Chapter 4 Camera Calibration Implementation Based on Zhang Zhengyou Plane Method

Pingping Lu, Qing Liu and Jianming Guo

Abstract Camera calibration is a crucial step in computer vision, the main determinant of the visual measurement effect, laying the basis for three-dimensional reconstruction. In order to know calibration's precision exactly, pinhole model and relations of the four coordinates are used, camera's internal and external parameter matrices can be solved by Zhang Zhengyou plane calibration method, camera'^s distortion coefficients are then easily solved, further considered radial distortion and tangential distortion. The paper establishes a simple but clear error assessment system to evaluate the accuracy of the results and compares them with MATLAB toolbox. The experiment demonstrates that the method has high accuracy, establishing a foundation for seeking depth by binocular stereo vision.

Keywords Camera calibration ⋅ Zhang zhengyou calibration method ⋅ Image distortion ⋅ MATLAB toolbox

4.1 Introduction

In computer vision research which is rapidly developing, we could extract geometric information (such as the position, shape, location, etc.) of objects in 3D space in accordance with images the camera captured, then reconstruct and perceive objects according to this information [[1\]](#page-11-0). A point on the surface of the three-dimensional object can be mapped to the corresponding point on the image by the geometric model (such as pinhole model) of the camera. The geometric parameters are named camera parameters [[2\]](#page-11-0). Usually under most conditions they can be calculated by experiments, the process is camera calibration [[3\]](#page-11-0).

Those current camera calibration techniques can be roughly classified into two categories: traditional calibration and self-calibration [[4\]](#page-11-0).

P. Lu [⋅] Q. Liu (✉) [⋅] J. Guo

School of Automation, Wuhan University of Technology,

Wuhan 430070, Hubei, China

e-mail: qliu2000@163.com

[©] Springer-Verlag Berlin Heidelberg 2016

Y. Jia et al. (eds.), *Proceedings of the 2015 Chinese Intelligent*

Systems Conference, Lecture Notes in Electrical Engineering 359, DOI 10.1007/978-3-662-48386-2_4

Traditional calibration: based on camera imaging model (such as pinhole or fish-eye model), under these conditions that the shape and size of the calibration target is fixed and must have been known, through image processing and a series of mathematical transformations (linear calculation and nonlinear optimization), to calculate camera parameters [\[5](#page-11-0)]. Self-calibration: using the corresponding relationships between some specific quantities of the two images imaged before and after the camera rotation or translation, to complete camera calibration. Because the camera's main point and effective focal length have inherent constraints on the basis of certain camera imaging model, and these constraints usually have nothing to do with the surrounding environment and the movement of the camera. Therefore, self-calibration can take advantage of this [\[6](#page-11-0)].

Camera calibration has a long history, as early as 1986, R. Tsai [\[7](#page-11-0)] has created a classical Tsai's camera model, putting forward the two-step calibration strategy which belongs to traditional calibration. This camera model can compensate camera radial distortion. The two-step calibration strategy [[8\]](#page-11-0) establishes equations by using RAC (radial alignment constraint), through direct linear operation and nonlinear optimization, to seek for the internal and external parameters. But this method has complex calculation process and high equipment accuracy requirement so that not fit with simple experimental conditions. Moreover, it is hard to detect feature points and measure data [\[9](#page-11-0)].

Professor Zhang [[10\]](#page-11-0) improved the two-step calibration strategy and proposed method based on planar template. At first, a set of images are obtained by observing a planar template at a few (at least two) different orientation. Then, the procedure consists of a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion.

Compared the above-mentioned methods, we adopt the second. This paper is organized as follows: Sect. 4.2 describes the basic principle of camera calibration. Section [4.3](#page-4-0) describes the calibration procedure. We make the planar template in front of a camera rotate or translate two times or above; or camera rotate or translate two times or above while planar template is fixed. Specific parameters of planar template moving need not be known. Section [4.4](#page-8-0) provides the experimental results. Finally, Sect. [4.5](#page-10-0) presents a brief summary.

4.2 The Basic Principle of Camera Calibration

4.2.1 Four Kinds of Coordinate Systems

There are many kinds of coordinate systems in computer vision. The following four kinds of coordinate systems are often used while calibrating.

(1) image pixel coordinate system

As shown in Fig. [4.1,](#page-2-0) there is a two-dimensional orthogonal coordinate system O_0 *uv*, whose origin O_0 is on the top left corner of the digital image $[2]$ $[2]$. (u, v) is the coordinate of each pixel, indicating the *v*-th row and the *u*-th column element in the array. $O_0 \mu \nu$ is called image pixel coordinate system [\[11](#page-11-0)].

(2) image physics coordinate system

As shown in Fig. 4.1, an orthogonal coordinate system O_1xy is built on the image plane, whose origin O_1 is the main point of the image [\[1](#page-11-0)]. Its *x* and *y* axes parallel to *u* and *v* axes respectively. Assumed the image pixel coordinate of the origin O_1 is (u_0, v_0) , the distances between every two pixel along *x* and *y* axes direction are *dx* and *dy* respectively (The unit can assume to be *mm*), so the relation between image pixel coordinate (u, v) and image physics coordinate (x, y) of any point in image is:

$$
u = \frac{x}{dx} + u_0 \quad v = \frac{y}{dy} + v_0 \tag{4.1}
$$

We represent the relation with homogeneous coordinates and the matrix. The expression is:

$$
\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1/dx & 0 & u_0 \\ 0 & 1/dy & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$
(4.2)

(3) camera coordinate system

The geometric relationship about camera imaging is shown in Fig. 4.2. The camera coordinate system $O_C X_C Y_C Z_C$ has its origin O_C at the center

Fig. 4.1 Image pixel coordinate system and image physics coordinate system

Fig. 4.2 Camera coordinate system and world coordinate system

of projection, its Z_C axis along the optical axis, and its X_C and Y_C axes parallel to the *x* and *y* axes of image physics coordinate system respectively [[5\]](#page-11-0). The distance from image plane to the core of camera $O_CO₁$ is the camera's focal length *f* [[5\]](#page-11-0).

(4) world coordinate system

Because the position of camera and target is not fixed in the space, we can only describe their relative position through the establishment of reference coordinate system. This is the world coordinate system $O_W X_W Y_W Z_W$, as shown in Fig. [4.2](#page-2-0) [[2\]](#page-11-0). The relationship between the camera coordinate and the world coordinate can be represented by a rotation matrix \vec{R} and a translation vector t [\[2](#page-11-0)]. Assumed that a specific point **P** in 3D space, whose camera coordinate and world coordinate are $(X_C, Y_C, Z_C)^T$, $(X_W, Y_W, Z_W)^T$, the relationship between them is:

$$
\begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = M_2 \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix}
$$
(4.3)

R is 3×3 unit orthogonal matrix and *t* is translation vector, $\mathbf{0} = (0, 0, 0, 0)$ $(0)^T$, M_2 is 4 × 4 matrix, standing for the relationship between the camera coordinate and the world coordinate.

4.2.2 The Camera Model

4.2.2.1 The Linear Camera Model

A 3D point **P** projected to the corresponding point **p** on the image plane can be expressed by pinhole model approximately. As shown in Fig. [4.2](#page-2-0), an image point **p** is intersection between image plane and the attachment of camera's core O_C and **P** [[2\]](#page-11-0). We call this model center projection or perspective projection [\[2](#page-11-0)]. According to the principle of similar triangles, we get:

$$
x = \frac{fX_C}{Z_C} \qquad y = \frac{fY_C}{Z_C} \tag{4.4}
$$

The relationship between image physical coordinate and camera coordinate is:

$$
Z_C \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix}
$$
(4.5)

We put Eqs. [4.2](#page-2-0) and [4.3](#page-3-0) into Eq. [4.5](#page-3-0), then get the relationship between world coordinate and image pixel coordinate:

$$
Z_C\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f/dx & 0 & u_0 & 0 \\ 0 & f/dy & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & t \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \mathbf{M}_1 \mathbf{M}_2 \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \mathbf{M} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix}
$$
(4.6)

 $Fx = f/dx$, $fy = f/dy$ are expressed as the effective focal length of the camera in the *x* and *y* axes. *M* is 3×4 matrix, matrix M_1 is only related to the camera's internal structure, which is called camera intrinsic parameters matrix defined by fx, fy, u_0, v_0 . While matrix M_2 is only related to the camera's external parameters, which is called camera extrinsic parameters matrix.

4.2.2.2 The Non-Linear Camera Model

When the camera lens are wide-angle lens or the production of camera is not standard, it occurs the distortion on the edge of image, what's more, the more close to the edge, the more serious distortion phenomenon. Therefore, if to use linear model to calibrate camera, image point **p** will deviate from the original position and produce very large error. So the non-linear camera model is used [\[2](#page-11-0)], as follows:

$$
\begin{cases} \n\delta_x = k_1 x (x^2 + y^2) + k_2 x (x^2 + y^2)^2 + k_3 x (x^2 + y^2)^3 + p_2 (3x^2 + y^2) + 2p_1 xy \\ \n\delta_y = k_1 y (x^2 + y^2) + k_2 y (x^2 + y^2)^2 + k_3 y (x^2 + y^2)^3 + p_1 (3x^2 + y^2) + 2p_2 xy \n\end{cases}
$$
\n(4.7)

where k_1 , k_2 are radial distortion coefficients, p_1 , p_2 are tangential distortion coefficients, δ_x , δ_y are distortion errors along x and y axes.

4.3 Camera Calibration Method

4.3.1 Solving Internal and External Parameters

At first, to capture 3 or more images in the camera's view. Planar template can be translated or rotated at random, but the Z_W of the world coordinate system of each image is chosen to perpendicular to the plane, the first corner detected is perceived as the world coordinate system's origin, and $Z_W = 0$ in the plane.

Let's denote the *i*-th column of the rotation matrix \bf{R} by r_i . So Eq. [4.5](#page-3-0) can be written as:

$$
\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = sM_1[r_1 \quad r_2 \quad r_3 \quad t] \begin{bmatrix} X_W \\ Y_W \\ 0 \\ 1 \end{bmatrix} = sM_1[r_1 \quad r_2 \quad t] \begin{bmatrix} X_W \\ Y_W \\ 1 \end{bmatrix} = H \begin{bmatrix} X_W \\ Y_W \\ 1 \end{bmatrix} \quad (4.8)
$$

where *s* is an arbitrary scale factor. Let

$$
\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix}
$$
 (4.9)

$$
\boldsymbol{h} = [h_{11} \quad h_{12} \quad h_{13} \quad h_{21} \quad h_{22} \quad h_{23} \quad h_{31} \quad h_{32} \quad 1]^T \tag{4.10}
$$

Then obtain:

$$
\begin{pmatrix} X_W & Y_W & 1 & 0 & 0 & 0 & -uX_W & -uY_W & -u \ 0 & 0 & 0 & X_W & Y_W & 1 & -vX_W & -vY_W & -v \end{pmatrix} h = 0 \tag{4.11}
$$

Each point can be shown above the two equations. One image has *N* points, and $2 \times N$ equations can be obtained, written as $\mathbf{S}h = 0$. The solution of h is the corresponding eigenvector to the minimum eigenvalue of the equation $S^T S = 0$ [[2\]](#page-11-0), *H* can be solved after the vector *h* is normalized. Nonlinear least square method can be used to solve maximum likelihood estimation of *H*, here is Levenberg-Marquardt algorithm [[12\]](#page-11-0).

Each image has homography matrix *H*, written as column vectors, $H = [h_1 h_2 h_3]$, where each *h* is 3×1 vector. $H = [h_1 h_2 h_3] = sM_1[r_1 r_2 t]$ can be decomposed as:

$$
h_i = sM_1r_i \text{ or } r_i = \lambda M_1^{-1}h_i \tag{4.12}
$$

where $\lambda = 1/s$, $i = 1, 2, 3$.

Rotation matrix \vec{R} is unit orthogonal matrix, so r_1 and r_2 are orthogonal. We have two constraints:

$$
h_1^T M_1^{-T} M_1^{-1} h_2 = 0 \tag{4.13}
$$

$$
h_1^T M_1^{-T} M_1^{-1} h_1 = h_2^T M_1^{-T} M_1^{-1} h_2
$$
\n(4.14)

Let $B = M_1^{-T} M_1^{-1}$, we can get:

$$
\boldsymbol{B} = \boldsymbol{M}_1^{-T} \boldsymbol{M}_1^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \tag{4.15}
$$

In fact, there is general closed-form of matrix *B*:

$$
\boldsymbol{B} = \begin{bmatrix} \frac{1}{f_x^2} & 0 & \frac{-u_0}{f_x^2} \\ 0 & \frac{1}{f_y^2} & \frac{-v_0}{f_y^2} \\ \frac{-u_0}{f_x^2} & \frac{-v_0}{f_y^2} & \frac{u_0}{f_x^2} + \frac{v_0}{f_y^2} + 1 \end{bmatrix}
$$
(4.16)

The two constraints have their general form $h_i^T B h_j$ via matrix *B*. We could obtain each element just making sure the six elements of \bf{B} as matrix \bf{B} is symmetric matrix. The six elements are written as a column vector:

$$
h_i^T B h_j = v_{ij}^T b = \begin{bmatrix} h_{i1} h_{j1} \\ h_{i1} h_{j2} + h_{i2} h_{j1} \\ h_{i2} h_{j2} \\ h_{i3} h_{j1} + h_{i1} h_{j3} \\ h_{i3} h_{j2} + h_{i2} h_{j3} \\ h_{i3} h_{j3} \end{bmatrix} \begin{bmatrix} B_{11} \\ B_{12} \\ B_{22} \\ B_{13} \\ B_{23} \\ B_{33} \end{bmatrix}
$$
(4.17)

The two constraints can be written as Eq. 4.18 via the definition of v_{ij}^T

$$
\begin{bmatrix} v_{12}^T \\ (v_{11} - v_{22})^T \end{bmatrix} \mathbf{b} = 0
$$
 (4.18)

If we get *K* images at the same time, we can get $Vb = 0$ where *V* is a 2 $K \times 6$ matrix.

If $K \geq 2$, it has solution. At last, we compute the internal parameters.

$$
f_x = \sqrt{\lambda / B_{11}} \tag{4.19}
$$

$$
f_{y} = \sqrt{\lambda B_{11} / (B_{11} B_{22} - B_{12}^{2})}
$$
\n(4.20)

$$
u_0 = -B_{13} f_x^2 \wedge \tag{4.21}
$$

$$
v_0 = (B_{12}B_{13} - B_{11}B_{23})/(B_{11}B_{22} - B_{12}^2)
$$
\n(4.22)

$$
\lambda = B_{33} - \left[B_{13}^2 + v_0 (B_{12} B_{13} - B_{11} B_{23}) \right] / B_{11}
$$
 (4.23)

Then we compute the external parameters by *B*:

$$
r_1 = \lambda \mathbf{M}_1^{-1} h_1 \tag{4.24}
$$

$$
r_2 = \lambda \mathcal{M}_1^{-1} h_2 \tag{4.25}
$$

$$
r_3 = r_1 \times r_2 \tag{4.26}
$$

$$
t = \lambda \mathcal{M}_1^{-1} h_3 \tag{4.27}
$$

$$
\lambda = 1 / \| \mathbf{M}_1^{-1} \mathbf{h}_1 \|
$$
\n(4.28)

But if let r_1 , r_2 , r_3 combine into rotation matrix \mathbf{R} , there may be a large error because \vec{R} is not a positive definite matrix in the actual process, that is to say $R^T R = R R^T = I$ is not founded.

To solve this problem, we can choose to use singular value decomposition (SVD), to make $\mathbf{\overline{R}} = \mathbf{U}\mathbf{D}\mathbf{V}^T$ is founded. \mathbf{U} and \mathbf{V} are orthogonal matrices, \mathbf{D} is diagonal matrix. Moreover, because r_1 , r_2 , r_3 are orthogonal to each other, matrix *D* must be identity matrix *I* so that $R = UIV^T$. So we first solve singular value decomposition of \vec{R} , then \vec{D} to be set as identity matrix, finally multiply \vec{U} and \vec{V} to solve rotation matrix R' which is matching the requirement.

4.3.2 Maximum Likelihood Estimation

We adopt Levenberg-Marquardt algorithm to optimize these parameters after they are solved. Evaluation function is expressed as:

$$
C = \sum_{i=1}^{N} \sum_{j=1}^{K} ||m_{ij} - m(M_1, R_i, t_i, M_{ij})||^2
$$
 (4.28)

where N is the total number of image, K is the total number of points in each image, *m* is image pixel coordinate, *M* is world coordinate, $m(M_1, R_i, t_i, M_{ij})$ is image pixel coordinate computed by these known parameters

4.3.3 Solving Camera Distortion

We have not dealt with camera distortion so far. We use camera's internal and external parameters and all distortion coefficients set to zero as initial values to compute them. Let (x_p, y_p) is the location of point, (x_d, y_d) is the distortion location of point, so

$$
\begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} f_x X_W Z_W + u_0 \\ f_y Y_W Z_W + v_0 \end{bmatrix}
$$
\n(4.29)

Combine with Eq. [4.7,](#page-4-0) we get

$$
\begin{bmatrix} x_p \\ y_p \end{bmatrix} = (1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \begin{bmatrix} x_d \\ y_d \end{bmatrix} + \begin{bmatrix} 2p_1 x_d y_d + p_2 (r^2 + 2x_d^2) \\ p_1 (r^2 + 2y_d^2) + 2p_2 x_d y_d \end{bmatrix}
$$
(4.30)

where $r^2 = x_d^2 + y_d^2$. We can get a large number of equations and solve them to obtain distortion coefficients.

4.4 Analysis of Experimental Results

4.4.1 Error Assessment Method

Camera calibration results are difficult to evaluate whether accurate or not, there is no objective criteria. We can use world coordinate (X_W, Y_W, Z_W) and the camera parameter matrix of a 3D point **P**, to get the back-projection value (\tilde{u}, \tilde{v}) after matrix multiplication, then compare it with origin value (u, v) detected actually, get average error E_{uv} , standard deviation e_{uv} and maximum error e in the image pixel coordinate system.

$$
E_{uv} = \frac{\sum_{i=1}^{K} \left(|\widetilde{u_i} - u_i| + |\widetilde{v_i} - v_i| \right)}{K}
$$
(4.31)

$$
e_{uv} = \sqrt{\frac{\sum_{i=1}^{K} (|\widetilde{u_i} - u_i|^2 + |\widetilde{v_i} - v_i|^2)}{K}}
$$
(4.32)

$$
e = \min\left(\sum_{i=1}^{K} |\widetilde{u}_i - u_i|, |\widetilde{v}_i - v_i|\right)
$$
\n(4.33)

4.4.2 The Introduction of Experiment System

The calibration template is checkerboard with nine corners along length and six corners along width. The size of each black square is 20 mm \times 20 mm, there are 54 angular points, the first corner of each image will be origin of the world coordinate system, and world coordinates of other corners will be computed.

Adopting microscopical industrial camera, whose resolution is 640×480 pixels, and focal length is 12 mm, the physical size is 1/3″, the distance of every two pixel is 3.2 μ m. The flow chart of camera calibration algorithm is shown in Fig. [4.3.](#page-9-0)

Fig. 4.3 The flow chart of camera calibration

4.4.3 Analysis of Experimental Results

To complete calibration through 5, 10, 15 and 20 images respectively, using Eqs. [4.31](#page-8-0)–[4.33](#page-8-0) to get average error, standard deviation and maximum error (See Fig. [4.4](#page-10-0)).

With the number of calibration image increasing, the average error and standard deviation gradually decrease and stabilize to a certain value. We should take 20–²⁵ image at least. In order to make the errors seem more convenient, we describe them as a scatter plot (See Fig. [4.4\)](#page-10-0).

We choose an image to calculate the camera parameters. The rotation matrix and translation vector of 13-rd image are:

In order to verify the accuracy of the calibration results, we use MATLAB toolbox to calibrate the 20 images (See Tables [4.1](#page-10-0) and [4.2](#page-10-0)).

Table 4.1 The error comparison of calibration precision

4.5 Conclusion

Table 4.2 The comparison of calibration results

Experiments show that Zhang Zhengyou plane calibration method not only has low requirement to the experimental equipment just with a camera and a calibration template, but also has high precision, which is a transition method between the traditional calibration and the self-calibration. In simple experimental conditions, we can accurately obtain the camera parameters, and use the theory of binocular vision to compute depth of field. At the same time, the software implementation compared with MATLAB toolbox is simple, do not needs to extract angular point manually.

Acknowledgments This work is supported by the National Natural Science Foundation under Grant No. 51279152 and the Foundation of postgraduate innovation of Wuhan University of Technology under Grant No. 155211005.

References

- 1. Shujing W (2007) Research on passive vision aided location of multiple observers [D]. Northwestern Polytechnical University, College of Automation, Xi'an
- 2. Zhenxian Y (2011) Multi-camera calibration technique research and its application [D]. Hunan University: School of Information Science and Engineering, Changsha
- 3. Jianlan Z (2008) Research on some problems of camera calibration [D]. East China Normal University: Electronic Science and Technology, Shanghai
- 4. Ming C, Xiuxia S, Shu L (2014) An accurate and real-time focal-length self-calibration method based on infinite homography between vanish points [J]. Acta Optica Sinica 34(5):1–⁶
- 5. Qinglei Q (2010) Research and implementation on three-dimensional position technology based on binocular stereo vision [D]. Northeastern University: School of Information Science and Engineering, Shenyang
- 6. Qianqian Y (2010) The research on automatic finding the box-parts' position and processing based on the machining center [D]. Lanzhou University of Technology: Electromechanic Engineering College, Lanzhou
- 7. Tsia RY (1986) An efficient and accurate camera calibration technique for 3D machine vision [J]. Proc CV PR 364–³⁷⁴
- 8. Jianfei M, Xiyong Z, Jing Z (2004) Improved two-stage camera calibration from a plane [J]. J Image Graph 9(7):846–⁸⁵²
- 9. Bo L, Xiao-tong W, Xiao-gang X (2006) A linear three-step approach for camera calibration [J]. J Image Graph 11(7):928–⁹³²
- 10. Zhengyou Z (1999) A flexible camera calibration by viewing a plane from unknown orientation [J]. Comput Vis, the proceedings of the seventh IEEE international conference 1:666–⁶⁷³
- 11. Ruxin G, Junmeng W (2014) Target recognition and location based on binocular vision [J]. J Henan Polytech Univ (Nat Sci) 33(4):495–⁵⁰⁰
- 12. Shiqi Y, Ruizhen L (2009) Learning OpenCV [M]. Tsinghua University Press, Beijing, pp 406–⁴⁴⁰