Problems Trump Methods: A Somewhat Different Mathematics from a Somewhat Different Institute

Dieter Prätzel-Wolters and Helmut Neunzert

This book is dedicated to mathematics-based topics that are driven by practical problems whose solutions generate innovation. The formulations of these problems have arisen in the context of projects carried out at the Fraunhofer Institute for Industrial Mathematics (ITWM), and the majority of the authors of this book are either employed at the Fraunhofer ITWM or are closely affiliated with it. Fraunhofer Institutes dedicate their work to problems in industry; a mathematical Fraunhofer Institute therefore makes "Industrial Mathematics".

The book's editors originally suggested "Fraunhofer Mathematics" as a title. This suggestion was discarded, however, since it found no consensus among the authors. A book about "Fraunhofer Mathematics" might have also had a polarizing effect. For many mathematicians, it would have also been a provocative title, one that generates confusion about what exactly "Fraunhofer Mathematics" refers to.

Mathematics is the science with the highest degree of abstraction; there is virtually one hundred percent agreement about what is recognized as mathematics; and mathematical results are highly objective, intrinsically verifiable, and formulated in a largely standardized language. Mathematics is divided into the categories of pure and applied, although making even this basic distinction is somewhat difficult. It also happens occasionally that the works of important mathematicians become identified with their originators, so that one then speaks, for example, of Hilbertian or Riemannian mathematics. There are also schools that have developed particular structural edifices of mathematical thought and whose works are then cited, for example, as Bourbaki or constructivist mathematics.

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How then is Fraunhofer mathematics to be fitted into such a classification system? As the mathematics of Joseph von Fraunhofer, perhaps? Hardly. Although he too produced mathematically oriented works, Joseph von Fraunhofer (1787–1826) was not a mathematician. He was a very successful scientist who discovered the lines in the solar spectrum that were subsequently given his name and who was extremely well versed in physics and in the design of lenses and optical equipment. At the same time, he was a successful businessman who, at the tender age of 22, was made director of the glassworks in Benediktbeuren, which he then successfully managed (the telescope pictured below, manufactured for the University in Dorpat (today: Tartu, Estonia) was the largest and best of its day) (Fig. 1). Joseph von Fraunhofer also became the namesake of the Fraunhofer-Gesellschaft—after MIT, the second largest institution for applied science in the world.

The identity of Fraunhofer research is characterized by proximity to application, industrial relevance, and innovation. The Fraunhofer-Gesellschaft has recognized that research in applied mathematics not only serves as an aid to other scientific disciplines in the search for solutions to practical, in particular, technical and organizational problems. Mathematics also represents a discipline that is indispensible for maintaining economic competitiveness and meeting the challenges faced by society. It has evolved from being a key to

Fig. 1 Joseph von Fraunhofer: researcher, inventor, and businessman (© Fraunhofer-Gesellschaft)

basic research and technology to being an enabling force for virtually every economically significant key technology.

The Fraunhofer-Gesellschaft, remaining cognizant of this evolutionary development, has added three new mathematics-based institutes to its ranks in the past decade:

- The Fraunhofer Institute for Industrial Mathematics ITWM, in Kaiserslautern,
- The Fraunhofer Institute for Algorithms and Computational Science SCAI, in Sankt Augustin, and
- The Fraunhofer Institute for Medical Image Computing MEVIS, in Bremen.

These institutes are dedicated, in terms of their research mission and focus, to application-oriented mathematics and the implementation of mathematics in society and industry.

Our book is dedicated to the mathematics practiced in the ITWM, whose spirit also prevails at the other institutes. It is problem-driven, model-based and solution-oriented. We will have more to say about this elsewhere in the book. If the goal is to highlight a unique feature associated with a particular "brand" of mathematics, then this is certainly the description "problem-driven, not method-driven." The style and structure of this book have been influenced by this attribute.

Beyond this, another motive was certainly to share the "success story" of the Fraunhofer ITWM. We want to illustrate how innovation in mathematics and the transfer of its results into the marketplace and society at large can be effectively carried out in a large research institute receiving relatively little basic funding. The success of the "ITWM model," as proven also by the formidable role played by mathematics in contemporary industry, might also serve as a motivating force for establishing similar institutions in other locations and other countries, adapted to the regional and national circumstances found there.

1 "Industrial Mathematics" Versus "Applied Mathematics"[1]

Many scientific disciplines profit from the solutions to practical problems developed through research in applied mathematics. As a rule, however, traditional, academically-oriented applied mathematics only examines and numerically treats problems that are also accessible to rigorous mathematical analysis; that is, problems for which existence and

[1] Portions of this introduction have been taken from the following publications:
H. Neunzert, U. Trottenberg: Mathematik ist Technologie – Ein Beitrag zur Innovations-Initiative aus Fraunhofer-Sicht, Fraunhofer ITWM und Fraunhofer SCAI, Kaiserslautern und Sankt Augustin, 2007
D. Prätzel-Wolters, U. Trottenberg: Rechnen für Fortschritt und Zukunft – Innovationen brauchen Mathematik, Jahresbericht der Fraunhofer-Gesellschaft 2007, S. 47ff., München 2008.

uniqueness statements for the solution and convergence statements for the applied numerical method, for example, can be proved. As a result, the problems treated in the mathematical literature are often highly idealized and not especially realistic.

The effective solution of large-scale, real-life problems only became the object of intensive mathematical research after technomathematics, economathematics, and computational science established themselves as new mathematical disciplines.

This practice-oriented mathematics, which further develops mathematical methods for the solution of specific problems and whose models and algorithms form the basis for simulating and optimizing complex products and processes, is at the heart of the mathematically oriented Fraunhofer Institutes. The fact that this research is far more than mere mathematics transfer is frequently underappreciated in the more academically-minded world found in universities. Here, one sometimes encounters the notion that such industrial-oriented mathematics is not "real" mathematics at all, or that the truly "new" mathematics is developed in universities—decoupled from practical application—and only after a time delay finds industrial application. The experience and expertise of the mathematically oriented Fraunhofer Institutes, gathered in extensive, long-term collaborations with industry, contradict these views.

Modeling and simulating the behavior of complex materials, for example, results in mathematically challenging problems involving the coupling of very diverse differential equations, such as those of fluid mechanics and Maxwell equations. This coupling represents a significant challenge, not only numerically, but also theoretically. The high-dimensional partial differential equations arising from the risk evaluation of financial securities require entirely new methods of numerical solution. The transition from smaller to larger scales can be tackled with homogenization methods, but only when the essential scales are well separated. When this is not the case—as happens in many practical applications, such as those involving turbulence or crack formation in materials under stress and in rocks—then there are currently only a few fruitful approaches for simplifying the models and/or the numerics. The digital interconnection of control systems demands new procedures for analyzing and synthesizing hybrid systems with continuous and discrete dynamics and logic based switching functions.

These few examples illustrate that substantial momentum for the development of "new" and "real" mathematics arises from treating complex, practical problems.

Nevertheless, the transfer of mathematics to the marketplace is a vital mission of the mathematically oriented Fraunhofer Institutes. Here, however, they don't restrict themselves to merely preparing general mathematical aids for the solution of practical problems, thus leaving the actual problem-solving to the users or to other technical software companies. Instead, they get involved themselves—in close cooperation with the users—to work towards a complete solution through the development of appropriate software modules. The goal of demonstrating a direct benefit to the economy, that is, of putting research results directly into practice with their industrial partners, is part of their identity and their mission. Here, it is accepted that the relevance of their research results is also reflected in the fact that the businesses making use of those results contribute substantially and directly to financing the costs of the research efforts. The Fraunhofer financing model assumes that an Institute will cover at least one third of its operating budget through business revenues.

However, to ensure long-term success in mathematics transfer, it is also essential to maintain contact with the frontlines of basic research and to actively pursue new mathematics oneself. Practical problems represent a wonderful source of new questions and methods that can then feed basic research in the Institutes.

In this context, the Institutes' joint ventures with other research institutions and universities, as well as with industrial partners in connection with projects publicly sponsored by the BMBF (Federal Ministry for Education and Research), the DFG (German Research Foundation), or the EU, for example, play a very significant role. They serve to build up new research areas and establish a trusting and cooperative working atmosphere with the participating institutions. The results of this research create innovation in economically and societally relevant fields of application and help finance the Institutes' knowledge-oriented basic research.

2 Problem-Driven or Method-Driven?

This view is reflected in the stereotype, still frequently encountered in public opinion, that mathematics is a difficult, dry, ivory-tower sort of endeavor. Mediocre or worse grades in school mathematics classes are accepted in society, where they are met with a shrug of the shoulders and commented upon sympathetically.

This attitude captures neither the fascination of mathematics as a playground for the mind nor its significance as a crucial instrument for shaping technological progress.

The mathematician himself is seen as a person who—cut off from the real world—performs his researches upon questions he has thought up himself, within the confines of his own system of thought. His research is driven by the methods and structures intrinsic to mathematics; solving practical problems doesn't interest him particularly. The ideal location for this endeavor is indeed the ivory tower, an intellectual refuge, inviolate and untouched by the world.

The ivory tower stands for the isolation of the scientist, who retreats from the events of the world and dedicates himself exclusively to pure research, paying no heed to either the practical uses or consequences of his investigations, but simply losing himself in his passionate pursuit of answers.

This image of the mathematician no longer fits into the research landscape of the 21st century. Applied mathematics has long-since abandoned the ivory tower, seized the computer as a tool of the trade, and addressed itself to the solution of practical, relevant problems. But it is a shortsighted view to assume that it was only through the computer that mathematics was finally rendered able and willing to solve practical problems.

Mathematics was always both: It was problem driven and it was method driven. It helped to solve practical problems, and it created culture, by following its own evolutionary path.

For the active participants in this process—mostly mathematicians—the past hundred years were dominated by the continued development of methods. This happened either in the pursuit of answers to questions that arose within mathematics—as in pure mathematics, such as with algebraic geometry—or in the pursuit of solutions to problems that typically manifest when dealing with practical questions—as in applied mathematics, such as with inverse problems. University mathematicians had, and still have, the privilege of being able to deeply immerse themselves for long periods of time in a particular class of mathematical problems.

Things were once different, however. Earlier, one's income depended on the successful treatment of problems posed from outside. Typical examples are the fluid dynamic problems that Euler needed to solve or the geodetic problems tackled by Gauss. And it is again different today; mathematics, with the aid of its tool, the computer, has become a technology in its own right, and a host of practical problems are standing in line, so to speak, outside its office door.

The doors of the Fraunhofer Institutes are opened wide to receive such problems, and the mathematics practiced there is driven very significantly by the need to solve them. This means that the focus of research is not on the further development of existing mathematical methods, but on the development of new methods for formulating and solving problems or the adaptation of known methods to the particular problem being addressed. The goal of solving the problem determines the direction in which the methods are developed and extended.

3 Model-Based and Solution-Oriented

Efficient mathematical treatment of practical problems calls for the preparation of "economical" mathematical models, as well as the development of efficient algorithms. A model is "economical" when it is as complex as necessary and as simple as possible. Often, the simplicity is also imposed by a desire for real-time simulations or because the simulations calculate the values of objective function(s) for an optimization task. Algorithms are efficient when they achieve maximal exactness on the computers at hand in the limited processing time available.

For most problems confronted in industrial practice, physics provides models. These are frequently continuum mechanical, thermodynamic, or electromagnetic equations, which very precisely describe the manufacturing processes of industrial goods or their behavior. Naturally, it is possible to describe the behavior of thousands of polymer fibers in the transition from fluid to solid phase in turbulent airflow. Or one can model very precisely at the particle scale the flow of a gas and the absorption of entrained particles by a porous medium.

However, even using high performance computers and the most modern algorithms, it is not possible to arrive at even a rough solution for these very complex equations. Presumably, this will not be possible decades from now either. But this isn't necessary, since one can simplify and reduce the models and still meet the specified precision requirements. The algorithms then have to be adapted to the model reductions, and vice versa: the first approximations in iterative solvers may work with simpler models; then, as the precision increases, the models themselves also become more precise. This interplay between model and algorithm is especially important for optimization tasks. Model reductions often deal with asymptotic analysis or multi-scale approaches, where small parameters are replaced by the limit value zero. Or they rely upon projection methods on lower-dimensional subspaces. It is also quite possible, however, that entirely new models based on a different mathematical theory are employed, for example, the use of stochastic models for very complex, deterministic behavior.

Because it is important when dealing with "real-life" problems to find usable solutions, the development of efficient algorithms, as already mentioned, also comes into consideration. Thus, multi-core approaches from modern computer architecture fit well together with multi-grid approaches, which, in turn, are often coupled with multi-scale models. Parallel algorithms also currently represent an important field. All of this, however, as is usually the case elsewhere, is not to be understood as "method for method's sake"—we repeat it once again here—but as problem-driven.

4 Mathematics as a Motor for Innovation in Technology and Society

The potential for applying mathematics is enormous. The scope of the mathematics that has found its way into industrial practice has grown explosively over the past 40 years. This can be explained for the most part by the fact that work with real models has been replaced by simulations, that is, by work with mathematical models. This development has been accompanied by the automation of work processes, sensory perceptions, and experiences in the form of algorithms, computer programs, or expert systems. Mathematics has become a key technology, one that can and should be mentioned in the same breath as nanotechnology or biotechnology.

At first blush, this may appear a rather audacious statement. It requires, at least, an explanation. To be sure, for thousands of years, natural scientists have used mathematics as a resource and as a language in which to formulate their theories, and it has formed the basis for the computations of the world's engineers. Thus, it is at least a raw material—the raw material of models that are then converted into technology. But simply being a raw material is not enough to qualify as a key technology. It is the computer that has elevated mathematics to the rank of technology. In a certain respect, the computer is the purest form of mathematics-turned-technology. Mathematics has taken on a material form in the guise of the computer, and it represents the intellect behind every computer simulation. Simulations need models and algorithms to evaluate and visualize their results. On closer inspection, it is always mathematics serving as the basis, as the "source code," so to speak, for these critical work steps.

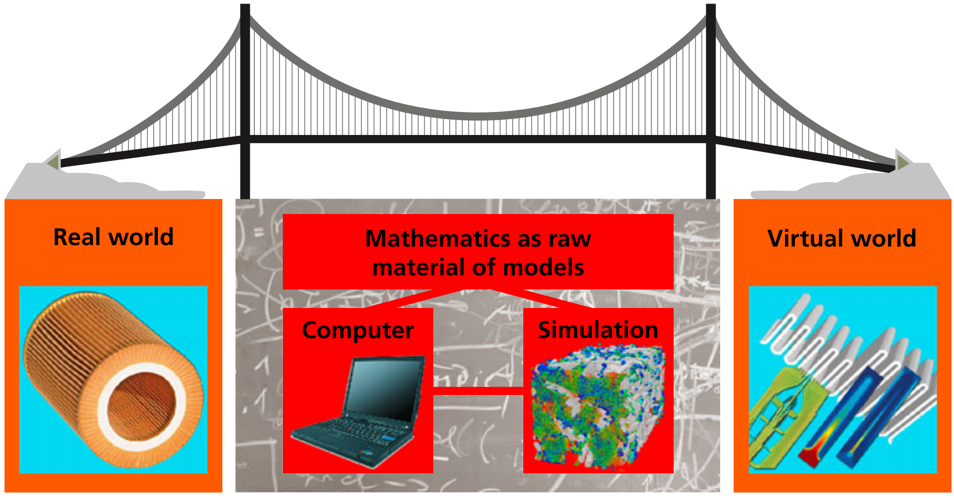

Fig. 2 Mathematics is a key technology (Graphic: S. Grützner, Fraunhofer ITWM)

The computer has altered our world. In the view of the cultural philosopher Ivan Illich (1926–2002), it has become a universal and convivial tool. Computer simulations—and thus also mathematics—represent the essential tool for shaping and optimizing products and work processes.

Real models are being replaced by virtual models. Mathematics, as raw material and key technology, forms the foundation for a bridge to this second world—the world of virtual simulations—which has found a foothold in almost every area of our society and economy (Fig. 2).

5 Mathematics Is Universally Applicable, Because It Traverses Boundaries

This universal applicability stems from the fact that mathematical methods and tools developed for one sphere of reality or science can also be made useful in other areas of application, either directly or in an analogous form. Mathematical models fit horizontally into a landscape of scientific disciplines and technological applications that are arranged vertically. This transverse quality of mathematics makes it a "generic" technology.

The ideas developed in one area can bear fruit in others. In keeping with this motto, mathematics creates cross-links between disciplines and makes comprehensive insights possible. "Out-of-the-box thinking," as a characteristic of the mathematical approach to work, creates innovation by layering different levels of reference.

Mathematical models are in demand; solutions require simulations. As a rule, there is not just one solution, and mathematical optimization is also required to find the best ones. The abbreviation for this triad of Model-Simulation-Optimization is MSO. MSO is anchored in the research and development departments of today's large technology companies as its own field of competence, and is occasionally even part of the organizational structure of such companies. In practically all mathematics-based, practically-oriented research projects, MSO is an integral component of project work.

All in all, one may easily speak of a quantum leap over the past decades in the visibility of mathematics as an engine for innovation in technology and society.

There is a great deal of evidence for this development—a development that, in the interim, has become entrenched in the fields of politics, science, and industry.

5.1 Committee for Mathematical Modeling, Simulation, and Optimization (KoMSO)

This committee was established in connection with the "Strategic Dialog on Mathematics," an initiative of the German Ministry for Education and Research (BMBF). Its goal is…

> "*…to anchor the triad of mathematical modeling, simulation, and optimization in research and development as a new field of technology, in order to strengthen the innovative power of the technology nation Germany. Research and innovation are the foundation of prosperity for all of society. Therefore, the potential of MSO, which has remained undiscovered or only partially exploited up to this time, must be tapped into and made visible.*"

And, as also found in the strategy paper of the BMBF Strategy Commission:

> "*Improved mathematical methods and continuously improving computer performance make increasingly complex physical-technical, economic, or medical questions accessible to description with mathematical modeling, virtual simulation in computers, and optimization relative to a given technological goal. In this way, the most diverse simulation techniques have become as thoroughly established—as a third pillar of knowledge acquisition—as theory and experiment for optimizing automation and decision-making in an increasingly complex and interconnected world.*"

5.2 "Mathematics—Engine of the Economy"

The book "Mathematik – Motor der Wirtschaft"[2], published in the Year of Mathematics in 2008, and produced in close cooperation with the Oberwolfach Stiftung (Oberwolfach Foundation) and the Mathematisches Forschungsinstitut Oberwolfach (Oberwolfach Research Institute), contains, among other things, a series of contributions from prominent

[2] *G.-M. Greuel, R. Remmert, G. Rupprecht (Eds.): Mathematik – Motor der Wirtschaft, Springer-Verlag, Berlin, Heidelberg, 2008.*

representatives of German industry. The book illustrates that mathematics is today of great significance in virtually all branches, in all areas of industry, business, and finance. For example, Peter Löscher, former chairman of the board of Siemens, Inc., writes:

> "*Mathematics—this is the language of science and technology. This makes it a driving force behind all high technologies and, thus, a key discipline for industrial nations. Without mathematics, there is no progress and no technical innovation.*"

Or, to quote Dieter Zetsche, Chairman of the Board of Daimler, Inc.:

> "*As does no other science, mathematics helps us in our branch to solve the most varied sorts of problems—and it is exactly this universal applicability that makes it the royal discipline.*"

Of course, not all the companies surveyed by the Oberwolfach Stiftung responded. Nor was the vast economic sector of small and medium-sized businesses included, although these companies are responsible for the lion's share of German economic power. Nevertheless, there can be absolutely no doubt about the general validity of these statements.[3]

There are indeed other studies that confirm them completely: For example, "MACSI-net roadmap," published in 2004 by ECMI (European Consortium for Mathematics in Industry); "Mathematics: Giving Industry the Edge," published in 2002 by the Smith Institute at Oxford; and "Forward Look: Mathematics in Industry," a report prepared in 2010 by the European Science Foundation in cooperation with the EMS (European Mathematical Society). The experience of the mathematical Fraunhofer Institutes, whose mission is, after all, research cooperation with industrial partners, also lends support to the argument.

5.3 ECMI

Since 1986, the "European Consortium for Mathematics in Industry ECMI," to which many European institutions belong—including groups in Barcelona (E), Dresden (D), Eindhoven (NL), Florence (I), Glasgow (GB), Gothenburg (S), Graz (A), Grenoble (F), Kaiserslautern (D), Lappeenranta (FIN), Limerick (IRL), Linz (A), Lund (S), Lyngby (DK), Madrid (E), Milan (I), Oxford (GB), Sofia (BG), Trondheim (N), and Wroclaw (PL)—has endeavored to emphasize the significance of mathematics for European industry and organize the training and cooperation of European "industrial mathematicians."

Germany's applied mathematics enjoys an outstanding position internationally; it is one of the few areas in which Germany ranks globally among the top three nations. In industrial mathematics, Europe as a whole and Germany in particular are also at the forefront; the

[3] Cf. H. Neunzert: Mathematik ist überall – Anmerkungen eines Mathematikers zu den Beiträgen der Wirtschaftsunternehmen in G.-M. Greuel, R. Remmert, G. Rupprecht (Eds.): Mathematik – Motor der Wirtschaft, Springer-Verlag, Berlin, Heidelberg, 2008.

USA and Asia orient themselves for the most part on European examples. Here, once again, there is ample evidence to back up this claim.

The DFG sponsors a collection of graduate schools, memberships in excellence clusters, and collaborative research centers that have a strong link to applied mathematics. Applied mathematics is also strongly represented in the BMBF's large flagship projects, such as the Leading-edge Clusters and the Research Campus Program.

5.4 Berlin

In the past few decades, Berlin has evolved into a nationally and internationally recognized center of excellence in the area of applied mathematics. The Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB) is one of the most successful institutes in scientific computing and has an excellent global network. ZIB is also home to the only mathematically oriented research campus "MODAL—Mathematical Optimization and Data Analysis Laboratories." Along with the graduate school "Stochastic Analysis with Applications in Biology, Finance and Physics" and the "Berlin Mathematical School," the DFG Research Center "Matheon—Mathematics for Key Technologies: Modeling, Simulation, and Optimization of Real Processes" is the German applied mathematics center having the widest international recognition. Matheon is supported by the mathematics institutes of the Technische Universität Berlin (TU Berlin), the Humboldt-Universität zu Berlin (HU Berlin), and the Freie Universität Berlin (FU Berlin), as well as by the ZIB and the Weierstrass Institute for Applied Analysis and Stochastics (WIAS) (see, also[4]).

Since 2010, Berlin, with the WIAS, has also been the permanent headquarters of the Internationale Mathematische Union (IMU), an umbrella organization for 77 national mathematical societies. Among other activities, it supports education and research in developing countries and organizes the International Congress of Mathematicians (ICM), the largest conference in the field of mathematics and venue for awarding the Fields Medals.

5.5 Kaiserslautern

The mathematics department at the TU Kaiserslautern (Technical University of Kaiserslautern) has acquired an outstanding global reputation by virtue of its research activities in theoretical and practical mathematics and its innovations in education. The curriculum of technomathematics was "invented" and conceived in Kaiserslautern, and the department here was one of the first in Germany, after Ulm, to introduce the economathematics

[4] P. Deuflhard, M. Grötschel, D. Hömberg, U. Horst, J. Kramer, V. Mehrmann, K. Polthier, F. Schmidt, C. Schütte, M. Skutella, J. Sprekels (Eds.): MATHEON—Mathematics for Key Technologies; EMS Series in Industrial and Applied Mathematics 1, European Mathematical Society Publishing House, Zürich 2014.

curriculum. Both fields of study have become successful curricula within Germany and developed into especially strong focal points in Kaiserslautern. The DFG has been a past sponsor of two graduate schools in mathematics in Kaiserslautern and a third, "Stochastic Models for Innovations in the Engineering Sciences," has just been approved.

With regard to its mathematics programs, the TU Kaiserslautern is among Germany's elite universities. This is evidenced by the university rankings, compiled by the CHE (Center for Higher Education Development) and the magazines Focus, Stern, Spiegel, and Zeit since 2003, in which mathematics in Kaiserslautern has always been placed in the top group.

Over the past five years, in connection with a mathematics initiative sponsored by the State of Rheinland-Pfalz, the TU Kaiserslautern, and the Fraunhofer ITWM, urgently needed specialists in differential-algebraic equations, image processing, biomathematics, and stochastic algorithms have been brought to Kaiserslautern.

The Fraunhofer ITWM emerged from the technomathematics working group and was the first mathematics institute to join the Fraunhofer-Gesellschaft. Today, with its yearly industrial revenues of more than 20 million euros and some 260 full-time employees and doctoral students, it is one of the largest applied mathematics institutes in the world.

The Institute is continually receiving new impulses for innovation from its cooperative efforts within the mathematical department of the TU Kaiserslautern. By the same token, the department is closely affiliated with the ITWM by virtue of third-party projects and doctoral programs, and research within the department is stimulated by the project-driven topics of the ITWM. Unfortunately, this close affiliation is not always perceived publicly and we encounter the misconception that there is a mathematics department very nicely situated in basic research and a Fraunhofer Institute that successfully transfers mathematics to industry, but the two have little to do with each other. The supposed separation into basic research within the department and mathematics transfer at the ITWM does not correspond to reality. The ITWM performs its own basic research within applied mathematics on a large scale. Between 2000 and 2013, for example, 150 PhDs and habilitations were successfully completed in the Institute and its immediate environment. Naturally, these degrees were granted by the TU Kaiserslautern.

In order to further strengthen the connection between the mathematical department and the ITWM, the "Felix-Klein-Zentrum für Mathematik" (FKZM) was founded in late 2008, in connection with the Rheinland-Pfalz "Mathematics Initiative." The center was named after the important mathematician and scientific promoter Felix Klein (1849–1925). This name was selected, because Felix Klein united Germany's pure and applied mathematics like no other mathematician in history, reorganized mathematics in Germany 100 years ago, built a solid bridge to industry, linked academic and school mathematics, and celebrated and promoted the history of science—all activities that have served and still serve as pole stars for Kaiserslautern's mathematicians. Therefore, the FKZM offers a platform and provides infrastructure for joint research projects, guest programs, scholarships, and school outreach activities. Last, but not least, the FKZM represents a forum for cooperation with other departments and industry.

5.6 Other Activities in Germany

It would exceed the scope of this introduction to offer detailed descriptions of all the sites in Germany at which applied mathematics plays a prominent role.

Heidelberg is certainly another exceptional location for applied mathematics, where great emphasis is placed on cooperation with industry. Along with the Heidelberg Graduate School of Mathematical and Computational Methods for the Sciences (HGS MathComp), the Interdisciplinary Center for Scientific Computing (IWR)—a research institute of the Ruprecht-Karls-Universität Heidelberg—is among the world's largest university-based centers for scientific computing. The previously mentioned Fraunhofer Institutes for Algorithms and Computational Science SCAI, in Sankt Augustin, and for Medical Image Computing MEVIS, in Bremen, along with the Max Planck Institute for Mathematics in the Sciences, in Leipzig, are all German centers of applied mathematics. In addition, there are numerous locations with well-funded chairs, state institutions, and special research areas that have also helped carve out Germany's applied mathematics landscape. Some examples are Bremen, Paderborn, Munich, Erlangen, Bonn, Stuttgart, Freiburg, Saarbrücken, Wuppertal, and Dresden—and this list is far from complete.

To conclude this introduction, we would like to offer the reader a bit of information about the design of this book and the various areas of focus in the individual chapters.

6 The Design of This Book

In structuring our book, we have kept in mind various groups of potential readers:

- Practitioners and interested laypeople who want to inform themselves—without having to dive into the technical details—about what today's mathematics can offer toward the solution of practical problems.
- Professional mathematicians and university-level mathematics students who want to understand the mathematics developed at the ITWM.
- Teachers, younger students of mathematics, and instructors or tutors who want to understand how to integrate the new image of mathematics into their school systems.

The triad "problem-driven–model-based–solution-oriented," has determined this book's design. The section entitled "The Concepts" (Part 2) is dedicated to the following superordinated topics:

- Mathematical modeling
- Computation
- Data analysis
- Optimization processes

These chapters serve to provide an overview of the essential questions, methodological approaches, strengths, and potentials—along with the weaknesses and limitations—of each topic. They are addressed to both practitioners and interested laypeople, as well as to professional mathematicians and university-level mathematics students. The aim here is not to offer a mathematical representation of specific models or algorithms. Instead, the chapters comment upon and give structure to the work of the Institute, work that has culminated in the research descriptions found later in the book. For this reason, these chapters tend to be written in more of a "prose" style.

This approach can be attributed to the fact that the mathematics of the ITWM is problem-driven, which means that the reality described by our models is much more complex than that forming the foundation of academic works. There are more complicated boundary conditions, the materials are non-homogeneous, the objective functions are not immediately clear, and the models must be simplified in order to make their application really practicable. All these aspects are discussed in the overview chapters. In addition, important models or algorithms that don't happen to appear in the "research" chapters presented later are also addressed briefly.

Significant results from the mathematics originating in the various ITWM departments are then introduced in the following five chapters under the rubric "The Research." These department-related chapters serve as prototypes of the model-based, problem-driven, and results-oriented mathematical research of the ITWM. They come nowhere near to providing a complete overview of the projects and results achieved during the past 20 years of research at the ITWM. Rather, they serve as examples from working areas that are especially suited to illustrate the unique flavor of industrial mathematics. All five chapters are structured in a similar fashion. The first three sections of each chapter are written so as to also be understandable to interested laypeople having no pertinent knowledge of mathematics.

In contrast, the fourth section is aimed principally at mathematicians. It comprises a "self-contained," compact mathematical presentation for one or two problem areas, addresses the mathematical challenges, and describes the significant results, including their relevance for the "problem solution." The remaining sections discuss simulations based on the previous results and round off each research chapter with descriptions of specific examples arising from "practice" that have been addressed in the joint projects.

The various chapter sections focus on the following questions:

Basic structure of the research chapters

1. Why is the industrial partner coming to us?
   – What are the industrial problems and challenges found in a particular area of focus in the department?
2. What are the mathematical challenges?
   – Which mathematical methods are needed and which results are available for solving these problems?
   – Why is the existing mathematics very often insufficient; i.e., why is it not simply a question of mathematics transfer?
3. What was achieved in the department?
   – What are the primary topics focused on in the department and what results were achieved?
   – What is the impact of doctoral dissertations and graduate theses and who are the visible cooperation partners and customers?
4. What problem-oriented mathematical results were achieved?
   – What results were achieved and to what extent are they relevant for the "problem solution"?
   – What works and what doesn't work?
5. How do the results apply in actual practice?
   – What is handed over to the customer in the end for his specific problems?
   – Are there simulation tools offered?

The final part, "The Training," containing the chapter entitled "Applied mathematics in schools—made in Kaiserslautern," is aimed primarily at high school teachers and students. As mentioned at length previously, recent decades have seen a quantum leap in the visibility of mathematics as an engine for innovation in technology and society. Unfortunately, the new role of mathematics as a key technology has not yet been recognized in our school systems. Mathematical modeling, computing for the solution of existing, practical problem, and interdisciplinary projects are hardly ever found in schools. Of course, one does find so-called "word problems," but these very rarely describe authentic problems whose relevance is clear to learners and who might thus be excited about finding solutions. Algorithms are introduced in schools, granted, but hardly any that have been developed in recent years to tackle large-scale challenges.

The MINT subjects (from German M = Mathematics, I = Informatics, N = Natural Sciences, T = Technology; in English maybe better STEM) are not sufficiently integrated into either the curriculum or into classroom practice. Learners perceive lessons in the various MINT subjects as sequences of contents and tools that often fail to make clear overarching relationships—even within a given subject. The linking of the subjects with each other happens even less often.

One reason for this is the way teachers are trained: to date, applied mathematics has been assigned a rather humble position in the teacher training curriculum. Neither modeling nor work with algorithms—which, in practice, have widely replaced the use of complicated formulas—play a role in school. Similarly, within the education curriculum, the interdisciplinary interplay of mathematics, computer science, the natural sciences, and technology is neither discussed nor trained adequately.

In the final chapter of our book, we want to present some ways and means for reforming instruction, both in our schools and in the training and continuing education programs for teachers, as they have been practiced for several years in Kaiserslautern.

After a short, application-oriented introduction into mathematical modeling, we will point out which measures can be adopted to bring learners into closer contact with applied mathematics and interdisciplinary work. Here, we will present both intracurricular and extracurricular events.

Activities such as "modeling week," "modeling day," and competitions can be used to offer pupils the opportunity, within the framework of a compact project, to become more closely acquainted with the role of mathematics, to actively and creatively practice mathematics, and to witness interdisciplinary connections. The sample problems serve as invitations to interested teachers to integrate modeling into their lessons. In the "Junior Engineer Academy" and the nation-wide "Fraunhofer MINT-EC Math Talents" program, participants have a chance to experience, over a longer period of time, a new philosophy of linking education with practical application.

We also offer pointers for the education and continuing education of future teachers that will help them to structure their lessons and additional intracurricular activities accordingly. For this purpose, prepared lesson material is far less important than the necessary technical training and a positive attitude towards using new methods and ways to address questions that have no clear-cut right or wrong answers. The information about the didactical integration of new instruction methods is designed to explain the impact and point out ways to connect new with traditional instruction.

7 A Brief Portrait of the Fraunhofer Institute for Industrial Mathematics ITWM

The Fraunhofer ITWM was founded by the working group "Technomathematics" from the University of Kaiserslautern. As a research institution belonging to the State of Rheinland-Pfalz, it was, from the beginning, under Fraunhofer administration. After a successful evaluation in 1999, it advanced to the status of the first mathematical research institute of the Fraunhofer-Gesellschaft, thus, becoming part of one of the world's largest and most successful research organizations (Fig. 3).

Fig. 3 Institute building of the ITWM at the Fraunhofer Center in Kaiserslautern (Photo: G. Ermel, Fraunhofer ITWM)

As a mathematics institute, the ITWM has remained committed to one of civilization's oldest sciences while, at the same time, developing into one of the most successful institutes in the Fraunhofer-Gesellschaft, as measured by its economic revenues. The basis for this balancing act has been the previously mentioned, dramatic increase in the relevance of mathematics for all production, service, and communication processes in modern industry.

After 20 years of effort, the vision with which the ITWM began—to transport mathematics out of the ivory towers and cathedrals of pure science and transform it into a key technology for innovation in technology and business—has become realized to a significant extent. This vision was not always universally applauded. Hardly anyone would have believed at the time of the Institute's founding that, in so short a span of time, such a large and successful Fraunhofer Institute of Mathematics would develop out of the seeds of technomathematics and economathematics from the University of Kaiserslautern.

The warnings often had the ring of "modern technology needs mathematics, but not mathematicians; it remains the domain of engineers and scientists." In the interim, a reversal in thinking has taken place here.

In the past 30 years, the scope of the mathematics that has found its way into industrial practice has grown exponentially. The essential reason for this is that work on real models has been replaced by simulations, that is, by work on mathematical models. Augmenting this development has been the automation of work processes, cognitive capabilities, sense perceptions, and experiences in the form of algorithms, computer programs, and expert systems. The materialization of mathematics in computers and software programs has also played a role. As a raw material for models and the core of every simulation program, mathematics serves as a key technology and forms the foundation of the bridge to the world of simulations—a world based on the highly efficient assistance of the computer, a tool that has gained a foothold in nearly every sphere of our society and economy.

Research and development projects with industry, preparation of customized software solutions and systems, and support with the use of high performance computing technology are integral building blocks of this transformation. The projects of the ITWM reflect a broad range of clients, from low tech to high tech companies, from small and mid-sized companies to industrial heavyweights, from regional businesses to customers throughout Europe and overseas. Industry appreciates and needs the Institute's modeling competence, its algorithms, and its software products. Significant economic revenues, coupled with a strong emphasis on research—62 doctoral students are working on their dissertations at the Institute in 2014—form the basis for sustainable success and continuous growth.

Since its founding in late 1995, the ITWM has attracted more than 81 million euros' worth of industrial projects and almost 51 million euros' worth of publicly sponsored projects. In the past three years alone, more than 700 industrial projects have been successfully completed.

This is proof that there is a great demand on the part of industry for innovative mathematics and, simultaneously, that industrial problems can serve as a driving force for developing innovative mathematical methods and tools.

The ITWM budget has grown continuously since the Institute's founding and reached a total of more than 22 million euros in 2014; almost half of that is financed by industrial projects. This establishes the ITWM as one of the world's largest institutes in the area of applied and industry-oriented mathematics.

One quarter of ITWM business revenues come from contracts with small and mid-sized businesses. One third of ITWM business projects are contracted with regional businesses, and a further third with companies outside of Germany.

Analysis of ITWM's industrial projects reveals several trends that, in our view, are not attributable to local or regional effects, but have a general validity:

- Mathematical modeling, simulation, and optimization are in demand by large companies in all business sectors.
- The use of mathematical methods is also a significant innovation factor for small and mid-sized companies.
- The transfer of mathematics to industry is subject to globalization.
- Regional companies represent a large customer potential.
- Small batch sizes predominate in the projects.

The ITWM boasts a broad customer spectrum: the main sectors involved are plant and machine construction; the automobile industry; the plastics, metal, and mineral processing industries; information and communication technology; the wood, paper, and printing industries; microelectronics; medical technology; the pharmaceutical industry; the chemical industry; technical textiles; banks; and the insurance industry. Many of the projects involve large companies traded on the German Stock Exchange (DAX). In the automobile industry, the ITWM cooperates with all of the domestic companies and many foreign manufacturers as well. The ITWM works with a problem-oriented approach in a project landscape that encompasses the most varied business sectors and that allows, due to the cross-linking character of mathematics, an efficient transfer of methods. This results in structural stability and makes the Institute resilient in the face of economic downturns in any given industrial branch.

Many small and mid-sized companies are subject to enormous competitive pressures and take advantage of the ITWM's modeling and simulation competence to help them cope. The vanguards in this development have profited in the marketplace by using simulations as proof of innovation and quality assurance in their products. The fact that computing power can be purchased more cheaply every year has helped small and mid-sized companies, who have more limited financial resources. Here, it is not investments in computers, but in the relatively expensive software, that is the bottleneck. Moreover, technically qualified personnel must also be hired to support the ever more complex software programs. Because small and mid-sized companies often have little or no in-house R and D, the use of simulations frequently means hiring additional staff, which results in permanent costs. Along with this economic factor, the psychological challenge of giving up tried and tested, mainly experiment-based procedures—where one can always see and measure the results—and replacing them with simulations—where one must put faith in the computer and software tools—still occasionally impedes progress in project business. However, when implemented correctly, simulations are extraordinarily reliable. This, along with their almost limitless flexibility, will sooner or later convince everyone, which means that the potential for cooperative ventures is enormous.

Businesses located here and in the surrounding region use the ITWM's competences. In 2013, almost one third of the projects were carried out with cooperation partners from Kaiserslautern and environs, although these were predominantly small and mid-sized companies. This shows that a mathematics-based research institute can also significantly support the regional economy in the field of R and D and promote innovation.

The globalization of the economy is reflected in the ITWM's contracting partners. The portion of industrial revenues from projects with foreign partners has grown to more than one third. Many customers are based in Europe, but cooperative ventures with companies in the USA and Asia are becoming more and more significant.

With respect to the associated marketing efforts, the long-term planning of staff utilization and competence development, and the minimization of administrative costs, the ITWM's ideal partner is one who signs a multi-year contract with us that covers the execution of individual projects. Existing customers that meet this profile are very valuable to the Institute.

Small and mid-sized companies usually contract for single, smaller projects. Many companies which, when taken together, provide us with a large volume of contracts, have various R and D departments that sign separate contracts with the ITWM for their specific projects. New customers tend to first test the competence and capabilities of the Institute by means of smaller feasibility studies and computational jobs. As a result, the number of projects being processed yearly at the ITWM has grown to more than 250, and the average economic volume of the projects in 2013 was just under 40 000 euros. The large number of follow-up projects is a clear sign of the quality of our project work and a source of great satisfaction.

7.1 Which Competences and Structures Are Needed to Successfully Transfer Mathematics to the Market Place?

The cornerstones for successfully transferring mathematics are the classical disciplines of applied mathematics: numerics, optimization, stochastics and statistics, differential equations, and mathematical modeling. These are augmented by such strongly mathematically-oriented theoretical fields as 3D differential geometry, continuum mechanics, electrodynamics, system and control theory, financial mathematics, inverse problems, and image and signal processing, which have evolved into boundary fields between mathematics and technology over the past decades (Fig. 4). They are indispensible constituents for successfully carrying out application projects.

The ITWM's main field of activity consists of transforming mathematics that is applicable into mathematics that is actually applied: We adapt theorems and algorithms to models that come from actual practice and we convert optimal solutions that exist in theory into practicable solutions that can exist in reality. However, this transformation requires specific competences above and beyond the aforementioned cornerstones in order to build actual bridges to the virtual world. In relation to the processing of available experimental and observational data, they consist of setting up the mathematical model; transforming the mathematical solution of the problem into numerical algorithms; combining data, models, and algorithms in simulation programs; optimizing solutions in interaction with the simulation; and, finally, visualizing the simulation runs in the form of images and graphics. The

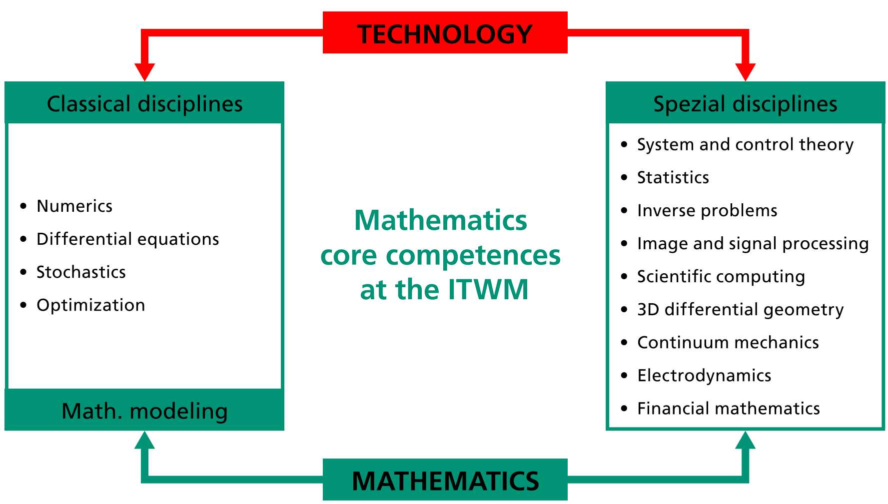

Fig. 4 Mathematics core competences at the ITWM (Graphic: S. Grützner, Fraunhofer ITWM)

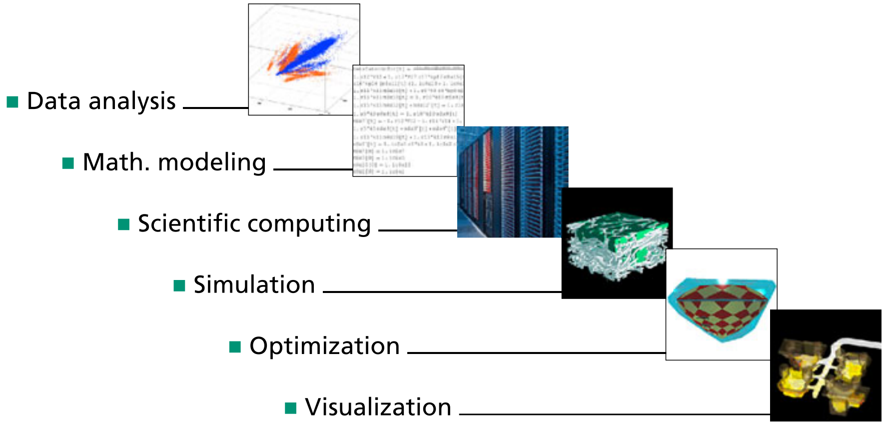

Fig. 5 Process chain at the ITWM (Graphic: S. Grützner, Fraunhofer ITWM)

competences needed to build this process chain represent the ITWM's core competences (Fig. 5).

This entire process chain is frequently subsumed under the term "numerical simulation." In the past 20 years, the increasing performance capacity of computers has opened up entirely new possibilities for industrial simulation tasks. More and more, computer networks are achieving central significance. There has been a dramatic paradigm change with regard to generating the highest computational performance for industrial applications. PC clusters, multi-core systems, and cloud computing are replacing super computers. Parallel computing systems, which just a few years ago were found only in a few meteorological research centers, have now made their way into industrial settings. Adapting numerical algorithms to these rapid changes in hardware configuration is still a troublesome bottleneck in the complete realization of the performance potential of these new computing systems.

The full process chain is illustrated in very many ITWM projects. One of the Institute's great advantages is that all these competences are available in-house and their utilization in projects can be centrally planned. The original team of 34 scientists, PhD students, and staff in the centralized areas has grown into the current team of 260 employees. All in all, 170 scientists, most of them with doctorates in mathematics, but also coming from the fields of physics, engineering, and informatics, process a multitude of topics and develop simulation software (cf. Appendix: The Fraunhofer Institute in numbers). In contrast to project execution in university settings, there is no need for coordination and reconciliation of content and timing between working groups from different chairs. Zones of responsibility and authority, along with schedules and delivery of work packages, are already clearly defined during bid preparation.

At the same time, in order to maintain contact with the frontlines of research and remain competitive with other research institutions in the marketplace, it is necessary to continuously reflect on how our own focal points, ideas, and goals mirror events in research and development outside the ITWM microcosm (Fig. 6).

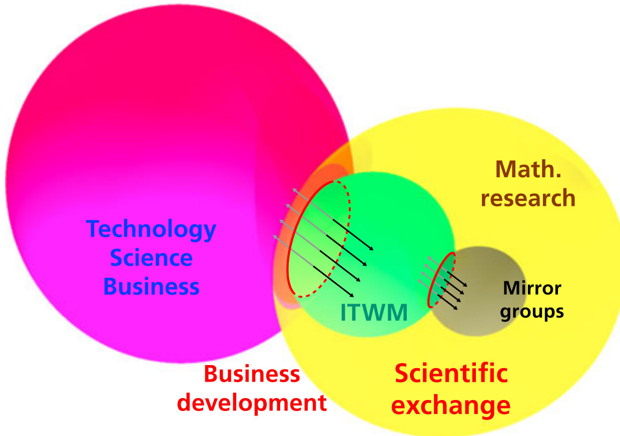

Fig. 6 Scientific exchange (Graphic: S. Grützner, Fraunhofer ITWM)

The research at the ITWM is very tightly integrated with the research in the TU Kaiserslautern Department of Mathematics. At the University, there are counterparts to the groups working in the Institute's primary areas of focus. The University also participates in the State's research focus area CM2 and the graduate school "Stochastic Models for Innovations in the Engineering Sciences." Beyond this, there are cooperative projects with many chairs in the Informatics, Mechanical and Process Engineering, Civil Engineering, Electrical Engineering, and Information Technology Departments, including, for example, projects in the innovation center "Applied System Modeling" and in the Kaiserslautern "Science Alliance."

The bridging technology of mathematics is also reflected in a multitude of cooperative projects between the ITWM and other Fraunhofer Institutes. The ITWM is one of the most profitable members in the Fraunhofer ICT Group (Information and Communication Technology) and also enjoys the status of a guest institute in the Fraunhofer Group for Materials. Moreover, the Institute is a member of the Fraunhofer Alliances Automobile Production, Batteries, Big Data, Cloud Computing, Lightweight Construction, Numerical Simulation of Products and Processes, Transportation, and Vision, as well as of the Fraunhofer Innovation Cluster "Digital Commercial Vehicle Technology."

In association with other institutes, the ITWM participates in a series of larger in-house Fraunhofer cooperative projects. Here, we contribute our mathematically oriented competences, which, as a rule, complement those of the partner institutes. All told, the ITWM is one of the best-connected institutes in the Fraunhofer-Gesellschaft.

The ITWM's international network manifests itself also in the current research cooperative ventures with many foreign universities and research institutions, in the numerous foreign guest scientists working here, and in the extensive participation of ITWM scientists in scientific committees and in the publication of technical journals.

7.2 Departments, Business Areas, and Customers

The departments serve to structure the Institute's business areas, not always with perfectly sharp divisions, but with sufficient specificity for differentiation purposes. The matrix structure found in many institutes was consciously avoided in order to have few hierarchic levels in the ITWM and to minimize the internal coordination processes necessary in business development and project work. As a rule, the departments have at their disposal the relevant competences needed to serve the business areas they address.

It is beyond the scope of this introductory chapter to offer detailed descriptions of the competence and customer profiles of the various departments. Five departments have made significant contributions to this book, and the chapters in "The Research" that were prepared by these departments offer a glimpse into the work they have conducted. The ITWM's pallet of customers is also far too extensive to offer a complete accounting.

From 2009 to 2013, the ITWM processed 1070 industrial projects. The following short overview illustrates, using 2013 as an example, the Institute's diverse branch and customer pallet.

- Business sectors:
  Vehicle industry, general mechanical engineering, energy and raw materials, chemicals, financials, manual trades, information and communication technology, medical technology, and textiles.
- Customers:
  Accenture CAS GmbH, Assyst GmbH, AUDI AG, AUTEFA (A), BASF SE, BMW Group, BPW Bergische Achsen Kommanditgesellschaft, ClusterVision (NL), Daimler AG, DZ-Bank (L), ebm papst, FLSmidth Wadgassen GmbH, Freudenberg Filtration Technologies, Görlitz AG, IBS FILTRAN GmbH, John Deere, Johns Manville Europe GmbH, K + S Kali, Klinikum Essen, Liebherr, LONZA Group AG (CH), Lundin (N), M + W Process Industries GmbH, Marathon Oil (USA), Math2Market GmbH, MTU Aero Engines GmbH, Paul Wild OHG, proALPHA Software AG, Procter & Gamble (USA), Progress Rail inspection & information systems, Repsol (USA), Robert Bosch GmbH, Seismic Image Processing Ltd (GB), SGL Carbon, SIEDA GmbH, Siemens AG, Statoil (N), Teckpro AG, Voith Hydro, Volkswagen AG, Volvo CE (S), Woltz GmbH.

7.3 Cooperation with the Fraunhofer-Chalmers Center for Industrial Mathematics FCC

The ITWM was one of the first Fraunhofer Institutes to implement the recommendation of the Fraunhofer Board to promote internationalization in Europe. In Gothenburg, Sweden, the Fraunhofer-Chalmers Center for Industrial Mathematics FCC was successfully established as a joint venture between the Chalmers Technical University and the ITWM

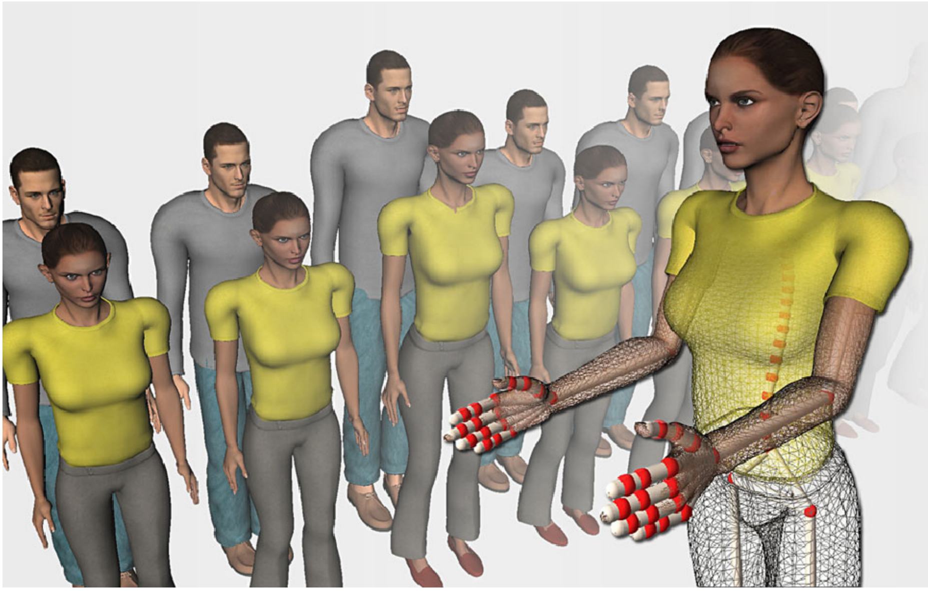

Fig. 7 The Software IMMA^{TM}—Intelligently Moving Manikins—utilizes families of manikins in order to accommodate the majority of the population. The manikins are used for evaluating assembly ergonomics (Graphic: FCC, geometry mesh: Poser®)

(Fig. 7). Today, 51 employees generate an operating budget of almost 41 million Swedish kronor (approx. 4.3 Million euros). The Institute, with its departments

- Geometry and Motion Planning
- Computational Engineering and Design
- Systems and Data Analysis

was founded in 2001, and since that time has developed into one of Sweden's most renowned centers for "industrial mathematics."

8 Summing up the ITWM

Today, the ITWM already numbers among the largest institutes in the field of applied and industry-oriented mathematics. Its mission is to be the spearhead of mathematics in industry, with particular focus on small and mid-sized companies.

It will strengthen and enlarge this position and continue to contribute its part to making mathematics a key technology in industry and business. The outstanding connection with the TU Kaiserslautern in research and education guarantees proximity to current research topics, particularly (but not only) in applied mathematics and represents an important resource for attracting talented, young scientists. The ITWM's integration in the

Fraunhofer-Gesellschaft, its participation in a number of international cooperative ventures, and the close collaboration with its affiliated institute FCC in Gothenburg are also among the Institute's strengths.

The horizontal structures, with autonomous departments and a small, efficient administration, allow for simple operational procedures, operational flexibility without complex matrix structures, and direct coupling of ITWM competences with customers. A good working environment, a minimum of hierarchical friction, and a climate of mutual respect and appreciation contribute significantly to our employees' high level of commitment to and identification with their work and the Institute as a whole. Last, but not least, our straightforward dealings with our cooperation partners, based on the motto "promise only those things that you can really deliver," is an important element in the Institute's on-going economic success.

This is not to say that there is no room for improvement. We want to increase our cooperation with top national and international researchers in applied mathematics in order to ensure the quality of our research and to further develop and add to our competences. The publication activities in the departments vary widely; overall, an increase here is desirable, both for its own sake and to strengthen the Institute's visibility within the scientific community. The Institute addresses numerous application topics in almost all branches. This provides a degree of structural stability and helps ensure that economic downturns in individual branches have only a modest impact on the Institute's revenues. On the other hand, this high level of diversification is frequently associated with small project size, and the ITWM is the premium partner for MSO in only a few branches, such as commercial vehicles and the oil and gas industry. Moreover, there is a potential for further focusing, for example, in process technology, the energy sector, or the IT industry, which we want to promote more strongly in the future.

In addition, the Institute operates in a competitive environment: there are engineering offices offering companies R and D consulting with commercial software; there are software companies who are members of the ITWM's contract research pool offering commercial solutions for problems; there are university chairs pushing their way into the marketplace in response to the increasing market and third-party-funding orientation in academia; and there are also other Fraunhofer Institutes expanding their own modeling and simulation competences in their particular application domains. Of course, this competition is also directed toward attracting the best minds available in the employment marketplace. Naturally, the restrictions imposed by the TVöD (public service wage agreement) represent a competitive disadvantage. Attracting highly qualified new personnel and maintaining high employee motivation levels, while the team is increasing in age, will be one of the Institute's biggest challenges in the coming years.

The ITWM participates in many BMBF (Federal Ministry for Education and Research) projects as a partner for MSO. Although the innovation initiatives in Germany and the EU, when compared globally, may be viewed as providing a positive overall framework, it must nonetheless be admitted that mathematics does not fit squarely into the BMBF's funding channels. The BMBF mathematics program is certainly an important resource for applied mathematics in Germany, and mathematics funding also has a high priority for the DFG, as evidenced by its inclusion in all DFG subsidy programs. However, mathematics, as an independent technology, still has no funding program of its own, and the financial support of the BMBF program is exceedingly modest in comparison to the funding provided to other key technologies. The significance of applied mathematics as a driver of innovation is still not taken seriously in political circles. Thus, mathematically oriented research institutes and university chairs must continually rely on successfully docking their competences onto domain-oriented projects. However, they receive little dedicated funding for developing methodologies and expanding their core competences. There is no funding program for larger network projects with industry, in which methodological development, oriented on industrial needs, is expedited under the consortium management of the mathematics partners, and in which the companies themselves can also receive funding.

We do not, however, wish to conclude this introduction with what needs to be improved upon. The fact is that applied mathematics has experienced great growth in Germany within the past decades and has become a "motor of innovation," firmly anchored in the economy and society. The ITWM has made an important contribution to this development and is among the most renowned institutes of applied mathematics today. A significant part of the Institute's success is owed to the authors of this book. They have contributed to the project with great enthusiasm and attempted to identify the main elements of a problem-driven, model-based, and solution-oriented mathematics in the context of the Fraunhofer ITWM. Whether they have succeeded, we leave it for you to decide, dear reader. In any case, we wish you an interesting journey through this book, and look forward to receiving both positive feedback and constructive criticism.

Appendix: The Fraunhofer ITWM in Numbers

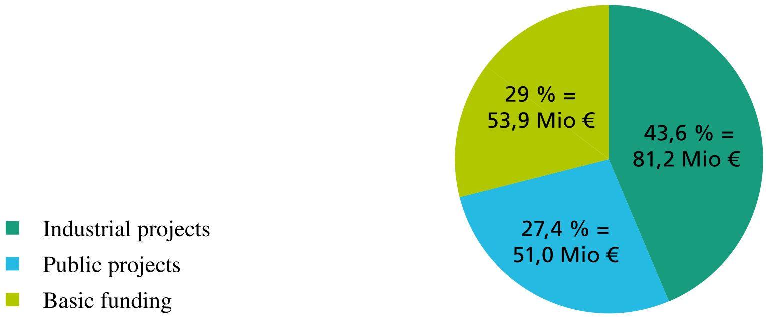

Fig. 8 Total operating budget 1996–2013

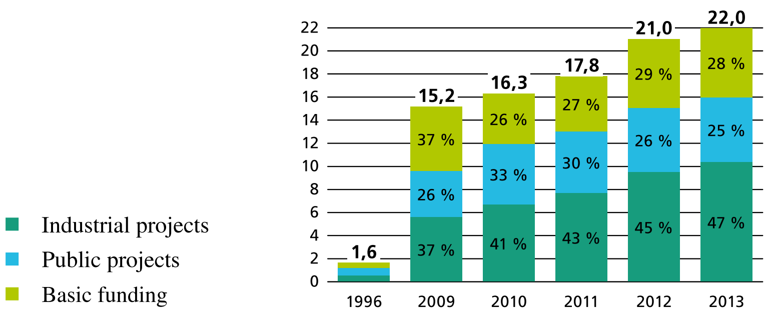

Fig. 9 Development of operating budget in millions of euros

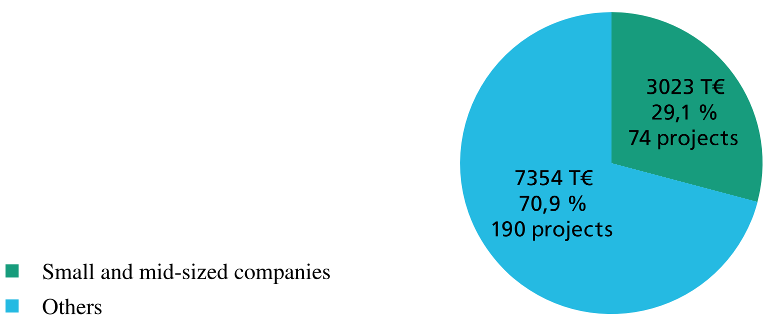

Fig. 10 Breakdown of industrial revenues, 2013: small and mid-sized companies portion

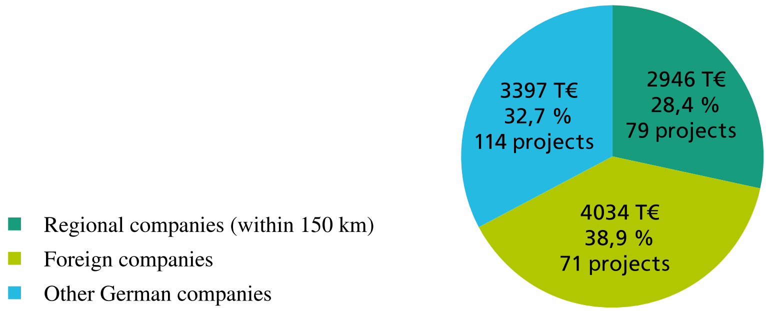

Fig. 11 Breakdown of industrial revenues, 2013: distribution of industrial customers

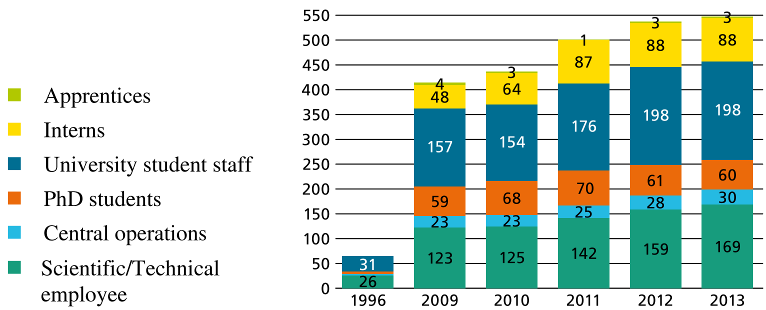

Fig. 12 Staff levels 1996–2013

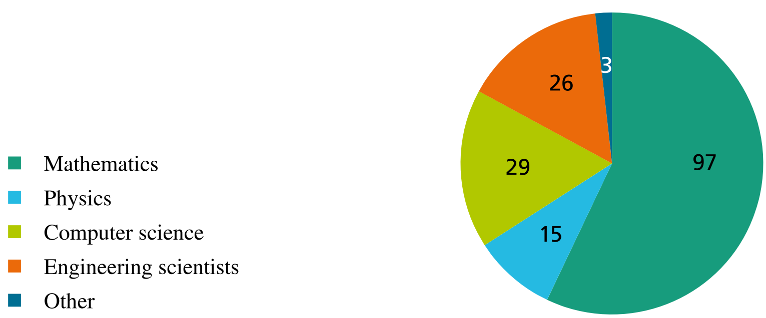

Fig. 13 Breakdown of scientific/technical staff acc. to field, 2013