

Chapter 2

Examples

Abstract This chapter illustrates the main problems of diagnosis and fault-tolerant control by means of three examples, which will be used later in the text.

2.1 Two-Tank System

Description of the system. As the first example, consider the two-tank system depicted in Fig. 2.1. The pump causes a liquid flow q_P into Tank 1 where the input $u(t)$ describes the pump velocity. u is determined by a security switch-off, which prevents an overflow of Tank 1. The inputs to the tank system prescribe the valve positions V_a and V_{12} . The only measured signal is the outflow q_M . Hence, the tank system results in the simple block diagram shown in Fig. 2.2.

In the faultless case, the valve V_a is closed and the valve V_{12} is used to control the level of Tank 2. Only in case of a valve fault, the upper pipe is used for this purpose.

The control aim results from the requirement of a batch process, in which the outflow of Tank 2 is used in succeeding parts of cascaded vessels and reactors. The valve V_{12} is used to fill and refill Tank 2 accordingly, where Tank 1 is a storage tank, which is to be filled to the height h_{\max} , at which the security switch-off stops the pump.

Faults. Two faults are considered. First, a leakage in Tank 1 may occur, which causes the additional flow q_L out of Tank 1. The “size” of the leakage is given by the parameter c_L (Table 2.1). The different approaches to fault diagnosis presented in the following chapters use this two-tank example with different notions of this fault. Either a parametric fault is considered where c_L denotes the fault size to be identified or a symbolic fault f is used which represents the faulty situation with the outflow $q_L = c_L \sqrt{h_1}$ out of Tank 1 where c_L is the parameter given in Table 2.1.

The second fault is a blockage of the valve V_{12} in the closed position. This fault can be modelled by setting the valve constant c_{12} to zero.

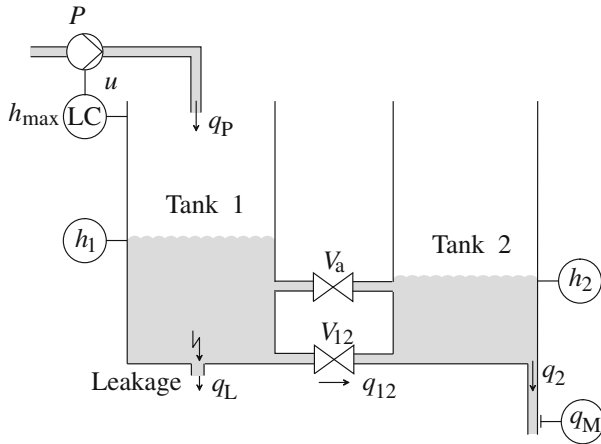
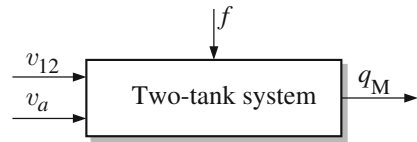


Fig. 2.1 Two-tank system

Fig. 2.2 Block diagram of the tank system



The example is used for illustrating the following diagnostic and fault-tolerant control problems:

- **Fault detection:** Determine whether a fault has occurred. The valve fault can be detected due to the decreasing outflow from Tank 2, which eventually vanishes. For the leakage the problem is more involved because neither the inflow q_P nor the level h_1 is assumed to be measured. Hence, the stationary outflow from Tank 2 is the same as before and the leakage and the fault can only be found by small dynamical effects that are “visible” in the outflow measurement just after the leakage occurs.
- **Fault isolation:** Determine which part of the system is faulty.
- **Fault identification:** Determine the size of the leakage.
- **Fault accommodation:** Design a fault-tolerant level controller that maintains the liquid level in Tank 1 at a given set-point independently of whether the leakage is present or not.
- **Control reconfiguration:** In case of the valve fault, the auxiliary valve V_a has to be used. The reconfiguration problem includes to *automatically* find that switching to the second pipe is the strategy to apply.

These problems are considered under different circumstances where the tank levels or the outflow from Tank 2 are measured numerically or in a quantised way. Therefore, different models are appropriate to describe the tank system and different methods have to be used to solve the diagnostic problem.

For this simple example, it is obvious under what conditions and how the given problems can be solved. If the pump is controlled according to level measurement h_1 , a static tank level will be reached. However, the leakage can only be found if the dynamical changes of the tank levels or of the outflow from Tank 2 are taken into account, because the pump is assumed to be strong enough to maintain the level of Tank 1 at the prescribed value even in case of the leakage. Hence, the system has the same static behaviour with and without the fault.

The diagnostic result will certainly be different if the outflow is the only measurement compared with the case in which all tank levels are measured. Also, the diagnostic problem becomes more difficult if instead of the numerical values of the tank levels only quantised measurements are possible.

If q_P is an additional measurement, the fault can be detected by comparing the mean value of q_P with its nominal value. If this value is increased, more liquid flows out of Tank 1 which under the given circumstances can only occur if the tank has a leakage.

The fault-tolerant controller is simply found as a PI-feedback of the tank level h_1 towards the pump velocity u_P . For reasonable leakages (reasonable values of q_L) the controller is able to hold the level at the prescribed value even if the fault occurs.

Model of the tank system. The two tanks have the liquid levels $h_1(t)$ and $h_2(t)$, which are used as state variables in the model given below. The liquid flows are

Table 2.1 Signals and parameters of the tank system

Parameter	Value and unit	Meaning
h_1, h_2	[m]	Tank levels in meters
q_M	[l/min]	Measured outflow in litres per minute
u	[1]	Control input to the pump
q_{12}, q_2, q_P, q_L	[m ³ /s]	Volume flows in cubic metres per second
A	$1.54 \cdot 10^{-2} \text{m}^2$	Cross-section area of both tanks
h_{\max}	0.60 m	Height of both tanks
u_{nom}	1.0	Nominal pump velocity
u_{\max}	5.0	Maximal pump velocity
c_{12}	$6.0 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of valve V_{12}
c_2	$2.0 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of the outlet of Tank 2
c_L	$8.0 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of a leakage in Tank 1
c_M	$12.0 \text{l}/(\text{min} \cdot \text{m}^{1/2})$	Constant of outflow sensor
\bar{q}_P	$1.5 \cdot 10^{-4} \text{m}^3/\text{s}$	Flow constant of the pump

denoted by q , the ground area of the cylindric tanks by A . The parameters used in the example are summarised in Table 2.1.

The following Eqs. (2.1), (2.2) describe the mass balance, where the tank levels h_1 and h_2 are related to the liquid flows indicated in Fig. 2.1 as follows:

$$\dot{h}_1(t) = \frac{1}{A}(q_P(t) - q_L(t) - q_{12}(t)) \quad (2.1)$$

$$\dot{h}_2(t) = \frac{1}{A}(q_{12}(t) - q_2(t)). \quad (2.2)$$

The measured signal q_M is proportional to the outflow q_2 :

$$q_M = c_M \cdot q_2. \quad (2.3)$$

The different flows used in the equations above can be obtained by TORICELLI's law:

$$q_{12}(t) = \begin{cases} c_{12} \text{ sign}(h_1(t) - h_2(t)) \sqrt{|h_1(t) - h_2(t)|} & \text{if } V_{12} \text{ is open} \\ 0 & \text{else} \end{cases} \quad (2.4)$$

$$q_2(t) = \begin{cases} c_2 \sqrt{h_2(t)} & \text{if } h_2(t) > 0 \\ 0 & \text{else,} \end{cases} \quad (2.5)$$

$$q_P(t) = \begin{cases} u(t) \cdot \bar{q}_P & \text{if } h_1(t) \leq h_{\max} \\ 0 & \text{else,} \end{cases} \quad (2.6)$$

$$q_L(t) = \begin{cases} c_L \sqrt{h_1(t)} & \text{if } h_1(t) > 0 \text{ and Tank 1 has a leakage} \\ 0 & \text{else.} \end{cases} \quad (2.7)$$

The pump is controlled by the security switch-off included in the level controller LC shown in the figure such that the level in Tank 1 is maintained below the height h_{\max} . The pump velocity is given by the control input u_P . Its nominal value is given by $u = u_{\text{nom}}$, and its maximal value by u_{\max} . If a control problem should be illustrated in the later chapters, then Eq. (2.6) is supplemented with an equation describing the control law $u = k(h_1)$.

The equations given above are hybrid because they include differential and algebraic equations as well as switching conditions, which result from the physical laws and from a security switch installed at Tank 1. Therefore, the differential equation includes several inequalities that describe the validity range of the given functions.

The tank will be used in many places to illustrate methods and results. For simplicity, often the parameter A is set to one so that the model gets the simpler form

$$\dot{h} = q_i(t) - q_o(t),$$

where q_i and q_o denote the input and the output flow.

2.2 Three-Tank System

Consider the three coupled tanks depicted in Fig. 2.3. These tanks are connected by pipes which can be controlled by different valves. Water can be filled into the left and right tanks using two identical pumps. Measurements available from the process are the continuous water levels h_i of each tank and, additionally, from tank T_2 discrete signals from two capacitive proximity switches signalling whether the water level in the tank is above or below the position of the sensor.

In the nominal case (Fig. 2.4), only the left tank T_1 and the middle tank T_2 are used. The right tank T_3 and pump P_2 act as redundant hardware. The purpose of the system is to provide a continuous water flow $q_2(t) = q_N$ to a consumer. Therefore, the water level in the middle supply-tank T_2 has to be maintained within the interval $h_{2L} < h_2 < h_{2H}$, i. e. between the two discrete level sensors of tank T_2 .

Water flows between the tanks can be controlled by several valves (V_{12L} , V_{12H} , V_{23L} , V_{23H}). All valves can only be completely opened or completely closed (on/off valves). The connection pipes between the tanks are placed at the bottom of the tanks (pipes with valves V_{12L} , V_{23L}) and at a height of h_H (pipes with valves V_{12H} , V_{23H}). One of the considered faults is a leakage in tank T_1 (see below). If such a leakage occurs, there is an additional outflow q_L of tank T_1 (cf. Fig. 2.3).

Dynamical model. Depending on the water levels and the position of the valves, different nonlinear state-space models are valid. In general, the water flow q_{ij} from Tank i to Tank j can be calculated using TORICELLI's law

$$q_{ij} = c_{ij} \cdot \text{sign}(h_i - h_j) \cdot \sqrt{|h_i - h_j|},$$

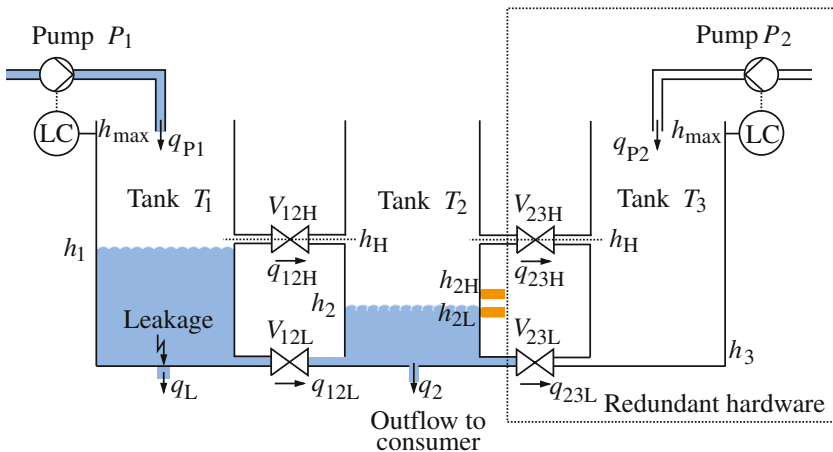


Fig. 2.3 Three-tank system

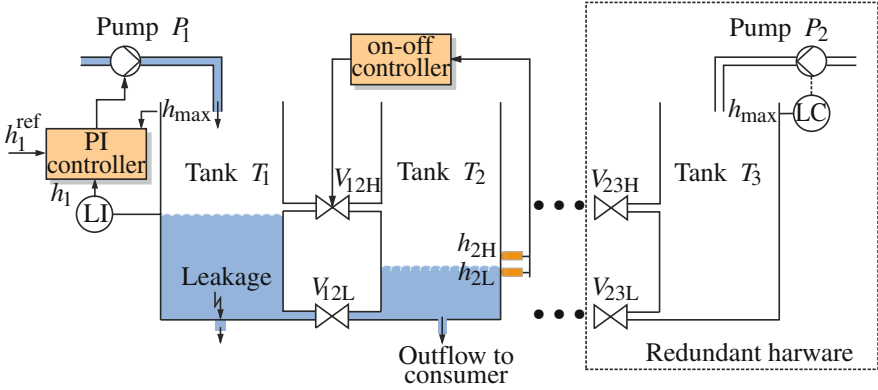


Fig. 2.4 Nominal configuration of the three-tank system

where c_{ij} is a constant depending on the geometry of the connecting pipe and the valve and h_i, h_j are the water levels. The change of water volume V in a tank is described by

$$\dot{V} = A \cdot \dot{h} = \sum q_{in} - \sum q_{out}, \quad (2.8)$$

where $\sum q_{in}$ is the sum over all water inflows and $\sum q_{out}$ the sum over all water outflows of the tank. In (2.8), A is the cross-section area and h the water level in the cylindrical tank. For the three tanks Eq. (2.8) yields:

$$\dot{h}_1 = \frac{1}{A} (q_{P1} - q_{12L} - q_{12H} - q_L) \quad (2.9)$$

$$\dot{h}_2 = \frac{1}{A} (q_{12L} + q_{12H} - q_{23L} - q_{23H} - q_2) \quad (2.10)$$

$$\dot{h}_3 = \frac{1}{A} (q_{P2} + q_{23L} + q_{23H}). \quad (2.11)$$

The flows in Eqs. (2.9)–(2.11) depend on the levels h_1, h_2 and h_3 as well on the position of the valves and the commands u_{P1}, u_{P2} given to the pumps. For example, the existence of the flow q_{12H} depends on the water levels h_1 and h_2 and the position of the valve V_{12H} . The flow is only non-zero if the valve is open and at least one liquid level exceeds the height h_H of the upper connecting pipe.

More precisely, the following expressions are obtained for the flows, with the parameters given in Table 2.2:

$$q_{P1} = \begin{cases} c_{P1} \cdot u_{P1} & \text{if } h_1 \leq h_{\max} \text{ and } c_{P1} \cdot u_{P1} < q_{P1}^{\max} \\ q_{P1}^{\max} & \text{if } h_1 \leq h_{\max} \text{ and } c_{P1} \cdot u_{P1} \geq q_{P1}^{\max} \\ 0 & \text{otherwise,} \end{cases}$$

$$\begin{aligned}
q_{P2} &= \begin{cases} c_{P2} \cdot u_{P2} & \text{if } h_3 \leq h_{\max} \text{ and } c_{P2} \cdot u_{P2} < q_{P2}^{\max} \\ q_{P2}^{\max} & \text{if } h_3 \leq h_{\max} \text{ and } c_{P2} \cdot u_{P2} \geq q_{P2}^{\max} \\ 0 & \text{otherwise,} \end{cases} \\
q_{12L} &= \begin{cases} c_{12L} \operatorname{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} & \text{if } V_{12L} \text{ open} \\ 0 & \text{otherwise,} \end{cases} \\
q_{12H} &= \begin{cases} c_{12H} \sqrt{|h_1 - h_H|} & \text{if } h_1 > h_H, h_2 \leq h_H, V_{12H} \text{ open} \\ -c_{12H} \sqrt{|h_2 - h_H|} & \text{if } h_1 \leq h_H, h_2 > h_H, V_{12H} \text{ open} \\ c_{12H} \operatorname{sign}(h_1 - h_2) \sqrt{|h_1 - h_2|} & \text{if } h_1 > h_H, h_2 > h_H, V_{12H} \text{ open} \\ 0 & \text{otherwise,} \end{cases} \\
q_{23L} &= \begin{cases} c_{23L} \operatorname{sign}(h_2 - h_3) \sqrt{|h_2 - h_3|} & \text{if } V_{23L} \text{ open} \\ 0 & \text{otherwise,} \end{cases} \\
q_{23H} &= \begin{cases} c_{23H} \sqrt{|h_2 - h_H|} & \text{if } h_2 > h_H, h_3 \leq h_H, V_{23H} \text{ open} \\ -c_{23H} \sqrt{|h_3 - h_H|} & \text{if } h_2 \leq h_H, h_3 > h_H, V_{23H} \text{ open} \\ c_{23H} \operatorname{sign}(h_2 - h_3) \sqrt{|h_2 - h_3|} & \text{if } h_2 > h_H, h_3 > h_H, V_{23H} \text{ open} \\ 0 & \text{otherwise,} \end{cases} \\
q_2 &= \begin{cases} c_2 \sqrt{h_2} & \text{if } h_2 > 0 \\ 0 & \text{otherwise,} \end{cases} \\
q_L &= \begin{cases} c_L \sqrt{h_1} & \text{if } h_1 > 0 \text{ and leakage in tank 1} \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

Nominal configuration. In the nominal case, valves V_{12L} , V_{23H} , V_{23L} are closed and not in use. Valve V_{12H} is used to control the water level in tank T_2 , pump P_1 to control the level in tank T_1 . To control the water levels in the reservoir-tank T_1 and the supply-tank T_2 , a conventional PI-controller and an discrete (on–off) controller are used (Fig. 2.4):

$$\begin{aligned}
u_{P1}(t) &= k(h_1(t), h_1^{\text{ref}}) \\
&= K_P \cdot (h_1^{\text{ref}} - h_1(t)) + K_I \cdot \int_0^t (h_1^{\text{ref}} - h_1(\tau)) d\tau \quad (2.12)
\end{aligned}$$

$$V_{12H} = \begin{cases} \text{open} & : h_2 \leq h_{2L} \\ \text{close} & : h_2 \geq h_{2H} \\ \text{no change} & : h_{2L} < h_2 < h_{2H}, \end{cases} \quad (2.13)$$

where K_P and K_I are controller parameters and h_1^{ref} is the set-point for tank T_1 . Equation(2.13) describes under what conditions the on–off controller changes the position of the valve from opened to closed or vice-versa. All parameters of the controllers are given in Table 2.2.

Table 2.2 Parameters and variables of the three-tank system and the controllers

h_1, h_2, h_3	[m]	Tank levels in meters
q_{P1}, q_{P2}, q_2, q_L	[m ³ /s]	Volume flows in cubic metres per second
q_{12L}, q_{12H}	[m ³ /s]	Volume flows in cubic metres per second
q_{23L}, q_{23H}	[m ³ /s]	Volume flows in cubic metres per second
A	$1.54 \cdot 10^{-2} \text{m}^2$	Cross-section area of the three tanks
h_{\max}	0.60 m	Height of the three tanks
h_H	0.60 m	Height of the three tanks
c_{12L}	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of valve V_{12L}
c_{12H}	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of valve V_{12H}
c_{23L}	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of valve V_{23L}
c_{23H}	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of valve V_{23H}
c_2	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of the outlet of tank 2
c_L	$1.6 \cdot 10^{-4} \text{m}^{5/2}/\text{s}$	Flow constant of a leakage in tank 1
c_{P1}	$1.0 \cdot 10^{-4} \text{m}^3/\text{s}$	Flow constant of pump 1
c_{P2}	$1.0 \cdot 10^{-4} \text{m}^3/\text{s}$	Flow constant of pump 2
q_{P1}^{\max}	$1.0 \cdot 10^{-4} \text{m}^3/\text{s}$	Maximum flow of pump 1
q_{P2}^{\max}	$1.0 \cdot 10^{-4} \text{m}^3/\text{s}$	Maximum flow of pump 2
h_1^{ref}	0.50 m	Set point of PI controller
K_P	10.0 1/m	Proportional gain of PI controller
K_I	$5.0 \cdot 10^{-2} 1/\text{ms}$	Integral gain of PI controller
h_{2L}	0.09 m	Position of lower discrete level sensor
h_{2H}	0.11 m	Position of upper discrete level sensor

In summary, the nominal behaviour is characterised by the following:

- Only the left tank and middle tank are in use, water level h_2 must be medium, the set-point for h_1 is chosen to h_1^{ref} .
- Valves V_{12L} , V_{23L} , V_{23H} are closed.
- No leakage occurs ($q_L = 0$).
- The PI-controller (2.12) controls the level h_1 of tank T_1 with pump P_1 using a continuous level sensor.
- The on–off controller (2.13) controls the level h_2 of tank T_2 with valve V_{12H} using discrete level sensors.

Reconfiguration problem. Three different fault scenarios are given:

1. Fault f_1 : Valve V_{12H} is closed and blocked.
2. Fault f_2 : Valve V_{12H} is opened and blocked.
3. Fault f_3 : A leakage in Tank T_1 occurs ($q_L \neq 0$).

The reconfiguration task is to find *automatically* a new control configuration of the three-tank system such that

- the water level h_2 remains between h_{2L} and h_{2H} for all scenarios, i. e. the relation

$$[h_2(k)] = \text{medium} \tag{2.14}$$

should hold for $k \geq \bar{k}$ for a possibly small \bar{k} .

- for scenario 3, the loss of water is minimal, i.e.

$$[h_1(k)] = \text{empty} \tag{2.15}$$

should hold for $k \geq \bar{k}$ for a possibly small \bar{k} .

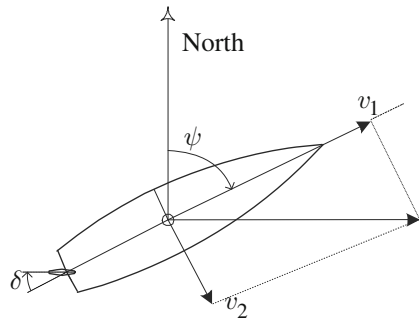
The reconfiguration task consists in finding a new control structure by selection of actuators and sensors, new control laws and new set-points for the control loops, such that the control aims above are met. If needed, the use of redundant hardware components is possible. Obviously, the idea of reconfiguration cannot be satisfied by simply changing the parameters K_P or K_I , but a structural change of the system is necessary.

2.3 Ship Steering and Track Control

Ship navigation and steering is used as an example to illustrate different methods in both diagnosis and fault-tolerant control. A ship is illustrated in Fig. 2.5. The ship is steered by its rudder, the angle of which is δ . The ships heading angle is denoted ψ , the turn rate ω_3 . The ship velocity ahead is v_1 , velocity sideways is v_2 .

To navigate a ship, information is needed on its position and heading angle as a minimum. In confined waters, distance is needed to a desired track that the ship is supposed to follow.

Fig. 2.5 Motion of a ship steered by its rudder. A rudder angle to port side (*left*) generates a turn to the port side of the ship. When turning to port, there is also a side velocity towards starboard (*right*)



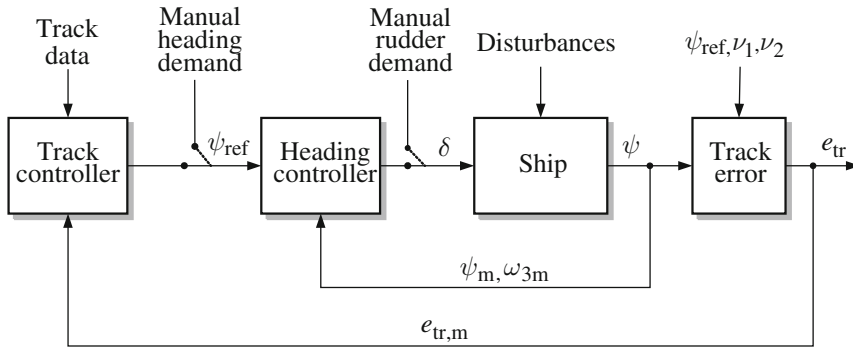


Fig. 2.6 Cascaded architecture of controllers for ship steering. The innermost loop is manual steering with rudder demand as input. The second loop provides automatic heading control, the third implements automatic track control

Should navigation data be wrong, ships may collide with banks or with other vessels. As unexpected manoeuvres can have fairly serious consequences, natural performance requirements exist to diagnosis and fault-tolerant control algorithms. Requirements are derived from the maximal motion the ship could make before a fault was diagnosed and a remedial action taken.

Control modes. In our ship steering example, three levels of steering control are considered:

- Hand steering. The rudder demand is manually set by a helmsman.
- Course control. An autopilot sets the rudder demand according to the deviation between instantaneous heading and a demanded course (heading reference). The ship's turn rate is used for derivative control action.
- Track control. A set of way-points specify a desired track for the ship to follow. The distance of the ship to the track is calculated and used by the track controller to command a heading reference to the heading controller. This reference is updated in each sampling cycle by the track controller.

A block diagram of the ship with the above-mentioned controllers is shown in Fig. 2.6.

Instrumentation. The ship motions and position are measured using dedicated sensors. The ship's heading is measured by some form of gyro compass, distance to a desired track is calculated from a position measurement, with the position measured by a GPS (Global Positioning System) receiver. Two identical gyro compasses are commonly available due to the critical nature of the heading measurement. In the sequel, we will consider the following types of instruments:

- Instrumentation with gyro compass and rate gyro as two separate units. The two measurements are independent.

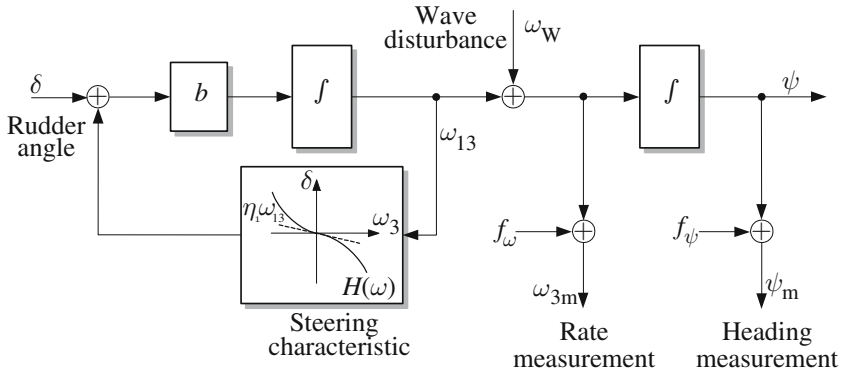


Fig. 2.7 A simple dynamical model of a ship steered by the rudder. Waves act as unknown input and measurement faults are considered on turn rate and heading angle measurements

- Measurement of track error by a navigation computer that measures ship’s position using a GPS receiver.

Faults. For the example we consider four possible faults. These faults and the consequences they will have in the example are as follows:

- **Fault in the heading measurement:** In heading control mode, this fault will cause the ship to steer a wrong course. In track control mode, there will be a permanent track error present.
- **Fault in the turn rate measurement:** In heading control mode, this fault will cause a transient error in the heading, but will then be compensated by the controller. A similar behaviour will be seen in track control mode.
- **Fault in the measurement of distance to the desired track:** This has no effect in heading control. In track control mode, there will be an offset equal to the size of the fault.
- **Fault in the track controller:** It causes the heading demand output from this controller to remain at the value it had when the fault occurred. With heading demand being input to the heading controller, this will sooner or later cause the ship to steer away from the desired track.

The sensor faults are modelled as additive faults. The rate gyro measures ω_{3m} and the gyro measures the heading angle ψ_m . This is illustrated in the block diagram in Fig. 2.7.

Dynamics of the ship. On a ship, a desired turn rate is obtained by turning the rudder to a certain angle. The input variable is hence rudder angle and the output is turn rate. Waves act as a disturbance to the turn rate, and the combined signal is integrated to give the actual heading of the ship. This dynamics is illustrated in the block diagram in Fig. 2.7. Turn rate and heading angle are measured variables, the sensors are subject to faults. These are added as fault signals in Fig. 2.7.

The following equations describe the steering problem using the simple model. Waves contribute to turn rate by ω_w . The control input is the rudder angle δ . The measured signals are ψ_m and ω_{3m} .

In sections dealing with the stochastic case, measurement noise is present on sensor signals. If $\nu_\omega(t)$ and $\nu_\psi(t)$ are noise signals on the turn rate or heading measurements, respectively,

$$\begin{aligned}\dot{\omega}_3(t) &= b(\delta(t) + H(\omega_3)) \\ \dot{\psi}(t) &= \omega_3(t) + \omega_w(t) \\ \psi_m(t) &= \psi(t) + f_\psi(t) + \nu_\psi(t) \\ \omega_{3m}(t) &= \omega_3(t) + \omega_w(t) + f_\omega(t) + \nu_\omega(t)\end{aligned}\tag{2.16}$$

where $H(\omega)$ is the steady-state relation between turn rate and rudder angle. In the literature, this is the steering characteristic of the ship.

In the example, we treat the steering characteristic as linear such that

$$H(\omega_3) = \eta_1 \omega_3$$

The sign convention is that angles are taken positive around the third axis, which points downwards as seen from a surface ship. A positive rudder angle (clockwise) will turn the ship counter-clockwise, which corresponds to a negative value of turn rate. Hence, η_1 is negative for a ship that is directionally stable.

In the real world, the relation between a rudder angle and the turn rate is not linear.

$$H(\omega_3) \approx \eta_0 + \eta_1 \omega_3 + \eta_2 |\omega_3| \omega_3$$

Large tankers or container ships may be directionally unstable in a region around zero turn rate angle. This is a consequence of a balance between hydrodynamical forces on the hull. As turn rate builds up, a directionally unstable ship eventually becomes stable. A directionally unstable ship will enter into a steady turn and move in a circle if the rudder is left in neutral position. A directionally unstable ship will be used to illustrate diagnosis techniques for unstable physical systems.

The variables and parameters in the ship example are listed in Table 2.3.

Heading control. The autopilot to control the ship heading in this example is a linear quadratic design, equivalent to a PD controller without any filtering, signal smoothing or integral action

$$\delta(t) = L_\omega \omega_{3m} + L_\psi (\psi_{\text{ref}} - \psi_m).\tag{2.17}$$

A block diagram of the autopilot loop is shown in Fig. 2.8.

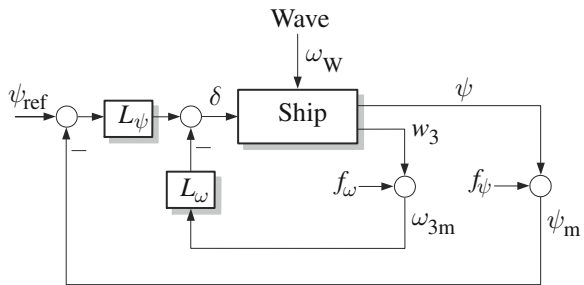
Track control. Track control means that the ship is commanded to follow a line (great circle) over the sea bottom. The desired track is specified to the controller, and position instruments provide the track error. The control architecture for track control was shown in Fig. 2.6.

Requirements. The requirement to fault-tolerant control for the ship steering example are the following:

Table 2.3 Signals and parameters of the ship steering example

Parameter	Typical value and unit	Meaning
ω_3, ω_w	[deg/s]	Turn rate (angular velocity in yaw)
ψ	[deg]	Ship's heading angle
δ	[deg]	Rudder angle
L_ψ	1 [deg]/[deg]	Gain in heading control
L_ω	2 [deg]/[deg/s]	Rate gain heading control
L_e	1 [deg]/[m]	Gain in track controller
b	2.2 [deg/s ²]/[deg]	Gain factor for ship
η_0	0.0 [deg]	Rudder bias
η_1	-10.0 [deg · s ⁻¹]/[deg]	Slope of steering characteristic (Stable ship)
η_2	-20.0 [deg]/[deg ² /s ²]	2nd order parameter in steering characteristic
v_1	10 [m/s]	Ship's forward speed (surge)

Fig. 2.8 Simple heading controller (autopilot) for the ship example



- An undesired alteration in the ship heading (ψ) must not exceed 5 deg.
- An undesired alteration in the ship turn rate (ω_3) must not exceed 0.2 deg/s
- An undesired alteration in the ship position relative to the track (e) must not exceed 5 m
- An undesired alteration in the ship velocity perpendicular to the track (\dot{e}) must not exceed 0.5 m/s

These requirements can be used as objective measures for requirements capture, including detection delay and time to reconfigure.