Applied School Mathematics—Made in Kaiserslautern

Wolfgang Bock and Martin Bracke

1 Why Applied School Mathematics?

The mathematical modeling week as an event format was introduced in 1993 by Helmut Neunzert in Kaiserslautern. Its direct successor, the Felix Klein Modeling Week, is thus the most venerable event of its type in Germany. In the interim, this successful event has inspired others far beyond the borders of Rheinland-Pfalz and led to modeling weeks in Aachen, Graz, Hamburg, and a number of other German cities. In addition to sponsoring the Felix Klein Modeling Week, the Felix Klein Center for Mathematics in Kaiserslautern also acts as supporting partner for a modeling week in Tramin, Italy.

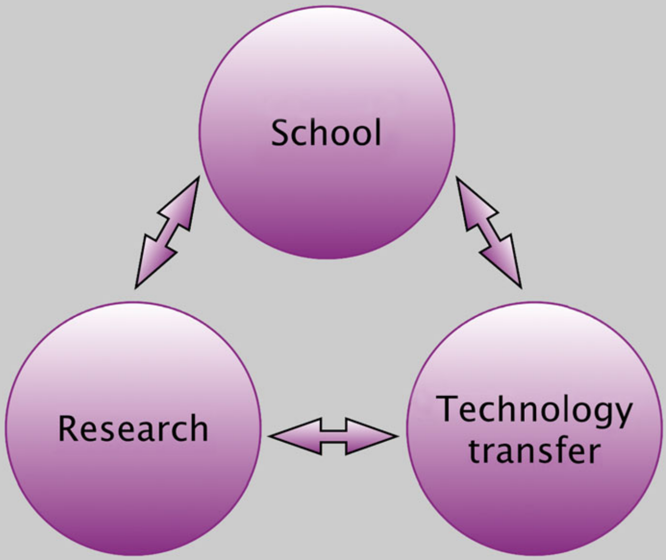
The demand for modeling events in schools reflects the need for more applied school mathematics, that is, mathematics that deals with visible, authentic, concrete problems. This demand has been the driving force behind more than 20 years' worth of modeling activities aimed at young learners.[1] The success of this concept is verified by booked-out events with waiting lists and by the receipt of the "School Meets Science" prize, received from the Robert Bosch Foundation in 2011.

[1] For the sake of readability, we will dispense with the simultaneous use of both masculine and feminine singular pronouns; in general, the masculine form will be used. All references to persons, however, may be understood to apply to both genders.

W. Bock · M. Bracke (✉)
Fachbereich Mathematik, Technische Universität Kaiserslautern, Gottlieb-Daimler Straße 48, 67653 Kaiserslautern, Germany
e-mail: bracke@mathematik.uni-kl.de

H. Neunzert, D. Prätzel-Wolters (eds.), *Currents in Industrial Mathematics*,
DOI 10.1007/978-3-662-48258-2_11

The Felix Klein Center for Mathematics, e.V, in Kaiserslautern In conjunction with the State of Rheinland-Pfalz's "Mathematics Initiative," the Felix Klein Center for Mathematics was founded in late 2008. It is named after the important mathematician and scientific promoter Felix Klein (1849–1925) and establishes an institutional connection between the Mathematics Department of the TU Kaiserslautern and the Fraunhofer Institute for Industrial Mathematics ITWM, two organizations who have worked together closely and successfully for many years.



The importance of each of the Felix Klein Center's three cornerstones—"Research," "School," and "Technology transfer"—is underscored by the diverse activities offered to students. All three are necessary to allow them to gain a genuine look inside the working methods and worldview of modern applied mathematics.

An important reason for the increased interest in applied mathematics and its impact on everyday life is that, over the past four decades, mathematics has found itself playing an entirely new role in society. The causes for its new status are closely related to the rapid development of the computer. Today's computers are some 10 000 times faster than their counterparts of 20 years ago. Moreover, powerful algorithms can deliver solutions for such applications as large equation systems—another factor of even greater significance. Today, this combination of computer and algorithm allows one to solve typical problems many millions of times faster than was possible two decades ago. This has made it possible to evaluate even complex mathematical models for industrial products and manufacturing procedures, as well as for scientific, economic, and medical processes, with acceptably high precision and speed. As a consequence, one can simulate the behavior of an automobile engine, the progress of a bypass operation, and the weather for tomorrow by preparing the appropriate mathematical models, developing the corresponding algorithms, and evaluating them on the suitable computers (cf. The Concepts—Modeling). Today, virtually every industrial development and every prediction is based on these mathematical activities. In other words, mathematics has evolved into a key technology.

At the same time, this new and crucial role that mathematics plays throughout society appears not to be fully appreciated in our schools. The linking of the MINT[1] subjects is not sufficiently evident in either the teaching curriculum or in practical lesson designs. One reason for this is the structure of the teacher training programs. Here, applied mathematics still plays a quite modest role. Neither modeling nor work with algorithms—which, in practice, have largely supplanted the use of complicated formulas—are given a position in schools commensurate with their significance. Nor is interdisciplinary cooperation between mathematics, computer science, the natural sciences, and technology adequately demonstrated or trained.

Obviously, with the means available to even an advanced high-school mathematics class, it is not possible to generate models or algorithms suitable for simulating a bypass operation. But one can develop the same abilities using other "authentic problems" that are indeed amenable to school mathematics. For example, one can measure the quality of fleeces, optimize simple radiation therapy plans, and identify turtles on the basis of their shell patterns. However, these foundational activities—modeling, calculating, and interdisciplinary work—are hardly practiced in schools. There are, of course, so-called word problems. But these very seldom reflect "authentic problems," whose relevance is clear and interesting to the students. To practice ratios, for example, one poses the following problem: "Six bulldozers need 24 hours to level a building site. After six hours, three more bulldozers come to help. How long does it take to level the building site?" In posing the problem, it is assumed that one is to work only with ratios to find a solution. However, it would be much more interesting to ask whether it even makes sense to add three more dozers, since the three extra machines might result in a "traffic jam" that actually slows down the work. It is clear to anyone who stops to think about it that arbitrarily increasing the number of dozers on the building site won't necessarily lead to a reduction in the time required for leveling.

Algorithms do appear in school, but hardly any like those developed in recent years to handle large-scale problems. An applied mathematician cannot really understand why school computers are only allocated to computer science classes. Most students have computers at home, with Internet access, and use them for playing video games, for chatting, for writing, and for surfing—for everything except the mathematics that the computer actually represents.

[1] MINT (Mathematics, Information technology, Natural sciences and Technology) is the abbreviation used in Germany. In English speaking countries, the common equivalent is STEM (Science, Technology, Engineering, and Mathematics).

Although algorithms can be taught, instructed, and learned from books, they must be implemented and tested to be truly understood. Modeling cannot be taught at all; the only valid approach is learning by doing. The goal, here, is "meta-strategic knowledge," erected on a solid foundation of mathematics expertise. Above all, however, one must learn how to structure problems from the real world, separate the important data from the trivial, and then transform it all into mathematics. Interdisciplinary thinking and working must also be learned in the course of practical activity; knowledge alone is no guarantee that information and methods from the fields of science and technology can be profitably applied.

The adoption of authentic MINT projects into mathematics instruction currently fails on two accounts. First, most teachers simply have no experience with it and therefore don't trust themselves to tackle such authentic, open-ended problems with their students. This cannot be done in the traditional lecture format, that is, "face to face," but only in a cooperative format, that is, "side by side." Often, the transition from face to face to side by side is somewhat tentative and uncertain. Second, finding suitable, authentic problems is difficult at the start, before teachers have attained "problem identification competence."

Mathematical modeling, calculating, and interdisciplinary work expose young people to new experiences and, above all, convey a more realistic and lively picture of the subjects involved. In the end, this promotes a better and deeper understanding of mathematics, science, and technology. The basic prerequisite for successfully combating the shortage of teachers trained in the MINT subjects—and this includes engineers—is a sufficiently large pool of students who are motivated to grapple with these disciplines. Solid technical skills, when coupled with the joy that arises naturally from successfully managing complex and interesting problems, will lead automatically to a greater interest in the subject areas that make such successes possible in the first place.

2 Modeling and the Search for Authentic Problems

To kick off all modeling events at the Felix Klein Center, a real problem is introduced. After years of experience with mathematical modeling weeks, modeling days, and other similar events, we can safely conclude that students are more interested in problems stemming from their real-life environments. We have thus arrived at the following definition (cf. [3]):

Definition 1 (Authentic problem) An authentic problem is one posed by a customer who is looking for a solution that can be applied to meet his needs. The problem is neither filtered nor reduced, but posed without manipulation in complete generality (i.e., posed exactly as perceived). A real-world problem (a.k.a. a realistic problem) is an authentic problem having elements that relate to the student's reality.

With this definition, we have identified the problem—the object, so to speak, upon which we want to practice "modeling." But what is "modeling" really? What must one do? There is extensive literature on mathematical modeling describing so-called modeling cycles, which have been continuously extended over the course of time (cf. [2] and the references it contains). Comparable cycles have also been applied in operations research (cf. [1]) since the 1950s. From the perspective of a problem solver, one can lay out a simple modeling procedure.

The four phases of modeling

1. Discuss the problem with the end-user (e.g., the customer)
2. Define the problem exactly
3. Design a mathematical model
    a. Analyze the problem
    b. Describe the problem mathematically
    c. Search for a suitable mathematical method
    d. Solve the mathematical problem
4. Interpret the solution in light of the end-user's original application

Define the Problem Exactly With Definition 1, we have already introduced the problem of the end-user, that is, the authentic problem. According to our definition, this problem description should be as unfiltered as possible. Naturally, this complicates Point 2. Defining the problem exactly represents a difficult challenge for students, for many working applied mathematicians, and for the customer alike. Plainly put, the real world is large and has many degrees of freedom, which allows a measure of latitude in defining any problem. The problem definition is also very dependent on the customer or problem supplier. It may even come to pass that the initial problem is formulated in such a way that it points toward a particular solution approach that later proves to be unsuited for solving the problem. Therefore, when defining the problem exactly, one must examine it carefully from all sides and, where needed, extract more information from the problem supplier. This allows one to clarify misunderstandings early on and to compensate for any missing data sets or specifications by means of skillful assumptions. For the latter, in particular, it is useful to ask oneself: What does the customer want to do with the solution? Is she perhaps also interested in something else, perhaps some issue or concern that lies hidden beneath the problem?

Design a Mathematical Model Designing a mathematical model, or mathematizing and solving the problem, surely represents the core of the mathematical modeling process. First, however, one must acknowledge that it is not easy in practice to draw a sharp boundary between "defining the problem exactly" and "designing a mathematical model." Both students and teachers, because they have purposefully enrolled in a mathematical modeling event, often get an idea immediately about which of the mathematical methods they are familiar with ought to be applied in a particular case. It is the same with mathematicians, who have many different mathematical models and techniques “in stock” in their heads and believe they recognize the appropriate ones. The expediency of such models or techniques is only revealed, however, when they are actually applied to the problem at hand.

Designing a mathematical model can be viewed as building a bridge from the real world to the mathematical world, where problems can be solved with the aid of mathematical techniques (cf. The Concepts—Modeling). However, depending on the type of problem, a model often requires more than just mathematics to be effective. A multitude of other disciplines may also be needed, such as the natural sciences, computer science, technology, geography, sports, economics, and many others. This can once again be traced back to the authenticity of the problem. For the “prediction of finish times in mountain races,” for example, in which the finish time of an athlete for a race with many ascents and descents is to be predicted on the basis of his finish times in flat-land races, models from the field of biomechanics and physics may be incorporated, depending on the modeling approach. These then have to be skillfully assembled into a suitable, calculable mathematical model. This example makes clear one basic characteristic of mathematical modeling: it is often interdisciplinary.

It is especially important at the start of the modeling process to keep in mind the above-mentioned calculability of the mathematized problem using a suitable model. Here, it is essential to simplify the problem by means of intelligent assumptions so as to attain an easily calculable basis model (cf. The Concepts—Modeling). In practice, this means concentrating initially on the essentials and trying to design a model “under laboratory conditions.” Naturally, such a model will have to be subsequently refined.

Once the model has been prepared and the problem translated into the language of mathematics, one can then commence to solve the mathematical problem. For authentic problems, this is a particular challenge for students. Either the techniques known from classroom instruction must be modified so as to be applicable to the problem or new techniques must be developed—on the basis of information gained in literature searches, for example. In the case of the latter, experience shows that students often cope better with the application of algorithms than teachers anticipate. Naturally, one cannot expect 12th graders to penetrate deeply into the mysteries of differential equation theory, for example. They can, however, learn to implement a Euler method.

The mathematical solution of the problem is often arrived at with the help of a computer—for example, when either very large data sets or suitable numerical methods are used. Of course, one also finds geometrical and analytical solutions on occasion, but we must confess that, today, this is more the exception than the rule. The same holds true, however, for applied mathematics at the university level and in industrial and research settings.

Thus, we sometimes hear the complaint from both students and teachers that modeling weeks have everything to do with computer science and nothing to do with mathematics. We, the authors, beg to differ. Certainly, one needs programming knowledge to implement the methods. However, those who express this complaint often overlook the fact that the method itself, as applied to a mathematical model that one has laboriously assembled, is also mathematics. If one wishes to use a computer to solve a problem, then one must also speak the language of the computer. And this language is—programming languages aside—mathematics.

Interpret the Solution in Light of the End-User's Original Application In order to re-cross the bridge from the world of mathematics and re-enter the real world, it is important to verify the solution's fitness for everyday use. This can be done by posing some simple questions: Does the solution I have just calculated make sense? How exact does it appear to be? Can it be improved in this regard?

At this point, the modeling procedure described at the start of this section develops into a cycle. In most cases, a comparison of the solution with reality reveals hints as to how to optimize the model or account for new influencing variables. Then, the model design process starts again from the top. If the initial model works crudely, it can perhaps be improved upon by making new assumptions, for example, or by incorporating new data sets, or by integrating previously ignored effects. It happens all too often, however, that the solution attained simply doesn't reflect reality—or doesn't reflect it accurately enough to be useful. Here, it may be that the approach selected simply will not lead to the desired goal. In this case, one must go back to the drawing board and design a new model.

The modeling sequence therefore illustrates only a single iteration of the modeling activities. These are repeated, as needed, until a satisfactory solution can be presented. One also observes, especially in student groups, that modelers don't necessarily follow the modeling sequence from top to bottom. Instead, they tend to jump back and forth from one phase to another—and this is often a quite reasonable approach. It can, in fact, be helpful during the model design phase to give some thought to the subsequent real interpretation, in order to avoid wasting time calculating unrealistic solutions, for example.

For longer modeling events in particular, one can—depending on the problem being addressed—add two more points to the four listed at the start of this section in our description of the modeling process. The extended list then takes the following form:

Extension to the four phases of modeling

1. Discuss the problem with the end-user (e.g., the customer)
2. Define the problem exactly
3. Design a mathematical model
   a. Analyze the problem
   b. Describe the problem mathematically
   c. Search for a suitable mathematical method
   d. Solve the mathematical problem

4. Interpret the solution in light of the end-user's original application
5. **Translate the solution into the language of the end-user**
6. **Generate a product prototype**

Both of these additional points could certainly be subsumed under Point 4 of our modeling recipe. Indeed, the time available for the modeling event ultimately determines whether Points 5 and 6 are feasible. Both the Felix Klein Modeling Week and the modeling days end with student presentations, which are delivered in the language of the end-user—that is, of the problem supplier or customer. One example of a product prototype, as mentioned in Point 6, might be a computer calculation program. At any rate, the presenters should strive to speak the language of the end-user—that is, one containing as little mathematics as possible—and to demonstrate in that language an applicable solution. Roughly speaking, the student groups explain to the end-user what he can do to manage his problem without his needing to understand the underlying "deeper mathematics". Experience shows that this is a difficult task for both the students and the teachers in the groups.

3 Modeling—The Attempt of a Didactic Classification

We saw in Sect. 2 what the term "modeling activities" encompasses. Namely, an authentic problem, taken whenever possible from the student's everyday life, is solved with the aid of the appropriate model design, mathematical terms, and methods, and the solution is then translated into the language of the end-user for purposes of comparing solution and reality. Here, wherever possible, the students should work out their own solutions; teachers and facilitators consciously keep a low profile. The problem suppliers are admonished to answer the students' questions exclusively from the perspective of the customers they are pretending to be. Depending on how much time is available, the facilitators can, however, attempt to help the student groups avoid getting lost in details by skillfully posing questions of their own. This gentle guidance is needed above all for groups with little or no previous experience in modeling. The goal of the modeling event for the students is to present the solution or product developed by their group to an audience consisting of the problem supplier and the students from the other groups.

In these modeling events, we find four of the five features of Jank and Meyer's activity oriented instruction (cf. [6]):

Orientation on Interests The active, hands-on investigation of various topics—a characteristic of activity oriented instruction—allows students to recognize, critically reflect upon, and further develop their own interests.

The same principle is applied at the modeling events sponsored by the Felix Klein Center for Mathematics. The problems posed during a modeling week are designed to represent a broad spectrum of topics, from which the students can then choose the ones that most appeal to their own interests.

Self-Initiative and Guidance In activity oriented instruction, the students receive as few guidelines as possible. They should remain free to explore, discover, and plan for themselves. This self-initiative poses the risk for the students, however, that the instruction degenerates into mere action and fun. Action and fun are well and good, but something sensible must come of the instruction as well. Here, one needs a dialectic comprising self-initiative and guidance.

As already indicated, much importance is attached to the student groups' generating their own solutions during modeling events. The primary goal here is that the students identify with and take ownership of their problem. By virtue of their own investigations, the students develop their own expertise and can be justifiably proud of their own accomplishments. The guidance provided by the facilitators or the teachers in attendance should come in the form of skillful questioning, done in such a way that the students critically reflect on their work and can recognize mistakes on their own. Experience shows that the student groups themselves take over guidance and leadership functions, since they know that they must later present their work in front of their peers.

Linking Head and Hand "The students' mental and manual labors achieve a dynamic equilibrium in the teaching-learning process" ([6]).

This means that, step by step, a culture of learning develops in which the material activities of the students are viewed as the expression of human development. Although genuine manual labor also takes place occasionally in the modeling events, in the form of building a functioning prototype, for example, this is certainly not the rule. In our opinion, however, creating simulation software, testing parameter sets with this same software, or conducting real experiments can also be considered examples of material activities. Since these very same activities have been carried out by almost all student groups in recent years, one might also identify this feature of activity oriented instruction in the modeling events, albeit in a less pronounced form.

Practicing Solidarity Drawing on Jörg Habermas's distinction between communicative and instrumental activity, Jank and Meyer define two forms of activity: first, linguistic-argumentative deliberation about the meaning and significance of activities and, second, purposeful, goal-oriented work. They supplement these with a third, namely, demonstrating solidarity in behavior, which is contained in both communicative and instrumental activity. Linguistic deliberation belongs to communicative activity. It consists of deliberation among the participants in the teaching-learning process about activities and possible approaches. Purposeful, goal-oriented work consists of students carrying out the activities. Solidarity in behavior arises when the first two forms are in balance with each other. It is oriented on mutual rather than individual benefit and relies on teamwork and cooperative teaching-learning forms.

Mathematical modeling—at least in the events sponsored by the Felix Klein Center for Mathematics—is a group process. Group dynamics have an enormous influence on the course of the modeling event. The degree to which the group can identify with its given (or chosen) problem also plays a role.

Orientation on the Product Activity oriented instruction is also product oriented. Students and teachers agree upon a product that is to be the goal of the students' efforts. The students can then identify themselves with this product and it can serve as a basis for criticism and for the students to evaluate the lesson period.

Here, there is a basic difference between activity oriented instruction and mathematical modeling. Only in modeling events of longer duration, such as the Felix Klein Center sponsored Junior Engineer Academy or the Fraunhofer MINT-EC Math Talents, do the participants work toward a product that has been decided upon in advance. Here, precisely because of the long duration and the planning latitude given to the students, the goal is not defined in detail in advance and can be altered as needed in the course of the event. The focus for these events, however, is on the process of modeling or, more exactly, on all the processes that belong to modeling. This having been said, most events do indeed conclude with project presentations.

Mathematical modeling is thus in accord with activity oriented instruction in many respects, perhaps even in accord with project instruction. This includes, of course, both the advantageous and the disadvantageous. We have already highlighted the advantages of this approach, and we would now like to examine the disadvantages.

Mathematical modeling places two demands on teachers that, under some circumstances, may be difficult to reconcile. First, the teachers must ensure that the guidelines for the solutions are appropriate to both the subject matter and the developmental stage of the students. Second, they must keep an eye on the presumptive motives of the students and offer them as much latitude as possible to play with and implement their (i.e., the students') own ideas. Moreover, they must also try to weave together the instructional goals and the articulated goals of the activity. As in activity oriented instruction, this can be accomplished in two ways: "Either one succeeds in getting the students to embrace the instructional goals as the goals of their activity (which becomes more likely as the instructional situation more closely involves problems, topics, and tasks that are of significance to the students), or one offers space to discuss differing interests, to argue out opposing opinions, and to develop and pursue mutual activity goals" (cf. [5]).

Schools themselves still evidence some resistance to mathematical modeling. For one thing, it is difficult to fit such an instructional unit into a 45-minute class period. Nor do the curricular pressures, compartmentalization of subjects, or often standardized classrooms necessarily have a conducive effect on activity oriented instruction.

In light of these obstacles, the introduction of this instructional concept requires at least those conditions, "that are possible with goodwill and without too much effort" (cf. [5]). It is often possible, for example, to schedule the class periods of related subjects (such as biology and physics or French and English) consecutively, thus gaining more time for activity oriented lessons. It is also advisable to consult with the other teachers, since the time-consuming and laborious project phases may indeed draw students' energies away from their other subjects—whether they like it or not—and students may behave differently in the class periods following a modeling lesson, due to its differing didactical framework. Students and teachers should also discuss the important topic of grades. Now and again, a project founders; students should be made aware of this and also that such an outcome will not be penalized with a poor grade.

4 The Felix Klein Modeling Weeks

As briefly described at the outset, the first modeling week for students and teachers was staged in 1993, and has been held every year since. The concept has been modified only slightly over the ensuing years. In this section, we will first discuss the organizational aspects and then present several projects from previous modeling week events. This should serve to give the reader an impression of the complexity of the problems being treated and also illustrate the very wide variety of topics that can be covered in this format.

4.1 The Event Concept

Since 2009, the Felix Klein Center for Mathematics has staged two modeling week events per year for some 40–48 students and 16 teachers in Rheinland-Pfalz. Once the event has been announced, teachers apply for participation, each with 2–4 students from grades 11–13. The recommendation is to select students who have very good math skills and are interested in interdisciplinary work and the use of computers. Programming skills are not required, but certainly do not hurt. We'll take a closer look at this point later. For the teachers, the event is accredited by the Ministry of Education, Science, Continuing-Education, and Culture as professional training.

The event is usually held at a youth hostel, where all participants and teachers lodge for the duration. On the one hand, this increases expenses. However, it also offers plenty of advantages over other possible venues, such as the University, for example, where the students come in the morning and go home in the afternoon, after putting in their prescribed hours.

In any case, the schedule is always the same: Participants arrive on Sunday evening and, after dinner, are welcomed officially and offered a brief introduction to the event. Then, a total of eight projects are introduced by the project tutors, who are normally lecturers, employees, or PhD students from the TU Kaiserslautern, the Fraunhofer Institute of Industrial Mathematics, or the University of Koblenz–Landau—with whom we have been collaborating for a number of years. The project introductions typically last 5–10 minutes each and, as described in Sect. 2, are formulated in the language of the end-user—that is, they contain virtually no mathematical terminology (Several examples of these project introductions follow this discussion of the event's organizational concept). After each introduction, the participants have a chance to ask questions. At this point, there is typically a conspicuous lack of curiosity; equally conspicuous, however, is how consistently this changes in the course of the week.

Once the introductions are finished, the participants can state their preferences regarding the eight projects. These are then logged into a software program developed expressly for this purpose.[2] Here, participants can choose up to three favorites and also register one project on which they definitely do not want to work. Once the preferences have all been logged, the software divides the participants into eight project groups of 5–7 students and two teachers. The most important criterion, of course, is the student preferences, but several other constraints are also taken into account:

- Participants from the same school are placed into different groups whenever possible. This practice is not usually greeted by the students with thunderous applause, since one's own schoolmates and teachers are generally the only people one knows before the event begins. However, in our experience, this approach greatly facilitates teamwork. For one thing, it prevents automatic sub-group formation within the project groups. Moreover, the participants communicate more openly with one another, since they don't know their fellow team members very well and therefore rarely make (possibly false) assumptions about strengths and weakness—a dynamic that tends to hinder progress.

  Because the participants very quickly transition to a first-name basis (and the familiar pronoun "Du"), it also makes life less complicated for the teachers if they are not working directly with the students they supervise daily in school.
- Programming skills are desirable for most projects. Therefore, when the groups are assembled, at least one person with such skills is assigned to each group, if possible. Depending on the requirements of the particular project, the project's tutor may request a larger number of programmers or students with other special skills. This might be familiarity with databases, for example, or experience with graphic programming or special expertise in a particular field of science.

After the project teams have been established, the first evening is concluded by a brief informational meeting with the project tutors and the teachers, in which the plan for the week is discussed and the teachers are informed about the role they play in the event (cf. Sect. 4.2).

On Monday, the project work begins. Each team is assigned its own room containing a flipchart or whiteboard. The program is easy to follow, since the work sessions are same each day, from 09:00–12:30 and 14:30 to 18:30, with fixed tea & coffee breaks during both the morning and afternoon sessions. Normally, however, the teams arrange their breaks to meet their own needs and often have to be reminded at break and lunch time that a change

[2] Interestingly, the genesis of this software was a modeling week project in 2009.

of pace now and then can also promote progress and that human beings need to eat and drink occasionally!

On Wednesday afternoon, we interrupt the routine for a joint field trip consisting of visits to the Fraunhofer Institute for Industrial Mathematics ITWM and the TU Kaiserslautern—provided the event location is close enough to permit this. If not, a visit to a business in the vicinity or a recreational activity, such as geo-coaching or rafting on the Rhine, serve as substitutes. This field trip is intended as an enforced time-out from work on one's own modeling project. It allows the participants to "power down" so to speak, and gives them a chance to exchange notes with members of the other groups. As a consequence of this exchange, Wednesday evening, after the participants' return, is often used for another work session, since students are eager to try out new ideas that have come to them during the day.

Officially, there are no work sessions scheduled for the evenings. As the week advances, however, the students begin to invest their supposed free time in their project work also. In future events, we plan to offer optional mini-workshops on such topics as *Using the scripting system LaTeX* or *Introduction to MATLAB*. On Thursday evening—their final evening together—the participants all work late. Since modeling week ends on Friday with presentations of the results from all project groups, and since a short written documentation of the work is expected, the first teams usually head off to bed around midnight. For some, the work continues deep into the night. Here, dry runs of the presentations—sometimes repeated dry runs—are also part of the (voluntary) program.

Friday revolves around the presentations of group results. Here, the students from each team present the solution they have developed. These presentations last 20–25 minutes; each is then followed by a brief discussion and concluding remarks from the project's combined tutor and "end-user." It is very important that each presentation be delivered in the language of the end-user—since it is designed to suit his needs—and that mathematical details are dispensed with as much as possible. Naturally, these details can be included in the written documentation. This non-technical approach allows all participants to understand the results and promotes lively discussion with active participation and many questions. One can observe here, first, that the students in the audience have the self-possession to ask the questions that are really on their minds and, second, that the presenters really have become experts in their respective project fields and can in almost all instances respond with cogent and intelligent answers.

Guests attending the final presentations, who have had no chance to observe the work in the individual groups, confirm this observation, which is a very important outcome of the modeling week and other similar events involving students. At the end, all participants are awarded certificates of participation, of course, and the event closes with a joint "coffee and cake" buffet.

One further organizational note involves computers. In the event advertisement and invitation, participants are encouraged to bring their own laptops or tablets with them. There is also a pool of laptops available, so that each team can be allocated at least one powerful device. In the early days of the modeling week, a central computing room was usually set up housing eight desktop computers with Internet access and printers. Today, however, the laptops in the eight group workrooms suffice. Internet access comes via WLAN, and there is even the chance to exchange data and execute printing jobs over a separate network. Whereas the eight desktop computers in the computing room used to be the only devices present, today, it is not unusual to find three or more laptops in each group.

As far as software is concerned, we make no specifications, merely offers: the allocated devices run standard software, such as Office, along with tools for graphics processing and various programming environments (usually Java, Free Pascal/Lazarus, Python, and C/C++). In addition, we also offer participants the use of the scientific programming environment MATLAB. This is commercial software, but the company Mathworks issues us temporary licenses for the duration of the events, which can also be used on the participants' own computers. MATLAB has the tremendous advantage that its comprehensive documentation and function catalog make it possible even for programming neophytes to implement their mathematical models and algorithms with the computer in their search for solutions. On the basis of excellent results in other school projects, we also plan to introduce LaTeX, since the students generally get up to speed quickly with it and are proud when they can produce professional-looking documents containing mathematical and scientific content.

4.2 The Role of the Teachers

As described above, the teachers work with the students in their groups during the modeling week. Here, they take on various roles. First and foremost, on the basis of their legal capacity, they serve as chaperones for the students. On the other hand, they are also to take part in the modeling process. Often, this is also the first time the teachers have attempted to solve an everyday problem with the aid of mathematical modeling. The important thing, however, is that teachers don't force their own solutions onto the students. Rather, they should assume the role of just another modeler in the group and, working "side by side" with their fellow modelers, discuss the project. In every instance, it should be the students themselves that chart the course the work is to take.

Nonetheless, if the students are grappling with the implementation of unknown techniques, teachers can serve as sounding boards and provide them with the needed support.

We emphasize the importance of explaining to the teachers in advance that "failure" is also part of the modeling process. Teachers are advised to let the students fail, if it comes to that. Thereafter, they can support the students with their troubleshooting and in the search for new ideas or workarounds.

4.3 Selected Modeling Projects

This section presents five very different projects that have been undertaken in connection with prior modeling weeks. The descriptions are composed in the language of each end-user and represent written drafts of the project presentations that the project tutors give at the start of an event. When possible, the oral presentations are supplemented with short video sequences or practical demonstrations.

Our goal here is not to give a detailed description of the selected projects with various possible solutions. Instead, we want to give the reader an impression of the complexity and diversity of the questions that have been addressed (and which therefore *can* be addressed) in modeling events. After each of the descriptions, there are several remarks about the general solution strategies developed by the working groups, as well as the data and software used.

Example 1 (Is the penguin's waddle energy efficient?) Penguins are very agile in the water. Their top underwater speed of more than 10 km/h is ample evidence of their skill as swimmers. With their torpedo-shaped bodies and very short legs, however, they don't seem to be designed for walking. The style of locomotion resulting from their anatomy is the familiar waddle. It is known that penguins require more energy for walking than other terrestrial animals of the same weight. For this reason, one might assume that penguins do not use energy very efficiently for walking. This would appear strange, however, in light of the fact that some penguin species travel great distances over land to reach their breeding and nesting grounds. And, in fact, some research results suggest that penguins have developed the ideal walking style for their body shape, and that their waddle even saves energy! This conclusion is supported by the research results of Rodger Kram, which he presents in his article "Penguin waddling is not wasteful":

http://spot.colorado.edu/~kram/penguins.pdf

Your task is to get to the bottom of this question, from a mathematical perspective, by finding a way to describe the waddle and assess the amount of energy it requires. Can you confirm the biologist's research conclusions?

In 2000, Rodger Kram published his research results in the renowned scientific journal *Nature*. He conducted experiments to verify that the penguin's waddle uses energy very efficiently for terrestrial locomotion, given that the penguin's anatomy is optimized for the world of water.

The first thought of the applied mathematician is of course, is it possible to verify the efficiency of these creatures' strange-looking gait without conducting experiments with them? In this project, which has already been carried out twice in modeling week events, it is not easy to develop a suitable model. The significant biomechanical idiosyncrasies of the penguin must be duplicated, but excessive detail can lead very quickly to too much complexity—at least for a further analysis with the means available in school mathematics (cf. Fig. 1).

Nonetheless, one arrives very quickly at a model based on differential equations that can be subsequently solved numerically. Obtaining realistic data for the dimensions and mass distribution of penguins is not easy, but with the appropriate values, one can in fact also show in the simulation that land-going penguins, in comparison to other animals, use their energy quite well.

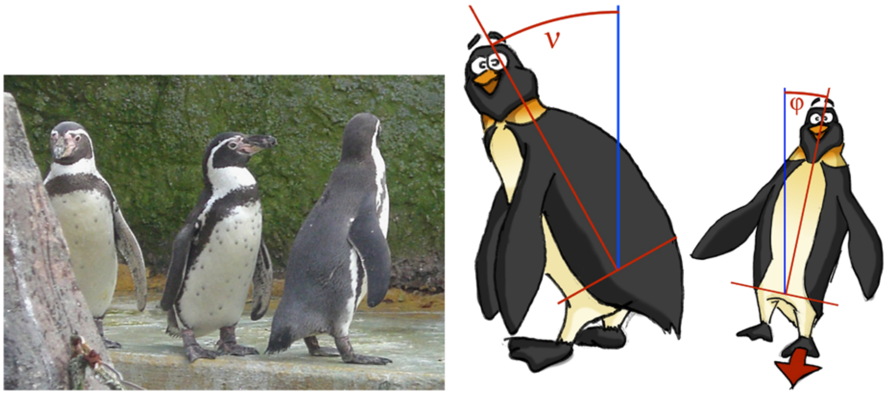


Fig. 1 Humboldt penguins and physical model (Photo, Drawing©Andreas Roth)

One might note here that the students' attempts to imitate the penguin's waddle—so as to conduct their own experiments and gain valuable insights for preparing a good model—provided first rate entertainment!

Example 2 (Judging the Laufladen Cup fairly) In 2006, the *Laufladen Cup* was inaugurated in the West Pfalz, and it has been a yearly event ever since. In 2013, the series consisted of the following 10 amateur races:

Date	Event	Length
24.03.	The Laufladen Half-marathon of the TSG Kaiserslautern	21.1 km
07.04.	Trail Run Rodenbach	10 km
21.04.	Fun Run *Auf der Platte* (Pirmasens)	10 km
03.05.	41st Intern. Fun Run Höheinöd	10 km
31.05.	Mölschbacher Wildsaulauf	10 km
07.06.	Intern. Evening Fun Run (Frankenstein)	15.6 km
08.06.	9th Kuseler Hutmacherlauf	10 km
04.08.	Udo-Bölts-Lindenparkfestlauf (Heltersberg)	12.8 km
10.08.	Residenzfestlauf Kirchheimbolanden	10 km
31.12.	41st Intern. New Year's Eve Road Race (Kottweiter)	10 km

At the end of the year, each participant who has competed in at least 6 of the 10 events receives a combined score. For every competition, each participant was assigned a point score according to the following formula:

$$P_i = 550 - 250 \cdot \frac{z_i}{z_s}, \quad P_i: \text{ point score for runner } i, \tag{1}$$

$$z_i: \text{ time for runner } i \text{ in seconds } z_s: \text{ winner's time in seconds} \tag{2}$$

For foot and bicycle races that are conducted in stages, it is typical that all participants compete in each stage; the times are then simply added together to obtain a combined result. This means of scoring is simple and seems fair. In contrast, however, several questions arise for the method of combined scoring described above:

- Do the above point values accurately reflect the differences in performance within a race?
- Does a total score consisting of the sum of the best 6 results yield an objective ranking?
- Is it possible for a racer to optimize his own point total by cleverly choosing his events? If so, how must the system be changed to eliminate this possibility?

The questions posed here seem quite unremarkable and, if one looks only superficially at the solutions presented at the end of the week, one might get the idea that they don't amount to much. This impression is deceptive, however. One need only consider the new point awarding system introduced for ski jumping competitions before the 2009–2010 season, in which the influence of the wind and the length of the approach are incorporated in a complicated manner. A complex model is used here that renders the judging of the events difficult for both athletes and spectators to understand.

In view of this somewhat dubious "standard" from the world of winter sports, it was important from the start to create an improved point awarding system that the athletes could understand and, yet, was still relatively easy to calculate. There were many discussions about what constitutes a *fair scoring system* which is of course essential for developing a model.

One recognizes quite early on that always awarding 300 points to the winner of a race is the heart of the problem: If the winner posts a comparably slow time, then all other racers receive more points for their performance then they normally would. Since only 6 of 10 events are considered for the overall ranking, participating in events with relatively high point awards offers a significant advantage over direct competitors who were not able to compete in such events.

But how can one redress this problem without making the point awarding system unnecessarily complicated at the same time? The project group that worked on this problem found a very convincing solution that was implemented in 2014 as the official judging system for the Laufladen Cup, and which is not much more difficult to calculate than the old system. Moreover, one of the participating students developed a web-based software that will permit automatic evaluation of future Cups and offer many possibilities for racers to make comparisons and predictions. This was an example where the modeling process yielded a product that was actually implemented by the end-user—an especially strong confirmation of the modelers' capabilities.

Example 3 (Optimal spread pattern for road salting vehicles) The moment snow and icy roads are forecast, road salting vehicles take to the streets and highways. Within city limits, where the vehicles travel at an average of 40 km/h, they represent no great hindrance to other traffic. On the expressways, however, where their average speed is 60 km/h, they slow down other drivers considerably. According to traffic regulations, on-duty salting vehicles have right of way and may only be passed when road conditions safely permit such a maneuver. Why, then, don't salting vehicles travel faster?

It is essential for road maintenance departments to know whether they need to buy special vehicles to service expressways, or whether the city vehicles are also suitable for use at the higher speeds. To plan the use of the vehicles effectively, road maintenance departments and vehicle manufacturers must know exactly which factors influence the distribution of salt on the road surface:

- What is the optimal speed for spreading salt on roads?
- What are the effects of changes in the throwing angle?
- How can winter road maintenance vehicles achieve an optimal spread pattern?

In dealing with these questions, one must be very careful not to get tangled up in the details: What effect exactly does salt have on the roads? How densely must it be spread to achieve the desired effect? What are the environmental impacts? Which chemical reactions—with which complex interrelationships—must be considered? To further exacerbate the problem, empirical data from actual practice is very rare.

Another important influencing factor is the method with which the salt is dispensed from the vehicle via a rotating disc (cf. Fig. 2). Here, one must investigate how rotational speed, plate inclination, frictional forces, and external factors, such as wind and turbulence behind the vehicle, impact salt distribution.

If one can find a good compromise between adequate simplicity and the needed details, then the simulations offer revealing relationships between the speed of the vehicle, the frequency of the rotating plate, and the amount of salt distributed per unit time in relation to the width of the street.

Example 4 (Optimizing the quality of fleece fabrics) In the textile industry, needling technology is used to convert fluffy, light fleece into inexpensive, durable, and tear-resistant fabric that can then be used in carpeting, lining, and insulating materials, for example (cf. Fig. 3).

During the needling process, fleece material on a conveyor belt is fed slowly under a needle board, which is pressed on the material for a fixed period of time (cf. Fig. 4). In this manner, the loose fleece material is converted into a stable fabric. The holes punched by the needles produce a pattern in the fabric that depends on the needle distribution and the belt feed rate. These patterns determine the applicability and quality of the final product. Because stripes, grids, and variations in hole density equate to diminished quality in the market, our client wishes to produce an especially homogeneous distribution of needle holes. How can he perfect his needling technology toward this end?

Fig. 2 Salting vehicle on duty (Foto: glasseyes view (flikr), Creative-Commons License)

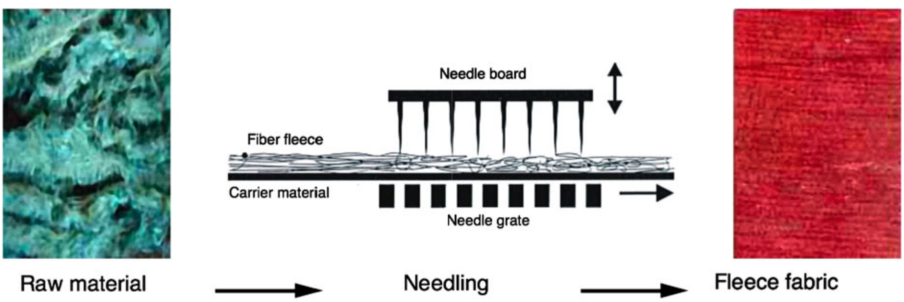


Fig. 3 Schematic depiction of the needling process (Source: Fraunhofer ITWM)

This problem confronts the students with the challenge of using a pre-designed needle board to generate a fleece needling pattern that is as random as possible. The parameters that can be varied are the frequency of the needle board pressing and the (constant) speed at which the raw fleece passes under the needle board.

An essential task is to find a suitable definition for a "non-perceivable" needling pattern. Intense structures, such as lines and regularly repeating areas of higher needling density can be easily detected. But how can one establish a continuous measure for the degree of patterning or regularity that can then be used to assess the quality of alternative configura-

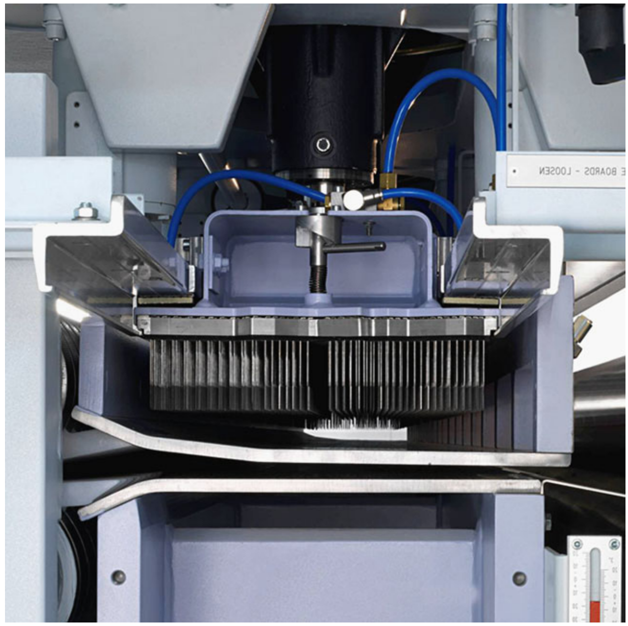


Fig. 4 Machine for needling fleeces (Photo: AUTEFA SOLUTIONS GERMANY GMBH)

tions? One approach used by some students is to first specify a fixed needle arrangement on the board and then use simulations to study the homogeneity and randomness of the resulting patterns when the two available parameters are varied. Here, however, defining a random pattern presents the students with a problem. Moreover, they must find a way to systematically investigate and appraise various parameter combinations.

To date, this challenge has inspired a wide variety of approaches: Some student groups invested a great deal of time in mathematically describing randomness and producing genuine random numbers. Others experimented with mechanical alterations to the machinery that allowed horizontal movements of the needle board to supplement the constant frequency vertical movements. On occasion, the students' attempts stimulated some very unexpected questions. One group of 10th graders wanted to place needles equidistantly along a sine curve. The group's teachers suddenly found themselves trying to answer the question of how to define and calculate the arc length of a curve—a process that necessitated explaining to their charges the previously unfamiliar derivative terms…

Along with the mathematical approach, this problem also lends itself to the use of experimentation in the search for a solution. One might build one's own needle board out of cork or Styrofoam and thin needles, for example, and then use it to simulate the real process by punching holes in a steadily moving sheet of wallpaper. The resulting hole pattern can then be analyzed and evaluated. This experimental approach allows even quite young students to delve into the underlying problem without the use of computer simulations. When this kind of experimentation succeeds in awakening the students' interest, they are often motivated to learn programming skills so as to transport the laborious real experiments into the virtual world, where new and exciting possibilities then present themselves.

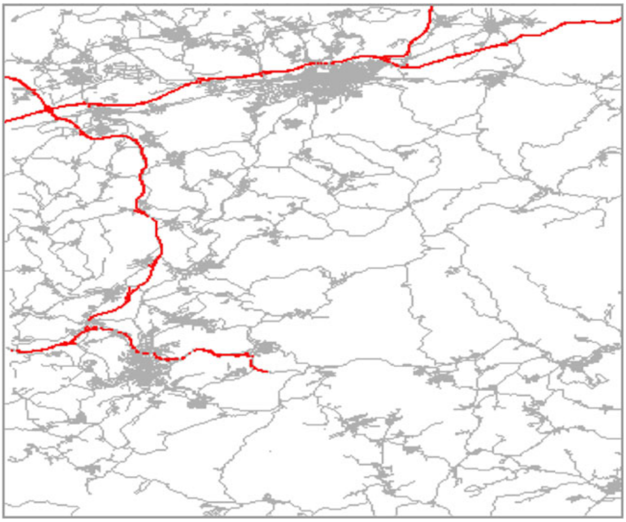


Fig. 5 Street map of Kaiserslautern and environs (OpenStreetMap data—visualized by MATLAB®)

Example 5 (Street evaluation based on route data) In this project, one is to investigate how existing route data can be used to evaluate an individual street or an entire street network. The students use a dataset for Kaiserslautern, which was provided by the *Mathematical Methods in Dynamics and Durability* am Fraunhofer-Institut für Techno- und Wirtschaftsmathematik ITWM was provided by the *Mathematical Methods in Dynamics and Durability* Department of the Fraunhofer Institute for Industrial Mathematics ITWM. This data is based on the open source database *OpenStreetMap*.

Characterizing a street network allows one to predict typical stresses for the chassis, brakes, clutches, and transmissions of motorized vehicles, such as trucks. This renders time-consuming test drives partially or entirely unnecessary, since information for areas with similar characteristics is already known and can be transferred.

Some of the project goals are:

- characterizing individual streets on the basis of individual data points,
- describing a street network,
- comparing streets or entire street networks on the basis of characteristic features.

At first glance, this project seems rather unremarkable, and it is not clear just how mathematical methods might be helpful. The students find challenges in several areas, however. For example, interpreting and processing *OpenStreetMap* data about the streets within a relatively small region of some 50×50 km (see Fig. 5) proves unexpectedly laborious. The dataset contains information about individual intersections, geographic data (longitude and latitude must first be converted prior to creating the graphical presentation), altitude, street category (highway, expressway, etc.), speed limits, and much more. A nontrivial problem, for example, is to identify from the almost 30 000 intersections in the map excerpt, those streets and junctions that must be known for the remaining processing steps.

Subsequently, various street network characteristics were discussed in the search for ways to ascertain corresponding values from the existing data. The first approaches involved the junction density of an area, the (maximal) incline of a street section, or its *curviness*. After several reasonable features were selected, the next goal was to rate a region on the basis of these features.

The selected features were then used to define a rating index that can be determined for a given street network.

The students quickly recognized, however, that average values alone are not enough for characterization purposes; they must also account for the possibility of driving around zones exhibiting poor characteristics. Thus, the more general idea arose of producing a navigation algorithm that investigates, on the basis of self-selected street network features, those connections between a region's various important points having the best possible rating. The distribution of these indices for a given region's important connections could then serve as a measure for rating and comparing different regions. Within the framework of a five-day modeling week, however, this idea could not be brought to fruition. It will be pursued as an extracurricular project over a longer time period of several months.[3]

5 Modeling Days

Modeling days are a short-format offshoot of modeling weeks. Every year, the Felix Klein Center for Mathematics stages up to 10 such events in schools. In this section, we describe the event concept in more detail. We also discuss the evolution of modeling days from the perspective of teacher training.

5.1 The Event Concept

Upon request, the Felix Klein Center for Mathematics stages several modeling days each year in schools in Rheinland-Pfalz. Twenty to thirty students, as well as members of the technical teaching staff (i.e. teachers of MINT subjects), participate in such an event. The inquiries come from the schools themselves. Because the Felix Klein Center's mission is to establish mathematical modeling in schools, the selection process considers whether a particular school already has modeling experience—either through its own modeling activities or previous participation in modeling days—and wishes to expand on its own offerings or gain more modeling experience. The modeling teams frequently comprise 11th and 12th graders in advanced college prep courses, but modeling days are also held

[3] At the time this chapter was written, the extracurricular continuation of the modeling week project was in the planning stage.

for 9th and 10th graders with equal regularity. Moreover, modeling days are also staged for other student groups, even in elementary schools. The duration of the modeling days generally depends on the schedules of the schools and varies from one to three days.

Depending on the school and the age of the participants, four to eight class periods per day are dedicated to the modeling event. Here, along with the student groups, teachers are to be present during each class period. Ideally, the teachers run the modeling days themselves, with the support of Felix Klein Center employees. The schedule is like that of a modeling week (cf. Sect. 4) in compact form: First, the students receive a short introduction to the work of applied mathematicians and mathematical modeling. Then, depending on how many students are participating, various themes are introduced for investigation in the project groups. As described in Sect. 3, the themes are introduced using the language of the end-user. Because modeling days follow the same principles as modeling weeks, the same problems are generally suitable for both types of events. In our experience, the problem sponsors need not worry about the feasibility of solving the problems within the shorter time period. Problems involving very large datasets, that is, problems for which students must invest a good deal of time in preparing the data, should be avoided or modified, however. Here, it can be useful for the problem sponsor to reduce the size of the data set in advance, or to subject it to some preliminary processing.

Directly after establishing the project groups, the members begin collecting ideas for treating their respective problems. Due to the shorter work period, it makes sense for problem sponsors to offer their groups a bit more guidance than is necessary during a modeling week. Intervening with critical questioning or encouraging the students to critically examine their own approach or model can shorten or even eliminate periods of stagnation. It is important, however, not to intervene so much that the students' motivation for solving the problem suffers.

On the final day, the student groups share their results via short presentations. This can be done at different scales. One can form an audience merely from the teachers, the problem sponsors, and the other students. More commonly, however, the audience also includes other students from the same grade. Consequently, it is important here also for presenters to use the language of the end-user, so as to make the results comprehensible to as wide an audience as possible. This comprehension then forms the basis for a further scientific discussion.

When planning a modeling day, there are a few organizational issues to bear in mind. One should try to have a computer room available or at least a laptop trolley. One should also provide Internet access. Should the above facilities not be available, however, one can also take advantage of the Felix Klein Center's laptop pool—just as in the modeling week events. Because the modeling days' relatively short time frame makes introductory programming sessions impracticable, the participants should rely on familiar programming languages and tabular calculation programs. It also makes good sense, before the modeling day commences, to get a feel for how much programming expertise the participants possess.

5.2 Modeling Days as Training Platforms for Teachers

In our view, one must practice and experience modeling oneself in order to learn it. During modeling weeks, one repeatedly hears teachers say that the concept is good, but implementing it in schools is hardly possible, due to time pressure and curricular demands, and also due to the lack of experience among the teachers. To counteract this situation, teacher training during modeling days was introduced, a concept designed to prepare teachers for integrating mathematical modeling into their own lesson plans. Here, along with mathematical modeling, some time is also devoted to practical issues, such as dealing with the 45-minute cycle time in modeling events or coping with the lack of suitable rooms.

At the moment, these training measures are in an experimental stage. If they are positively evaluated, they will then be offered to a wider audience. Training part or all of a school's technical teaching staff means that modeling teams can then be formed in which the teachers help one another in planning and carrying out modeling in the school. Experience shows that modeling is indeed possible in a school; in order to institute it on a broader basis, one needs several teachers to initiate it and serve as champions.

6 Teacher Trainings

Again and again, we see great willingness among teachers to take a chance on "Modeling and MINT at school." There is often great confusion as well, however, especially with regard to finding suitable test cases and designing lessons around real problems, which are often interdisciplinary in nature. The fact that modeling problems and implementing solutions can only be learned by doing places the question of finding appropriate instructional material in an unusually critical light. It is not possible to deliver complete solutions to teachers, since there is always a multiplicity of solution paths, which the students (and the teachers) themselves are supposed to discover. In practice, the teachers must often react to unforeseeable approaches, as well as to difficulties that suddenly arise in the search for solutions. This cannot be done with prepared materials. Instead, it requires intensive training to be able to respond appropriately and spontaneously. The design of training seminars for teachers should take this into account. Here it is not enough for teachers to listen passively to how a specially selected problem should be solved. They must actively sharpen their own problem-solving skills—by solving problems. This represents meta-strategic knowledge, which one can indeed learn, but which is quite difficult to teach. Naturally, teachers must also be given the appropriate tools and resources. Their knowledge and experience base must be extended so that they are capable of finding suitable problems, modeling them, and calculating approximate solutions. They must also learn this by doing—together with students, for example, "side by side"—in the course of a modeling week, or in intensive teaching sessions. Recently conducted in-house training events for several technical teaching staffs—conducted in the style of a modeling week—have proven effective. These trainings take into account that teachers, of course, have a deeper mathematical understanding than students. However, they usually cannot put this knowledge to work in a suitable fashion when confronted with complex, real-life problems. Our initial impression is that having the chance to use fellow teachers as a sounding board makes it noticeably easier to later implement modeling in the classroom. One such training concept might be for a teacher to first participate in a modeling week, then conduct project days in his own school (with the support of the Felix Klein Center), and finally, design and support his own project during a modeling week. Teachers trained in this manner are then in a position to pass along their knowledge and experience to their own colleagues, as well as to teachers from neighboring schools. In this way, local centers of problem solving—and also problem finding—competence are slowly built up, which require less and less support from outsiders as time goes by. Eventually, the impulse to bring modeling into schools reaches a critical mass and becomes self sustaining. For some time, teacher training opportunities have been evolving in Kaiserslautern: continuing education seminars at the Pädagogische Landesinstitut (formerly, IFB Speyer); the Felix Klein modeling weeks (twice per year in Rheinland-Pfalz and once per year in South Tyrol, together with the German school board in Bozen); in-house trainings in modeling for the technical teaching staffs of various schools; and the Junior Engineer Academy at the Heinrich Heine High School in Kaiserslautern.

7 Junior Engineer Academy

Since 2010, the Felix Klein Center for Mathematics and the Heinrich Heine High School in Kaiserslautern have jointly conducted a Junior Engineer Academy (JEA). From year to year, the JEA concept has been enhanced and refined. In this section, we will take a closer look at this developmental process and discuss the special features of this instructional form.

7.1 The Event Concept

The Junior Engineer Academy consists of a three-year project, in which an interdisciplinary topic encompassing the subjects of mathematics, computer science, and at least one natural science is offered as an alternative compulsory subject in the form of a weekly, three-period lesson.

The goal of this instructional form is not only to offer a new concept for mathematics and physics lessons, but also to help establish new organizational structures in schools. The Heinrich Heine High School, which has a gifted-and-talented curriculum, was seen as a promising partner for the JEA, since they had already introduced an alternative compulsory MINT course for students in the 7th grade and upward.

In this predecessor course, the subject was sub-divided into three sections: In the first year, students were instructed in computer science; in the second year, mathematics lessons were added; and in the third year, there was instruction in one of the natural sciences, that is, biology, chemistry, or physics (in rotating order). During this 3-year course, the class had three lesson periods of MINT per week. It is important to emphasize that the lesson contents were not supposed to overlap with the normal curriculum of the 7th to 10th grades. In selecting Heinrich Heine as partner, the main criterion was not the school's gifted-and-talented curriculum, but its already existing organizational structures. Similar projects had already been conducted by the TheoPrax® Center in Pfinztal for normal classes in grades 8–10 (cf. [4]). The teaching-learning method TheoPrax® was developed in the 1990s by Peter Eyerer, Bernd Hefer, and Dörthe Krause at the Fraunhofer Institute for Chemical Technology ICT. The goal is to use activity and practice oriented instructional concepts—implemented in cooperation with external partners in industry, research, and the services sector—to create an "interface between the school and the marketplace."

The school administrators and MINT teachers developed the new MINT course together. Naturally, it remained an alternative compulsory class with three lesson periods per week in the 7th, 8th, and 10th grades (The 9th grade was skipped, due to the BEGYS program—Gifted-and-Talented Support in Academic High Schools). However, there is now a common topic for the entire three-year cycle, along with weekly instruction in mathematics, computer science, and a natural science. In the first round, which ran from 2010 to 2013, the common topic was *location planning for wind farms*, and physics joined mathematics and computer science as third subject. The second round, begun in 2011, addressed the topic of *batteries, power packs, and fuel cells: the search for the super storage device*, and included chemistry as the natural science component. The round begun in 2012 tackled the topic *bioacoustics: automatic recognition of birdsongs*, and added biology to the trio of underlying subjects. The recently commenced Junior Engineer Academy has taken on the challenge of re-designing one of Kaiserslautern's former industrial parks, and is learning geography in the process. What makes this project special is that the topic is being shaped into its final form during the course of the Academy, which gives individual students a chance to pursue their particular interests. In the meanwhile, the group has decided to address questions from the very current field of electric mobility.

Some readers, simply from scanning these brief topic descriptions, might conclude that these are very ambitious projects for this age group, perhaps even impossible—especially for the 7th and 8th graders. Our idea was to work on real-world MINT problems that capture the interest of the students. The course is not simply about learning concepts and solving problems, and the teachers are not simply project leaders, but project partners exploring the MINT topic together with the students. For each round, there is a team of regular teachers and external teachers. The external teachers are computer scientists (for computer science instruction in rounds 1 and 2) and mathematicians (for mathematics instruction in round 2 or for mathematics and computer science instruction in rounds 3 and 4). In addition, each subject has an expert from the TU Kaiserslautern or the Fraunhofer Institute for Industrial Mathematics ITWM. The Academy includes the regular 3 hours per week of classroom instruction plus additional field trips and workshops, which are embedded in the 3-year course. Field trips include visits to the University's laboratories (not just for sight-seeing, but also for doing experiments) or other institutes or businesses. The workshops cover such subjects as *team building*, *time and project management*, *creativity* *training*, and *conflict management*. Financing the pilot project is another important consideration. For the first round, the Felix Klein Center and the Heinrich Heine High School were awarded a so-called Junior Engineer Academy grant from the Deutsche Telekom Foundation. This covered the costs of the field trips and workshops, as well as materials and equipment not included in the regular school budget. Because of clear successes visible in the first year, the two partners decided to extend the program to at least three rounds, that is, five years. The fourth round was begun in the 2013/14 school year, and the JEA has in the interim become a regular feature of the Heinrich Heine High School.

7.2 Features of the Junior Engineer Academy

Self-initiated planning increases motivation and a sense of ownership in the results. The above-mentioned workshop *time and project management* is designed to enable students to plan as many phases of the instruction for themselves as possible. Here, small projects and/or lesson content are to be assigned as often as possible by the class itself. This requires becoming conscious of where one is missing knowledge and includes bringing in expertise from outside, where needed. The practice of incorporating student ideas makes changes in lesson plans unavoidable. As a result, the planning horizon is much shorter than for regular instruction. A compensating virtue, however, is that students identify more strongly with the project and are more highly motivated to grapple with the lesson material. It is permissible—perhaps even desirable—for students to make one plunge down a blind alley, that is, to think a bad idea through to its logical conclusion.

Students Should Be Given Time to Gather as Much Experience for Themselves as Possible During the MINT lessons, students should be allowed enough time to investigate the topic from a variety of angles. One way to do this is to arrange for as much small-group work as possible. Here, the group members take on various tasks, such as timekeeper, materials supervisor, group speaker, and group secretary. Many experiments are also carried out, either with the computer or, when necessary, with physical equipment. From the very beginning, emphasis is placed on regularly presenting and discussing results. Starting with the 8th graders in round three of the JEA, the class was divided into “expert teams” so as to take better account of the students’ individual interests and strengths. Granted, this brings with it the extra challenge of suitably integrating the various teams, so as to keep the focus on the overall goal.

The Path to the Product Is the Goal The problems addressed in the JEA stem from the real word and, therefore, a real product is expected at the end of the project. Working towards a significant goal serves to chart the course through the three years. It motivates the students to keep trying when they find themselves at a dead-end. When the whole group considers the question “Where do we want to go from here?” new aspects and perspectives often come to light. The end product is also important for strengthening group identification with the project.

Use of Technology In round three, during the 2013/14 school year, tablets were introduced into the class. Put briefly, their subsequent use can be divided into two categories: First, they were used for such traditional purposes as documentation, communication, presentation, and, of course, calculation. In addition, they were also used as "black box" tools to replace challenging (mathematical) techniques with the appropriate computer programs. For example, the computer can perform a Fourier analysis of an audio signal. Or, an oscilloscope can be used to illustrate that noise consists of vibrations, which one can recognize and process again as an audio signal.

7.3 Setting and Development

Even before the Heinrich Heine High School began its collaboration with the Felix Klein Center for Mathematics, it had an alternative compulsory MINT subject in its gifted-and-talented curriculum for junior high school students. Here, students had the choice between a 3-year MINT course with three class periods per week or a course in Japanese. Because the class is categorized as an alternative compulsory subject, its mandate was to cover MINT material that supplemented the regular instruction material, that is, it was to avoid, where possible, treating material included in the regular curriculum of the 7th, 8th, and 10th graders (9th grade is skipped). This was designed to prevent the MINT students from gaining an advantage over the Japanese students—an important point, since positive grades in alternative compulsory subjects count towards graduation.

7.3.1 Evolution of the JEA (Since the 2012/13 School Year)

Time Allocation IFor the first two rounds of the Junior Engineer Academy, the three subjects (mathematics, computer science, and a natural science) were each taught one period per week. Given the course's project format, this allocation was not especially helpful. Starting in the 2012/13 school year, it was possible to reorganize the class scheduling; subsequently, the natural science lesson in the 7th grade was still given as a single period lesson, but mathematics and computer science were team taught as a double period. Since the start of the 2013/14 school year, the biology class in round 3 is taught by a teacher who also participates in the mathematics and computer science teaching team. This flexible allocation provides the freedom needed for the project with respect to scheduling and content. The joint team teaching approach greatly improves coordination among the teachers and facilitates lesson planning and reflection.

Small-Group Instruction As mentioned previously, students involved in project work have a product as their goal. This focus on the product serves as the leitmotiv for the 3-year project. Within the project, however, students also work on small or mini-projects in order to acquire the knowledge and skills needed for the large goal at the project's end. Here, the students themselves should dictate which mini-projects should be worked on and what knowledge is needed. Since the start of the 2013/14 school year, the students in round 3 have been working in expert teams on various sub-areas of the project. The project goal of this round is to program an app that automatically recognizes birdsongs. Along with the algorithmic implementation, mathematical modeling, technical, and even design aspects must be considered. Accordingly, the small groups work in these sub-areas and confer with one another as often as possible, to ensure that they all keep their eyes on the common project goal. This approach is intended to foster the development of individual skills, but it also takes into account the interests of the various students. Moreover, the required coordination of the various groups forces the specialists of a given group to transmit their knowledge as clearly as possible to the non-specialists of the other groups. Initially, this was an unaccustomed challenge. Regular, short presentations starting in the 7th grade are designed to strengthen these communication skills. As a result, one observes that the class has become very critical of poor presentations and unintelligible explanations.

7.3.2 The Role of the Teachers—Team Teaching

During class periods, the teachers serve more frequently as moderators, and where needed, as experts. Discussions often extend beyond the instructional material, and it is therefore necessary for teachers to sometimes admit that they have no answer to the question posed and must seek out expertise and advice elsewhere. Managing experiments and software frequently takes more time than has been planned for. After all, the students are supposed to experiment and try things out as much as possible. Teaching in teams has proven to be very helpful in dealing with these circumstances. Team teaching makes it possible to individually address any one student's specific problem without abandoning the remainder of the class. The presence of two teachers for a double period also allows one to divide up the class—to learn programming skills, for example. This technique has proven very effective. For example, one part of the class can conduct physical experiments, while the other part carries out programming tasks on the computers. During the second half of the double period, the roles are then reversed. Another advantage of team teaching is that it makes lesson planning easier, especially in view of the project's interdisciplinary nature.

7.3.3 Challenges and Ideas

The open structure reduces the time available for lesson preparation. The goal of adopting the ideas of the students, where feasible, and giving them as much leeway as possible to do their own planning means that lesson contents are often not foreseeable or predictable. The introduction of tablets has improved access to information, but resulted in the problem of how to deal with the abundance of available information. Students often have trouble critically examining the quality of the information they find, working it into usable form, and summarizing it. The interdisciplinary nature and project format seem to deter many teachers from MINT project instruction. Team teaching surely offers a way around this problem, although the demands of teachers' day to day instruction routines often makes it difficult to convince colleagues in other subjects to join project teams as experts. This is a particular problem in non-MINT subjects. Team teaching and the formation of MINT teams in schools could strengthen communication channels between teachers in different subject areas. This is especially important for lesson planning and for dealing with questions that straddle the boundaries of the individual subjects.

8 Fraunhofer MINT-EC Math Talents

In the Autumn of 2011, the Fraunhofer MINT-EC Talents program was initiated as a cooperative venture between the Fraunhofer Gesellschaft and the MINT-EC Association. The program focuses on *mathematics* and *chemistry*, the latter under the direction of the Fraunhofer Institute for Production Technology and Applied Material Research, in Bremen. Following a selection process involving 10th (12-year high schools) and 11th (13-year high schools) graders from various MINT-EC schools throughout Germany, 12 students were chosen to participate in the two-year program. This began with a joint kickoff event for all participants in January of 2012, and the Felix Klein Center for Mathematics is providing support and consultation for the selected students.

8.1 *Math Talents* Projects

For the second round of the *Math Talents* program, the selection process was changed somewhat. Out of the group of applicants from MINT-EC schools throughout Germany, approximately 40 students were invited to participate in a Fraunhofer Talent School for *mathematics & athletics*, which took place in January of 2014. This event was closely modeled on the Felix Klein modeling week, although shortened to three days. The Talent School served, first, to determine the suitability of the candidates for, and their interest in, a two-year MINT program and, second, to provide them with a glimpse into the world of mathematical modeling in the context of MINT projects. This was valuable experience, even for those who did not make the final selection.

Due to the fine performance of many applicants, the number of participants in the second round of the *Math Talents* was increased to 24. The plan is for these students to work on four different MINT projects and convene for six multi-day workshops by the time they graduate from high school. In the intervening time, they can exchange intermediate results via an Internet platform and confer with the project sponsors.

In contrast to round one of the program, the participants were much more actively involved in selecting their projects. They were initially presented with only very general fields of investigation. In April 2014, in the course of the first workshop, the students then defined a total of four projects, which we will present briefly in this section. The projects chosen by the students are not necessarily from the field of “mathematics and athletics” and only partially overlap with the projects from the Fraunhofer Talent School.

Example 6 (Computer-supported analysis of pocket billiard strategies) In this project, students consider how to analyze and assess game situations in pocket billiards in order to derive a long-term strategy or, at least, recommend which shot makes the most sense at

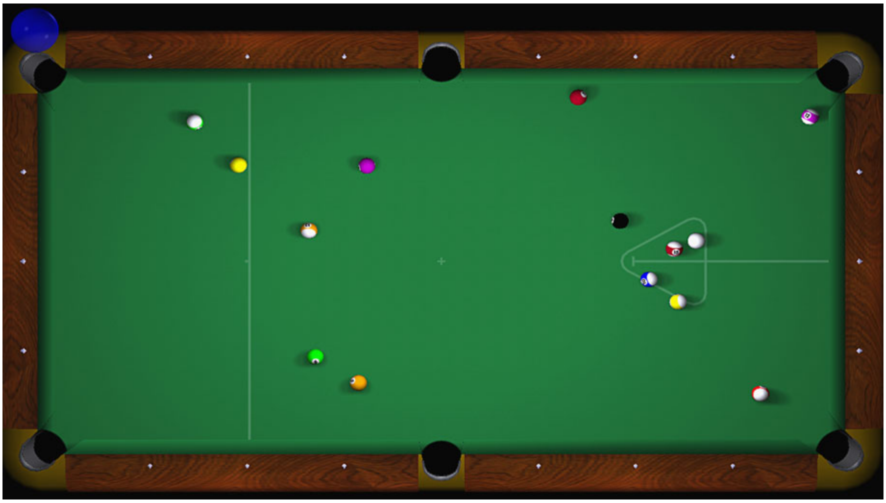


Fig. 6 Game situation in pocket billiards (GNU General Public License)

any given point in the game (cf. Fig. 6). Due to the complexity of game situations and the abundance of factors to consider, the data analysis is performed via computer. Thus, the challenge is to develop software that recommends one (or more) shots to the player that are either easy to pocket or make strategic sense. One might also account for the experience of the player regarding shots of differing degrees of difficulty (such as banked shots or shots with English) or even the personal tastes of the player. The latter would naturally require the development of player profiles.

The project members have established the following preliminary goals:

- Use photo/video to map the geometry of the table and the position of the balls; identify which balls are playable.
- Develop a suitable model to assess the difficulty of the possible shots (angles and distances are already captured in the model; further factors are to follow) and present the player with a top-3 list of suggested shots.
- Analyze the shot with respect to outcome (made/missed/fouled) via the video, so as to document the course of the game.
- Create a computer program or smartphone app with an appropriate user interface.

Example 7 (Self-piloting quadcopter) Small flying devices outfitted with cameras and sensor technology, such as quadcopters, are today readily available to everyone and have a potentially wide field of application. Their use for automatic delivery service in the logistics branch or for cheaply and effectively monitoring large public events both lie within the realm of possibility (cf. Fig. 7).

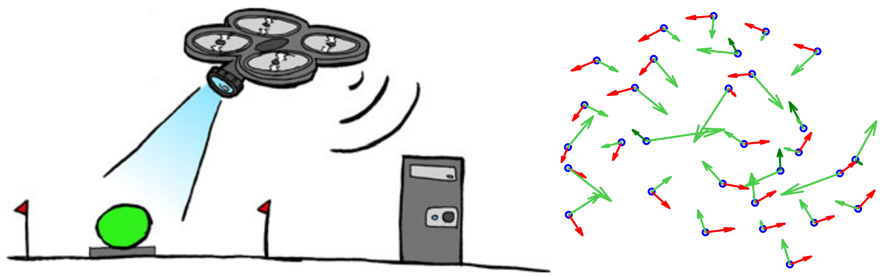


Fig. 7 Self-piloted flight with object recognition and a virtual swarm in MATLAB®

In order to realize these possibilities, however, the quadcopter must be able to pilot itself autonomously to a certain degree. Aside from the legal difficulties associated with such a procedure (Who is liable when a technical failure causes the device to fall on someone's head?), there are numerous technical challenges to overcome.

In this project, a quadcopter is to be first made capable of autonomous flight, that is, the device must be able to carry out a search within a specified area. Based on the data from its on-board camera, the quadcopter must identify color-coded objects and execute the flight maneuvers needed to reach them.

This is to be accomplished via a wireless connection between the drone and a guidance computer that manages the image processing and flight control functions. This permits the use of a light, programmable platform for the test phase. It would also be interesting to establish the coordinated flight of multiple drones with limited sensor technology by enabling them to inform one another about their surroundings. This goal of squadron flying is also to be pursued, as soon as the automated control system for a single drone functions as desired.

Example 8 (Optimized fitness training with electric mountain bikes) Electric bikes are gaining an ever-increasing share of the bicycle market. Their great advantage is that they enable riders to enjoy longer and more difficult tours than their fitness levels would normally permit. This trend is also visible in the mountain biking sector.

The main aim of this project is to generate a control system to optimize fitness training with an electric bike. To do so, one needs to simulate effects such as headwinds and slipstreams. Another possible feature is simulating special training tours on one's standard route by using the electric motor in either support or re-charge (i.e., braking) mode, as appropriate. Integrating performance data, such as pulse and pedaling frequency, also helps provide the rider with an optimized training program.

The required hardware and control system are both to be developed by the project team. The base hardware consists of a Hardtail mountain bike (see Fig. 8), over-sized disc brakes (the motor adds weight), and a hub motor for either the back wheel or the bottom bracket. The control system is to be based on a *Raspberry Pi* or an *Arduino* micro-controller, although there are also plans to design a smartphone based system.

Fig. 8 Standard mountainbike (Foto: Martin Bracke)

Example 9 (A self-navigating outdoor robot) The goal of this project is to develop a self-propelled, "all-terrain" robot that can independently find a designated target and advance over open land to reach it. The robot is to plot its own course to the target, while optimizing energy consumption and range, and recognize and evade both stationary and non-stationary obstacles.

The current design plan calls for a sturdy chassis with an electric drive system, and will possibly take the form of a tracked vehicle. The control system will use a *Raspberry Pi* mini-computer, and there will be several sensors, including a camera and a distance sensor.

The first challenges recognized by the students were technical in nature: How can one make the chassis robust enough to survive the rigors of traversing open land? How can sensor data be evaluated and the motors controlled via the *Raspberry Pi* One must also answer the question of how to use the sensor data to help the robot orient itself in its surroundings, so as to then set a course for the given target.

For the duration of the program, the participants will continue to develop their projects as independently as possible, although they can draw on the support of the Felix Klein Center as needed. Mini-workshops have been planned or requested on the topics of *Raspberry Pi* programming, mathematical image processing, simulation and hardware control with MATLAB/Simulink, and app programming for smartphones.

Because the development of a specific product plays an important role in all the projects, the Math Talents participants will be assisted at the end of the program, should they wish to enter their projects in the *Jugend forscht* (Young Researchers) competition. In this case, additional soft skills courses—*presentation techniques*, *project planning*, and *time management*—will be offered. Along these lines, all the participants were also introduced to LaTeX, at the start of the program, a typesetting and composition system very widely used in mathematics and the natural sciences. Thus, they had a powerful tool for documenting and designing the layout of their results at their disposal throughout the duration of the project.

We would also like to mention that the projects described in this section were not at all pre-defined; estimates of the challenge level and difficulty of implementation could thus only be made once the projects were underway. This resulted in a certain measure of uncertainty regarding their practicability, since the goals set by the students for themselves were very high—so high, in fact, that some aspects even touch upon current state-of-the-art research! This lack of pre-definition also presented a great opportunity, however: that of drawing on student capabilities that were hidden at the start and only developed over the course of time. The key, here, as is often the case, is for all participants to manage their expectations about the results. If very specific goals or even quantitative targets are set, this will quite likely lead to frustration and failure. If, instead, the work is viewed as a research project with an uncertain outcome (which, in fact, it is), then winding up in a blind alley does not automatically mean the end of the project. Rather, it means rethinking one's previous work and, if necessary, re-aligning one's goals. As with the countless successful projects from modeling weeks and days, here too, each group will ultimately have produced a functioning product in the form of hardware and/or software—although the originally defined features may be partially scaled-back or perhaps, as also happens, even extended.

9 Final Remarks

To conclude this chapter, in which we have presented an overview of the possibilities for implementing mathematical modeling with real-world applications in student projects, we would like to offer some encouragement to our readers.

Dare to enter the exciting world of application problems waiting to be discovered all around by those whose eyes are open to see them. Your point of entry for an intriguing modeling project might come in the form of a claim in a newspaper article that has no convincing evidence to back it up. Or it could be a conversation with a friend or acquaintance about a tricky problem he is facing in his company, for which there is no standard solution. Or a simple observation of a generally accepted procedure that suddenly triggers the thought, why does it have to be done exactly that way? Is there not perhaps some better alternative?

When you find yourself fascinated with such a problem and are able to dive into models and simulations of your own in the effort to solve it, then the optimal conditions have been established for you to also capture the fancy of young learners. And if the students themselves can then develop their own projects on the basis of your inspiring example, then it is almost certain that they too will be carried away on their own exciting journeys of invention, creation, and learning.[4]

[4]Information about current school projects at the Felix Klein Center for Mathematics in Kaiserslautern is available at the website of the *Kompetenzzentrum für Mathematische Modellierung in* MINT-*Projekten* in der Schule (KOMMS), http://komms.uni-kl.de.

Acknowledgements The authors wish to express their appreciation to many colleagues at the Felix Klein Center for the years of successful collaboration in carrying out school projects. We are especially grateful to all the students and teachers who, as the main actors, have enriched and shaped the project work with their creative ideas and concentrated efforts, and who have evidenced much joy in everything they undertook to accomplish.

References

1. Ackoff, C., Arnoff, R., Churchman, E.: Introduction to Operations Research. Wiley, New York (1957)
2. Blum, W., Borromeo Ferri, R., Maaß, K.: Mathematikunterricht im Kontext von Realität, Kultur und Lehrerprofessionalität. Springer, New York (2012)
3. Bock, W., Bracke, M.: Project teaching and mathematical modelling in stem subjects: a design based research study. In: Ubuz, B., Haser, C., Mariotti, M. (eds.) Proceedings of CERME 8, Lecture Notes in Computational Science and Engineering, pp. 1010–1020. Middle East Technical University, Ankara (2013)
4. Eyerer, P.: TheoPrax®-Projektarbeit in Aus- und Weiterbildung. Bausteine für Lernende Organisationen. Klett-Cotta, Stuttgart (2000)
5. Gudjons, H.: Handlungsorientiert lehren und lernen: Projektunterricht und Schüleraktivität. Klinkhardt, Bad Heilbrunn (1986)
6. Jank, W., Meyer, H.: Didaktische Modelle. Cornelsen Scriptor, Berlin (2003)