(\omega, q)E_q$, where the conductivity of the 2D-liquid is determined by the formula

$$\sigma(\omega, q) = -\frac{i\omega d_c}{4\pi}\left(-1 + \varepsilon_c\left(1 + \sum_a \frac{q_D^2}{q^2 - i\omega/D_a}\right)\right). \tag{9.49}$$

For the (2D electron)–(2D liquid) configuration (with the same parameters as in Sect. 9.3 for electron system (with high electron density)) at $d = 175$ Å, and with $\varepsilon_c = 80$, $N_0 = 10^{24}\,\mathrm{m}^{-3}$, $D = 10^{-9}\,\mathrm{m}^2/\mathrm{s}$, $d_c = 100$ Å we get $E = 4.6 \times 10^{-8} v$ V/m, which is one order of magnitude larger than for the (2D electron)–(2D electron) configuration (with high electron density). Figure 9.6 shows the dependence of the effective electric field in the channel on the velocity of the liquid flow in the adjacent channel, with the same liquid, for the (2D liquid)–(2D liquid) configuration.

Compared on the (2D-electron)–(2D-electron) and (2D-electron)–(2D-liquid) configurations, for the (2D-liquid)–(2D-liquid) configuration the effective electric field is many orders of magnitude larger, and depends nonlinearly on the liquid flow velocity v.

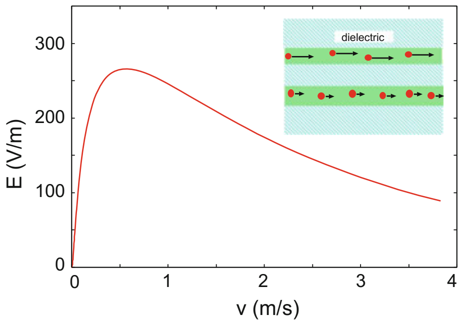

Fig. 9.6 The effective electric field in a 2D channel with liquid as a function of the flow velocity in a second 2D channel for identical liquids in both channels. The temperature $T = 300$ K, the ion concentration in the liquid $N_0 = 10^{24}\,\mathrm{m}^{-3}$, the thickness of the channels $d_c = 100$ Å, the diffusion coefficients of ions $D = 10^{-9}\,\mathrm{m}^2/\mathrm{s}$ and the dielectric constant of the liquid $\varepsilon_c = 80$. The dielectric constant of the dielectric in the gap between channel is $\varepsilon = 10$, and the separation between the channels $d = 175$ Å

9.4.2 Casimir Frictional Drag in a 2D System Induced by Liquid Flow in a Semi-infinite Chamber

Let us consider a 2D electron system, isolated from a semi-infinite liquid flow by a dielectric layer with the thickness d. For the 2D electron system, the reflection amplitude is given in Appendix M. To find the reflection amplitude for the interface between the dielectric and the liquid, we will assume that the liquid fills the half-space $z \geq 0$, and that the half-plane with $z < 0$ is filled by a dielectric with the dielectric constant ε. Let us study the reflection of an electromagnetic wave from the surface of the liquid in the nonretarded limit, which formally corresponds to the limit $c \to \infty$. In the region $z < 0$ the potential can be written in the form

$$\varphi_q = \left(e^{-qz} - Re^{qz}\right)e^{i\mathbf{q}\cdot\mathbf{x}-i\omega t}, \tag{9.50}$$

where q is the magnitude of the component of the wave vector parallel to surface. We will assume that the liquid consists of ions of two types a and b. The equation of continuity for the ions

$$-i\omega n_i + \nabla \cdot \mathbf{J}_i = 0, \tag{9.51}$$

where $i = a, b$, $n_i = N_i - N_0$, where N_i and N_0 are the concentration of ions in the presence and absence of the electric field, respectively. To linear order in the electric field

$$\mathbf{J}_i = -N_0\mu_i Q_i \nabla\varphi - D_i \nabla n_i, \tag{9.52}$$

where D_i is the diffusion coefficient, μ_i is the mobility and Q_i is the charge for ions of type i. The diffusion coefficient and the mobility are related to each other by the Einstein relation: $D_i = k_B T \mu_i$. We consider the case of different ion mobilities differing considerably. In this case, in the calculation of dielectric response it is possible to disregard the diffusion of the less mobile ions. Omitting the index i for the more mobile ions, after substitution of (9.52) in (9.50), we obtain

$$i\omega n + D\Delta\left(n + \frac{N_0 Q}{k_B T}\varphi\right) = 0. \tag{9.53}$$

This equation must be supplemented with the Poisson's equation

$$\nabla^2\varphi = -\frac{4\pi Q n}{\varepsilon_0}, \tag{9.54}$$

where ε_0 is the dielectric permeability of the liquid. The general solution of equations (9.53) and (9.54) can be written in the form

$$\varphi = \left(C_1 e^{-\lambda z} + C_2 e^{-qz}\right)e^{i\mathbf{q}\cdot\mathbf{x}}, \tag{9.55}$$

where $\lambda = \sqrt{q^2 + q_D^2 - i\omega/D}$ and $q_D = \sqrt{4\pi N_0 Q^2/\varepsilon_0 k_B T}$. At the interface ($z = 0$), the electric potential and the normal component of the electric displacement field must be continuous, and the normal component of the flow density must vanish. From these boundary conditions, we obtain

$$C_1 + C_2 = 1 - R, \tag{9.56}$$

$$-\varepsilon_0(\lambda C_1 + qC_2) + \varepsilon q(1 + R) = 0, \tag{9.57}$$

$$i\omega\lambda C_1 + Dq_D^2 qC_2 = 0. \tag{9.58}$$

From (9.56) to (9.58) we get

$$R_{lp} = \frac{\epsilon - 1}{\epsilon + 1}, \tag{9.59}$$

where

$$\epsilon = \frac{\varepsilon_0\lambda\left(Dq_D^2 - i\omega\right)}{\varepsilon\left(Dq_D^2 q - i\omega\lambda\right)}. \tag{9.60}$$

For $v \ll v_F$, the frictional drag force acting on the electrons in the 2D system, due to the interaction with the ions in the liquid, increases linearly with the fluid velocity v. In particular, for $N_0 = 10^{24}\,\mathrm{m}^{-3}$, $T = 273\,\mathrm{K}$, $\varepsilon_0 = 80$, $D = 10^{-9}\,\mathrm{m}^2/\mathrm{s}$, for a high electron density ($n_s = 10^{19}\,\mathrm{m}^{-2}$) in the 2D electron system, $E = 1.4 \times 10^{-6} v$ V/m. This effective electric field is three orders of magnitude larger than that obtained for two 2D electron systems with high electron concentration, and of the same order of magnitude as friction between two 2D-electron systems with a low electron concentration.

Let us replace now the 2D electron structure with the 2D channel with polar liquid. Figure 9.7 shows the dependence of the effective electric field in the 2D-channel on the velocity of the liquid flow in the semi-infinite chamber, for identical liquid in the channel and in the chamber. We have used the same parameters as above for the liquid, with the separation between the channel and chamber $d = 1$ nm. The effective electric field in the channel initially increases with the fluid flow velocity, reaches a maximum, and then decreases. The position of the maximum decreases when the density of ions decreases. The frictional drag force induced by the liquid flow in the narrow channel is nine orders of magnitude larger than the frictional drag force induced in a 2D electron system.

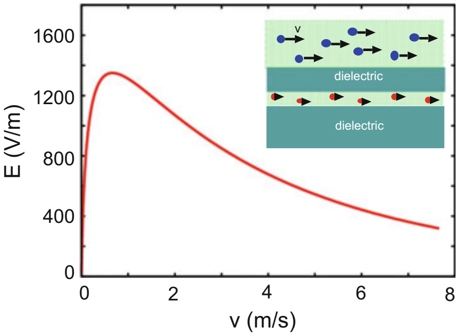

Fig. 9.7 The effective electric field in a 2D channel with liquid induced by liquid flow in a semi infinite chamber as a function of the flow velocity for identical liquids in the channel and in the chamber. The temperature $T = 300$ K, the ion concentration in liquid $N_0 = 10^{24}\,\mathrm{m}^{-3}$, the thickness of the channel $d_c = 10$ nm, the diffusion coefficients of ions $D = 10^{-9}\,\mathrm{m}^2/\mathrm{s}$ and the dielectric constant of the liquid $\varepsilon_0 = 80$. The dielectric constant of the dielectric in the gap between channel $\varepsilon = 10$, and the separation between the channel and semi-infinite chamber $d = 1$ nm

9.4.3 Casimir Frictional Drag in Low Dimensional Structures Induced by Liquid Flow in Infinite System

As a limiting case of the situation considered above, let us consider a 2D system immersed in a flowing liquid in an infinite chamber. We assume that the liquid flows along the x-axis, and that the plane of the 2D system coincides with the xy-plane. Let us calculate first the spectral function of the fluctuations of electric field in the quiescent infinite liquid without the 2D-system. We will examine the non-retarded limit, where only longitudinal fluctuations matters. According to the fluctuation-dissipation theorem (see Sect. 3.1), for an infinite medium the correlation function for the Fourier components of the longitudinal current density is determined by

$$\left|j^f_k\right|^2_\omega = \frac{\hbar}{(2\pi)^2}\left[n(\omega) + 1/2\right]\omega^2 \mathrm{Im}\varepsilon(\omega, k), \tag{9.61}$$

where $\mathbf{k} = (\mathbf{q}, k_z)$ is the wave vector. Using the continuity equation, the longitudinal current density is connected to the charge density via $\rho_k = kj_k/\omega$. Thus,

$$\left|\rho^f_k\right|^2_\omega = \frac{\hbar}{(2\pi)^2}\left[n(\omega) + 1/2\right]k^2 \mathrm{Im}\varepsilon(\omega, k). \tag{9.62}$$

Poisson's equation for the electric potential gives

$$\varphi^f_k = \frac{4\pi\rho_k}{k^2\varepsilon(\omega, k)}. \tag{9.63}$$

In the xy-plane, the **q**-component of the electric potential is determined by

$$\varphi_q^f = \int \frac{dk_z e^{ik_z 0^+}}{2\pi} \varphi_k^f, \tag{9.64}$$

where $k = \sqrt{q^2 + k_z^2}$. Taking into account (9.62) and (9.63) we get

$$\left|\varphi_q^f\right|_\omega^2 = 4\hbar\big(n(\omega) + 1/2\big)\mathrm{Im}\,\Sigma(\omega), \tag{9.65}$$

where

$$\Sigma(\omega, q), = -\int_{-\infty}^{\infty} \frac{dk_z e^{ik_z 0^+}}{2\pi} \frac{1}{k^2\varepsilon(\omega, k)}. \tag{9.66}$$

Taking into account that $E_q = iq\varphi_q$ we get $|E_q^f|_\omega^2 = q^2|\varphi_q^f|_\omega^2$. From the diffusion and Poisson's equations we get

$$\frac{i\omega}{D}\rho_k = k^2\left(\rho_k + \frac{N_0 Q^2}{k_B T}\varphi_k\right), \tag{9.67}$$

$$\varepsilon_0 k^2 \varphi_k = 4\pi\rho_k + 4\pi\rho_k^f. \tag{9.68}$$

From (9.65) and (9.68) we get the dielectric function of the Debye plasma

$$\varepsilon(k) = \varepsilon_0\left(1 + \frac{q_D^2}{k^2 - i\omega/D}\right). \tag{9.69}$$

Substituting (9.69) in (9.66) gives

$$\Sigma(\omega, q) = -\frac{1}{\varepsilon_0\big(q_D^2 - i\omega/D\big)}\left[-\frac{i\omega/D}{2q} + \frac{q_D^2}{2\lambda}\right]. \tag{9.70}$$

According to (9.6), the correlation function for the Fourier components of the fluctuating surface charge density in the 2D system is determined by

$$\left|\tau_q^f\right|_\omega^2 = \frac{\hbar q^2}{\pi\omega}\big(n(\omega) + 1/2\big)\mathrm{Re}\,\sigma(\omega, q). \tag{9.71}$$

If the 2D system is surrounded by liquid flow, the electric field created by the fluctuations of the charge density in the fluid will induce surface charge density fluctuations in the 2D system. The spectral correlation functions (9.65) and (9.71) are determined in the rest reference frame of the liquid, and of the 2D system, respectively. In order to find the connection between the electric fields in the different reference frames, we use the Galileo transformation, which leads to the Doppler frequency shift of the electrical field in the different reference frames. The electric field in the plane of the

2D system, due to the fluctuations of the charge density in the liquid, will take the form

$$E(\mathbf{x},t) = e^{-i(\omega+q_xV)t+i\mathbf{q}\cdot\mathbf{x}}E_q^{\mathrm{I}}, \tag{9.72}$$

where E_q^I is the sum of the electric fields created by the fluctuations of the charge density in the fluid and the induced charge density in the 2D system:

$$E_q^{\mathrm{I}} = E_q^f + 4\pi iq\Sigma(\omega,q)\tau_q^I(\omega^+), \tag{9.73}$$

where $\omega^+ = \omega + q_xV$ and τ_q^I is the surface-induced charge density. According to Ohm's law

$$j_q^I = \sigma_q^+ E_q^I = \sigma_q^+\left(E_q^f + 4\pi iq\Sigma(\omega,q)\tau_q^I(\omega^+)\right), \tag{9.74}$$

where $\sigma_q^+ = \sigma(\omega^+, q)$ is the longitudinal conductivity for the 2D system. The continuity equation for the surface charge density gives $j_q^{ind} = \omega^+\tau_q^{ind}/q$ and from (9.74) we get

$$\tau_q^{\mathrm{I}} = \frac{q}{\omega^+}\frac{\sigma_q^+ E_q^f}{1 - 4\pi iq^2\sigma_q^+/\omega^+\Sigma(\omega,q)} \tag{9.75}$$

and

$$E_q^{\mathrm{I}} = \frac{E_q^f}{1 - 4\pi iq^2\sigma_q^+/\omega^+\Sigma(\omega,q)}. \tag{9.76}$$

In order to find the electric field created by the charge density fluctuations in the 2D system, it is necessary to solve Poisson's equation in the rest reference frame, of the liquid. In this reference frame the charge density takes the form

$$\tau(\mathbf{x},t) = e^{-i(\omega-q_xV)t+i\mathbf{q}\cdot\mathbf{x}}\tau_q^{\mathrm{II}}, \tag{9.77}$$

where the surface charge density is composed from the fluctuating τ^f and induced τ^{ind} charge density: $\tau_q^{II} = \tau_q^f + \tau_q^{ind}$. In the presence of the liquid flow, the electric field in the plane of the 2D system, due to the fluctuating surface charge density, is determined by

$$E_q^{\mathrm{II}} = 4\pi iq\Sigma(\omega^-,q)(\tau_q^{ind} + \tau_q^f), \tag{9.78}$$

where $\omega^- = \omega - q_xV$. From Ohm's law, we get the following expression for the induced charge density

$$\tau_q^{ind} = \frac{4\pi iq^2\sigma(\omega,q)\Sigma(\omega^-,q)}{\omega}\frac{\tau^f}{1 - 4\pi iq^2\sigma_q\Sigma(\omega^-,q)/\omega}. \tag{9.79}$$

Substituting (9.79) in (9.78), we get

$$E_q^{\mathrm{II}} = \frac{4\pi iq\Sigma(\omega^-, q)\tau^f}{1 - 4\pi iq^2\sigma_q\Sigma(\omega^-, q)/\omega} \tag{9.80}$$

and

$$\tau_q^{\mathrm{II}} = \frac{\tau^f}{1 - 4\pi iq^2\sigma_q\Sigma(\omega^-, q)/\omega}. \tag{9.81}$$

The friction force per unit area of the 2D system is given by

$$\sigma_\| = \int_{-\infty}^{\infty} d\omega \int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{q_x}{q}\left\langle E_q\tau_q^*\right\rangle_\omega, \tag{9.82}$$

where $E_q = E_q^{\mathrm{I}} + E_q^{\mathrm{II}}$, $\tau_q = \tau_q^{\mathrm{I}} + \tau_q^{\mathrm{II}}$. Substituting (9.75), (9.76) and (9.80), (9.81) in (9.82) we get

$$\sigma_\| = \frac{2\hbar}{\pi^2}\int_{-\infty}^{\infty} dq_y \int_0^{\infty} dq_x q_x q^2 \Bigg\{ \int_0^{\infty} d\omega \left[n(\omega) - n(\omega + q_x v)\right] \times$$
$$\times \left(\frac{\mathrm{Re}\sigma(\omega + q_x v)\mathrm{Im}\Sigma(\omega, q)}{(\omega + q_x v) \mid 1 - 4\pi iq^2\sigma(\omega + q_x v)\Sigma(\omega, q)/(\omega + q_x v) \mid^2} + (\omega + q_x v \leftrightarrow \omega) \right) -$$
$$- \int_0^{q_x v} d\omega \left[n(\omega) + 1/2\right] \left(\frac{\mathrm{Re}\sigma(\omega - q_x v)\mathrm{Im}\Sigma(\omega, q)}{(\omega - q_x v) \mid 1 - 4\pi iq^2\sigma(\omega - q_x v)\Sigma(\omega, q)/(\omega - q_x v) \mid^2} + \right.$$
$$\left. + (\omega - q_x v \leftrightarrow \omega) \right) \Bigg\}, \tag{9.83}$$

where $(\omega \pm q_x v \leftrightarrow \omega)$ denotes the terms that are obtained from the preceding terms by permutations of the arguments $\omega \pm q_x v$ and ω. With the same parameters used above for the liquid, and for the high density 2D electron system, we get $E = 8.1 \times 10^{-6} v$ V/m. For a 1D-electron system, we obtained a formula that is similar to (9.83).

Figure 9.8 shows the result of the calculations of the effective electric field for a 1D-electron system with the electron density per unit length $n_l = 3 \times 10^9\,\mathrm{m}^{-1}$, the temperature $T = 300\,\mathrm{K}$, and with the same parameters for the liquid as used above. For the 1D-electron system, we obtained a slight deviation from the linear dependence of the frictional drag on the liquid flow velocity. The frictional drag for the 1D-electron system is one order of magnitude larger than for the 2D electron system.

Figure 9.9 shows the dependence of the effective electric field in the liquid in the 2D channel on the liquid flow velocity in the infinite chamber, assuming identical liquid in the channel and in the chamber. Qualitatively, we obtained the same results for a 1D channel.

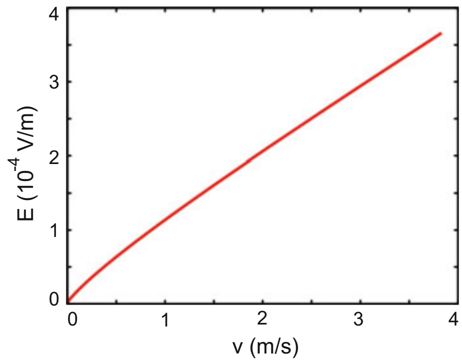

Fig. 9.8 The effective electric field in a 1D-electron system induced by liquid flow in a infinite chamber, as a function of the flow velocity. For the same parameters for the liquid as in Fig. 9.7. The electron concentration per unit length in the 1D-system $n_l = 3 \times 10^9\,\mathrm{m}^{-1}$, and the electron relaxation time $\tau = 4 \times 10^{-14}\,\mathrm{s}$

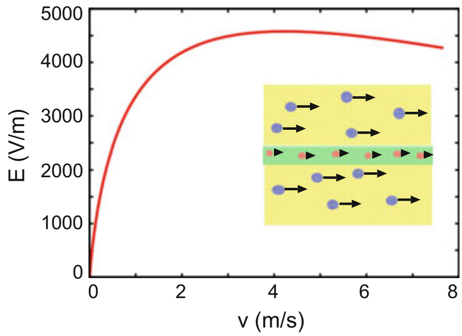

Fig. 9.9 The same as Fig. 9.7 but for infinite chamber

For a channel with open ends, the frictional drag force will induce a drift motion of the ions in the liquid with the velocity $v_d = D_c QE/k_B T$. The positive and negative ions will drift in the same direction. If ions have different mobility then the drifting ions will lead to an electric current whose direction will be determined by the current created by the ions with the largest mobility. For a channel with closed ends, the frictional drag force will lead to a change in ion concentration along the channel. In the case of ions with the different mobilities, the friction force will be different for the ions with the opposite charges. As a result, the ions of opposite charges will be characterized by different distribution functions, which, as for electronic systems, will result in an electric field and an induced voltage that can be measured. Let us write the friction force acting on the ions of different type in the form: $F_a = QE_a$ and $F_b = QE_b$. From the condition that, in the static case, the flux density in the channel must vanish, we get

$$n_a = -\frac{Q}{k_B T}(\varphi - E_a x)\,, \tag{9.84}$$

$$n_b = \frac{Q}{k_B T}(\varphi + E_a x)\,. \tag{9.85}$$

These equations must be supplemented with Poisson's equation

$$\frac{d^2\varphi}{d^2x} = -\frac{4\pi Q}{\varepsilon_c}(n_a - n_b). \tag{9.86}$$

Substituting (9.84) and (9.85) in (9.86), we get

$$\frac{d^2\varphi}{d^2x} = q_D^2\left(2\varphi - \Delta E x\right), \tag{9.87}$$

where $\Delta E = E_a - E_b$. The solution of (9.87) with boundary condition

$$\left.\frac{d\varphi}{dx}\right|_{x=\pm L/2} = 0, \tag{9.88}$$

where L is the channel length, has the form

$$\varphi(x) = \frac{\Delta E}{2}\left(x - \frac{1}{\sqrt{2}q_D}\frac{\sinh\sqrt{2}q_D x}{\cosh\sqrt{2}q_D L/2}\right). \tag{9.89}$$

The voltage between the ends of the channel is determined by

$$U = \varphi(L/2) - \varphi(-L/2) = \Delta E\left(\frac{L}{2} - \frac{1}{\sqrt{2}q_D}\tanh\sqrt{2}q_D L/2\right). \tag{9.90}$$

For $q_D L \gg 1$, the voltage, which appears as a result of the frictional drag, will be approximately equal to $U \approx \Delta E L/2$. Furthermore, the frictional drag will induce a pressure difference $\Delta p = nLeE$. For example, if $N_0 = 10^{24}\,\mathrm{m}^{-3}$, $L = 100\,\mu\mathrm{m}$ and $E = 1000\,\mathrm{V/m}$, we get the pressure difference $\Delta p = 10^4$ Pa, which should be easy to measure. Assume now that one type of ions are fixed (adsorbed) on the walls of the channel and an equal number of mobile ions of opposite sign are distributed in the liquid phase. In this case, the motion of the polar liquid in the adjacent region will lead to frictional drag force acting on the mobile ions in the channel. For a channel with closed ends, this frictional drag will induce a voltage, which can be measured.