An Adaptive Method of Signal Separation Based on Spatial Filter

Biao Cheng, Hong-yi Yu, Zhi-xiang Shen and Yun-peng Hu

Abstract This paper addresses the problem of parameter estimation and adaptive separation by antenna arrays. The technique of Sparse Bayesian Learning (SBL), with remarkable performance in low SNR and limited snapshots, is introduced to estimate Direction-of-Arrival (DOA), as no information about the statistical property or deterministic property is known in advance. The spatial filter is designed based on the DOA estimates to separate signals from different directions. It is shown that the spatial filter can separate the signals with the noise power decreased. To enhance the performance of separation, an iteration processing is utilized until satisfying the convergence criterion. Experimental results are used to evaluate the performance of the spatial filter.

Keywords Array signal processing · Sparse bayesian learning · Spatial filter · Diversity processing · Direction-of-Arrival

1 Introduction

As an important technique of signal processing, spatial filtering is widely applied in communication system, biomagnetic source imaging [1], surface electromyography [2], electroencephalogram analysis [3] and fiber transmission [4]. It is well-known that it can obtain signal from certain direction and suppress unwanted signals from other directions, with SINR increased and system performance improved. As the electromagnetic environment becomes more congested, the effective spatial filtering of communication signals seems to be necessary and essential. Consequently in this paper we present a new method to spatially filter communication signals.

The existing spatial filtering techniques generally consist of two main structures: classical spatial filter [5–7] and adaptive spatial filter [8, 9]. The former is computed as an optimization problem and derives closed-form solution according to the prior

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information. A null phase-shift spatial filter (NPSF) in [5] is designed by subspaces of target signal and interference which aims to cover a narrow band rather than a single direction, and can avoid the amplitude and phase distortion caused by the filters. A spatial filter is designed in an efficient manner by formulating the design procedure as a rank-deficient linear least-squares problem in [6]. The combination of the subspace-based bearing-estimation and spatial filter algorithms is capable of resolving sources that are below the resolution limit. Spatial filter can also separate the ultra-wideband signals. A spatial filter [7] is proposed for one-dimensional time-of-arrival localization and its performance is comparable to basic time-reversal systems.

The adaptive spatial filter is computed without prior parameters; however, it needs to utilize the characteristics of signals. An adaptive blind spatial filter [8] is proposed by utilizing a constant modulus criterion and Kalman filter without prior information about the signals. An algorithm with less computational complexity [9] is proposed to filter signals with cyclostationary characteristic and this subspace projection-based method achieves better performance than the original algorithms.

In this paper, we propose a method to design adaptive blind spatial filter without prior information or signal characteristic. The proposed method generally combines direction estimation and spatial filter design. The direction can be estimated by Sparse Bayesian Learning (SBL) and then spatial filter based on direction estimates is proposed to separate the signals. Generally the algorithm can blindly separate the signals and adaptively adjust filter parameter to changing directions.

This paper is organized as follows. Section 2 presents a review of the array output model. Section 3 introduces the estimation of sparse matrix and exploitation of spatial filters, and the overall scheme of proposed method is also illustrated in Sect. 3. Section 4 contains numerical simulations to examine the performance of the proposed method, and conclusions are given in Sect. 5.

2 Model Formulation

Suppose that K independent far-field stochastic and stationary signals impinge onto an M-element array from directions of $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_K]$ simultaneously, the array output $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T$ at time t is

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \boldsymbol{\upsilon}(t) \tag{1}$$

where the array responding matrix to all the incident signals is denoted by $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$, with each column being the array responding vector of the kth incident signal $\mathbf{a}(\theta_k) = [e^{j\varphi_{k,1}}, \ldots, e^{j\varphi_{k,M}}]^T$. $\varphi_{k,m}$ is the phase shift of the kth signal propagating from the reference antenna to the mth antenna. $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$ is the complex waveform vector of the signal. $\boldsymbol{\upsilon}(t) \sim N(0, \sigma^2\mathbf{I}_M)$ is the zero-mean white Gaussian noise with power σ^2. In the scenario of N snapshots, the array output formulation presented in (1) can be extended and rewritten as $\mathbf{Y} = \mathbf{A}(\boldsymbol{\theta})\mathbf{S} + \mathbf{V}$, where $\mathbf{Y} = [\mathbf{y}(t_1), \ldots, \mathbf{y}(t_N)]$, $\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_N)]$, $\mathbf{V} = [\boldsymbol{\upsilon}(t_1), \ldots, \boldsymbol{\upsilon}(t_N)]$.

In fact, the signals only impinge on the array from limited directions. In order to utilize the sparsity characteristics, we partition the potential space of the incident signals with $\boldsymbol{\Psi} = [\theta_1, \ldots, \theta_P]$ and the direction set is $\boldsymbol{\Phi} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_P)]$. Generally, the cardinality of the direction set is much larger than the antenna number and the dictionary is overcomplete. When substituting $\boldsymbol{\Phi}$ for $\mathbf{A}(\boldsymbol{\theta})$, (1) can be rewritten as

$$\mathbf{Y} = \boldsymbol{\Phi}\mathbf{X} + \mathbf{V} \tag{2}$$

where $\mathbf{X} = [\boldsymbol{\rho}(t_1), \ldots, \boldsymbol{\rho}(t_N)] \in C^{P \times N}$ is sparse matrix with K non-zero rows. Then signal direction can be derived by scanning the location of non-zero rows in sparse matrix. When $P \gg K$ the waveform matrix $\mathbf{X}$ presents sparsity characteristics which can be solved by sparsity recovery.

3 Spatial Filtering for Signal Separation

3.1 Sparse Matrix Estimation

The likelihood function of $\boldsymbol{\rho}(t_n) \in C^{P \times 1}$ is

$$P\left(\mathbf{y}(t_n)|\boldsymbol{\rho}(t_n), \sigma^2\right) = \left(2\pi\sigma^2\right)^{-M/2} \exp\left[-\frac{1}{2\sigma^2}(\mathbf{y}(t_n) - \boldsymbol{\Phi}\boldsymbol{\rho}(t_n))^H(\mathbf{y}(t_n) - \boldsymbol{\Phi}\boldsymbol{\rho}(t_n))\right] \tag{3}$$

To complete the structure of hierarchical prior, we define the individual hyperparameter $\alpha_p (p = 1, \ldots, P)$ to independently moderate the strength of each row in sparse matrix $\mathbf{X}$. Suppose that the column $\boldsymbol{\rho}(t_n)$ is Gaussian distributed, i.e., $\boldsymbol{\rho}(t_n) \sim CN(0, \boldsymbol{\Gamma})$ and $\boldsymbol{\Gamma} = \mathrm{diag}\left(\alpha_1, \ldots, \alpha_P\right)$. Then the probability of $\mathbf{X}$ with respect to $\boldsymbol{\Gamma}$ and σ^2 can be derived by Bayesian criterion as follows

$$P\left(\mathbf{X}|\mathbf{Y}, \boldsymbol{\Gamma}, \sigma^2\right) = (2\pi)^{-NP/2}|\boldsymbol{\Sigma}|^{-N/2} \exp\left[-\frac{1}{2}\sum_{n=1}^{N}(\boldsymbol{\rho}(t_n) - \boldsymbol{\mu}_n)^H \boldsymbol{\Sigma}^{-1}(\boldsymbol{\rho}(t_n) - \boldsymbol{\mu}_n)\right] \tag{4}$$

where $\boldsymbol{\Sigma}_t = \sigma^2\mathbf{I} + \boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Phi}^H$, the posterior covariance and mean are, respectively

$$\boldsymbol{\Sigma} = \left(\sigma^{-2}\boldsymbol{\Phi}^H\boldsymbol{\Phi} + \boldsymbol{\Gamma}^{-1}\right)^{-1} = \boldsymbol{\Gamma} - \boldsymbol{\Gamma}\boldsymbol{\Phi}^H\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\Phi}\boldsymbol{\Gamma} \tag{5}$$

$$\boldsymbol{\Lambda} = [\boldsymbol{\mu}_1, \ldots, \boldsymbol{\mu}_N] = \boldsymbol{\Gamma}\boldsymbol{\Phi}^H\boldsymbol{\Sigma}_t^{-1}[\mathbf{y}_1, \ldots, \mathbf{y}_N] \tag{6}$$

We regard posterior mean $\mathbf{\Lambda}$ as the sparse matrix estimate which is directly influenced by $\mathbf{\Gamma}$. SBL estimates $\mathbf{\Gamma}$ by maximizing the marginal likelihood known as type-II maximum likelihood method. The EM algorithm is an iterative method to implement the maximum likelihood estimation. The E-step step is derived by calculating the conditional mean according to $E_{\mathbf{X}} \ln P(\mathbf{Y}, \mathbf{X}|\mathbf{\Gamma}, \sigma^2)$. The M-step is expressed via the update rule [10]

$$\alpha_p^{(\text{new})} = \frac{1}{N} \left\| \mathbf{\Lambda}_p \right\|_2^2 + \mathbf{\Sigma}_{pp}, \quad \forall p = 1, \ldots, P \tag{7}$$

$$\left(\sigma^2\right)^{(\text{new})} = \frac{\frac{1}{N} \| \mathbf{Y} - \mathbf{\Phi\Lambda} \|_F^2}{M - P + \sum_{p=1}^{P} \frac{\Sigma_{pp}}{\alpha_p^{(\text{new})}}} \tag{8}$$

where $\mathbf{\Lambda}_p$ is the pth row of $\mathbf{\Lambda}$ at last iteration and $\mathbf{\Sigma}_{pp}$ is the pth diagonal value of $\mathbf{\Sigma}$. Essentially the learning processing is composed of two steps. Firstly we calculate posterior covariance and mean based on (5) and (6), then update parameters based on (7) and (8). The iteration will stop when reaching the final convergence criterion.

3.2 *Spatial Filters Design*

The K spatial filters aim to separate the signals in space domain based on the DOA estimates. Define the kth spatial filter by $\mathbf{T}_k (k = 1, \ldots, K)$. The most intuitive expression of output filtered by $\mathbf{T}_k$ can be written by

$$\mathbf{T}_k \mathbf{X} = \mathbf{T}_k \mathbf{A}(\boldsymbol{\theta}) \begin{bmatrix} \mathbf{s}_1(t) \\ \vdots \\ \mathbf{s}_K(t) \end{bmatrix} + \mathbf{T}_k \mathbf{V} = \begin{bmatrix} \mathbf{s}_k(t) \\ \vdots \\ \mathbf{s}_k(t) \end{bmatrix} + \mathbf{T}_k \mathbf{V} \tag{9}$$

From the above equation, we aim to obtain sufficient samples of signal $\mathbf{s}_k(t)$ for further processing. In this subsection, we utilize the subarray composed of K successive antennas to filter each sample of the kth signal. Denote the partial responding matrix consisting of the mth to $(m + K - 1)$th rows of the original matrix $\mathbf{A}(\boldsymbol{\theta})$ by $\mathbf{A}_m(\boldsymbol{\theta})$ $(m = 1, \ldots, M - K + 1)$, and inverse matrix by $\mathbf{B}_m(\boldsymbol{\theta}) = [\mathbf{A}_m(\boldsymbol{\theta})]^{-1}$, with its kth row denoted by $\mathbf{b}_m^{(k)} \in C^{1 \times K}$, then

$$\mathbf{B}_m(\boldsymbol{\theta}) \mathbf{A}_m(\boldsymbol{\theta}) \mathbf{S} = \begin{bmatrix} \mathbf{b}_m^{(1)} \\ \vdots \\ \mathbf{b}_m^{(K)} \end{bmatrix} \mathbf{A}_m(\boldsymbol{\theta}) \mathbf{S} = \mathbf{S} \tag{10}$$

where $\mathbf{b}_m^{(k)} \mathbf{A}_m(\boldsymbol{\theta}) \mathbf{S} = \mathbf{s}_k$, indicating that kth row of $\mathbf{B}_m(\boldsymbol{\theta})$ can filter kth signal from the sources impinging onto array. Then the kth spatial filter $\mathbf{T}_k \in C^{(M-K+1) \times M}$ can be designed by $(\mathbf{T}_k)_{m\bullet} = \left[\mathbf{0}_{1\times(m-1)}, \mathbf{b}_m^{(k)}, \mathbf{0}_{1\times(M-K+1-m)}\right]$. The kth spatial filter output $\mathbf{u}_k \in C^{(M-K+1)\times N}$ is

$$\mathbf{u}_k = \mathbf{T}_k\mathbf{A}(\boldsymbol{\theta})\begin{bmatrix}\mathbf{s}_1(t)\\ \vdots \\ \mathbf{s}_K(t)\end{bmatrix} + \mathbf{T}_k\mathbf{V} = \begin{bmatrix}\mathbf{b}_1^{(k)}\mathbf{A}_1(\boldsymbol{\theta})\\ \vdots \\ \mathbf{b}_{M-K+1}^{(k)}\mathbf{A}_{M-K+1}(\boldsymbol{\theta})\end{bmatrix}\begin{bmatrix}\mathbf{s}_1(t)\\ \vdots \\ \mathbf{s}_K(t)\end{bmatrix} + \mathbf{T}_k\mathbf{V} = \begin{bmatrix}\mathbf{s}_k(t)\\ \vdots \\ \mathbf{s}_k(t)\end{bmatrix} + \mathbf{T}_k\mathbf{V} \tag{11}$$

It can be inferred from the above equation that the output $\mathbf{u}_k$ through kth spatial filter $\mathbf{T}_k$ contains the kth signal and effectively eliminates the other signals. Furthermore, each row in $\mathbf{u}_k$ represents the identical kth signal $\mathbf{s}_k(t)$, indicating that every subarray consisting of K successive antennas can spatially filter the signal. It ends up with $M - K + 1$ identical signals and then we formulate the kth signal estimation as

$$\tilde{\mathbf{s}}_k(t) = \frac{1}{M-K+1}\sum_{j=1}^{M-K+1}(\mathbf{u}_k)_{j\cdot} \tag{12}$$

where $(\mathbf{u}_k)_{j\cdot}$ is the jth row of the output $\mathbf{u}_k$. Similar to diversity processing, it can be concluded that (12) can decrease noise power and improve filtering performance.

3.3 Overall Scheme of the New Method

During the above procedure, the design of the spatial filter relies on accurate matrix estimate and directly influences the algorithm performance. Therefore, a refined DOA estimation should be introduced to obtain the accurate value iteratively until it satisfies the convergence criterion.

Remark 1 Denote the signal passed through the kth spatial filter in qth iteration by $\tilde{\mathbf{s}}_k^{(q)}(t)$. The iteration satisfies the convergence criterion when the difference of signal filtered in the qth iteration and $(q + 1)$th iteration is smaller than a fixed threshold ε_1. We set the convergence criterion as

$$\sum_{k=1}^{K}\left\|\tilde{\mathbf{s}}_k^{(q+1)}(t) - \tilde{\mathbf{s}}_k^{(q)}(t)\right\|^2 < \varepsilon_1 \tag{13}$$

Remark 2 To obtain coarse DOA estimates, the potential space of the incident signals is divided into P samples in the initialization step. However, the coarse sampling cannot promise to include the actual DOA into the direction set which results in inaccurate DOA estimates. In the simulation, the mismatch of direction set leads to cluster peak emerging beside the real value. Therefore, we propose a probabilistic method to redesign the direction set. Denote the DOA estimate of the

kth signal in qth iteration by $\theta_k^{(q)}$ and the direction set in $(q + 1)$th iteration is updated by $\boldsymbol{\Psi}^{(q+1)} = \left[\boldsymbol{\beta}_1^{(q+1)}, \ldots, \boldsymbol{\beta}_K^{(q+1)}\right]$. $\boldsymbol{\beta}_k^{(q+1)} \sim \mathrm{N}\left(\theta_k^{(q)}, \sigma_\varphi^2\right)$ is a sub-dictionary with σ_φ^2 being the perturbation variance.

4 Simulation Results

Consider a uniform linear array with 16 half-wavelength spaced antennas and two signals modulated with binary-phase-shift-keying (BPSK) arrive from 30° and 60°, respectively. The carrier frequency is set to 5 MHz and the sampling frequency is set to 15 MHz. The baud rates are 1 MHz and 0.6 MHz, respectively. In the initialization step, the sparsity-inducing space is sampled form 0° to 180° with 2° interval to obtain the direction set $\boldsymbol{\Psi}$. The threshold ε_1 and the perturbation variance σ_φ^2 are set to 0.01 and 0.05 respectively.

In the initialization, we sample the potential space uniformly to obtain the coarse estimates and refine the dictionary set for the next iteration. The performance of DOA estimates versus iteration index is demonstrated to prove the effectivity of proposed method. 300 independent simulations are carried out. The average root-mean-square-error (RMSE) in qth iteration is defined as

$$\mathrm{RMSE}^{(q)} = \sqrt{\sum_{w=1}^{W} \sum_{k=1}^{K} \left(\theta_k^{(w,q)} - \theta_k\right)^2 /KW} \tag{14}$$

where $\tilde{\theta}_k^{(w,q)}$ is the DOA estimate in qth iteration and θ_k is the actual value. The signal-to-noise ratio (SNR) is set to 0 dB for both of the signals. In Fig. 1, the simulation result indicates that the estimation performance improves with respect to

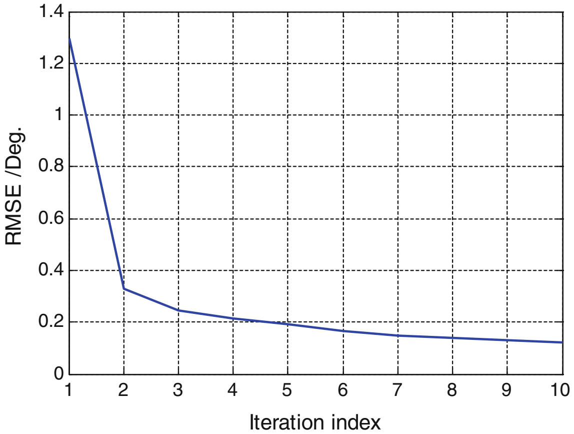

Fig. 1 DOA estimation RMSE versus iteration index

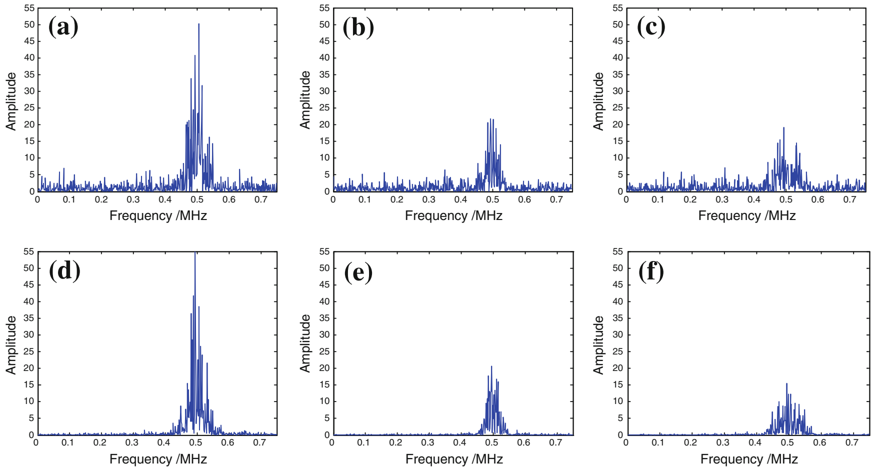

Fig. 2 Power spectrum of signals: **a** original signals; **b** original signal 1; **c** original signal 2; **d** filtered signals; **e** filtered signal 1; **f** filtered signal 2

increased iteration index. The RMSE of DOA becomes stable when iteration index reaches to 7.

Figure 2 demonstrates the spectrum of original signals and filtered signals when satisfying the stop criterion. Figure 2b, c show two original signals that arrive at the array and Fig. 2a is the sum of two original signals. From Fig. 2e, f, it is obviously that the spatial filters based on final direction estimates can effectively separate two signals and as a result of (12) the noise power is decreased distinctly.

The performance of signal separation by different algorithms is to be demonstrated. The signal-to-interference-plus-noise ratio (SINR) of all the incident signals is used for precision evaluation. In this subsection SNR is changing from −30 to 30 dB for both of the signals. 75 snapshots are collected and 300 independent simulations are carried out in each scenario. We define signal from $\theta_e = 30°$ as expected signal $\mathbf{s}_e$ and signal from $\theta_i = 60°$ as interference $\mathbf{s}_i$. Then we evaluate the performance of spatial filter $\mathbf{T}_e$ designed for expected signal. The SINR of the proposed spatial filter is defined as

$$\mathrm{SINR} = \frac{\left\| \sum_{j=1}^{M-K+1} [\mathbf{T}_e \, \mathbf{a}(\theta_e) \, \mathbf{s}_e]_{j\cdot} \right\|^2}{\left\| \sum_{j=1}^{M-K+1} [\mathbf{T}_e \, \mathbf{a}(\theta_i) \, \mathbf{s}_i]_{j\cdot} \right\|^2 + \left\| \sum_{j=1}^{M-K+1} (\mathbf{T}_e \mathbf{V})_{j\cdot} \right\|^2} \tag{15}$$

Figure 3 demonstrates the performance of the proposed method based on the spatial filter (SF), Stimulate Covariance Matrix Inversing (SMI) algorithm, and Eigenspace-Based (ESB) algorithm. The SNR of expected signal and the interference is identical in each scenario. The performance of SMI deteriorates because the power of expected signal is high. The performance of ESB under high SNR surpasses SMI. However, the performance of ESB under low SNR is not satisfactory.

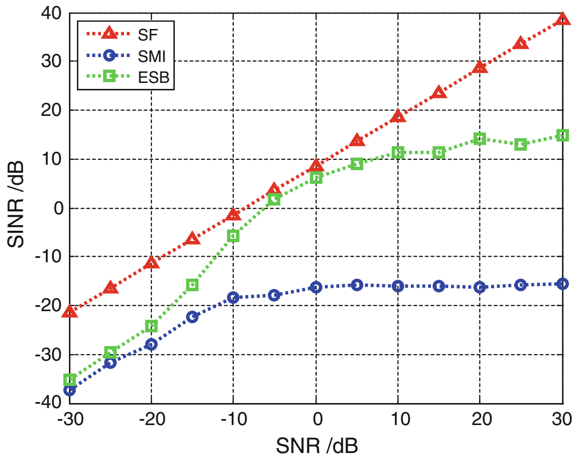

Fig. 3 Performance evaluation of different algorithms

By contrast, the spatial filter can increase SINR and outperforms the subspace-based algorithms remarkably.

5 Conclusions

In this paper, we proposed a novel method to separate signals devised by the combination of SBL algorithm and spatial filters. The SBL algorithm is utilized to estimate the directions without any prior information or characteristic of signals. We design spatial filters based on direction of each signal to achieve efficient separation. The filtering outputs obtain sufficient samples which can be used to calculate the mean values of each signal with the noise power reduced extremely. The simulation results indicate that the proposed algorithm can spatially filter the signals and outperform existing subspace-based algorithms.

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