A General Framework for Modal Correspondence in Dynamic Epistemic Logic

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Abstract. We introduce a unified framework for dynamic epistemic logics, which in particular encompasses Public Announcement Logic (**PAL**), Epistemic Action (**EA**) and Preference Upgrade (**PU**). Our framework consists of a generic language, in which some of the known reduction axioms are expressible, together with relational and algebraic semantics. We then establish correspondences between generic reduction axioms and semantic properties, in both relational and algebraic settings. This leads to alternative proofs of the completeness of **PAL**, **EA**, **PU** with respect to their relational semantics and algebraic semantics (for the former two).

1 Introduction

Dynamic Epistemic Logic (DEL) is a branch of modal logic for reasoning about knowledge changes or belief revisions caused by communication. This is technically materialised by adding, to a static epistemic logic, dynamic operators that express actions of communication. These operators are interpreted as transformations of Kripke models (*model transformations*). The pioneer study on DEL is *Public Announcement Logic* (**PAL**) [5]. Then *Epistemic Action* (**EA**) [2] was proposed for reasoning about a greater variety of communication, including public announcements, and this has made the research area much more active. Until now, many DELs for various kinds of actions of communications have been proposed and studied: *Update Model* [11], *Command Logic* [14], *Belief Change* [6], *Preference Upgrade* (**PU**) [8], *Evidence Dynamics* [9] and *Manipulative Update* [12]. **PAL** and **EA**, among others, have been studied well: recently, algebraic counterparts of model transformations were proposed as the algebraic semantics of **PAL** [4] and **EA** [3].

Modal Correspondence in DEL: We aim at developing a modal correspondence theory for DEL in general, which establishes a link between axioms and properties of model transformations. There are some precedents in the literature [6–8]. Among these, van Benthem [7] gives a quite comprehensive account based on the concept of *update universe* to **PAL**, **EA**, Belief Change and Evidence Dynamics. In this paper we further this line of research, proposing a more general framework. We consider a wider class of dynamic operators than those studied in [7], which are generally expressed by formulas. In addition, we give not only

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frame/model correspondences but also soundness-completeness-type correspondences for model transformations in general.

Organisation and Novel Contribution: The main contribution of our paper is in proposing a general framework for modal correspondence in DEL. We proceed as follows:

- Section 2: Language. We first propose a generic DEL language that is defined by using abstract action expressions. The languages of **PAL**, **EA** and **PU** (without the auxiliary universal modality) can be obtained by substituting their action expressions for abstract ones.
- Section 3: Relational Semantics. We then propose the notion of a two-layered relational model in which model transformations are expressed by abstract update relations instead of ordinary operational ways. The language and the two-layered models allow us to develop a general modal correspondence theory for the generic fragment. We then give correspondence results specific to **PAL**, **EA** and **PU**. As corollaries, we obtain alternative proofs of completeness of these logics. While the ordinary proofs are based on translation of dynamic formulas into purely static ones by reduction axioms, our new proofs consist in matching each reduction axiom with a corresponding semantic property. Thus our proofs are modular.
- Section 4: Algebraic Semantics. We undertake a similar analysis in an algebraic setting: we propose an algebraic notion of model; give general correspondence results and ones specific to **PAL** and **EA**; and also obtain alternative modular proofs of the completeness of these two logics.
- Section 5: Duality. To conclude the paper, we give several results on the duality between our relational and algebraic models.

2 Generic DEL Language

Let us begin by proposing a generic language for DEL.

Definition 1 (Generic DEL Language). *Let $\mathcal{P}$ be a set of* atomic propositions*, E a set of* epistemic expressions *and $\mathfrak{A}$ a set of* action expressions*. We define a* generic DEL language $\mathcal{L}(E, \mathfrak{A})$ *by the following rule:*

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \vee \psi \mid \langle e \rangle \varphi \mid \langle\!\langle \alpha \rangle\!\rangle \varphi$$

where p ranges over $\mathcal{P}$, e over E and α over $\mathfrak{A}$.

In other words, the language $\mathcal{L}(E, \mathfrak{A})$ is the multimodal language with modalities $\langle e \rangle$ and $\langle\!\langle \alpha \rangle\!\rangle$ ($e \in E$, $\alpha \in \mathfrak{A}$). The individual languages of *Public Announcement Logic* (**PAL**), *Epistemic Action* (**EA**) and *Preference Upgrade* (excluding the universal modality) (**PU**) can be seen as special cases of $\mathcal{L}(E, \mathfrak{A})$:

Example 1 (Specific DEL Languages). Let Ag be a given set of agents.

- The language of **PAL** can be expressed as $\mathcal{L}_{\mathsf{PAL}} = \mathcal{L}(Ag, \mathfrak{A}_{\mathsf{PAL}})$, where $\mathfrak{A}_{\mathsf{PAL}} = \{!\varphi \mid \varphi \in \mathcal{L}_{\mathsf{PAL}}\}$. As $\mathfrak{A}_{\mathsf{PAL}}$ depends on $\mathcal{L}_{\mathsf{PAL}}$, these two sets are actually defined by simultaneous induction; however, the resulting language fits the pattern of $\mathcal{L}(E, \mathfrak{A})$. The same remark applies to the other two examples below. The intended meaning of $[n]\varphi := \neg\langle n\rangle\neg\varphi$ is 'agent n knows φ', while $[[!\varphi]]\psi := \neg\langle\langle !\varphi\rangle\rangle\neg\psi$ means 'ψ holds after a truthful public announcement of φ'.

- The language of **EA** is $\mathcal{L}_{\mathsf{EA}} = \mathcal{L}(Ag, \mathfrak{A}_{\mathsf{EA}})$ where $\mathfrak{A}_{\mathsf{EA}}$ is the set of *action models* (U, s) [2]. An action model (U, s) consists of a finite Kripke frame $(U, \{\rightarrow_n\}_{n\in Ag})$ together with a precondition function $\mathrm{Pre} : U \to \mathcal{L}_{\mathsf{EA}}$ and s is a state of U: $[[(U, s)]]\varphi := \neg\langle\langle(U, s)\rangle\rangle\neg\psi$ is read as 'φ holds after an epistemic action (U, s)', and $[n]\varphi$ is as in **PAL**.
- The language $\mathcal{L}_{\mathsf{PU}}$ of **PU** can also be expressed as $\mathcal{L}_{\mathsf{PU}} = \mathcal{L}(E_{\mathsf{PU}}, \mathfrak{A}_{\mathsf{PU}})$ where $E_{\mathsf{PU}} = \{n, \bar{n} \mid n \in Ag\}$ and $\mathfrak{A}_{\mathsf{PU}} = \{\varphi!, \sharp\varphi \mid \varphi \in \mathcal{L}_{\mathsf{PU}}\}$. $[n]\varphi$ and $[[\varphi!]]\psi := \neg\langle\langle\varphi!\rangle\rangle\neg\varphi$ are as $[n]\varphi$ and $[[!\varphi]]\psi$ in **PAL**, while $[\bar{n}]\varphi := \neg\langle\bar{n}\rangle\neg\varphi$ and $[[\sharp\varphi]]\psi := \neg\langle\langle\sharp\varphi\rangle\rangle\neg\psi$ express 'all the worlds which agent n considers at least as good as the current one satisfy φ' ([8]) and 'ψ holds after suggestion of φ', respectively.

A common feature of dynamic epistemic logics is the use of *reduction axioms*, which are intended to transform any dynamic formula (involving dynamic modalities $\langle\langle\alpha\rangle\rangle$) into a purely static one. Some reduction axioms are already expressible in the generic DEL language $\mathcal{L}(E, \mathfrak{A})$:

Definition 2 (Generic Reduction Axioms). *We call the following axioms* generic reduction axioms*:*

$$\mathsf{R_N} : \langle\langle\alpha\rangle\rangle\neg\varphi \leftrightarrow \langle\langle\alpha\rangle\rangle\top \wedge \neg\langle\langle\alpha\rangle\rangle\varphi \qquad \mathsf{R_P} : \langle\langle\alpha\rangle\rangle p \leftrightarrow \langle\langle\alpha\rangle\rangle\top \wedge p$$
$$\mathsf{R_K} : \langle\langle\alpha\rangle\rangle\langle e\rangle\varphi \leftrightarrow \langle\langle\alpha\rangle\rangle\top \wedge \langle e\rangle\langle\langle\alpha\rangle\rangle\varphi \qquad \mathsf{R_A} : \langle\langle\alpha\rangle\rangle\top \leftrightarrow \top$$

Notice that $\mathsf{R_P}$ refers to atomic propositions p, thus logics involving $\mathsf{R_P}$ are not closed under uniform substitution.

We can give proof systems $\mathsf{PAL}, \mathsf{EA}$ and PU to the above three logics—**PAL**, **EA**, **PU**—by choosing a suitable set of generic reduction axioms and adding some extra ones: here we consider the multimodal logic K (without the substitution rule) in the language $\mathcal{L}(E, \mathfrak{A})$ as the base logic and use the symbol $\oplus$ for the addition of axiom schemata.

Example 2

- $\mathsf{PAL} = \mathsf{K} \oplus \mathsf{R_N R_K R_P} \oplus \mathsf{R_T} : \langle\langle !\varphi\rangle\rangle\top \leftrightarrow \varphi$.
- $\mathsf{EA} = \mathsf{K} \oplus \mathsf{R_N R_P} \oplus \mathsf{Pre} : \langle\langle(U, s)\rangle\rangle\top \leftrightarrow \mathrm{Pre}(s) \oplus \mathsf{A_{EA}} : \langle\langle(U, s)\rangle\rangle\langle n\rangle\varphi \leftrightarrow \langle\langle(U, s)\rangle\rangle\top \wedge \bigvee\{\langle n\rangle\langle\langle(U, t)\rangle\rangle\varphi \mid s \rightarrow_n t\}$.
- $\mathsf{PU} = \mathsf{K} \oplus \mathsf{R_N}$ (for $\varphi!$ and $\sharp\varphi$) $\oplus\, \mathsf{R_P}$ (for $\varphi!$ and $\sharp\varphi$) $\oplus\, \mathsf{R_K}$ (for $(\varphi!, n)$, $(\varphi!, \bar{n})$, $(\sharp\varphi, n)$) $\oplus\, \mathsf{R_T}$ (for $\varphi!$) $\oplus\, \mathsf{R_A}$ (for $\sharp\varphi$) $\oplus\, \mathsf{A_{PU}} : \langle\langle\sharp\varphi\rangle\rangle\langle\bar{n}\rangle\psi \leftrightarrow (\neg\varphi \wedge \langle\bar{n}\rangle\langle\langle\sharp\varphi\rangle\rangle\psi) \vee (\langle\bar{n}\rangle(\varphi \wedge \langle\langle\sharp\varphi\rangle\rangle\psi))$.

We can easily see that these proof systems are equivalent to the original ones in [2, 5, 8].

3 Relational Semantics

3.1 Model Transition System

Usually, the language of a DEL is interpreted by using a Kripke model $\mathbf{M} = (S, \{R_e\}_{e\in E}, V)$. The effect of an action α is explained in terms of *model* *transformation*: $\mathbf{M}$ is transformed into another model $\mathbf{M}^\alpha$ and a state v in $\mathbf{M}$ is sent to a state w in $\mathbf{M}^\alpha$ (cf. Baltag [1]). Since we want to treat a family (property) of model transformations in one general framework, it is convenient to consider a family of Kripke models, linked to each other by dynamic action relations. Hence, we consider the following novel system, which modifies the update universe (see Remark 1 infra) in [7]:

Definition 3 (Model Transition System). *A* model transition system (MTS) *for* $\mathcal{L}(E, \mathfrak{A})$ *is a triple* $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ *such that*
1. $\mathfrak{M}_I$ *is a family of Kripke models* $\mathbf{M}_i = (S, \{R_e\}_{e \in E}, V)$ *indexed by* $i \in I$ ($\mathbf{M}_i$ *is allowed to be an empty structure*),
2. $\Phi : I \times \mathfrak{A} \to I$ *is a function* (*notation:* $\mathbf{M}_i^\alpha := \mathbf{M}_{\Phi(i,\alpha)}$),
3. $\mathfrak{R}$ *assigns a binary relation* $\mathfrak{R}_i^\alpha \subseteq \mathbf{M}_i \times \mathbf{M}_i^\alpha$ *to each* $(i, \alpha) \in I \times \mathfrak{A}$.

Analogously, a *frame transition system* (*FTS*) $\mathfrak{F} = (\mathfrak{F}_I, \Phi, \mathfrak{R})$ is defined by using indexed Kripke frames instead of indexed Kripke models. We say that $\mathfrak{F} = (\mathfrak{F}_I, \Phi, \mathfrak{R})$ is the *underlying* FTS of an MTS $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ and write $\mathfrak{F} = \mathbf{U}(\mathfrak{M})$ if $\mathbf{F}_i$ is the underlying frame of $\mathbf{M}_i$ for each $i \in I$. An MTS expresses model transformations: a model $\mathbf{M}_i$ is transformed into $\mathbf{M}_i^\alpha$ by action α, and the state v in $\mathbf{M}_i$ is sent to w in $\mathbf{M}_i^\alpha$ if $v\mathfrak{R}_i^\alpha w$. As a result, a pointed model $(\mathbf{M}_i, v)$ is transformed into $(\mathbf{M}_i^\alpha, w)$ that satisfies $v\mathfrak{R}_i^\alpha w$, if such a w exists.

The generic DEL language $\mathcal{L}(E, \mathfrak{A})$ is interpreted by an MTS:

Definition 4. *Suppose that* $\mathbf{M}_i = (S, \{R_e\}_{e \in E}, V)$ *is a Kripke model in an MTS* $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ *and* v *is a state in* $\mathbf{M}_i$. *We inductively define the notion of a formula* φ *being* satisfied *at state* v *in* $\mathbf{M}_i \in \mathfrak{M}_I$ (*notation:* $\mathfrak{M}, \mathbf{M}_i, v \models \varphi$) *as follows:*

$\mathfrak{M}, \mathbf{M}_i, v \models \top$	*iff*	*always*
$\mathfrak{M}, \mathbf{M}_i, v \models p$	*iff*	$v \in V(p)$
$\mathfrak{M}, \mathbf{M}_i, v \models \neg\varphi$	*iff*	$\mathfrak{M}, \mathbf{M}_i, v \not\models \varphi$
$\mathfrak{M}, \mathbf{M}_i, v \models \varphi \vee \psi$	*iff*	$\mathfrak{M}, \mathbf{M}_i, v \models \varphi$ *or* $\mathfrak{M}, \mathbf{M}_i, v \models \psi$
$\mathfrak{M}, \mathbf{M}_i, v \models \langle e \rangle \varphi$	*iff*	*for some* $w \in S$, $vR_e w$ *and* $\mathfrak{M}, \mathbf{M}_i, w \models \varphi$
$\mathfrak{M}, \mathbf{M}_i, v \models \langle\langle \alpha \rangle\rangle \varphi$	*iff*	*for some* $w \in \mathbf{M}_i^\alpha$, $v\mathfrak{R}_i^\alpha w$ *and* $\mathfrak{M}, \mathbf{M}_i^\alpha, w \models \varphi$

We say that $\mathfrak{M}$ *validates* φ if $\mathfrak{M}, \mathbf{M}_i, v \models \varphi$ for any Kripke model $\mathbf{M}_i$ in $\mathfrak{M}$ and state v in $\mathbf{M}_i$. Validity in an FTS is defined analogously.

The model transformations of the three logics—**PAL**, **EA**, **PU**—are expressed by MTSs $(\mathfrak{M}_I, \Phi, \mathfrak{R})$ by defining Φ and $\mathfrak{R}$ as follows (the families $\mathfrak{M}_I$ have to be given so that Φ and $\mathfrak{R}$ are well-defined):

Example 3 (Specific Model Transition Systems)

- *PAL-MTS*: for every $!\varphi \in \mathfrak{A}_{\mathsf{PAL}}$ and $\mathbf{M}_i = (S, \{R_n\}_{n \in Ag}, V) \in \mathfrak{M}_I$,
  - $\mathbf{M}_i^{!\varphi} = (S', \{R'_n\}_{n \in Ag}, V')$ is the submodel of $\mathbf{M}_i$ whose carrier set is $S' = \{v \in S \mid \mathfrak{M}, \mathbf{M}_i, v \models \varphi\}$, and
  - $\mathfrak{R}_i^{!\varphi} = \{(v, v) \in \mathbf{M}_i \times \mathbf{M}_i^{!\varphi} \mid v \in \mathbf{M}_i^{!\varphi}\}$.
- *EA-MTS*: for every action model $(U, s) \in \mathfrak{A}_{\mathsf{EA}}$ with $U = (U, \{\to_n\}_{n \in Ag})$ and $\mathbf{M}_i = (S, \{R_n\}_{n \in Ag}, V) \in \mathfrak{M}_I$,

- $\mathbf{M}_i^{(U,s)} = (S', \{R'_n\}_{n\in Ag}, V')$ is given by
  - $S' = \{(v,t) \mid v \in \mathbf{M}_i, t \in U$ and $\mathfrak{M}, \mathbf{M}_i, v \models \mathrm{Pre}(t)\}$,
  - $(v,t)R'_n(w,u)$ iff vR_nw and $t \to_n u$ for any $n \in Ag$,
  - $(v,t) \in V'(p)$ iff $v \in V(p)$,
- $\Phi(i,(U,s)) = \Phi(i,(U,t))$ for any $i \in I$ and $(U,s),(U,t) \in \mathfrak{A}_{\mathsf{EA}}$, and
- $\mathfrak{R}_i^{(U,s)} = \{(v,(v,s)) \in \mathbf{M}_i \times \mathbf{M}_i^{(U,s)} \mid (v,s) \in \mathbf{M}_i^{(U,s)}\}$.

– *PU-MTS*: for every $\varphi!$, $\sharp\varphi \in \mathfrak{A}_{\mathsf{PU}}$, and $\mathbf{M}_i = (S, \{R_n, R_{\bar{n}}\}_{\bar{n}\in Ag}, V) \in \mathfrak{M}_I$,

- $\mathbf{M}_i^{\varphi!} = (S, \{R'_n, R_{\bar{n}}\}_{n\in Ag}, V)$ is given by
  - $R'_n = \{(v,w) \in R_n \mid \mathfrak{M}, \mathbf{M}_i, v \models \varphi$ iff $\mathfrak{M}, \mathbf{M}_i, w \models \varphi\}$,
- $\mathbf{M}_i^{\sharp\varphi} = (S, \{R_n, R'_{\bar{n}}\}_{n\in Ag}, V)$ is given by
  - $R'_{\bar{n}} = \{(v,w) \in R_{\bar{n}} \mid \mathfrak{M}, \mathbf{M}_i, v \models \neg\varphi$ or $\mathfrak{M}, \mathbf{M}_i, w \models \varphi\}$,
- $\mathfrak{R}_i^{\varphi!} = \{(v,v) \in \mathbf{M}_i \times \mathbf{M}_i^{\varphi!} \mid \mathfrak{M}, \mathbf{M}_i, v \models \varphi\}$, and $\mathfrak{R}_i^{\sharp\varphi} = \{(v,v) \in \mathbf{M}_i \times \mathbf{M}_i^{\sharp\varphi} \mid v \in \mathbf{M}_i^{\sharp\varphi}\}$.

Usually bounded morphisms are defined between Kripke models. We extend them to morphisms between MTSs as follows:

Definition 5. *Let* $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ *and* $\mathfrak{N} = (\mathfrak{N}_J, \Psi, \mathfrak{Q})$ *be MTSs. A* bounded morphism $\mathfrak{f} : \mathfrak{M} \to \mathfrak{N}$ *is a pair* $(f, \{f_i\}_{i\in I})$ *of a function* $f : I \to J$ *and bounded morphisms (in the ordinary sense)* $f_i : \mathbf{M}_i \to \mathbf{N}_{f(i)}$ *that satisfies the following conditions for any* $i \in I$ *and action expression* $\alpha \in \mathfrak{A}$*: (Here* f_i^α *denotes* $f_{\Phi(i,\alpha)}$*.)*

1. $f(\Phi(i,\alpha)) = \Psi(f(i),\alpha)$, *2. if* $v\mathfrak{R}_i^\alpha w$ *then* $f_i(v)\ \mathfrak{Q}_{f(i)}^\alpha\ f_i^\alpha(w)$,

3. if $f_i(v)\ \mathfrak{Q}_{f(i)}^\alpha\ w'$ *then* $v\mathfrak{R}_i^\alpha w$ *and* $f_i^\alpha(w) = w'$ *for some* $w \in \mathbf{M}_i^\alpha$*.*

Items 2 and 3 in the definition correspond to the homomorphic condition and the back condition in the definition of ordinary bounded morphisms. Item 1 is their precondition. As expected, we have:

Proposition 1. *Let* $(f, \{f_i\}_{i\in I}) : \mathfrak{M} \to \mathfrak{N}$ *be a bounded morphism between MTSs. Then, for any* $\mathbf{M}_i$ *in* $\mathfrak{M}$ *and state* v *in* $\mathbf{M}_i$*,* $(\mathfrak{M}, \mathbf{M}_i, v)$ *and* $(\mathfrak{N}, \mathbf{N}_{f(i)}, f_i(v))$ *satisfy exactly the same formulas.*

We call a bounded morphism $(f, \{f_i\}_{i\in I}) : (\mathfrak{M}_I, \Phi, \mathfrak{R}) \to (\mathfrak{N}_J, \Psi, \mathfrak{Q})$ *surjective* if for any $j \in J$ there is an $i \in I$ such that $f(i) = j$ and $f_i : \mathbf{M}_i \to \mathbf{N}_j$ is surjective, and we say that $\mathfrak{N}$ is a *bounded morphic image* of $\mathfrak{M}$ if there is a surjective bounded morphism from $\mathfrak{M}$ to $\mathfrak{N}$. Similar notions are defined for FTSs.

Remark 1. Our MTSs generalise the idea of *update universe* [7] to the generic DEL language $\mathcal{L}(E, \mathfrak{A})$. In particular, PAL-MTSs for the specific language $\mathcal{L}_{\mathsf{PAL}}$ (Example 3) roughly correspond to the original. The difference is that [7] considers relations $(\mathbf{M}, s)R_P(\mathbf{N}, t)$ with P a subset of the carrier set of $\mathbf{M}$. These relations, for example, interpret an announcement $!\varphi$ as $R_{[\![\varphi]\!]}$, which may be called 'extensional' in the sense that $R_{[\![\varphi]\!]} = R_{[\![\psi]\!]}$ whenever $[\![\varphi]\!] = [\![\psi]\!]$. In comparison with this interpretation, our interpretation is 'intensional', since $\mathfrak{R}_i^{!\varphi}$ does not necessarily coincide with $\mathfrak{R}_i^{!\psi}$, even if φ and ψ are logically equivalent.

3.2 General Correspondence Results

We now give a correspondence between classes of MTSs (or FTSs) and the generic reduction axioms. The results below extend some of the observations made in [7].

Definition 6 (Deterministic FTS). *An FTS* $(\mathfrak{F}_I, \Phi, \mathfrak{R})$ *[or an MTS* $(\mathfrak{M}_I, \Phi, \mathfrak{R})$*] is* deterministic *if for each* $(i, \alpha) \in I \times \mathfrak{A}$, $\mathfrak{R}_i^\alpha$ *is a partial function.*

This means that the result of each action is completely determined by the current state.

Proposition 2. *An FTS validates* $\mathsf{R_N}$ *iff it is deterministic.*

Definition 7 (Epistemic MTS). *An MTS* $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ *is* epistemic *if for each* $(i, \alpha) \in I \times \mathfrak{A}$, $v\mathfrak{R}_i^\alpha w$ *implies that* v *and* w *satisfy exactly the same atomic propositions.*

Proposition 3. *An MTS* $\mathfrak{M}$ *validates* $\mathsf{R_P}$ *and* $\mathbf{U}(\mathfrak{M})$ *validates* $\mathsf{R_N}$ *iff* $\mathfrak{M}$ *is deterministic and epistemic.*

Proposition 3 indicates that $\mathsf{R_N R_P}$ corresponds to the model transformations that are deterministic and preserve the facts (the valuations). Examples of this kind of action include suggestion [8], lying [12] and commanding [14].

Definition 8 (Eliminative FTS). *An FTS* $\mathfrak{F} = (\mathfrak{F}_I, \Phi, \mathfrak{R})$ *is called* eliminative *if for any* $(i, \alpha) \in I \times \mathfrak{A}$, $\mathbf{F}_i^\alpha$ *is a subframe of* $\mathbf{F}_i$ *and the inverse relation* $(\mathfrak{R}_i^\alpha)^{-1}$ *embeds* $\mathbf{F}_i^\alpha$ *into* $\mathbf{F}_i$.

Proposition 4. *An FTS validates* $\mathsf{R_N}$ *and* $\mathsf{R_K}$ *iff it is a bounded morphic image of an eliminative FTS.*

Proof. For convenience, we assume that E is a singleton. Suppose that FTS $\mathfrak{F} = (\mathfrak{F}_I, \Phi, \mathfrak{R})$ validates $\mathsf{R_N}$ and $\mathsf{R_K}$. It is easy to see that for each $(i, \alpha) \in I \times \mathfrak{A}$, $\mathfrak{R}_i^\alpha$ of $\mathfrak{F}$ is a partial bounded morphism between Kripke frames.

However, $\mathfrak{F}$ may not be an eliminative FTS. We construct an eliminative FTS $\mathfrak{F}' = (\mathfrak{F}_{I \times \mathfrak{A}^*}, \Phi', \mathfrak{R}')$ from $\mathfrak{F}$ thus: let us denote each Kripke frame $\mathbf{F}_i$ in $\mathfrak{F}_I$ by (S_i, R_i). We first extend the notation $\Phi(i, \alpha)$ and $\mathfrak{R}_i^\alpha$ ($\alpha \in \mathfrak{A}$) (cf. Definition 3) to $\Phi(i, \gamma)$ and $\mathfrak{R}_i^\gamma$ for each string $\gamma \in \mathfrak{A}^*$; $\Phi(i, \epsilon) = i$ and $\Phi(i, \gamma\alpha) = \Phi(\Phi(i, \gamma), \alpha)$ for $\gamma \in \mathfrak{A}^*$ and $\alpha \in \mathfrak{A}$, and $\mathfrak{R}_i^\epsilon = \{(x, x) \mid x \in \mathbf{F}_i\}$ and $\mathfrak{R}_i^{\gamma\alpha} = \mathfrak{R}_i^\gamma \circ \mathfrak{R}_{\Phi(i,\gamma)}^\alpha$. Then, each Kripke frame $\mathbf{F}_{(i,\gamma)} = (S_{(i,\gamma)}, R_{(i,\gamma)})$ in $\mathfrak{F}_{I \times \mathfrak{A}^*}$ is defined as follows: $S_{(i,\gamma)} := \{v \in \mathbf{F}_i \mid \text{there exists a state } w \in \mathbf{F}_{\Phi(i,\gamma)} \text{ such that } v\mathfrak{R}_i^\gamma w\}$; $R_{(i,\gamma)} := R_i \cap (S_{(i,\gamma)} \times S_{(i,\gamma)})$. Lastly, Φ' and $\mathfrak{R}'$ are defined to be $\Phi'((i, \gamma), \alpha) = \Phi(i, \gamma\alpha)$ and $\mathfrak{R}'^{\alpha}_{(i,\gamma)} = \{(v, v) \mid v \in \mathbf{F}_{(i,\gamma\alpha)}\}$.

A surjective bounded morphism $\mathfrak{f} = (f, \{f_{(i,\gamma)}\}_{(i,\gamma) \in I \times \mathfrak{A}^*})$ from $\mathfrak{F}'$ to $\mathfrak{F}$ can be defined as follows: $f : I \times \mathfrak{A}^* \to I$ maps (i, γ) to $\Phi(i, \gamma)$; $f_{(i,\gamma)} : \mathbf{F}_{(i,\gamma)} \to \mathbf{F}_{\Phi(i,\gamma)}$ maps $v \in S_{(i,\gamma)} \subseteq S_i$ to $w \in S_{\Phi(i,\gamma)}$ such that $v\mathfrak{R}_i^\gamma w$. Since each $\mathfrak{R}_i^\alpha$ is a partial bounded morphism, $\mathfrak{f}$ is indeed a bounded morphism. ◻

Proposition 4 means that $\mathsf{R_N R_K}$ corresponds to the actions that eliminate several possible states as **PAL** actions do, but may cause change of truth values of atomic propositions.

In the context of DEL, we often restrict our attention to epistemic actions, which do not change the truth values of atomic propositions. Thus here we name an epistemic MTS $\mathfrak{M}$ whose underlying FTS $\mathbf{U}(\mathfrak{M})$ is eliminative, an *eliminative MTS*. Eliminative updates appear in the literature not only in this *state-eliminating* style [5, 6] but in the *link-cutting* style as in PU-MTSs [6, 8]. By Proposition 3 and the construction of the proof in Proposition 4, this class is captured by $\mathsf{R_N}$, $\mathsf{R_K}$ and $\mathsf{R_P}$:

Proposition 5. *An MTS* $\mathfrak{M}$ *validates* $\mathsf{R_P}$ *and* $\mathbf{U}(\mathfrak{M})$ *validates* $\mathsf{R_N}$ *and* $\mathsf{R_K}$ *iff* $\mathfrak{M}$ *is a bounded morphic image of an eliminative MTS.*

Proposition 5 means that $\mathsf{R_N R_K R_P}$ corresponds to the actions that eliminate possible states, i.e. increase agents' knowledge in the epistemic case. Public announcements are a typical example of such actions, but these do not always have to be expressed by a formula. For example, think of the computer game Minesweeper. Each time the player clicks on a cell on the board, the player's knowledge about the mines' locations is updated. Thus ⟨⟨click⟩⟩ can be equally considered an action. The logic $\mathsf{K} \oplus \mathsf{R_N R_K R_P}$ could be useful to model such a situation.

Definition 9 (Unconditional FTS). *An FTS* $\mathfrak{F} = (\mathfrak{F}_I, \Phi, \mathfrak{R})$ [*or an MTS* $(\mathfrak{M}_I, \Phi, \mathfrak{R})$] *is called* unconditional *if for each* $(i, \alpha) \in I \times \mathfrak{A}$, $\mathfrak{R}_i^\alpha$ *satisfies the condition that for any* $v \in \mathbf{F}_i$, *there exists* $w \in \mathbf{F}_i^\alpha$ *such that* $v \mathfrak{R}_i^\alpha w$.

If we were to express this differently, there is no precondition to the action α and therefore α is always possible. $\mathsf{R_A}$ expresses this property:

Proposition 6. *An FTS validates* $\mathsf{R_A}$ *iff it is unconditional.*

Note that Propositions 2, 3 and 6 do not involve the construction of a bounded morphism, thus they can be freely combined.

We know that all the above combinations of generic reduction axioms are *canonical* in the ordinary sense, that is, their canonical models of the form $(S, \{R_e\}_{e \in E}, \{R_\alpha\}_{\alpha \in \mathfrak{A}}, V)$ satisfy their corresponding frame/model properties. Given this and all the correspondence results above, soundness and completeness hold for each of the following pairs:

1. $\mathsf{K}\oplus\mathsf{R_N}$ and the class of deterministic FTSs
2. $\mathsf{K}\oplus\mathsf{R_N R_P}$ and the class of deterministic and epistemic MTSs
3. $\mathsf{K}\oplus\mathsf{R_N R_K}$ and the class of eliminative FTSs
4. $\mathsf{K}\oplus\mathsf{R_N R_K R_P}$ and the class of eliminative MTSs
5. $\mathsf{K}\oplus\mathsf{R_A}$ and the class of unconditional FTSs
6. $\mathsf{K}\oplus\mathsf{R_A R_N}$ and the class of unconditional and deterministic FTSs
7. $\mathsf{K}\oplus\mathsf{R_A R_N R_P}$ and the class of unconditional, deterministic and epistemic MTSs

Remark 2. $\mathsf{R_N}$ plays an important role; for example, without $\mathsf{R_N}$ we cannot even prove that $\mathsf{R_P}$ is linked to the class of epistemic MTSs.

3.3 Specific Correspondence Results

So far we have been concerned with the generic DEL language $\mathcal{L}(E, \mathfrak{A})$ and MTSs/FTSs in general. We now proceed to the extra reduction axioms in Example 2, which are specific to the languages of **PAL**, **EA** and **PU**. Our modular approach leads to an alternative proof of completeness for each of the three logics.

Proposition 7

1. *Let $\mathfrak{M}$ be a deterministic and epistemic MTS for $\mathcal{L}_{\mathsf{PAL}}$. $\mathfrak{M}$ validates $\mathsf{R_T}$ and $\mathbf{U}(\mathfrak{M})$ validates $\mathsf{R_K}$ iff $\mathfrak{M}$ is a bounded morphic image of an eliminative MTS $\mathfrak{N}$ where each transformed model $\mathbf{N}_i^{!\varphi}$ is the submodel of $\mathbf{N}_i$ whose carrier set is $\{v \in \mathbf{N}_i \mid \mathfrak{N}, \mathbf{N}_i, v \models \varphi\}$, i.e. $\mathfrak{M}$ is a bounded morphic image of a PAL-MTS.*
2. PAL *is sound and complete with respect to the class of PAL-MTSs.*

As a PAL-MTS precisely expresses the intended model transformation of **PAL**, we obtain an alternative proof of the completeness of PAL with respect to the original semantics in [5].

Remark 3. An alternative and modular proof of the completeness of PAL has already been given in [13]. It also uses a canonical Kripke model of the form $(S, \{R_n\}_{n \in Ag}, \{R_{!\varphi}\}_{!\varphi \in \mathfrak{A}_{\mathsf{PAL}}}, V)$. However, our approach stresses the modular nature of the argument by starting from the generic framework.

For **EA**, we have the following result:

Proposition 8

1. *Let $\mathfrak{M} = (\mathfrak{M}, \Phi, \mathfrak{R})$ be an MTS for $\mathcal{L}_{\mathsf{EA}}$ that satisfies $\Phi(i, (U, s)) = \Phi(i, (U, t))$ for any $i \in I$ and $(U, s), (U, t) \in \mathfrak{A}_{\mathsf{EA}}$. Then, $\mathfrak{M}$ validates $\mathsf{R_P}$ and Pre while $\mathbf{U}(\mathfrak{M})$ validates $\mathsf{R_N}$ and $\mathsf{A_{EA}}$ iff $\mathfrak{M}$ is a bounded morphic image of an EA-MTS.*
2. EA *is complete with respect to the class of EA-MTSs.*

From these two results, we obtain an alternative proof of the completeness with respect to the semantics in [2], in a way analogous to the case of **PAL**.

For PU, we do not yet have an adequate characterisation of $\mathsf{A_{PU}}$. Nevertheless, the previous generic results turn out to be useful when proving the following result:

Proposition 9. PU *is complete with respect to the class of PU-MTSs.*

This also leads to an alternative proof of the completeness of PU with respect to the semantics in [8].

4 Algebraic Semantics

We discuss algebraic semantics of DEL and develop a similar correspondence theory as above.

4.1 Algebraic Model Transition System

We first introduce an algebraic counterpart of the notion of MTS (Definition 3). By *algebraic model*, we mean $\mathcal{M} = (\mathcal{A}, \theta)$, where $\mathcal{A} = (A, +, -, 1, \{f_e\}_{e \in E})$ is a *Boolean algebra with operators* (*BAO*) and $\theta : \mathcal{P} \to A$ is an *assignment*.

Definition 10 (Algebraic Model Transition System). *An* algebraic model transition system (AMTS) *for* $\mathcal{L}(E, \mathfrak{A})$ *is a triple* $\mathsf{M} = (\mathsf{M}_I, \Phi, \mathsf{F})$ *such that*

1. M_I *is a family of algebraic models* $\mathcal{M}_i$ *indexed by* $i \in I$,
2. $\Phi : I \times \mathfrak{A} \to I$ *is a function (notation:* $\mathcal{M}_i^\alpha := \mathcal{M}_{\Phi(i,\alpha)}$*),*
3. F *assigns a* 0*-preserving additive function*[1] F_i^α *from the carrier set of* $\mathcal{M}_i^\alpha$ *to that of* $\mathcal{M}_i$ *for each* $(i, \alpha) \in I \times \mathfrak{A}$ *(notation:* $\mathsf{F}_i^\alpha : \mathcal{M}_i^\alpha \to \mathcal{M}_i$*).*

Analogously, an *algebra transition system* (*ATS*) $\mathsf{A} = (\mathsf{A}_I, \Phi, \mathsf{F})$ is also defined by using indexed BAOs instead of indexed algebraic models. We say that $\mathsf{A} = (\mathsf{A}_I, \Phi, \mathsf{F})$ is the *underlying* ATS of an AMTS $\mathsf{M} = (\mathsf{M}_I, \Phi, \mathsf{F})$ and write $\mathsf{A} = \mathbf{U}(\mathsf{M})$ if $\mathcal{A}_i \in \mathsf{A}_I$ is the underlying BAO of algebraic model $\mathcal{M}_i$ for each $i \in I$.

Definition 11. *Let* $\mathsf{M} = (\mathsf{M}_I, \Phi, F)$ *be an AMTS for* $\mathcal{L}(E, \mathfrak{A})$ *and* $\mathcal{M}_i = (A, +, -, 1, \{f_e\}_{e \in E}, \theta)$ *in* M_I. *The* meaning $[\varphi]_{\mathcal{M}_i, \mathsf{M}}$ *of an* $\mathcal{L}(E, \mathfrak{A})$*-formula* φ *is inductively defined as follows:*

$$
\begin{array}{ll}
[\top]_{\mathcal{M}_i,\mathsf{M}} = 1 & [p]_{\mathcal{M}_i,\mathsf{M}} = \theta(p) \\
[\neg\varphi]_{\mathcal{M}_i,\mathsf{M}} = -[\varphi]_{\mathcal{M}_i,\mathsf{M}} & [\varphi \vee \psi]_{\mathcal{M}_i\mathsf{M}} = [\varphi]_{\mathcal{M}_i,\mathsf{M}} + [\psi]_{\mathcal{M}_i,\mathsf{M}} \\
[\langle e \rangle \varphi]_{\mathcal{M}_i,\mathsf{M}} = f_e([\varphi]_{\mathcal{M}_i,\mathsf{M}}) & [\langle\!\langle \alpha \rangle\!\rangle \varphi]_{\mathcal{M}_i,\mathsf{M}} = \mathsf{F}_i^\alpha([\varphi]_{\mathcal{M}_i^\alpha,\mathsf{M}})
\end{array}
$$

We say that an AMTS M *validates* φ if $[\varphi]_{\mathcal{M}_i,\mathsf{M}} = 1$ for any $\mathcal{M}_i$ in M. Validity in an ATS is defined analogously.

The algebraic semantics of **PAL** [4] and of **EA** [3] can be rephrased in terms of our AMTSs as follows. An algebraic semantics of **PU** has not been established. This shall be dealt with in our forthcoming work.

Example 4 (Specific Algebraic Model Transition Systems)

PAL: Let $\mathcal{A} = (A, +, -, 1, \{f_e\}_{e \in E})$ be a BAO and $\mathcal{M} = (\mathcal{A}, \theta)$ an algebraic model. For each $a \in A$, we define $\mathcal{A}{\downarrow}a = (A{\downarrow}a, +', -', 1', \{f'_e\}_{e \in E})$ and $\mathcal{M}{\downarrow}a = (\mathcal{A}{\downarrow}a, \theta_a)$ as follows:

$$
\begin{array}{ll}
A{\downarrow}a = \{x \in A \mid x \leq a\} & x +' y = a \cdot (x + y) = x + y \\
-'(x) = a \cdot (-x) & 1' = a \cdot 1 = a \\
f'_e(x) = a \cdot f_e(x) & \theta_a(p) = a \cdot \theta(p)
\end{array}
$$

A *PAL-AMTS* (for the language $\mathcal{L}_{\mathsf{PAL}}$) is then given as follows: for every $\mathcal{M}_i = (A, +, -, 1, \{f_n\}_{n \in Ag})$ and $!\varphi \in \mathfrak{A}_{\mathsf{PAL}}$, let $\mathcal{M}_i^{!\varphi} = \mathcal{M}{\downarrow}[\varphi]_{\mathcal{M}_i,\mathsf{M}}$, and $\mathsf{F}_i^{!\varphi}$ be the set inclusion function from $A{\downarrow}[\varphi]_{\mathcal{M}_i,\mathsf{M}}$ to A. It is easy to see that $\mathsf{F}_i^{!\varphi}$ is indeed 0-preserving and additive.

EA: Suppose that $\mathcal{M} = (A, +, -, 1, \{f_n\}_{n \in Ag}, \theta)$ is an algebraic model and that (U, s) is an action model with $U = (U, \{\to_n\}_{n \in Ag})$. We define $\prod_U \mathcal{M} = (\prod_U A, +', -', 1', \{f'_n\}_{n \in Ag}, \theta')$: $\prod_U A$ is the $|U|$-colored product (i.e. the power of A with each coordinate indexed by $u \in U$); $+'$, $-'$, $1'$ and θ' are defined

[1] That is, F_i^α such that $\mathsf{F}_i^\alpha(0) = 0$ and $\mathsf{F}_i^\alpha(x + y) = \mathsf{F}_i^\alpha(x) + \mathsf{F}_i^\alpha(y)$.

coordinatewise; and $f'_n : \prod_U A \to \prod_U A$ is defined by $f'_n(k)(s) = \bigvee\{f_n(k(t)) \mid s \to_n t\}$ for any $k \in \prod_U A$ and $s \in U$. An *EA-AMTS* M is then given as follows (here $[\mathrm{Pre}]_{\mathcal{M}_i,\mathsf{M}}$ denotes the element $\langle[\mathrm{Pre(s)}]_{\mathcal{M}_i,\mathsf{M}}\rangle_{s\in U}$ of $\prod_U \mathcal{M}$): each transformed model $\mathcal{M}_i^{(U,s)}$ is given by $(\prod_U \mathcal{M}_i)\!\downarrow\![\mathrm{Pre}]_{\mathcal{M}_i,\mathsf{M}}$; $\mathsf{F}_i^{(U,s)} : \mathcal{M}_i^{(U,s)} \to \mathcal{M}_i$ is defined to be the composition of the set inclusion $\mathcal{M}_i^{(U,s)} \hookrightarrow \prod_U \mathcal{M}_i$ and the s-th projection $\prod_U \mathcal{M}_i \to \mathcal{M}_i$; and we impose the condition that $\Phi(i,(U,s)) = \Phi(i,(U,t))$ for any $i \in I$ and (U,s), $(U,t) \in \mathfrak{A}_{\mathsf{EA}}$.

Homomorphisms for AMTSs are defined as follows:

Definition 12. *Let* $\mathsf{M} = (\mathsf{M}_I, \Phi, \mathsf{F})$ *and* $\mathsf{N} = (\mathsf{N}_J, \Psi, \mathsf{G})$ *be AMTSs. A* homomorphism h *from* M *to* N *is a pair* $(h, \{h_j\}_{j\in J})$ *of a function* $h : J \to I$ *and algebraic model homomorphisms*[2] $h_j : \mathcal{M}_{h(j)} \to \mathcal{N}_j$*, such that for any* $j \in J$ *and action expression* $\alpha \in \mathfrak{A}$*, 1.* $\Phi(h(j), \alpha) = h(\Psi(j, \alpha))$ *and 2.* $h_j \circ \mathsf{F}^\alpha_{h(j)} = \mathsf{G}^\alpha_j \circ h_{\Psi(j,\alpha)}$.

Proposition 10. *Let* $(h, \{h_j\}_{j\in J}) : \mathsf{M} \to \mathsf{N}$ *be a homomorphism between AMTSs. Then, for any* $\mathcal{N}_j$ *in* N *and* $\mathcal{L}(E, \mathfrak{A})$*-formula* φ, $h_j([\varphi]_{\mathcal{M}_{h(j)},\mathsf{M}}) = [\varphi]_{\mathcal{N}_j,\mathsf{N}}$.

We call a homomorphism $(h, \{h_j\}_{j\in j}) : (\mathsf{M}_I, \Phi, \mathsf{F}) \to (\mathsf{N}_J, \Psi, \mathsf{G})$ *injective* if for any $i \in I$ there is a $j \in J$ such that $h(j) = i$ and $h_j : \mathcal{M}_i \to \mathcal{N}_j$ is an injective homomorphism, and we say that M can be *embedded* into N if there is an injective homomorphism from M to N. Similar notions are defined for ATSs.

4.2 General Algebraic Correspondence Results

Let us now discuss correspondences between reduction axioms and algebraic properties.

Definition 13. *An ATS* $\mathsf{A} = (\mathsf{A}_I, \Phi, \mathsf{F})$ *is* deterministic *if each* $\mathsf{F}^\alpha_i : \mathcal{A}^\alpha_i \to \mathcal{A}_i$ *preserves all meets (i.e.* $\mathsf{F}^\alpha_i(x \cdot y) = \mathsf{F}^\alpha_i(x) \cdot \mathsf{F}^\alpha_i(y)$*).*

Proposition 11. *An ATS validates* $\mathsf{R_N}$ *iff it is deterministic.*

Correspondence results concerning $\mathsf{R_K}$ are expressed by the following notions:

Definition 14. *An ATS* $\mathsf{A} = (\mathsf{A}_I, \Phi, \mathsf{F})$ *is* eliminative *if for any algebraic model* $\mathcal{A}_i \in \mathsf{A}_I$ *and* $\alpha \in \mathfrak{A}$*, the transformed model* $\mathcal{A}^\alpha_i$ *is given by* $\mathcal{A}_i\!\downarrow\! a$ *for some* $a \in \mathcal{A}_i$*, and* $\mathsf{F}^\alpha_i : \mathcal{A}^\alpha_i \to \mathcal{A}_i$ *the set inclusion function. An* eliminative AMTS *is analogously defined by using* $\mathcal{M}_i\!\downarrow\! a$ *instead of* $\mathcal{A}_i\!\downarrow\! a$.

Proposition 12. *Let* A *be an ATS and* M *an AMTS.*
1. A *validates* $\mathsf{R_N}$ *and* $\mathsf{R_K}$ *iff it can be embedded into an eliminative ATS.*
2. M *validates* $\mathsf{R_P}$ *and* $\mathbf{U}(\mathsf{M})$ *validates* $\mathsf{R_N}$ *and* $\mathsf{R_K}$ *iff* M *can be embedded into an eliminative AMTS.*

[2] These are BAO homomorphisms that preserve assignments.

As a corollary, soundness and completeness hold for each of the following pairs:

1. $\mathsf{K}{\oplus}\mathsf{R_N}$ and the class of deterministic ATSs
2. $\mathsf{K}{\oplus}\mathsf{R_N R_K}$ and the class of eliminative ATSs
3. $\mathsf{K}{\oplus}\mathsf{R_N R_K R_P}$ and the class of eliminative AMTSs

4.3 Specific Correspondence Results

We next turn to correspondence results specific to **PAL** and **EA**.

Proposition 13
1. *An AMTS* M *for* $\mathcal{L}_{\mathsf{PAL}}$ *validates* $\mathsf{R_P}$ *and* $\mathsf{R_T}$ *while* $\mathbf{U}(\mathsf{M})$ *validates* $\mathsf{R_N}$ *and* $\mathsf{R_K}$ *iff* M *can be embedded into a PAL-AMTS.*
2. PAL *is sound and complete with respect to the class of PAL-AMTSs.*

The above results lead to an alternative and modular proof of the completeness of PAL with respect to the algebraic semantics in [4] since a PAL-AMTS expresses the intended algebraic model transformations of **PAL**.

Analogously, in the case of **EA**, the following results give its completeness with respect to the algebraic semantics in [3]:

Proposition 14
1. *Let* $\mathsf{M} = (\mathsf{M}_I, \Phi, \mathsf{F})$ *be an AMTS for* $\mathcal{L}_{\mathsf{EA}}$ *that satisfies* $\Phi(i,(U,s)) = \Phi(i,(U,t))$ *for any* $i \in I$ *and* $(U,s),(U,t) \in \mathfrak{A}_{\mathsf{EA}}$*. Then,* M *validates* $\mathsf{R_P}$ *and* Pre *while* $\mathbf{U}(\mathsf{M})$ *validates* $\mathsf{R_N}$ *and* $\mathsf{A_{EA}}$ *iff* M *can be embedded into an EA-AMTS.*
2. EA *is sound and complete with respect to the class of EA-AMTSs.*

5 On Duality between MTSs and AMTSs

The correspondence results in Section 4 were generated by the duality between MTSs and AMTSs. This can be summarised as follows.

First of all, all MTSs and all bounded morphisms, and all AMTSs and all homomorphisms constitute categories $\mathscr{MTS}$ and $\mathscr{AMTS}$. Here, the composition of morphisms is defined as follows: for bounded morphisms $\mathfrak{f} = (f, \{f_i\}_{i\in I}) : (\mathfrak{L}_I, \Phi, \mathfrak{P}) \to (\mathfrak{M}_J, \Psi, \mathfrak{Q})$ and $\mathfrak{g} = (g, \{g_j\}_{j\in J}) : (\mathfrak{M}_J, \Psi, \mathfrak{Q}) \to (\mathfrak{N}_K, X, \mathfrak{R})$, their composition $\mathfrak{g} \circ \mathfrak{f}$ is given by $(g \circ f, \{g_{f(i)} \circ f_i\}_{i\in I})$; and for homomorphisms $\mathsf{f} = (f, \{f_j\}_{j\in J}) : (\mathsf{L}_I, \Phi, \mathsf{F}) \to (\mathsf{M}_J, \Psi, \mathsf{G})$ and $\mathsf{g} = (g, \{g_k\}_{k\in K}) : (\mathsf{M}_J, \Psi, \mathsf{G}) \to (\mathsf{N}_K, X, \mathsf{H})$, their composition $\mathsf{g} \circ \mathsf{f}$ is defined by $(f \circ g, \{g_k, \circ f_{g(k)}\}_{k\in K})$.

Between these two categories, there are contravariant functors as follows: (Here, $\mathbf{M}^+$ and $\mathcal{M}_+$ denote the full complex algebra with the assignment of an ordinary Kripke model $\mathbf{M}$ and the ultrafilter model of an algebraic model $\mathcal{M}$.)

Definition 15

1. *A contravariant functor* $(-)^+ : \mathscr{MTS} \to \mathscr{AMTS}$ *is given as follows: its object function assigns to each MTS* $\mathfrak{M} = (\{\mathbf{M}_i\}_{i\in I}, \Phi, \mathfrak{R})$ *the AMTS* $\mathfrak{M}^+ = (\{\mathbf{M}_i^+\}_{i\in I}, \Phi, \mathfrak{R}^+)$ *where* $\mathfrak{R}^+(i,\alpha)$ *is given by* $(\mathfrak{R}_i^\alpha)^{-1} : (\mathbf{M}_i^\alpha)^+ \to \mathbf{M}_i^+$*; its arrow function assigns to each bounded morphism* $(f, \{f_i\}_{i\in I}) : \mathfrak{M} \to \mathfrak{N}$ *the homomorphisms* $(f, \{f_i^{-1} : \mathbf{M}_{f(i)}^+ \to \mathbf{M}_i^+\}_{i\in I}) : \mathfrak{N}^+ \to \mathfrak{M}^+$*.*
2. *A contravariant functor* $(-)_+ : \mathscr{AMTS} \to \mathscr{MTS}$ *is given as follows: its object function assigns to each AMTS* $\mathsf{M} = (\{\mathcal{M}_i\}_{i\in I}, \Phi, \mathsf{F})$ *the MTS* $\mathsf{M}_+ = (\{\mathcal{M}_{i+}\}_{i\in I}, \Phi, \mathsf{F}_+)$ *where* $\mathsf{F}_+(i,\alpha)$ *is defined by* $v{\mathsf{F}_+}_i^\alpha w \Leftrightarrow \mathsf{F}_i^\alpha[w] \subseteq v$*; its arrow function assigns to each homomorphism* $(f, \{f_j : \mathcal{M}_{f(j)} \to \mathcal{N}_j\}_{j\in J}) : \mathsf{M} \to \mathsf{N}$ *the bounded morphism* $(f, \{f_j^{-1} : \mathcal{N}_{j+} \to \mathcal{M}_{f(j)+}\}_{j\in J}) : \mathsf{N}_+ \to \mathsf{M}_+$*.*

In particular, surjective bounded morphisms of MTSs and injective homomorphisms of AMTSs are 'dual' via the above contravariant functors.

On the relationship between these two functors $(-)^+$ and $(-)_+$, the following result is immediate:

1. An MTS $\mathfrak{M} = (\mathfrak{M}_I, \Phi, \mathfrak{R})$ is 'embedded' into $(\mathfrak{M}_+)^+$ by $\epsilon_{\mathfrak{M}} = (Id_I, \{\Pi_i\}_{i\in I}) : \mathfrak{M} \to (\mathfrak{M}^+)_+$ where each embedding $\Pi_i : \mathbf{M}_i \to (\mathbf{M}_i^+)_+$ assigns the principal ultrafilter π_x to $x \in \mathbf{M}_i$.
2. An AMTS $\mathsf{M} = (\mathsf{M}_I, \Phi, \mathsf{F})$ is 'embedded' into $(\mathsf{M}_+)^+$ by $\eta_{\mathsf{M}} = (Id_I, \{r_i\}_{i\in I}) : \mathsf{M} \to (\mathsf{M}_+)^+$ where $r_i : \mathcal{M}_i \to (\mathcal{M}_{i+})^+$ is the canonical embedding.

All the $\epsilon_{\mathfrak{M}}$ meet the condition of a natural transformation from $Id_{\mathscr{MTS}}$ to $(-)_+ \circ (-)^+$and all the η_{M} meet that of a natural transformation from $Id_{\mathscr{AMTS}}$ to $(-)^+ \circ (-)_+$. However, placing a condition on $\mathfrak{M}$ and M is necessary for $\epsilon_{\mathfrak{M}}$ and $\eta_{\mathcal{M}}$ to be arrows in the categories $\mathscr{MTS}$ and $\mathscr{AMTS}$, and to obtain natural transformations ϵ and η. For instance, as an easy example, let us take the condition that all $\mathbf{M}_i \in \mathfrak{M}$ and all $\mathcal{M}_i \in \mathsf{M}$ are finite. Those objects satisfying this condition constitute full subcategories $Fin\mathscr{MTS}$ and $Fin\mathscr{AMTS}$ of $\mathscr{MTS}$ and $\mathscr{AMTS}$, which are equivalent via the restricted functors of $(-)^+$ and $(-)_+$ as η and ϵ become natural isomorphisms in this case. It is this duality that underlies our algebraic development: for example, the algebraic characterisation of the axiom $\mathsf{R_N}$ (Proposition 4.2) is obtained by the fact that $\mathfrak{R}_i^\alpha$ is a partial function (i.e. deterministic) iff $(\mathfrak{R}_i^\alpha)^{-1}$ preserves intersections (i.e. meets).

6 Conclusion

We have proposed a general framework for modal correspondence in Dynamic Epistemic Logic (DEL) in both relational and algebraic semantics. (i) We first introduced a generic DEL language and (ii) accordingly introduced model transition systems (MTSs) and algebraic model transition systems (AMTSs) as 'static' formalisations of model transformations. Using our framework, (iii) we gave general correspondence results for generic reduction axioms and (iv) extended them to specific reduction axioms defining **PAL**, **EA** and **PU**. (v) All these constitute modular proofs to the completeness of the three logics with respect to both relational and algebraic semantics.

An exception is the algebraic study of **PU**, which shall be addressed in our future work. It would be also interesting to study other DELs, such as **LCC** [10], and other operators, like common knowledge operators. In this paper we have only considered reduction axioms that already exist in the literature. However, since our language is generic, it is perhaps possible to treat a more general class of axioms and develop a 'dynamic' Sahlqvist theory for them. This too shall form the object of our future studies.

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