

Stackelberg Game-Based Dynamic Spectrum Access Scheme in Heterogeneous Network

Yunfeng Liao, Yong Chen, Aiwei Sun and Jianzhao Zhang

Abstract In this paper, we consider a problem of dynamic spectrum access in a heterogeneous network with spectrum database-assisted where spectrum operators (SOs) provide licensed spectrum and shared spectrum to secondary users (SUs) for maximizing their revenue. SUs can select a shared spectrum to transmit data with low price, but the quality of service (QoS) could be influenced by activities of primary users (PUs). SUs can also select licensed spectrum with high price for satisfactory QoS. We use the Stackelberg game to analyze the economic behavior of SUs and the optimal revenue of operators, respectively. Furthermore, we propose a price compensation scheme (PCS) to enhance the utility of SUs when PUs appear. Numerical results verify that the high activities of PUs can motivate SUs to purchase licensed spectrum and maximize revenue of spectrum operators. The proposed scheme could also enhance the utility of SUs several times than no compensation when channel condition is getting worse.

Keywords Dynamic spectrum access · Price compensation · Spectrum leasing · Cognitive radio

1 Introduction

With the concept of cognitive radio emerging, it is likely that the scarce spectrum resource could be utilized efficiently. With the development of research work, scientists find the existence of spectrum holes and propose a novel approach to access idle spectrum, Dynamic Spectrum Access [1]. In a typical dynamic spectrum

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access network, SUs are allowed to opportunistically exploit the unoccupied spectrum resource and utilize the idle channel to transmit data but not to cause harmful interference to PUs. However, the main challenge is how to exploit the idle spectrum efficiently with little consumption for SUs.

The most former researches are focused on spectrum sensing and channel estimation. However, a recent study shows that sensing is not an efficient approach since it has to pay high cost for the unsatisfied sensing performance. As an alternative, the Federal Communications Commission suggests to use geo-location database to obtain spectrum information [2]. With the assist of database, unlicensed device could get spectrum information instead of sensing wireless radio environment. In [3], Luo et al. investigate the white space ecosystem, and study the equilibrium behavior of secondary network operators. Since spectrum availability is determined by PUs' activities and changeful radio environment, Liu et al. [4] propose a joint local sensing and database scheme to confirm specific condition of channel and guarantee reliability for exploiting spectrum holes. Considering the stochastic and heterogeneous nature of SUs' demands, Jiang et al. [5] solve the revenue maximization of SOs with joint pricing of spectrum resources and admission control. The secondary market is a promising approach to provide different spectrum for variety of QoS demands [6]. In [7], the authors study the interaction between single primary spectrum owner and multiple unlicensed SUs, and design an optimal contract to maximize their profit, respectively. With the assist of spectrum database, SOs could design mechanism to provide shared spectrum for unlicensed users (such as SUs) with suitable price to satisfy their demands. However, consider the changeful wireless environment; it is not realistic that there is enough available spectrum resource to lease. Thus, operators would like to lease licensed spectrum from licensed users (such as PUs) with high price.

However, few papers have considered the activities of PUs in a dynamic access network with database-assisted. In that case, we use the Stackelberg game to investigate the interaction between SOs and SUs in a heterogeneous network where PUs would appear with stochastic probability. Furthermore, we study SUs' economic behavior with primary activities, and propose a price compensation scheme which could alleviate the loss of SUs when channel condition is getting worse.

The rest of this paper is organized as follows. In Sect. 2, we discuss the system model. And the problem analysis is given in Sect. 3. Section 4 provides the simulation results. Finally, Sect. 5 concludes the paper.

2 System Model

We consider a heterogeneous dynamic spectrum access network with database-assisted where SOs could provide two types of spectrum, the licensed spectrum and shared spectrum, for SUs. The licensed spectrum is owned by PU^L s who would like to lease portion of spectrum to SOs in exchange for additional reward, and the channel quality could be guaranteed. The shared spectrum owned by PU^S s who do

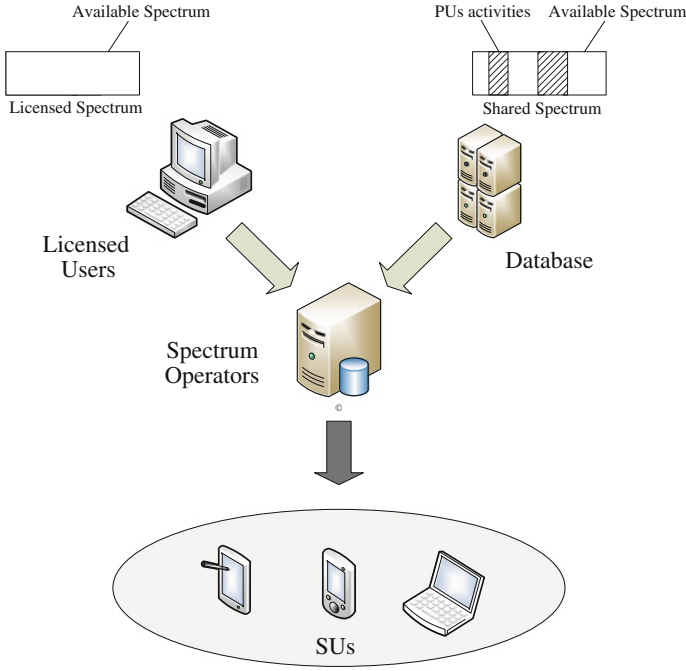


Fig. 1 System Model

not lease spectrum could be used by everyone if PU^S s do not occupy the channel, but the channel quality could not be guaranteed for PU^S s' activities. The spectrum database real-timely updates shared spectrum information from wireless environment and provides it to SOs. The SOs make corresponding licensed price according to the shared channel-occupied probability obtained from the database to attract more SUs for maximizing their revenue.

The system model is described in Fig. 1. We formulate their interaction with a three-stage Stackelberg game to investigate their optimal profit, respectively. In a duration T , PU^L s charge operators with price ρ_0 first, then operators determine their leasing bandwidth B in stage one, but the maximum leasing bandwidth of PU^L s provided is B^{\max} . Operators announce the licensed spectrum price ρ_l and shared spectrum price ρ_s to SUs in stage two. Simultaneously, operators send the channel-occupied probability obtained from database to SUs. According to the shared spectrum information and the two types of price, SU_i determines the licensed spectrum fraction $\theta_i \in [0, \theta]$ of the total bandwidth in stage three. It is obvious that the licensed bandwidth is finite, and SUs are willing to select shared spectrum if the price of licensed spectrum is too high. In that case, SOs have to make an optimal price strategy to motivate more SUs to purchase licensed spectrum for maximizing revenue.

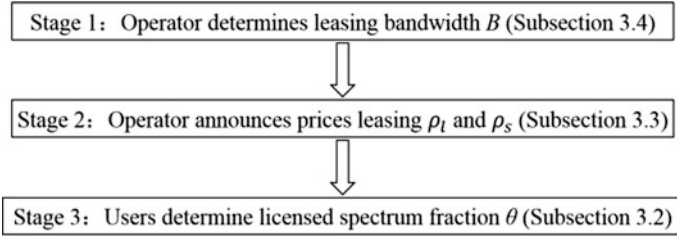


Fig. 2 Three-stage Stackelberg game

In this paper, we assume that there is one spectrum operator, and N number of SUs, $\mathcal{N} = \{1, 2, \dots, N\}$. The spectrum efficiency is denoted by η (Mbps/MHz), the traffic demand of SU_i is D_i and $\theta = \min\{\frac{\eta B}{D}, 1\}$ [8]. $D = \sum_{i=1}^N D_i$ is sum of all SUs' demands. Considering the activities of PUSs in shared spectrum, we use φ to denote channel-occupied probability.

3 Problem Formulation and Analysis

To investigate the optimal profit of SO and SUs, we formulate their interaction as a three-stage Stackelberg game as shown in Fig. 2. In this section, we will describe the price compensation scheme first and then analyze their optimal profit and PUS' impact on SUs' economic behavior with backward induction method.

3.1 The Price Compensation Scheme

It is worth noting that the utility function of SU_i is increasing with D_i and more SUs would like to select licensed spectrum with the θ_i increasing because the worse QoS on shared spectrum could not satisfy users' traffic demands.

The utility function of SU_i could be defined as follows [8]:

$$u_i = D_i \left[\alpha - \varphi e^{\beta(1-\theta_i)} \right] - [(1-\varphi)\rho_s(1-\theta_i)D_i + \rho_l\theta_i D_i] \quad (1)$$

The first term is the profit of SU_i and the second term is the corresponding cost. Considering the QoS influenced by primary activities at shared spectrum, it is not reasonable that the shared spectrum price is ρ_s . So, the operator should adjust the price to $(1-\varphi)\rho_s$ as a compensation for alleviating the loss of SUs. It is obvious that the shared spectrum price is decreasing with channel-occupied probability and the detail analysis is presented in Sect. 4.

3.2 The Optimal Fraction in Stage Three

In stage three, SUs need to make the decision of how much to purchase. If a user SU_i purchase spectrum from operator, then its utility function u_i is given in (1). The optimal fraction of licensed spectrum that maximizes the profit of SU_i is

$$\theta_i^* = \arg \max_{\theta_i \in [0, \theta]} u_i(\theta_i, \rho_l) \quad (2)$$

Then we will characterize the optimization problem as follows:

$$\begin{aligned} \text{(P1)} \quad \max \quad & u_i = D_i \left[\alpha - \varphi e^{\beta(1-\theta_i)} \right] - [(1-\varphi)\rho_s(1-\theta_i)D_i + \rho_l\theta_i D_i] \\ \text{s.t.} \quad & 0 \leq \theta_i \leq \theta \end{aligned} \quad (3)$$

Lemma 1 *The (P1) is a convex optimization problem with $\{\theta_i\}$.*

Proof It is clear that the Hessian matrix of function of u_i is negative, and constraints of (P1) are affine functions, so the (P1) is a convex problem [9].

Using the Lagrangian method, the Lagrangian function of u_i is

$$L(\theta_i, \lambda) = D_i \left[\alpha - \varphi e^{\beta(1-\theta_i)} \right] - [(1-\varphi)\rho_s(1-\theta_i)D_i + \rho_l\theta_i D_i] + \lambda(\theta_i - \theta) \quad (4)$$

with the KKT conditions [9], we could get the optimal fraction

$$\theta_i^* = \begin{cases} 1 - \frac{1}{\beta} \ln \frac{\rho_l - \rho_s(1-\varphi)}{\beta\varphi}, & \lambda = 0 \\ \theta, & \lambda \neq 0 \end{cases} \quad (5)$$

because of the $\theta_i \in [0, \theta]$, after algebra steps, we have

$$\rho_s(1-\varphi) + \beta\varphi e^{(1-\theta)\beta} \leq \rho_l < \rho_s(1-\varphi) + \beta\varphi e^\beta \quad (6)$$

3.3 The Optimal Price in Stage Two

In stage two, the operator will make the optimal price strategies to maximize its revenue according to the demands in stage three. From the analysis in stage three, the utility function of SO could be presented as

$$U_{SO} = \begin{cases} \sum_{i=1}^N (1-\phi)D_i\rho_s - \rho_0B & \rho_l \geq \rho^H \\ \sum_{i=1}^N \left[\frac{(1-\phi)\rho_s - \rho_l}{\beta} D_i \ln \frac{\rho_l - (1-\phi)\rho_s}{\beta\phi} + \rho_l D_i \right] - \rho_0B & \rho^L \leq \rho_l < \rho^H \\ \sum_{i=1}^N [(1-\phi)D_i\rho_s(1-\theta) + \rho_l\theta D_i] - \rho_0B & \rho_l < \rho^L \end{cases} \quad (7)$$

when $\rho_l \geq \rho^H$, the revenue of PU^Ls is zero, this situation should be ignored.

When $\rho^L \leq \rho_l < \rho^H$, we have the optimization problem as:

$$(P2.1) \quad \max \quad U_{SO}^1 = \frac{(1-\phi)\rho_s - \rho_l}{\beta} D \ln \frac{\rho_l - (1-\phi)\rho_s}{\beta\phi} + \rho_l D - \rho_0B \quad (8)$$

$$s.t. \quad \rho^L \leq \rho_l < \rho^H$$

(P2.1) is a convex problem as (P1). Using the same method, let the Lagrange multiplier equal to 0, the optimal licensed price is

$$\rho_l^* = \rho_s(1-\phi) + \beta\phi e^{\beta-1} \quad (9)$$

when $\rho_l < \rho^L$, the optimization problem is

$$(P2.2) \quad \max \quad U_{SO}^2 = (1-\phi)\rho_s D(1-\theta) + \rho_l\theta D - \rho_0B \quad (10)$$

$$s.t. \quad \rho_l < \rho^L$$

Similarly, the optimal solution of problem (P2.2) is

$$\rho_l^* = \rho_s(1-\phi) + \beta\phi e^{(1-\theta)\beta} \quad (11)$$

It is obvious that $\rho_s(1-\phi) + \beta\phi e^{\beta-1} > \rho_s(1-\phi) + \beta\phi e^{(1-\theta)\beta}$, so the fraction $\theta > \frac{1}{\beta}$. From the analysis above, we substitute (9) and (11) into (5). When $\theta \leq \frac{1}{\beta}$,

$$\begin{aligned} \rho_l^* &= \rho_s(1-\phi) + \beta\phi e^{(1-\theta)\beta} \\ U_{SO} &= (1-\phi)\rho_s D + \theta D\beta\phi e^{\beta(1-\theta)} - \rho_0B \\ \theta^* &= \theta \end{aligned} \quad (12)$$

when $\theta > \frac{1}{\beta}$,

$$\begin{aligned} \rho_l^* &= \rho_s(1-\phi) + \beta\phi e^{\beta-1} \\ U_{SO} &= (1-\phi)\rho_s D + D\phi e^{\beta-1} - \rho_0B \\ \theta^* &= \frac{1}{\beta} \end{aligned} \quad (13)$$

3.4 The Optimal Leasing Bandwidth in Stage One

In stage one, operator will determine its leasing bandwidth B , but the maximum leasing bandwidth that PU^{L} s could provide is B^{max} . It means that operator could not increase the leasing bandwidth infinitely. According the analysis in stage two, the utility function of SO could be presented as

$$U_{SO} = \begin{cases} (1 - \varphi)\rho_s D + \theta D\beta e^{\beta(1-\theta)} - \rho_0 B & \theta \leq \frac{1}{\beta} \\ \varphi D e^{\beta-1} + D\rho_s(1 - \varphi) - \rho_0 B & \theta > \frac{1}{\beta} \end{cases} \quad (14)$$

Since $\theta = \min\{\frac{\eta B}{D}, 1\}$, the utility function can be derived as follows:

$$U_{SO} = \begin{cases} (1 - \varphi)\rho_s D + \eta B\beta\varphi e^{\beta(1-\frac{\eta B}{D})} - \rho_0 B & B \leq \frac{D}{\eta\beta} \\ \varphi D e^{\beta-1} + D\rho_s(1 - \varphi) - \rho_0 B & B > \frac{D}{\eta\beta} \end{cases} \quad (15)$$

When $B \leq \frac{D}{\eta\beta}$, we have optimization problem as:

$$(P3) \quad \max \quad U_{SO} = \rho_s D(1 - \varphi) + \eta B\beta\varphi e^{\beta(1-\frac{\eta B}{D})} - \rho_0 B \\ \text{s.t.} \quad 0 \leq B \leq B^{\text{max}} \quad (16)$$

The (P3) is a convex optimization problem for the same character as (P1), we could get the Lagrangian function as

$$L(B, \lambda) = \rho_s D(1 - \varphi) + \eta B\beta\varphi e^{\beta(1-\frac{\eta B}{D})} - \rho_0 B + \lambda(B - B^{\text{max}}) \quad (17)$$

Giving the KKT conditions as follows:

$$\begin{aligned} \frac{\partial L}{\partial B} &= -\rho_0 + \eta\beta\varphi e^{\beta(1-\frac{\eta B}{D})} + \eta\beta B\varphi e^{\beta(-\frac{\eta B}{D})} \left(-\frac{\beta\eta}{D}\right) + \lambda = 0 \\ \lambda(B - B^{\text{max}}) &= 0 \\ \lambda &\geq 0 \end{aligned} \quad (18)$$

if $\lambda = 0$, substituting it to (18), we have

$$\eta\beta\varphi e^{\beta(1-\frac{\eta B}{D})} + \eta\beta B\varphi e^{\beta(-\frac{\eta B}{D})} \left(-\frac{\beta\eta}{D}\right) = \rho_0 \quad (19)$$

$$\left(1 - \frac{B\beta\eta}{D}\right) e^{1-\frac{\beta\eta B}{D}} = \frac{\rho_0}{\eta B\varphi e^{\beta-1}} \quad (20)$$

with the *Lambert W* Function [10],

$$1 - \frac{B\beta\eta}{D} = W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right) \quad (21)$$

$$B^* = \frac{D}{\eta\beta} \left(1 - W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right)\right), W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right) \in [0, 1] \quad (22)$$

if $\lambda \neq 0$,

$$B^* = B^{\max} \quad (23)$$

Thus, the optimal leasing bandwidth is

$$B^* = \min\left\{B^{\max}, \frac{D}{\eta\beta} \left(1 - W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right)\right)\right\} \quad (24)$$

when $B > \frac{D}{\eta\beta}$, from (15), utility function $\varphi D e^{\beta-1} + D\rho_s(1-\varphi) - \rho_0 B$ increases with B , the $B^* = \frac{D}{\eta\beta}$.

Considering $W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right) \in [0, 1]$, we obtain the optimal leasing bandwidth as

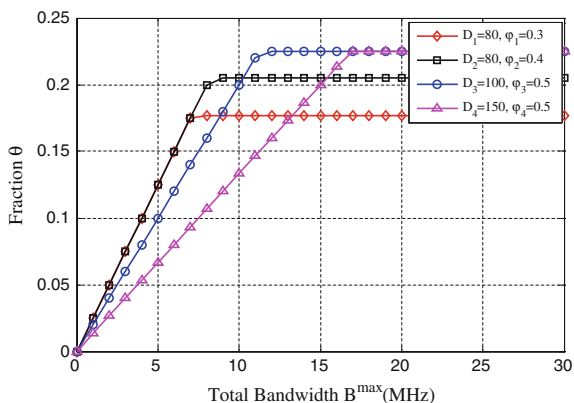
$$B^* = \min\left\{B^{\max}, \frac{D}{\beta\eta} \left(1 - W\left(\frac{\rho_0}{\eta\beta\varphi e^{\beta-1}}\right)\right)\right\} \quad (25)$$

4 Simulations

In this section, we analyze the numerical results to illustrate the performance of the system. In the simulation, we will find that profit of SUs and SO could achieve equilibrium solution. And the activities of primary users will impact secondary users' interest on licensed spectrum.

In this network, there are four SUs groups, the total traffic demands are $D_1 = 80, D_2 = 80, D_3 = 100, D_4 = 150$. When they access shared spectrum, the channel-occupied probability are $\varphi_1 = 0.3, \varphi_2 = 0.4, \varphi_3 = 0.5, \varphi_4 = 0.5$, respectively. The other parameters set as follows: $\alpha = 22, \beta = 3$, the licensed spectrum unit price determined by $\text{PU}^{\text{L}}_s \rho_0 = 1$.

Figure 3 shows the variation of licensed spectrum fraction θ under the heterogeneous network model. When channel condition of shared spectrum is getting worse, more users prefer licensed spectrum for high QoS. It denotes that the θ will increase with φ . Meanwhile, with θ increasing, the demands for licensed spectrum

Fig. 3 Variation in licensed spectrum fraction

from PU^L 's increases, it is obvious in curve D_1 and D_2 . That is to say, operator will lease more spectrum from PU^L 's to satisfy SU 's increasing licensed spectrum requirements. Furthermore, comparing with the two curve $D_3 = 100$ and $D_4 = 150$, we can find that the more SU 's traffic demands is required, the more licensed spectrum could be provided.

In Fig. 4 with the B^{\max} increasing, operator will reduce the licensed price ρ_l to attract more users to select licensed spectrum for maximizing operator's revenue. If the channel condition is better, the ρ_l is lower. Because the shared spectrum could satisfy SU 's demands, the operator has to adjust ρ_l much lower to draw attention to SU 's for achieving more revenue. It is worth noting that the price decreases faster in D_3 compared with D_4 ; the reason is that, operator adjusts the licensed price according to SU 's demands of licensed spectrum. For example, the traffic demands on licensed spectrum are 50 in D_3 and 75 in D_4 , respectively. Group D_4 must purchase more licensed spectrum to satisfy communication demands so that operator decreases the price slower.

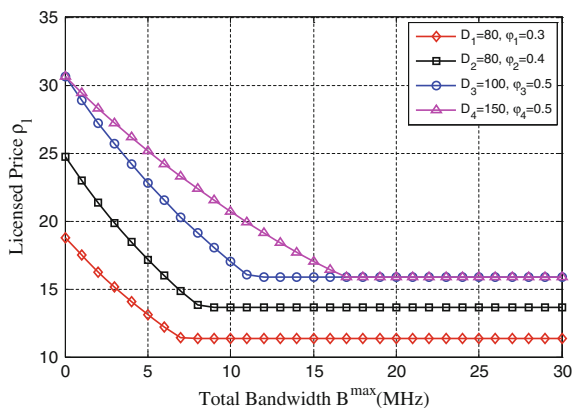
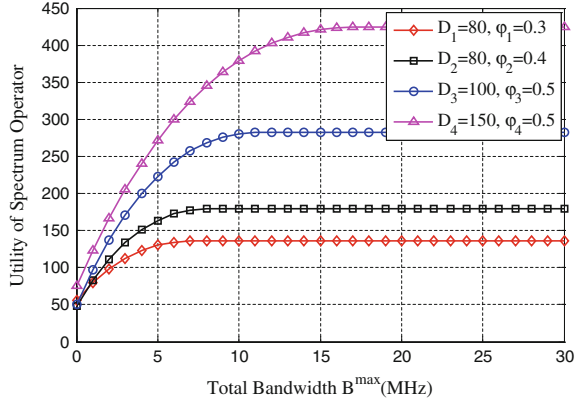
Fig. 4 Variation in licensed spectrum price

Fig. 5 Variation of utility of operator



From Fig. 5 the four curve denote when operator reduces the price ρ_l , he will achieve more revenue. The reason is due to when shared spectrum is getting worse and the licensed price is decreasing, the licensed spectrum becomes the most favorable choice. Simultaneously, operator will lease more licensed spectrum from PU^Ls.

From Fig. 6 the leasing licensed bandwidth B increases for the increasing traffic demands and worse shared spectrum (because of PU^Ss' activities). When SUs achieve their QoS demands, operator will not lease more licensed spectrum, for no user would like to purchase it. Thus, the leasing licensed bandwidth B will reach the equilibrium solution, meanwhile the revenue of operator could not increase in Fig. 5.

We can further see from Fig. 7 that under the proposed scheme, the SUs using worse channel will get more compensation to reduce their loss of profit. When $\varphi=0.9$, the channel condition of shared spectrum is getting worse than that at $\varphi=0.3$. But the utility of SUs could increase up 14.17 %, when $\varphi=0.9$. However, the utility just increase up 1.92 % when $\varphi=0.3$. Moreover, when shared spectrum

Fig. 6 Variation of leasing bandwidth

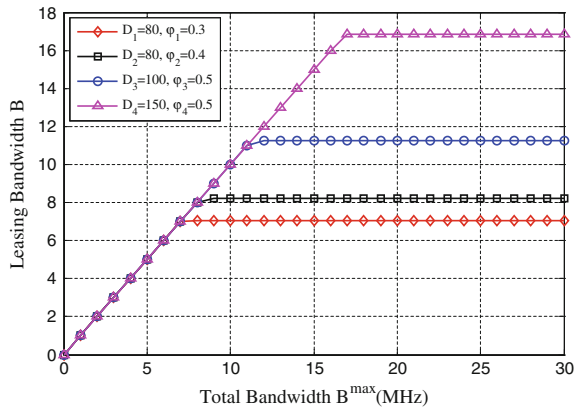
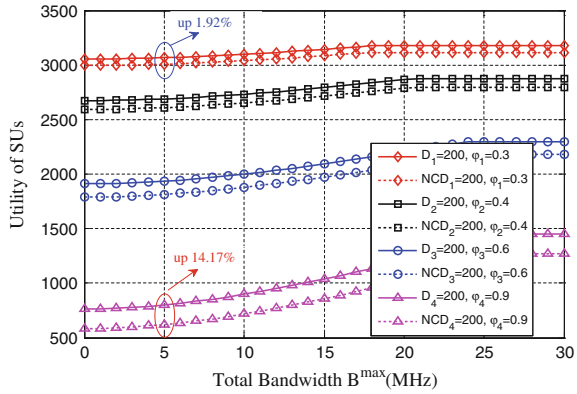


Fig. 7 Utility of SUs with price compensation and no compensation respectively



is worse, such as $\varphi = 0.9$, utility of SUs increases faster than that with $\varphi = 0.3$ under the PCS. Meanwhile, more licensed spectrum is required. From Fig. 7, the maximum leasing bandwidth for licensed spectrum is 24 MHz when $\varphi = 0.9$, and it is 18 MHz when $\varphi = 0.3$. Obviously, PU^L s will achieve more profit when shared spectrum is getting worse.

5 Conclusions

In this paper, we have studied the economic behavior of SUs under the impact of PU^S s' activities on shared spectrum. We use three-stage Stackelberg game to analyze the optimal revenue of operator and users, respectively. Furthermore, we propose a price compensation scheme (PCS) to enhance the utility of SUs when channel condition of shared spectrum is getting worse. Numerical results verify that the high activities of primary users could motivate SUs to purchase more licensed spectrum, which maximizes revenue of operator and PU^L s, respectively. The proposed scheme could enhance the utility of users up to 14.17 % when $\varphi = 0.9$, that is better than the case of $\varphi = 0.3$.

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