Chapter 9
Summary and Concluding Remarks

9.1 Introduction

The previous chapters have developed several probabilistic approaches for geotechnical site characterization and slope stability analysis, including a general Bayesian framework for geotechnical site characterization, a subjective probability assessment approach for determining prior distribution, an equivalent sample approach using limited site observation data, a Bayesian approach using a relatively large number of test data, a probabilistic slope stability analysis approach, and a probabilistic failure analysis approach. This chapter summarizes the major conclusions drawn from previous chapters and provides some recommendations for future studies.

9.2 Uncertainty Propagation During Geotechnical Site Characterization

This book revisited geotechnical site characterization from an uncertainty propagation point of view. Geotechnical site characterization was divided into six stages as follows: desk study, site reconnaissance, in situ investigation, laboratory testing, interpretation of site observation data, and inferring soil properties and underground stratigraphy. Desk study and site reconnaissance provide prior knowledge (i.e., site information available prior to the project) about the site. The prior knowledge is not perfect information but is combined with some uncertainties, such as inherent spatial variability and measurement errors in existing data and uncertainties in engineers' expertise. Then, project-specific test data can be obtained from in situ investigation work and laboratory testing but it fluctuates because of inherent spatial variability of soils, statistical uncertainty, and measurement errors. These uncertainties together with the transformation uncertainty are incorporated into the

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interpretation outcomes obtained from site observation data using a transformation model. Geotechnical engineers use both interpretation outcomes of site observation data and prior knowledge to estimate soil properties and underground stratigraphy for geotechnical analysis and/or designs. Estimations of soil properties and underground stratigraphy are, therefore, affected by both uncertainties in prior knowledge and uncertainties (i.e., inherent spatial variability of soils, statistical uncertainty, measurement errors, and transformation uncertainty) in interpretation outcomes of project-specific test data. These uncertainties are taken into account rationally by the Bayesian framework developed in this book, as discussed in the next section.

9.3 Bayesian Framework for Geotechnical Site Characterization

A Bayesian framework was developed for geotechnical site characterization, which integrates systematically prior knowledge and site observation data to characterize probabilistically soil properties and boundaries of statistically homogenous soil layers. The Bayesian framework addresses directly the inherent spatial variability of the design soil property and models explicitly the transformation uncertainty associated with the transformation model. In addition, statistical uncertainty and measurement errors are incorporated into the Bayesian framework through site observation data. Based on the Bayesian framework, the most probable number of statistically homogenous soil layers is then determined through a Bayesian model class selection method. It is also noted that the Bayesian framework provides a rational vehicle to accumulate the knowledge of soil properties progressively as the site observation data increase.

The proposed Bayesian framework is general and equally applicable for different types of prior knowledge and different numbers of site observation data. It was applied to combine prior knowledge and sparse standard penetration test (SPT) data for probabilistic characterization of Young's modulus and to integrate prior knowledge with a relatively large number of cone penetration test (CPT) data for probabilistic characterization of effective friction angle, as discussed in Sects. 9.5 and 9.6, respectively.

9.4 Prior Knowledge and Prior Distribution

Under the Bayesian framework, the information provided by prior knowledge is quantitatively reflected by prior distribution in a probabilistic manner. As mentioned above, the Bayesian framework is equally applicable for different types of prior knowledge. When only a typical range of the soil parameter concerned is available as prior knowledge, a uniform prior distribution of the soil parameter that covers the typical range can be used in the Bayesian framework. As the information provided by prior knowledge improves, a more sophisticated and informative prior distribution can be estimated from the prior knowledge.

Based on a stage cognitive model of engineers' cognitive process, a subjective probability assessment approach was developed to estimate prior distribution from prior knowledge. The subjective probability assessment approach assists engineers in utilizing prior knowledge in a relatively rational way and expressing quantitatively their engineering judgments in a probabilistic manner. The assessment outcomes obtained from the proposed approach are then taken as the prior distribution in the Bayesian framework. The proposed subjective probability assessment approach consists of five steps as follows: specification of assessment objectives, collection of relevant information and preliminary estimation, synthesis of the evidence, numerical assignment, and confirmation of assessment outcomes. Several suggestions were provided for each step to assist engineers in reducing the effects of cognitive biases and limitations during subjective probability assessment.

The proposed subjective probability assessment approach was illustrated under two scenarios as follows: one with sparse prior knowledge and the other with a reasonable amount of prior knowledge. When prior knowledge is sparse, the prior distribution obtained from the proposed approach is relatively uninformative (e.g., uniform distributions). As the information provided by prior knowledge improves, the proposed approach provides informative prior distribution. The prior distribution obtained from the subjective probability assessment approach quantifies properly the information provided by prior knowledge and is readily used in the Bayesian framework.

9.5 Probabilistic Characterization of Young's Modulus Using SPT

The number of project-specific test results is generally too sparse to generate meaningful statistics (i.e., mean, standard deviation, and other high order statistics) of soil properties, particularly in projects with medium or relatively small sizes. For this case, a Markov Chain Monte Carlo simulation (MCMCS)-based approach (i.e., the equivalent sample approach) was developed for probabilistic characterization of soil properties. The proposed approach is equally applicable for various soil properties and different types of in situ or laboratory tests. As an illustration, the proposed approach was formulated for probabilistic characterization of the undrained Young's modulus E_u using SPT. Project-specific SPT data and prior knowledge are integrated probabilistically under the Bayesian framework developed in this book and are transformed into a large number, as many as needed, of equivalent samples of E_u. Then, conventional statistical analysis is carried out to estimate statistics of E_u. This allows a proper selection of characteristic value of the soil property in implementation of probabilistic design codes (e.g., Eurocode 7) and reliability analysis in geotechnical engineering practice. The equivalent sample approach effectively tackles the difficulty in generating meaningful statistics from the usually limited number of soil property data obtained during geotechnical site characterization.

Equations were derived for the proposed equivalent sample approach, and the proposed approach was illustrated and validated using real SPT data and simulated SPT data. It has been shown that based on the limited SPT data and relatively uninformative prior knowledge (i.e., reasonable ranges of soil parameters reported in the literature), the equivalent sample approach provides reasonable estimates of statistics and probability distribution of E_u. Such probabilistic characterization is used to require a large number of data from laboratory and/or in situ tests (e.g., pressure meter tests), which of course involve significant commitment of cost, man power, and time.

It is also noted that results of the equivalent sample approach are affected by both the number of project-specific test results and prior knowledge. When only limited project-specific test data is available, the equivalent sample approach improves significantly the probabilistic characterization of soil properties and reduces the effects of statistical uncertainty by incorporating reasonable ranges of soil parameters as prior knowledge. As the number of project-specific test data increases, the standard deviation of soil properties estimated from the equivalent sample approach gradually approaches its true value and mainly reflects the inherent variability itself. The proposed approach is general and applicable for different types of prior knowledge, although using relatively informative and consistent prior knowledge does improve the probabilistic characterization of soil properties.

9.6 Probabilistic Site Characterization Using CPT

When a large number of site observation data can be obtained directly from project-specific tests (e.g., near-continuous measurements during a cone penetration test (CPT)), the inherent spatial variability of soil properties can be explicitly modeled using the random field theory. In such a case, a Bayesian approach was proposed for probabilistic site characterization using the Bayesian framework developed in this book and the random field theory. The proposed Bayesian approach was formulated for probabilistic characterization of effective friction angle ϕ' using CPT. Project-specific CPT data and prior knowledge are integrated probabilistically under the Bayesian framework. The Bayesian approach addresses explicitly the inherent spatial variability of ϕ' using random field theory. It contains two major components as follows: a Bayesian model class selection method to identify the most probable number of soil layers and a Bayesian system identification method to estimate the most probable layer thicknesses/boundaries and soil properties simultaneously.

Equations were derived for the Bayesian approach, and the proposed approach was illustrated and validated using real CPT data and simulated CPT data. It has been shown that the proposed Bayesian approach correctly identifies the number and thicknesses/boundaries of the statistically homogenous soil layers and provides proper probabilistic characterization of soil properties. In addition, as the number of model classes increases, the Bayesian model class selection method identifies the statistically homogenous layers progressively, starting from the most statistically significant boundary and gradually "zooming" into local difference with improved "resolution". The Bayesian approach also contains a mechanism to determine when to stop further increasing the number of model class (i.e., the "zooming").

Furthermore, it is also found that results of the Bayesian approach are affected by the quality of both prior knowledge and project-specific test data. It is always prudent to rely more on the high quality project-specific test data, if available, and to start the Bayesian approach with relatively uninformative prior knowledge (i.e., low confidence level), particularly when the prior knowledge is not well justified.

9.7 Probabilistic Slope Stability Analysis

Inherent spatial variability of soils and various uncertainties arising during geotechnical site characterization affect probabilistic estimations of soil properties and underground stratigraphy, which subsequently influence probabilistic analysis and/or designs of geotechnical structures, such as probabilistic slope stability analysis. Monte Carlo simulation (MCS) provides a robust and conceptually simple way to account rationally for these uncertainties (including inherent spatial variability of soils).

A MCS-based probabilistic slope stability analysis approach was developed using an advanced MCS method called "subset simulation" in a commonly available spreadsheet environment, Microsoft Excel, with the aid of Visual Basic for Application (VBA). Excel worksheets and VBA functions/Add-In for deterministic slope stability analysis are deliberately decoupled from those for reliability analysis (e.g., random sample generations and statistical analysis) so that the reliability analysis can proceed as an extension of deterministic analysis in a non-intrusive manner. The Excel spreadsheet package was used to assess reliability of short-term stability of a cohesive soil slope, followed by a comparative study on different reliability methods, including the first-order second-moment method (FOSM), first-order reliability method (FORM), direct MCS using commercial software Slope/W, and direct MCS and subset simulation using the Excel package.

It has been shown that the MCS-based probabilistic slope stability analysis approach significantly improves the efficiency and resolution at relatively small probability levels. With the aid of improved efficiency, the MCS-based probabilistic slope stability analysis approach was used to explore the effects of inherent spatial variability of soil properties and the critical slip surface uncertainty. It is found that when the inherent spatial variability of soil properties is ignored by assuming perfect correlation, the variance of factor of safety (*FS*) is overestimated. Such overestimation of the *FS* variance may result in either overestimation (conservative) or underestimation (unconservative) of slope failure probability P_f. It is also noted that when the inherent spatial variability of soil properties is considered, the critical slip surface varies spatially. Using only one given critical slip surface significantly underestimates P_f, and it is unconservative. Thus, when the inherent spatial variability of soil properties is considered, the critical slip surface uncertainty should be properly accounted for.

9.8 Probabilistic Failure Analysis of Slope Stability

Based on failure samples generated in MCS, a probabilistic failure analysis approach was developed to shed light on relative contributions of various uncertainties to slope failure probability. The probabilistic failure analysis approach contains two major components as follows: hypothesis tests for prioritizing effects of various uncertainties and Bayesian analysis for further quantifying their effects.

A hypothesis test statistic was formulated to evaluate the statistical difference between failure samples and their respective nominal (unconditional) samples. The absolute value of the hypothesis test statistic is used as an index to measure the effects of the uncertain parameters on failure probability and to prioritize their relative effects on failure probability. A Bayesian analysis approach was developed to further quantify effects of the uncertain parameters that have been identified from the hypothesis tests as influential parameters. Equations were derived for the Bayesian analysis to estimate conditional failure probability of slope stability for a given value of an uncertain parameter. The resolution of the conditional failure probability obtained from the Bayesian analysis relies on the number of failure samples generated in MCS. Subset simulation was employed to improve efficiency of generating failure samples in MCS and resolution at small failure probability levels.

The probabilistic failure analysis approach was illustrated through a case study, and it was validated by independent sensitivity studies using repeated runs of MCS. It has been shown that the effects of various uncertainties on slope failure probability are properly prioritized and quantified by the proposed approach. The proposed failure analysis approach gives results equivalent to those from sensitivity studies, and hence, saves additional computational time and efforts for sensitivity studies. In addition, a cross-check between the hypothesis test results and the Bayesian analysis results showed that they agree well with each other.